REPORT 12

NATIONAL RADIO ASTRONOMY OBSERVATORY
Post Office Box 2
Green Bank, West Virginia 24944
Telephone Arbovale 456-2011

REPORT NO. 12
CONTRACT NO.
PAGE 1 OF
DATE July 22, 1966

PROJECT: LFST
SUBJECT:

Mirror for the Floating Sphere

S. von Hoerner

The floating sphere deforms if tilted by several inches, and the question was raised whether a homologously deforming mirror could be found under these circumstances. The present report does not give a final solution for such mirror, but it shows that the problem can be solved if wanted. A good way seems to be attaching the mirror at its rim to the stiffener ring, and preventing deformations of the ring by an arrangement of radial ropes.

I. Non-Deforming Ring

The stiffener ring at the opening of the sphere has two types of deformations: within the plane of the opening, and perpendicular to it.

1. Deformations Within the Plane

Otto Heine has given the deformations within the plane in Fig. 10 of his Report 10 B (5-12-66). The circle of the opening is deformed approximately into an ellipse with its long axis being horizontal. The ptp deviation (peak to peak) from the circle is 12 inches, and the rms deviation (root mean square) is 4.24 inches. In order to counteract this deformation, by tensioned ropes along the horizontal axis, one needs a force of F = 6070 tons according to O. Heine.

We make use of two principles of equivalence. First, if we construct the ring as a perfect circle in zenith position, it deforms into an ellipse if tilted, and the ropes then must pull along the horizontal axis. But we could as well have constructed the ring as a perfect circle in the tilted position; if brought to zenith, it then would deform the opposite way, and the ropes must pull along an axis perpendicular to the first one. Both methods are equivalent, and any combination is possible, too. Second, it does not matter whether we pull a rope in one direction, or release another rope in a perpendicular direction; and again, both things can be combined. Finally, we can combine all four ways of counteracting the deformations. But, no matter which method we choose, the ropes must always be radial and symmetrically distributed, for allowing accurate polarization measurements.

The arrangement of ropes suggested is shown in Fig. 1, where 24 ropes run from the center to the ring. This number of ropes is arbitrary and must be a compromise between distributing the force evenly along the ring (many ropes), and avoiding large aperture blocking (few ropes). The blocking, by the way, can be reduced by splitting up each rope into many thin ropes, one behind the other perpendicular to the plane (Fig. 1b).

These ropes meet at the center not in one point, but along a straight hub which is, say, 40 feet long in z-direction. The height Z (above the plane), where a rope of angle φ is attached to the hub, is, for an elliptical deformation, then given by

$$Z = -20 \text{ feet} \cos(2\varphi). \tag{1}$$

If the hub moves in z-direction by an amount Δz, then the end point of a rope at the ring moves radially by $\Delta r = Z\ \Delta z/r$ (neglecting terms of second order), which means the hub would move $\Delta z = 8.2$ feet, if the ring deforms by $\Delta r = 6$ inches, and if hub and ropes had no weight. In order to counteract this movement and deformation, the weight of the hub plus 1/2 that of all cables must be

$$W = 6070 \text{ tons } 2\ \frac{20 \text{ feet}}{328 \text{ feet}} = 740 \text{ tons}. \tag{2}$$

Taking high-strength ropes, the weight of all ropes turns out to be 180 tons; we subtract 1/2 of it and obtain for the weight of the hub:

$$W_{hub} = 650 \text{ tons}. \tag{3}$$

The hub, for example, could be a steel cylinder, 40 feet long and 13 feet in diameter,

PROJECT: LFST
SUBJECT:

filled with high-density ore and concrete. Its aperture blocking is only 0.02 per cent and completely negligible.

Formula (1) holds for an elliptical deformation. But any other deformation can as well be counteracted by the same arrangement of ropes and some other formula. For example, O. Heine's 5-term equation (letter of July 14) demands, instead of (1):

$$Z = -20 \text{ ft } (0.0033 + 0.0349 \cos \varphi + 1.000 \cos 2\varphi - 0.0567 \cos 3\varphi + 0.0068 \cos 4\varphi). \quad (4)$$

2. Deformations Normal to the Plane

The normal deformations are given by O.Heine in his letter of July 14. The ptp deformation, as given in his graph, is 4.70 inches. But in Heine's equation, we can omit the constant term (translation) as well as the term with cos φ (tilt); both are not really a deformation but only a rigid-body movement. The remaining true deformation is shown in Fig. 2; the ptp deformation now is 3.3 inches, and the rms deformation is 1.09 inches. The equation for this deformation is

$$z(\varphi) = 1.520 \text{ inch } (\cos 2\varphi - 0.1552 \cos 3\varphi + 0.0363 \cos 4\varphi). \quad (5)$$

This normal deformation is only 1/4 the tangential one, but it still is too large to be tolerated (for example, $\lambda = 10$ cm demands $\lambda/16 = 0.25$ inch). It could be reduced by widening the ring in z-direction, but we prefer to suppress it completely. The easiest way seems to be the following.

The ring in its present design provides for two things: stiffness (against tangential deformations) and counterweight (for balancing the sphere). With a rope arrangement as shown in Fig. 1, the stiffness of the ring can easily be reduced quite a bit, whereas the full counterweight still is needed (minus the weight of ropes and hub). But for the purpose as counterweight, any distribution of the weight along the ring will do, as long as its center of gravity coincides with the center of the ring.

PROJECT: LFST
SUBJECT:

We now construct the ring to be in a perfect plane when tilted, and we arrange the counterweight according to

$$W(\varphi) = W_0 - W_1 z(\varphi), \tag{6}$$

where W_0 is the present weight of the ring minus ropes and hub, W_1 is determined by the forces needed to counteract the normal deformation, and $z(\varphi)$ is given by (5). With this distribution of weight, together with our rope arrangement, all deformations are completely counteracted, normal as well as tangential ones, at least in first order.

In second order, we get some mutual distortions. The counterweight distribution will also give some tangential deformations because the walls of the sphere cut the plane not at a right angle; and since the ropes are not exactly in the plane, they will give some normal deformations, too. But since any tangential deformation can be suppressed completely by some formula similar to (4), and any normal deformation by a formula similar to (6) and (5), the problem definitely is solvable, which would need only a few iterative computer runs with Heine's analysis program. Thus:

The deformations of the ring can be brought exactly to zero. (7)

I would like to emphasize that this result is achieved without any extra costs. The steel needed for the ropes can be taken away from the stiffness-providing part of the ring, and the concrete for the counterweight is distributed in a different way but remains the same amount.

II. Mirror with Homologous Deformations

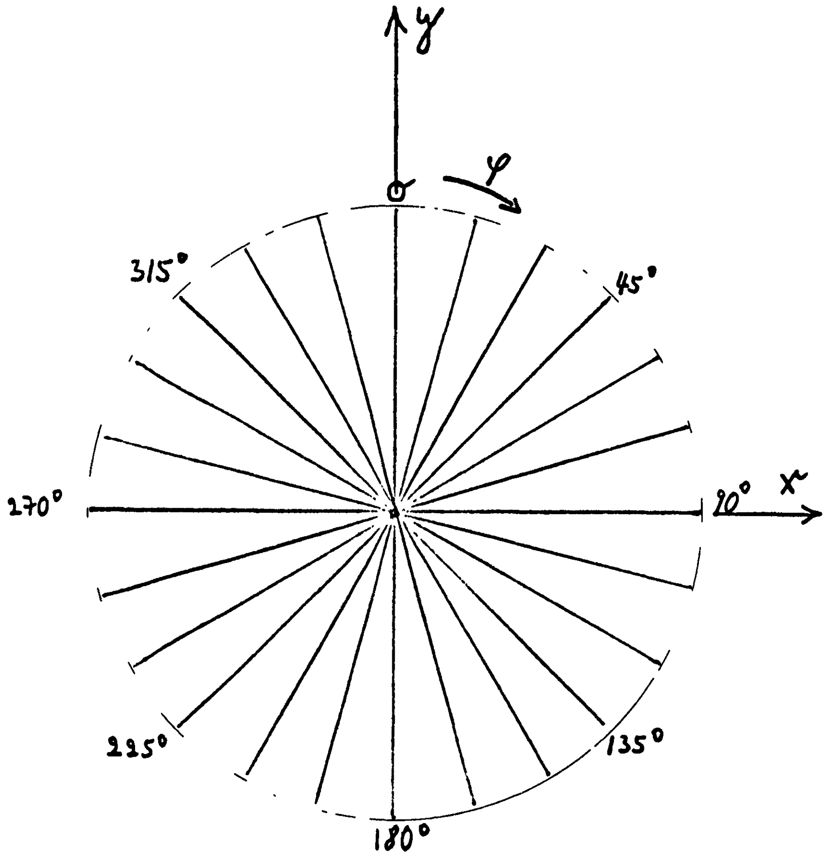
For a mirror attached to the non-deforming ring, the conditions of homology must be fulfilled in two positions, looking at zenith and at horizon. Looking at zenith, the problem is trivial: even a single, thin shell with variable thickness would do

PROJECT: LFST
SUBJECT:

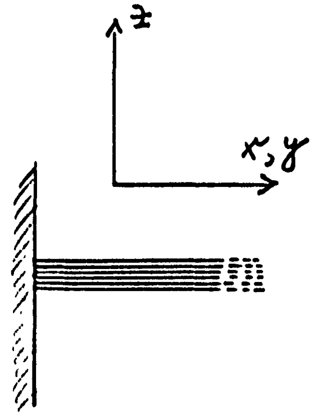
the job. As demonstrated in Fig. 3a and 3b, we can produce deviations of opposite type and thus can suppress them completely.

Looking at horizon, the problem is more difficult but still seems solvable. Fig. 3c shows the deformations of a single, thin shell. This shape could again be made parabolic (axis looking somewhat upwards) by a proper distribution of thickness; but this degree of freedom we have already used up for reaching homology in zenith-position. Now, a framework has more freedom than a shell because it consists of members of different direction which can be treated differently; but even if exact homology cannot be reached with a single shell or an equivalent framework, it might well be that a sufficient approximation still can be found. I have given to R. Jennings a structure, simulating a thin shell suspended in a non-deforming ring, for investigating this question; it is shown in Fig. 4. Here, the 12 holding points along the ring are not regarded as surface points by reasons connected only with our present program. First, any tilt of the axis of the best-fit paraboloid would demand a slight ellipticity of the ring, but a deformation of holding points cannot be handled by the present program; second, it cannot regard holding points as surface points. For the actual structure, however, this is no problem; any desired deformation of the ring can be achieved by a formula similar to (4).
Should it turn out that the single shell is not good enough an approximation, we must provide the mirror structure with the freedom to deform the opposite way, and one possibility of doing so is shown in Fig. 3d. A combination of both principles then should be able to deform any desired way. We will try structures of this type with our homology program as soon as possible.

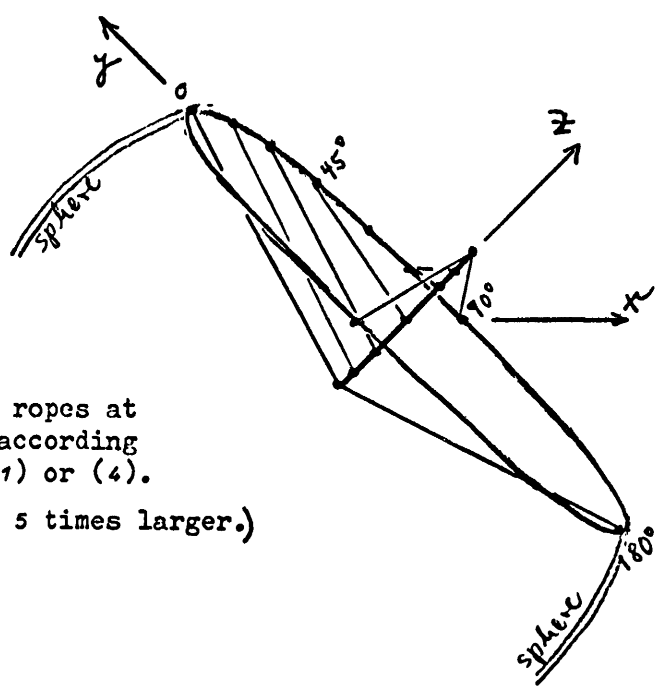
Although the problem of a homologous mirror has not yet been solved numerically, I think that the demonstrations of possibilities given in Fig. 3 show convincingly that it can and will be solved. The lower limit of observing wavelengths then is given entirely by thermal deformations and wind deformations, and might be of the order of a few centimeters.



1a) 24 radial ropes



1b) Each rope split up into many thin ropes, one behind the other in the line of sight, for reduction of apperture blocking.



1c) Attachment of ropes at central hub, according to equation (1) or (4).

(Scale of z is 5 times larger.)

Figure 1. Arrangement of ropes and central hub, for suppression of tangential deformations.

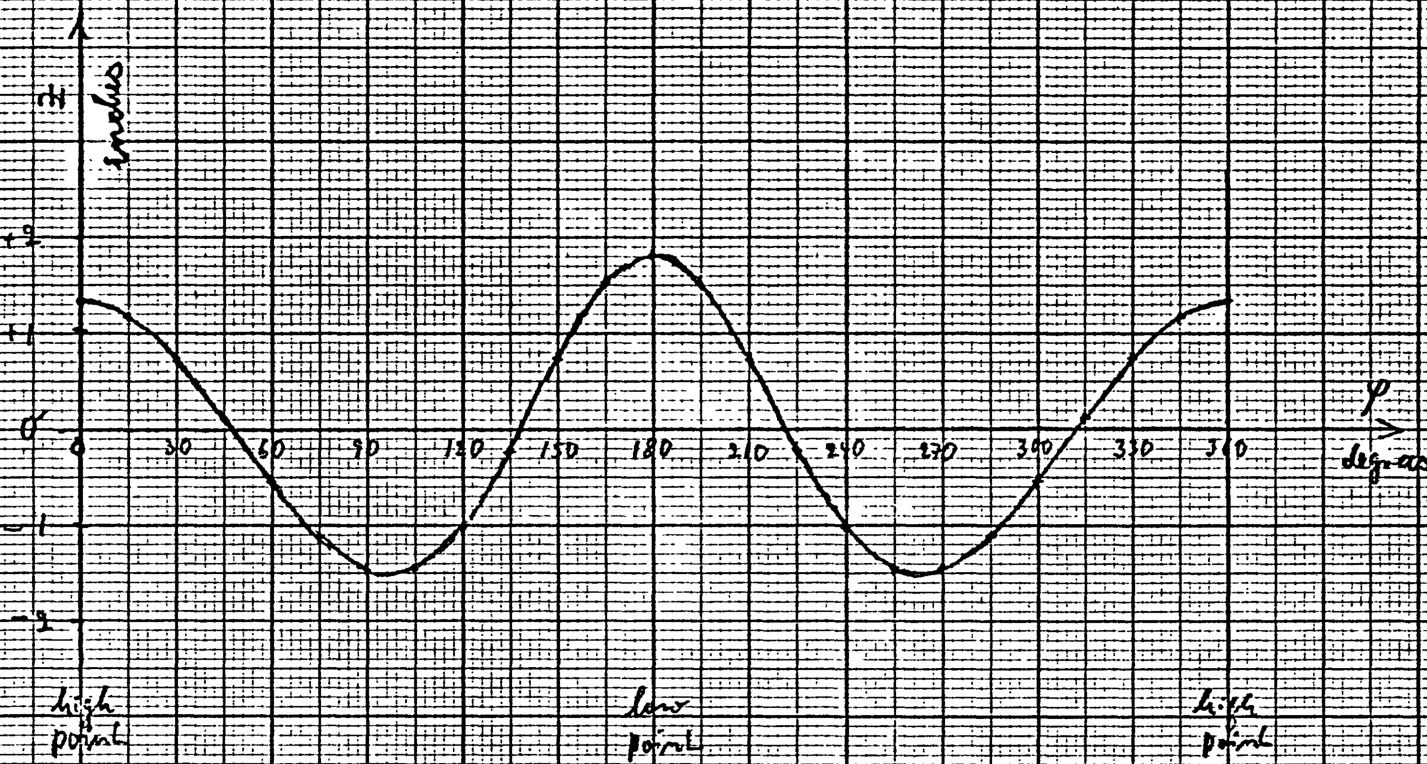


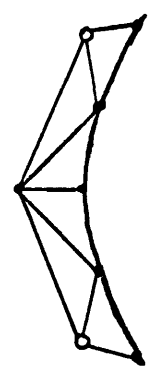
Figure 2. Deviations from best-fit plane, in normal direction.

3a) Small thick center:
too small curvature
near rim,

3b) Large thick center:
too large curvature
near rim.

3c) Single shell
bulges down,

3d) This suspension
bulges up,



3e) Combination can
bulge as wanted.

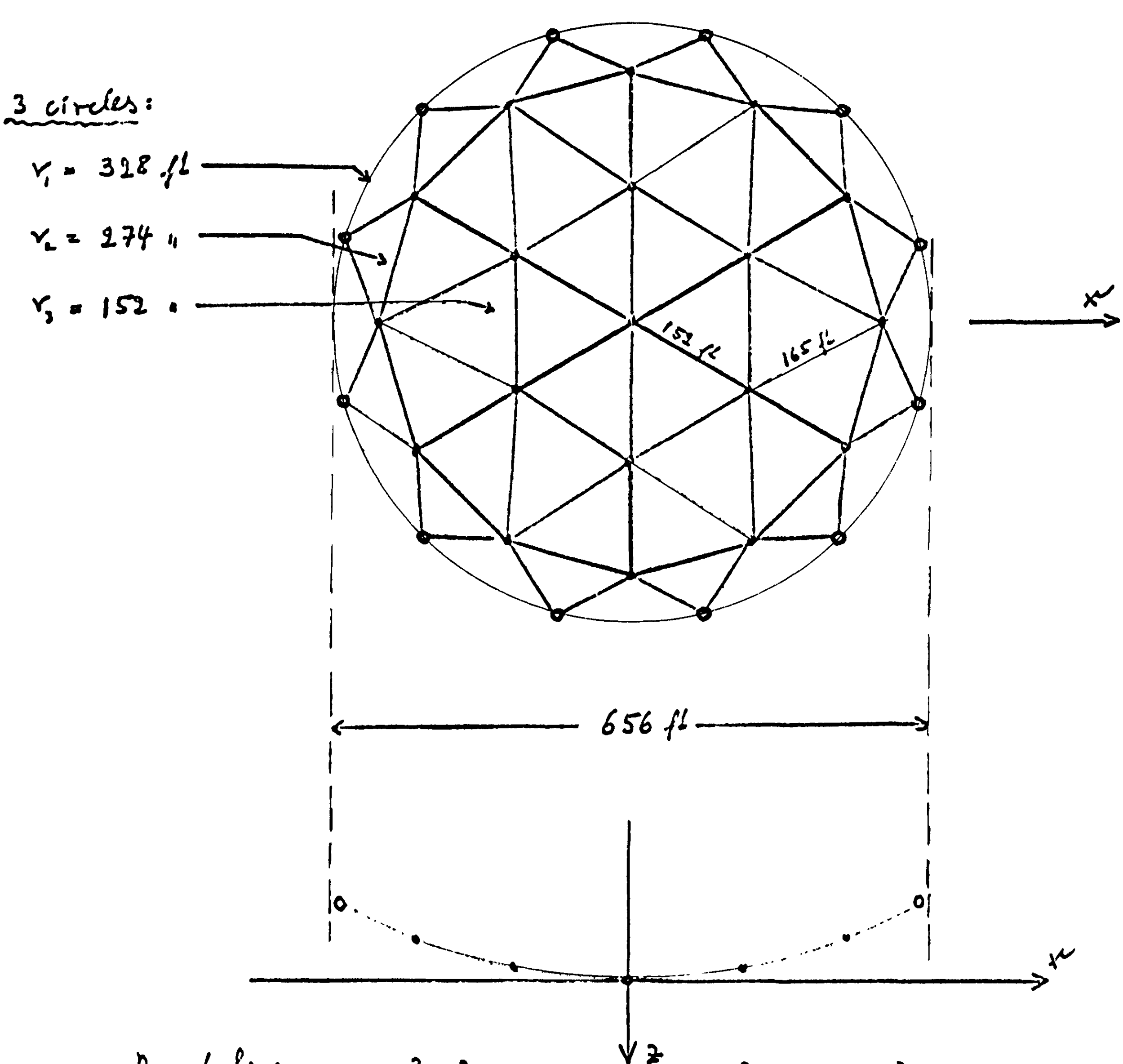
<u>Figure 3.</u> Deformations of rim-suspended mirror.

Structure FSM 1

(floating-sphere mirror)

July 21, 1966; S.v.H.

o 12 holding points
• 19 surface points
31



Parabola: $z = -\frac{x^2+y^2}{4f}$

$\left.\begin{matrix} f = 328 \text{ ft} \\ D = 656 \text{ "} \end{matrix}\right\} \frac{f}{D} = 0.5$

all Q = 80 inch^2

all W = 30 tons

|| p = 31 points
m = 66 members ||

This structure simulates a simple shell, suspended in a non-deforming ring. I think it will not have an exact homology solution, but maybe it is already a good approach.

Figure 4.