REPORT 27

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The Number of Observable Radio Sources for Large Single Telescopes

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Summary

Future large radio telescopes should reach as far out into space as possible, especially for cosmological studies. For comparing the range of various telescopes we calculate the maximum number N_m of observable sources per sky area, and N_2, the number of sources/area for which the spectrum can be observed through a factor of 2 in wavelength. These calculations are done allowing for future receiver improvement, assuming 40 ^oK system noise temperature.

The proposed 300-ft homologous telescope will reach $N_2 = 5.0 \times 10^4$ sources/steradian. The NEROC 440-ft reaches 1.5×10^4, the Bonn 100-m telescope 3.2×10^4, and the planned Jodrell Bank 450-ft telescope 0.25×10^4 sources/sterad. Of the existing large telescopes, the NRAO 140-ft reaches $N_2 = 0.81 \times 10^4$, the 300-ft 0.11×10^4, and the Goldstone 210-ft reaches 0.44×10^4. A homologous 410-ft telescope would reach 11.3×10^4 sources/sterad.

These results show again how important it is that future telescopes not only be large, but also be able to observe at short wavelengths. The shortest wavelength λ_o of the telescope should be smaller than λ_m, the crossing point of resolution limit and brightness limit. We find λ_m = 2.83 cm for D = 300 ft, 2.58 cm for 410 feet, and 2.45 cm for 500 feet diameter. All proposed homologous telescopes fulfill the condition $\lambda_o \leq 0.75\ \lambda_m$, which means that N_2 is not resolution-limited.

For cosmological studies one should build a homologous telescope of at least 500 feet diameter, observing down to λ_o = 1.80 cm wavelength.

1. Assumptions and Formulas

Brightness limit and resolution limit of radio telescopes have been treated in detail (von Hoerner 1961, NRAO Publications Vol. 1, No. 2; in the following called Paper I). At present we give only the basic assumptions and resulting formulas, including a few changes from Paper I.

For the brightness limit, we calculate the flux density of the faintest visible source from Paper I as

$$S_{min} = 1.70 \times 10^{-5} \frac{T_o \sqrt{\lambda}}{g(\lambda)} \left(\frac{300 \text{ ft}}{D} \right)^2 , \qquad \begin{array}{|l} S \text{ in flux units} \\ T \text{ in } {}^oK \\ \lambda \text{ in cm} \end{array} \qquad (1)$$

where T_o is the overall noise temperature of receiver, spillover, atmosphere and galactic background, λ is the observational wavelength, and $g(\lambda)$ is a somewhat arbitrary cutoff function representing slow atmospheric changes or scintillations for small λ, and man-made noise for large λ. The numerical factor of (1) assumes a bandwidth of 5% of the frequency and an integration time of 10 sec (or 1% and 50 sec, for example).

The cutoff function $g(\lambda)$ assumed in Paper I was much too pessimistic, as observations at 2 cm and below have shown meanwhile. A more realistic but still conservative one is now assumed and is given in Table 1.

For the number of visible sources, we assume a slope of -1.50 for the log N / log S relation, and we assume that all sources have the average spectrum index of -0.80. We normalize to the 3C catalog, and obtain

$$N_{vis} = 7.84 \times 10^4 \ \lambda^{1.2} \ (10^3 \ S_{min})^{-1.5} . \qquad \begin{array}{|l} N \text{ in sources/sterad} \\ \lambda \text{ in cm} \\ S \text{ in flux units} \end{array} \qquad (2)$$

For the resolution limit, we calculate N_{res}, the number/sterad of resolvable sources. With respect to fainter background sources we have from Paper I

$$N_{res} = 9.92 \times 10^5 \ \lambda^{-2} \ (D / 300 \text{ ft})^2 . \qquad \begin{array}{|l} N \text{ in sources/sterad} \\ \lambda \text{ in cm} \end{array} \qquad (3)$$

Finally, the number of actually observable sources is mostly the smaller one of N_{vis} and N_{res}; but in the neighborhood of their crossing point we must add quadratically the actual noise and the background irregularities, and one can show that, approximately,

$$N_{obs} = (1/N_{res} + 1/N_{vis})^{-1}. \quad (4)$$

The maximum number N_m then is observed at a wavelength λ_m approximately given by

$$\lambda_m \text{ where } N_{res}(\lambda) = N_{vis}(\lambda). \quad (5)$$

But just barely observing and counting sources is not enough. We should at least be able to observe the spectrum of a source over a range of, say, a factor of 2 in wavelength, in order to measure the spectrum index and to see any stronger curvature of the spectrum. For comparing the range of various telescopes, we thus define N_2 as the maximum number/steradian of sources which can be observed through a factor of 2 in wavelength.

For this purpose we plot for any given telescope the function $N_{obs}(\lambda)$ according to (4). Toward small λ we cut off at λ_o, the smallest observational wavelength of this telescope. We then find the maximum observable range of length 2 in wavelength, as shown in Fig. 1.

For completely resolution-limited telescopes, we have only

$$N_2 = 0.25\, N_o, \quad \text{if } \lambda_o \geq 1.6\, \lambda_m; \quad (6)$$

while it turns out that

$$N_2 = 0.82\, N_m, \quad \text{if } \lambda_o \leq 0.75\, \lambda_m. \quad (7)$$

2. Applications and Results

For planning future large telescopes, one must somehow take into account the improvement of future receivers to be expected. In Paper I, for example, we find for D = 300 ft, with a receiver of 900 ^{o}K, that λ_m = 14 cm and N_m =

3.2×10^3 sources/sterad; whereas a receiver of 20 ^{o}K demands $\lambda_m = 2.6$ cm and yields $N_m = 9.1 \times 10^4$. This means that future telescopes must be able to observe at very short wavelengths, or they will soon become badly resolution-limited with improving receivers.

For the following we assume a future receiver noise of 25 ^{o}K, plus a spillover of 15 ^{o}K. Atmospheric and galactic contributions are assumed the same as in Paper I. The resulting overall noise T_o is given in Table 1, together with the adopted cutoff function $g(\lambda)$. Then S_{min}, N_{vis} and N_{res} are calculated as functions of λ for a diameter of 300 feet.

Table 1. Brightness and resolution limit, as functions of λ.

λ	T_o	$g(\lambda)$	for D = 300 feet: S_{min}	N_{vis}	N_{res}
cm	^{o}K		10^{-3} flux units	10^4 sources/steradian	
.5	180	.15	14.4	.062	397
.7	160	.25	9.11	.185	202
1.0	130	.40	5.53	.603	99.2
1.5	80	.58	2.87	2.61	44.1
2	56	.70	1.92	6.78	24.8
3	48.5	.85	1.68	13.4	11.0
5	45.4	.96	1.80	22.3	3.97
7	44.5	.98	2.04	27.9	2.02
10	44.1	1	2.37	34.1	.992
15	44.3	1	2.92	40.4	.441
20	44.5	1	3.38	46.0	.248
30	45.8	1	4.26	48.6	.110
100	85.0	1	14.4	36.0	.0099

Figure 1 shows the number/sterad of observable radio sources, for the homologous telescopes as suggested in Report 26 (March 1969). In Paper I we have derived that it would be important for cosmological studies to reach or come close to $N_m = 3 \times 10^5$ sources/sterad; we see from Figure 1 that we should have at least 500 feet diameter for reaching this goal.

For comparison with other proposed or future telescopes we plot Figure 2. For the NEROC telescope we use 440 ft for the resolution limit, and 410 ft for the brightness limit; the Bonn telescope observes at 5 cm for its full size of 100 m, and at 3 cm for its inner part of 90 m diameter. We see from Figure 2 that all these telescopes will be resolution-limited, at least for future receivers. Especially the proposed Jodrell Bank telescope shows how much is lost if a large telescope does not observe at short wavelengths.

Figure 3 shows the maximum observable number of sources, N_m, and the wavelength λ_m of this maximum, as functions of the telescope diameter D. For the range $300 \leq D \leq 500$ ft, we find $2.83 \geq \lambda_m \geq 2.45$ cm. The demand that $\lambda_o \leq 0.75\ \lambda_m$ from (6) then leads to at least, say

$$\lambda_o < 2 \text{ cm} \tag{8}$$

for avoiding severe resolution limitations for future large telescopes.

Table 2 finally summarizes the observational limits of various homologous and other telescopes; λ_m is given only where it can be observed (if $\lambda_o \leq \lambda_m$); N_m and N_2 refer to the actually observable range above λ_o for resolution-limited telescopes.

In table 2 we see again the importance of short wavelengths. As compared to the NEROC telescope, N_2 of the homologous 300-ft telescope is larger by a factor of 3.40, in spite of the smaller telescope diameter; and a homologous telescope of 410 feet diameter would even yield a factor of 7.69 for N_2.

Table 2. Observational limits for various telescopes.

D = diameter of telescope;

λ_o = shortest wavelength of telescope;

λ_m = wavelength for maximum number of sources;

N_m = maximum number of observable sources;

N_2 = maximum number, where spectrum can be observed through factor 2 in wavelength.

		D	λ_o	λ_m	N_m	N_2
		feet	cm	cm	10^4 sources/sterad	
future	homologous	210	.68	3.12	2.48	2.1
	telescopes	250	.80	3.00	3.80	3.1
		300	.98	2.83	6.15	5.0
		350	1.17	2.69	9.20	7.6
		410	1.42	2.58	13.8	11.3
		500	1.80	2.45	23.0	18.2
	NEROC	440	6	-	5.8	1.47
	Bonn	328	5	-	4.6	3.25
		296	3	-	5.7	
	Jodrell Bank	450	15	-	1.0	.25
present	NRAO	140	2	3.50	.96	.81
		300	15	-	.43	.11
	JPL	210	5	-	1.73	.44

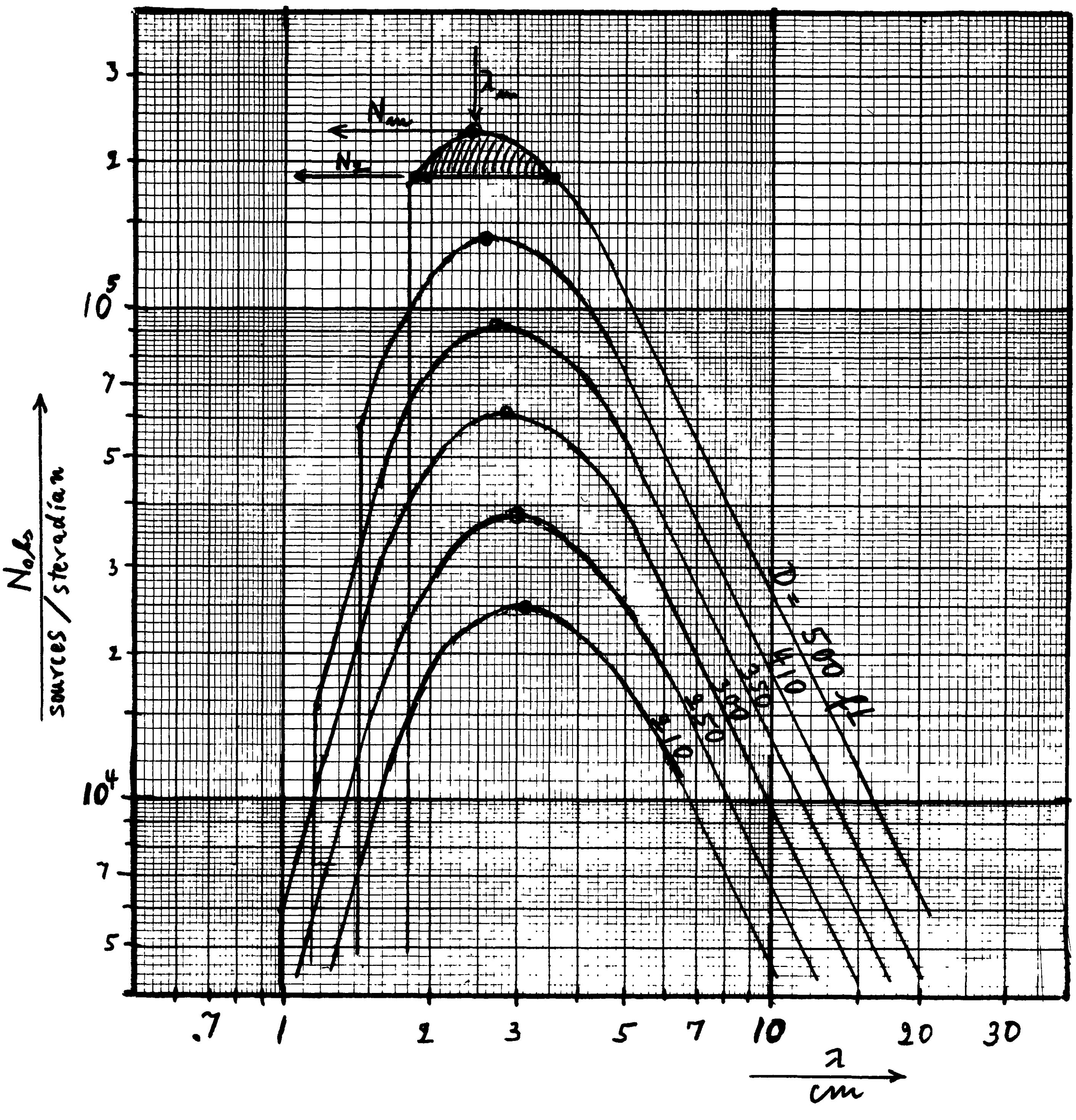

Fig.1: Number of observable radio sources, for homologous telescopes of various diameter D.

left-hand cutoff = shortest wavelength of telescope;
left-hand slope = brightness limit (system noise 40°K);
right-hand slope = resolution limit.

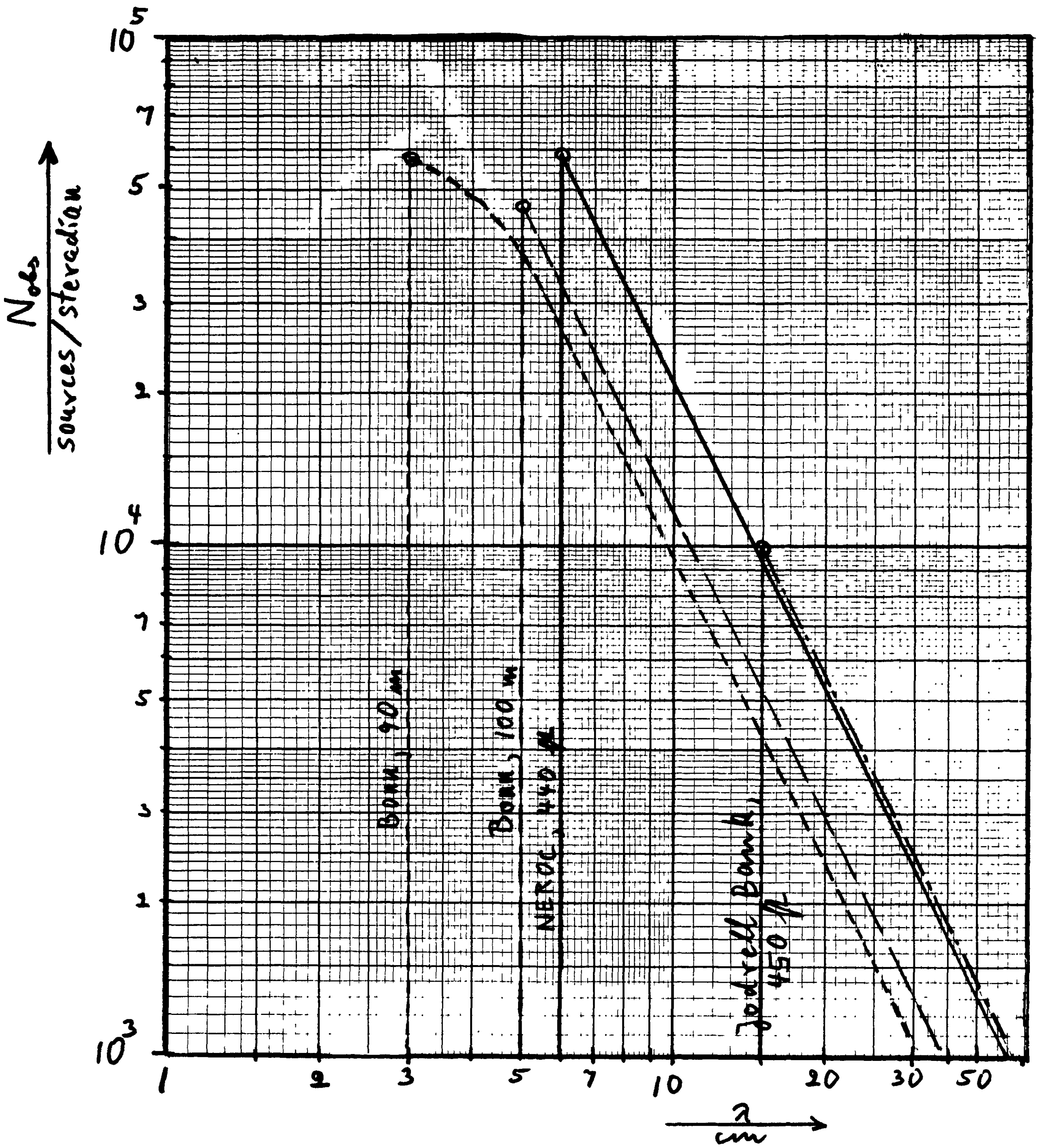

Fig.2: Number of observable radio sources, for various future telescopes.

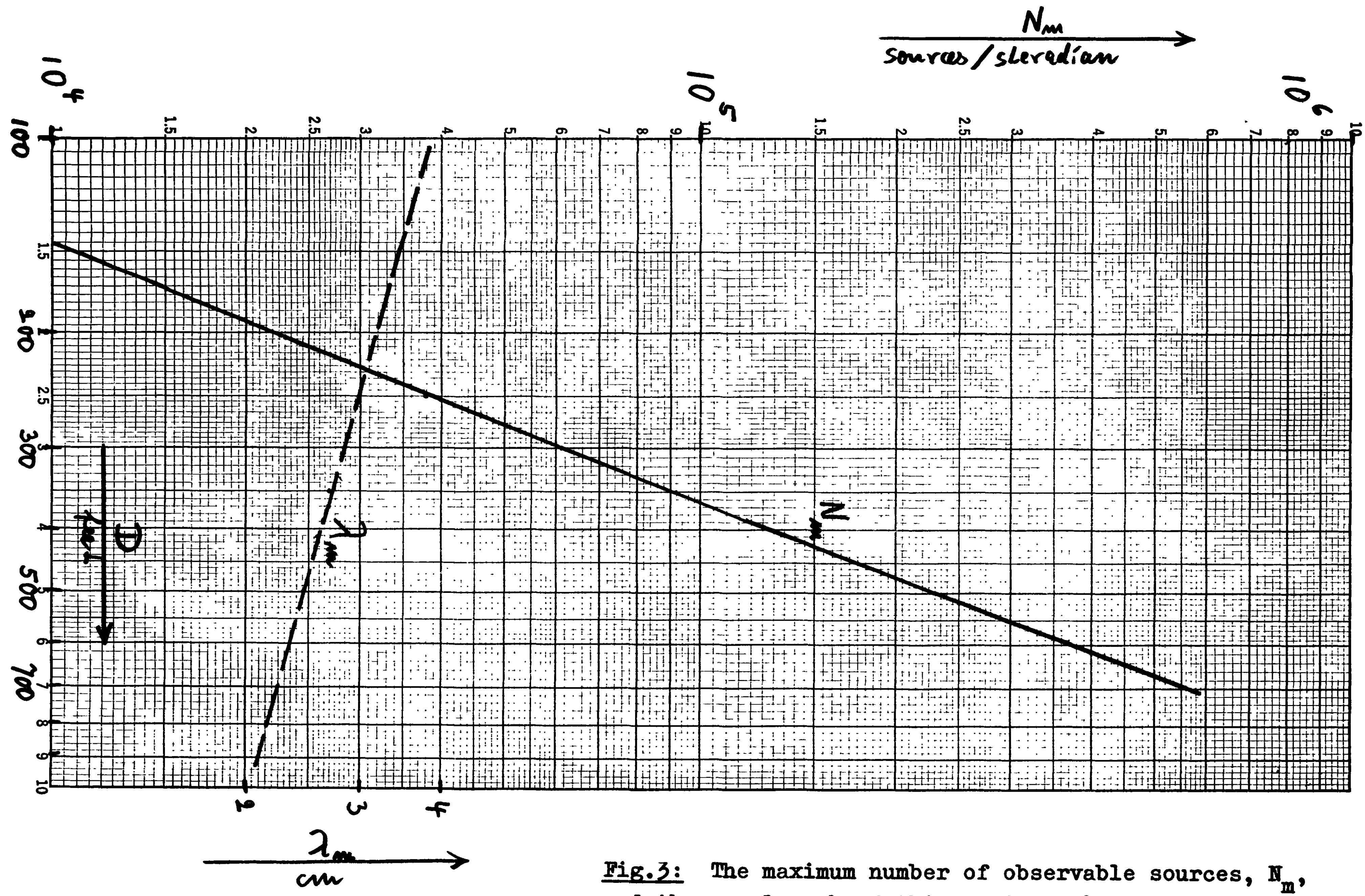

Fig.3: The maximum number of observable sources, N_m, and the wavelength of this maximum, λ_m, as functions of the telescope diameter, D.