May 1964

THE DESIGN OF VERY LARGE STEERABLE ANTENNAS

Sebastian von Hoerner
National Radio Astronomy Observatory*

Summary

There is a maximum size for a self-supporting structure made of any given material, no matter what its purpose, and a further limiting size for maintaining a given accuracy if the structure is tilted. This second limit is essential in antenna design.

A movable structure should have about the same diameter in every direction, since deflections increase with the square of the ratio of diameters. The regular octahedron is adopted as the best basic structural principle.

A large surface should not be supported, it should be pulled by cables. The spoked wheel is adopted as the best principle.

Temperature deformations can be neglected for diameters over 300 feet.

The number of compression members, as well as the weight of their internal braces, should be kept as low as possible. All remaining members being cables, the structure should be pre-stressed to half of its capacity.

For a given diameter, the connection between weight (price) and wavelength shows three characteristic points: first, the weight of a self-supporting structure for $\lambda \to \infty$. At the second point, the strength needed for safety just yields the rigidity needed for observation. At the third point, the shortest possible wavelength is approached with infinite mass. Between first and second point, the weight is entirely defined by safety against strongest winds, and between second and third point entirely by the

*Operated by Associated Universities, Inc., under contract to the National Science Foundation.

deflections tolerated for observation. Economical antennas ought to be close to the second point, any other point giving a waste of either strength or rigidity.

A model-design for fully steerable antennas is developed which can be scaled for both diameter and wavelength (within the limits given), and the total weight of dish plus towers is calculated as function of both.

One example of a complete price estimate including foundations showed that it should be possible to build a fully steerable antenna of 500 feet diameter, usable down to 20 cm wavelength, for a total price of 1.5 million dollars.

Introduction

The present investigation was inspired by the question of how to build a special antenna for lunar occultations as cheaply as possible. This antenna must be a fully steerable parabolic reflector; it should be at least 300 feet in diameter, but much more if possible; and it should be usable down to at least 50 cm wavelength, or to 20 cm if possible within a reasonable price. The antenna could be used for other purposes half of the time.

Since the specifications give only lower limits while the actual values will depend on price, and since, in general, economy will play a decisive role for any future very large antenna, the investigation is made as general as possible. The first step, then, is to find the basic principles involved and the general limitations they impose. The second step is to develop a certain overall antenna design supposed to be the most economical one, which can be scaled to any desired size and accuracy within the limits derived in the first step. The third step will go to some more details and will yield the price of such an antenna as function of both diameter and wavelength. Whereas the results up to here could be applied to any future antenna, the fourth step finally selects a special antenna for the purpose in mind, as a compromise between the opposing demands of price, diameter, and wavelength. Total costs for this antenna then are estimated.

I. Basic Principles

1. Gravity and Elasticity

Even with gravity as the only force (no wind, no load), one could not build indefinitely high structures. The limit is reached when the weight of the structure gives a pressure at its bottom equal to the maximum allowed stress of the material used. If we call

S = maximum allowed stress of material

ρ = density of material

h_o = maximum height of structure

we then have, for a structure of constant cross section, like a standing solid block or a hanging cable:

$$h_o = S/\rho \tag{1}$$

This is our first general limit, giving a maximum size for any structure, no matter what its purpose. A second limit comes from the accuracy desired (smallest wavelength in our case), since even a standing block will get compressed under its own weight, the lower parts more than the upper ones. We call

E = modulus of elasticity

h = height of structure

Δh = change of height under own weight

and find, by integrating the compression from ground to top:

$$\Delta h = \frac{1}{2} \frac{\rho}{E} h^2 \tag{2}$$

This second limit is especially severe since the deformation goes with the square of the size. For antennas of increasing size, the second limit is reached much earlier than the first one.

Both limits can easily be understood, since the weight goes with the volume, r^3, while the strength goes only with the cross section, r^2. The same limits apply to animals, too; if you want something considerably bigger than an elephant, you must fight gravity by floating it in water, like a whale. And the bigger an animal, the clumsier it gets. These limits are not ultimate but can be surpassed by special tricks. As to the first limit, one can start at the bottom with a large cross section and taper it toward the top, but this type of structure cannot be tilted. As for the second limit, one can adjust a surface in any position by servo motors, but this gets extremely expensive and awkward.

Both limits depend on the combination of only three material constants: maximum stress, density and elasticity. Table 1 gives some examples, together with an average price including erection; the coefficient of linear thermal expansion, C_{th}, is included for later use. As we see, the largest structure can be made from aluminum, over three kilometers high; all three materials give the same order of magnitude for this maximum height, which could be increased only by tapering, and we understand why even mountains cannot be higher than a few kilometers. All three materials are about equal with respect to deflections under their own weight, steel being slightly better, while for thermal deformation aluminum is worst and wood is best (but wood has too much deformation with humidity). Since the second limit will be reached first, there is no need to go to the more expensive aluminum, and we finally arrive at steel as the best material. The largest block of steel could be a mile high, but a block only 300 feet high is already compressed under its own weight by 4 millimeters.

Table 1

Some Material Constants

	Density ρ	Maximum stress S	Elasticity E	Maximum size S/ρ	Deflection ρ/E	Therm. exp. C_{th}	Price p
	g/cm^3	kg/cm^2	kg/cm^2	km	$cm/(100m)^2$	1/°C	$/kg
Steel	7.8	1400	2.1×10^6	1.79	0.37	12×10^{-6}	1.0
Aluminum	2.7	910	0.7×10^6	3.37	0.38	24×10^{-6}	4.5
Wood	0.5	133	0.12×10^6	2.77	0.40	3.5×10^{-6}	0.5

2. Shape Factor

We want some large structure, held at certain points, to be turned about in all directions, and the question is: what should be the over-all shape of the structure in order to have the smallest possible amount of deflection under its own weight plus wind force?

In Figure 1 we have two columns, held at their endpoints to a "base" of length 2b, while at their junction, at distance ℓ from the base, a force F_1 is applied parallel to the base. This will result in a deflection Δh. Question: if ℓ is given, what is the value of b which makes Δh a minimum? The answer is found by calculus as

$$b = \sqrt{2}\,\ell$$

Next, we apply a force F_2 perpendicular to the base. If now b is given, the best value of ℓ, making $\Delta\ell$ a minimum, is found as

$$\ell = \sqrt{2}\, b$$

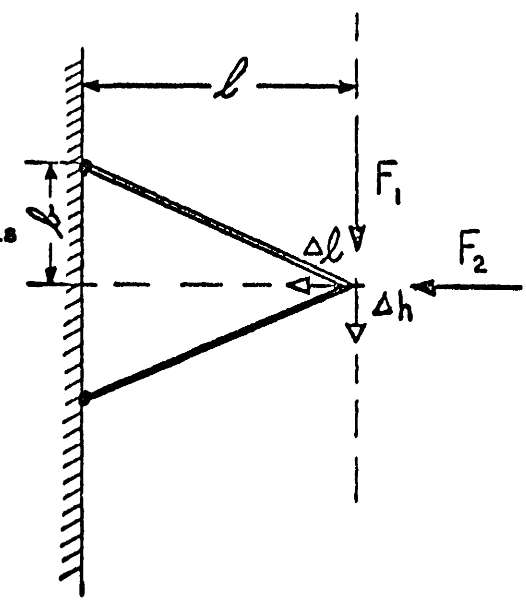

Figure 1.

Thus, the best solution for forces in all directions is

$$b = \ell \qquad (3)$$

and since the baseline should go through the center of gravity in order to avoid torques, equation (3) requires equal diameters in both directions. If this rule is not fulfilled, we get, for example, for the extreme case of F_1:

$$\Delta h \sim (\ell/b)^2 \quad \text{for} \quad b \ll \ell \qquad (4)$$

In summary, the structure should have about the same diameter in every direction. Small deviations from this rule do not matter much, but for greater deviations the deflection increases in proportion to the square of the ratio of the diameters.

3. The Octahedron

The simplest structure approaching requirement (3) which still is easy to hold and to turn, and which provides a flat surface through its center with a point normal to it for the focus, is the octahedron. It has the additional advantage that its deflections can easily be calculated. Thus we adopt the octahedron as the basic structural principle of our antenna, modifications like a curved surface being introduced later on.

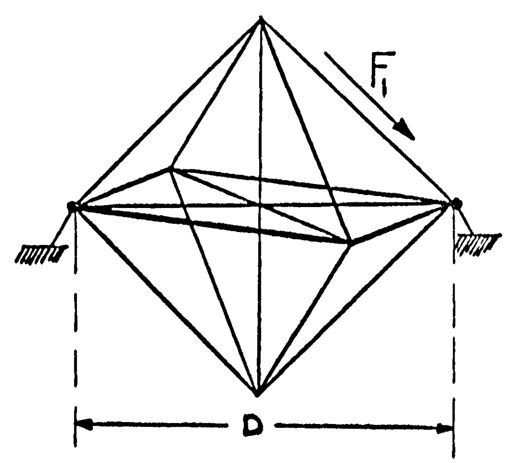

Figure 2. Octahedron with diagonals.

If all members shown in Figure 2 have equal cross sections Q, the force along one outer member, resulting from the weight of the whole structure, turns out to be $F_1 = 2.88\ DQ\rho$. The largest possible size D_o of an octahedron then is 1/2.88 of the value found in equation (1):

$$\begin{aligned} D_o &= 0.347\ S/\rho \\ &= 622 \text{ meter for steel} \end{aligned} \tag{5}$$

According to equation (2), the deflection should be

$$\Delta h = \gamma D^2 \rho/E$$

but the numerical value of γ depends on where we measure the deflection and with respect to which point. Measuring Δh in centimeters and D in units of 100 meters, we get the values shown in Figure 3, with respect to the focal point at the top. The rms deflection over the whole horizontal plane, as seen from the top, is given by $\gamma = 0.34$; but since we have neglected any deflections arising from lateral sagging of the members, and since we really want to be on the safe side, we multiply by a safety factor of 1.5 and obtain for the rms deflection:

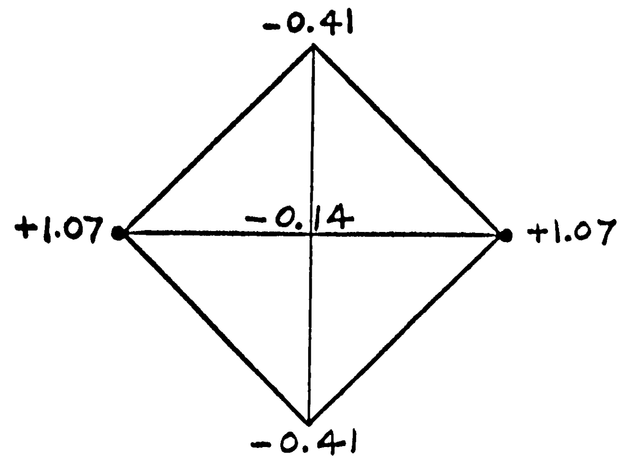

Figure 3. Deflections (in cm) in the horizontal plane of an octahedron of 100 meter diameter, as seen from the top point.

$$\Delta h/\text{cm} = 0.51\ (D/100\text{m})^2 \tag{6}$$

Finally, denoting the shortest wavelength to be used by λ and requiring that the rms deflection should be $\lambda/16$, we get

$$\lambda/\text{m} = 0.0816\ (D/100\text{m})^2 \tag{7}$$

Some examples are given in Table 2, but for actual antennas these values will be modified in two ways. First, the antenna surface can extend beyond the supporting octahedron, by an amount limited by equation (3). Second, the limiting wavelength will become larger than the one shown if all additional weight (surface and its supporting structure, braces, etc.) is not negligible compared to the main chords of the octahedron. Since the two modifications work in opposite directions, Table 2 still might be used for a first estimate of the limiting wavelength for large antennas.

4. Temperature

If C_{th} is the coefficient of linear thermal expansion as given in Table 1, and if one member in Figure 1 (with $b = \ell = D/2$) is ΔT degrees warmer than the other one, the resulting deflection is $\Delta h_{th} = C_{th} \Delta T D$. By some geometrical considerations one can show that the rms deflection of the surface, even in unfavorable cases, will not exceed 1/4 of this value, which gives for steel:

$$\Delta h_{th}/\text{cm} = 0.03 \; \Delta T \; D/100\text{m} \tag{8}$$

This thermal deflection is to be compared with the gravitational deflection from equation (6), and we see that temperature effects may dominate in small antennas but can be neglected in large ones, as is illustrated in the last column of Table 2. For antennas standing in the open, larger temperature differences can occur only through the combined influence of sunlight and shadow, and they will mostly be below 10°C. We thus conclude that thermal deflections may be neglected for antennas over 300 feet diameter. If an antenna is enclosed in a radome, a vertical temperature gradient will

Table 2
Shortest Wavelength λ for an Octahedral Structure of Diameter D, if Temperature Differences are Smaller than ΔT

D meter	λ centimeter	ΔT centigrade
25	0.51	4.3
50	2.0	8.5
100	8.2	17
150	18.4	25
200	32.6	34
300	73	51
500	204	85

exist, and the larger the antenna the larger the temperature differences will be. The thermal deflection thus becomes proportional to D^2, just as with the gravitational one, and the question of which deflection is larger then depends on the gradient, but not on the antenna size. An estimate showed that the two deflections are equal if the gradient is about 25°C/100m.

5. Active and Passive Weight

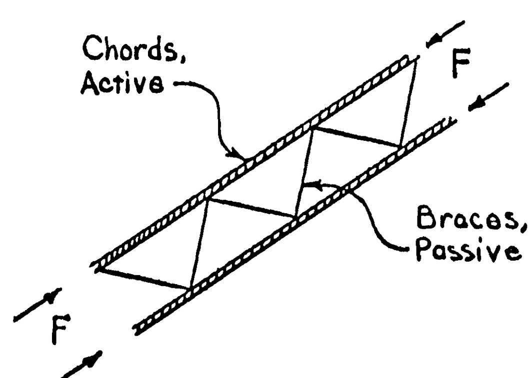

Figure 4.

Since the next point is a crucial one for large antennas, and since no suitable terminology seems to exist, I shall introduce my own:

Active weight = weight of those parts which oppose deflections of the structure to the same extent as they add weight to it. In our case, only the main chords of the octahedron members are active. If each member were simply a solid rod, or a single beam or pipe (all of the same cross section) and no other weight were added, we would have nothing but active weight, and the amount of gravitational deflection would be independent of the cross section and thus of the weight of the octahedron for given diameter.

Passive weight = weight of everything else, such as the braces in the octahedron members, the surface, any additional structures to hold the surface, as well as any parts of the drive mechanism which are fixed to the octahedron. Passive weight adds to the total weight without opposing the deflection it causes and thus increases the deflection of the structure. Furthermore, any asymmetry of the octahedron would add passive weight: if we double the cross section of just one of the members, most of the weight added would be passive.

Total weight = active weight plus passive weight.

Weight factor = K = (total weight) / (active weight) (9)

If any passive weight is present, the deflections calculated up to now must be multiplied by the weight factor K, and from equation (7) we get for the shortest wavelength to be used:

$$\lambda/m = 0.0816\ K\ (D/100m)^2 \tag{10}$$

We see how important it is to keep the passive weight down; but how do we accomplish this? A long compression member needs a certain minimum diameter in order to prevent sagging under its own weight and buckling under its longitudinal force, and for the same reasons the wall of a pipe must have a certain thickness. There are two possibilities: first, we might avoid all passive weight and build the members from single steel pipes of proper diameter and wall thickness. This gives us $K = 1$, at least for the octahedron itself, but for a diameter of, say, 400 feet, we obtain an octahedron weight of 2000 tons. Since this is much more than we want to pay for, and also much more than we need for holding the surface against any wind, we should compromise to some extent and go to the second possibility of splitting up the members into three or more chords connected by braces. In this way we sacrifice some accuracy and pay considerably less money. But this bargain is a good one only up to a point: even if we didn't care at all about accuracy or wind force, we still have to build a certain minimum structure just for a stable self-support. This problem, that the self-support of a structure tends to become more important than its purpose, seems to be a good example of Parkinson's Law.

In order to proceed in a general way, we now need a formula which gives the weight W of a member (split up into chords like a tower) as function of its length ℓ and of the force F it should be able to hold. Arguing that we need a minimum structure even without any force, and additional strength in the chords to hold the force, we might look for a formula of the type

$$W = A F \ell + B \ell^n$$

where the second term is the weight of the minimum structure. We see that $A = \rho/S$, but B and n should best be obtained empirically. Since exactly the same type of problem must arise in communication towers, I have taken the data quoted for 10 different towers, with a non-guyed length between 40 and 140 feet, and forces between 7 and 120 tons. From a best fit with these data I find

$$W = 0.06\, F\ell + 6.5\ell^2 \qquad \left\| \begin{array}{l} W \text{ and } F \text{ in tons} \\ \ell \text{ in 100 meters} \end{array} \right. \tag{11}$$

For further use we will always assume that the first term is active, the second one passive.

In order to keep the passive weight down, the number of compression members should be as small as possible, the remaining tension members being cables with no passive weight. A structure of this type, of course, must be "pre-stressed" for avoiding the sagging of cables, which gives the further advantage that we do not have any loose joints or rattling pieces. I suggest pre-stressing up to 1/2 of the full capacity of the material, the other half being left for taking up the wind forces.

6. Surface and Wind Force

For wavelengths above 10 cm we do not need a closed surface, and for the following we adopt galvanized wire mesh, with square openings, and with a wire diameter of 2 mm. The distance δ between neighboring wires then is given by the shortest wavelength to be used and by the transmission through the surface that can be tolerated. For the transmission we will demand "15 db down"; this reduces the gain by 3.2% and gives a noise contribution from the ground of 10 °K. We use a nomogram given by Jasik (Handbook of Antenna Design, Figure 25-40) and replace it by a formula

$$\delta/\text{cm} = 5.4\,(\lambda/\text{m})^{2/3} \tag{12}$$

which is a sufficiently good approximation in the range $0.1\text{ m} = \lambda = 2.0\text{ m}$.

From this formula, we calculate the weight of the surface. With respect to the model discussed later on, we assume that the diameter of the surface is 1.26 times the diameter of the octahedron; we multiply the weight by a factor of 2 in order to allow for wires or frames holding the mesh wire, and obtain

$$W_{surf} = 8.0\ \lambda^{-2/3}\ D^2 \qquad \left\{ \begin{array}{l} W_{surf} \text{ in tons} \\ \lambda \text{ in meter} \\ D \text{ in 100 meter} \end{array} \right. \qquad (13)$$

The next problem is the wind force. First, a general consideration. Even for a closed surface in the strongest storm, the wind pressure is about 10^5 times smaller than the maximum stress of steel, and this figure goes up to 10^6 or 10^7 for mesh wire. Thus, a small cross section of steel would be sufficient for holding a very large surface against the wind. But we must support a large surface at many points, and the small cross section of steel actually needed cannot be split up into many thin members because of the length to thickness ratio needed for preventing compression members from buckling. Thus, a lot of steel usually is wasted in the support of the surface, and the dish gets much more weight than needed. I think there is just one conclusion to be drawn: the surface should never be supported, it should be pulled by cables. Unlike compression members, cables can be split up unlimited, without any waste of material. Since we need a curved surface opening in forward direction, the cables could be kept within and behind the surface. Of course, the surface or its cables must be supported somewhere, and the most economical model seems to be one circular rim around the whole surface, all the rest being done by cables.

We want to know the amount of the wind force for two cases: stow position with 85 miles/hour true wind velocity (30 lb/ft^2), and observing positions with, say, 25 miles/hour. We use measurements of wind forces on flat mesh wire by K. N. Astill et al. (Aerodynamic and Radar Transmissivity Properties of Screen Materials; Cambridge, Mass. 1954). But since the wind force on curved surfaces might be more complicated, since we neglect all forces on the structure itself, and since our estimate should really be on the safe side, we multiply in most cases with a safety factor of 2, and in stow position we assume a closed surface for $\lambda \leq 0.2$ m. Omitting the details, we finally obtain (measuring again F in tons, λ in meters, and D in 100 meters)

<u>Stow position</u>

looking at zenith
$$F_{w,st} = \begin{cases} 310\ D^2 & \text{for } \lambda \leq 0.2 \text{ m} \\ 106\ \lambda^{-2/3}\ D^2 & \text{for } \lambda \geq 0.2 \text{ m} \end{cases} \qquad (14)$$

<u>Observing positions</u>

looking at horizon
$$F_{w,obs} = 29\ \lambda^{-2/3}\ D^2 \qquad (15)$$

maximum uplifting force, at 45°
$$F_{up} = 7.5\ \lambda^{-2/3}\ D^2 \qquad (16)$$

Another question which can be answered in general is the following: given a structure built for survival in the strongest wind, up to which wind velocity is observation possible for a given wavelength? Let ℓ be the length of a member and Q its cross section. At the safety limit we then have $F_{w,st} = QS$, and the deflection in observing position is $\Delta\ell = \ell F_{w,obs}/(QE)$;

or

$$\Delta \ell = \frac{S}{E} \frac{F_{w,obs}}{F_{w,st}} \ell \qquad (17)$$

We see another essential combination of material constants, S/E, which for steel is 6.67 x 10^{-4}. Omitting all details, and calling v the wind velocity during observation, we derive for our antenna, if $\lambda \geq 0.2$ m,

$$\frac{\lambda}{m} = 0.21 \frac{D}{100\ m} \left(\frac{v}{25\ mph}\right)^2 \qquad (18)$$

With an octahedron diameter of 300 feet, for example, built for survival at 85 mph, we can observe at 21 cm up to 25 mph.

II. Suggested Design

1. Summary of Basic Principles to be Followed

1) For given material and dismater, there is a shortest wavelength (Table 2) which cannot be surpassed within reasonable costs.

2) The structure should have about the same diameter in every direction. It should have at least three defined points where it should be held and about which it must turn, and it should provide support for a surface through its center. The best solution is a regular octahedron or some modification of it.

3) Thermal deformations can be neglected for diameters over 300 feet.

4) The number of compression members, as well as the weight of their internal braces, should be kept as small as possible, all remaining members being cables. The structure should be pre-stressed to half of its capacity.

5) The surface should be pulled by cables, being supported only by an outer rim.

2. The Spoked Wheel

The easiest way to get a stable rim surrounding the surface is a wheel with two systems of spokes to the two ends of a hub. In order to approach the octahedral shape, we make the hub as long as the diameter of the rim; a third system of spokes to the middle of the hub (representing two diagonals of the octahedron) would provide a flat surface through the center of the structure. Up to here, all weight could be made active if wanted by constructing hub and rim from single pipes. But two problems still remain to be solved: how to pull the surface into a parabolic shape, and where to hold and turn the structure; both should be solved with the least amount of passive weight.

First, the shape. We split up the rim and center spokes into two separate systems, and pull the center spokes together by short cables. One of the two curved systems of center spokes then holds the surface. Only the short cables and the surface provide passive weight, the connections between the two rims being part of the outer spokes. With only little more passive weight, the surface could be estended somewhat beyond the rim.

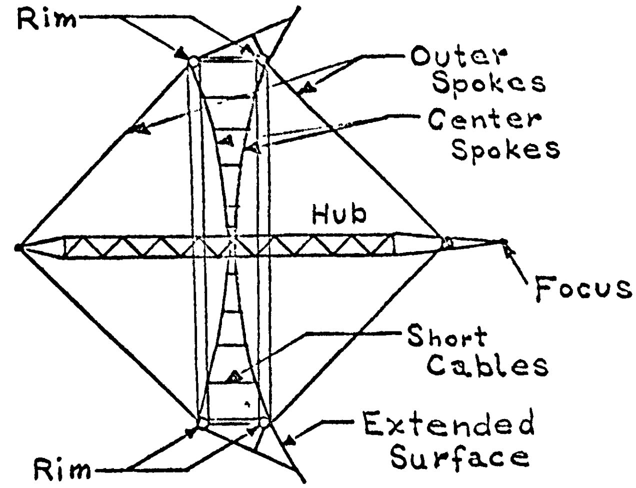

Figure 5. Spoked Wheel with Double Rim.

The spokes holding the surface are polygons instead of parabolas. The deviation should not exceed $\lambda/16$; from this we derive that the distance between these cables, as well as the distance between the short cables, should not exceed $0.57\sqrt{D\lambda}$. For a surface of 500 feet diameter and a wavelength of 20 cm, for example, we need 370 short cables and 66 outer cables. One might object that these outer cables are situated in front of the dish, disturbing the observation. But for the example given, the 66 outer cables block only 3% of the surface for $\lambda = 20$ cm, and 11% for $\lambda = 1$ m. Since they are distributed evenly and symmetrically, no serious side lobes could be introduced.

Second, the external support. The structure as shown in Figure 5 can be held at only three points without breaking its symmetry: at the center and at both ends of the hub. If there were some good way of using these points for support and drive, nothing would have to be added to the internal structure of Figure 5. Unfortunately, I was not able to find any safe and economical solution.

Let us decide to hold the structure at two opposite points of the rim, at equal height above ground. Only in one case would the structure of Figure 5 be sufficient: looking at the horizon without wind. For any other case we need an additional square of strong cablès, connecting both supporting points with both ends of the hub. The hub must get additional strength, and we need a second diagonal between the supporting points. The cross section of the strong cables should be equal to the active cross section of the rim. Since we have departed from the complete symmetry of Figures 2 or 5, we have introduced passive weight. A calculation showed that even if we make all compression members in Figure 6 from single pipes, the weight factor needed for equation (10) cannot be reduced below

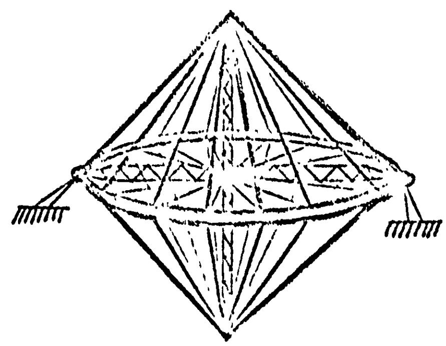

Figure 6. Strengthening of vertical plane.

$$K_6 = 1.27 \tag{19}$$

This will be sufficient for many cases. But if we are to reduce down to $K = 1$, we must go one step further by completing all additional parts of Figure 6 to a full octahedron. This gives a combination of the systems of Figures 2 and 5. If, in Figure 7, both systems have

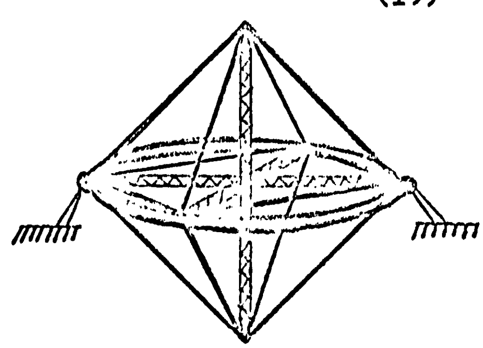

Figure 7. Octahedron plus wheel (spòkes not shown)

equal strength, we still get only $K = 1.27$; but now we can increase all cross sections of the octahedron until the weight of the wheel could be neglected, and can approach $K = 1$ in this way, although at considerable expenses.

3. Support, Drive, and Foundation

Having tried many other ideas without success, I suggest using the conservative approach: two supporting towers on wheels, running in a circle on tracks. This has the disadvantage that every point of the tracks must have the full strength for holding a tower, although only very few points actually hold the towers at any given position. But still it seems to be the best solution, if done as economically as possible.

Let us begin with the part where economy becomes most important: foundation and rails. These would become extremely expensive if built for special purpose and extreme conditions, especially in case of uplifting or lateral forces. For this reason we choose the opposite approach: we start with the usual, normal kind of railroad, with roadbed, ties and rails, and we try to adapt our structure to whatever conditions are given by this start. The usual railroad, for $80,000/mile, has a maximum load of 30 tons on one axle, and of 450 tons per 100 feet. It cannot restrain upward or lateral forces. Its accuracy after one year of use will be about ± 1/2 inch. Upward forces, then, must be avoided by counterweights over the wheels. For eliminating lateral forces we must have a strong pintle bearing at the center point of the circular track which takes up all horizontal wind force. As to maximum loads we suggest buying normal steel hoppers (without springs), mount the towers on just as many hoppers as needed for the combination of the weight of the structure plus downward component of the wind force plus counterweight against uplifts, and to get the counterweight by filling the hoppers with rock and gravel; since we drive our telescope extremely slowly as compared to a railroad train, we might surpass the load limits given by, say, 50%. As to the accuracy, we keep in mind that the circular track will have a diameter about equal to that of the dish; if we demand that the pointing accuracy of the telescope should be 1/10 of a beamwidth, we find that the accuracy of the rails must be 1/10 of a wavelength no matter what the diameter. Adopting ± 1/2 inch as the accuracy given for the rails, we arrive at a lower limit of 12 cm for the wavelength. Actually, this will even be better, since the tracks in the model I suggest will have a diameter larger than the dish by a factor of 1.39, which gives a lower limit of 9 cm.

For comparison I would like to show what happens if we go the other way. If we ask, for example, for a special single rail on concrete foundation, taking 300 tons down and 80 tons up and laterally, we arrive at $704,000/mile, almost ten times more than the usual railroad.

The next question is the shape of the towers. The most stable and most economical shape is a tetrahedron. Since railroad is expensive, we would like to do with only one circular roadbed, which can be done by putting one leg of the tetrahedron right at the center of the circle; strong connections to the pintle bearing at the center are needed anyway. This design causes a little trouble because of the clearance needed for the rim of our antenna wheel, but this can be solved as shown in Figure 8. The pintle bearing must hold a horizontal force equal to the full wind force in stow position; in addition, it has to hold a downward force equal to 1/3 of the total weight of dish and towers. Since the vertical components of the wind force from both towers always cancel at the pintle bearing, no uplifting

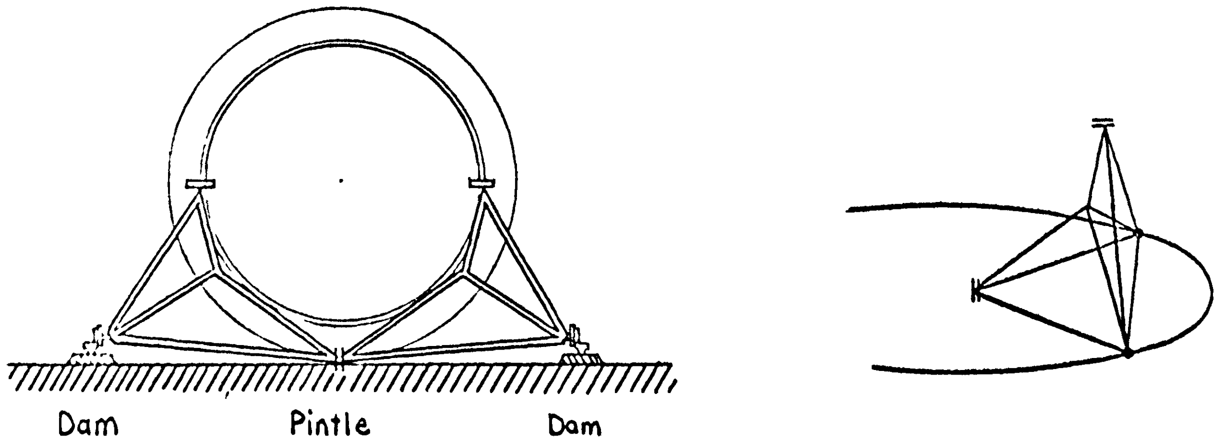

Figure 8. The supporting towers

force will result.

Next we have to decide about the drives. The azimuth drive is no problem and could be done with friction wheels on the rails. Since horizontal forces are taken care of at the pintle bearing, the only force acting on the drive could be a differential wind force, if the wind occasionally blows more on one side of the dish than on the other one. As to the elevation drive, we must be prepared for stronger and more permanent differential wind forces because of the great height of the dish when looking at the horizon; we thus want to have the drive point as far removed from the axle as possible. In general, the drive needs a round structure, at least a quarter of a full circle, and a point being guided along it. Since the round part would add a large amount of passive weight if mounted to the dish structure, we do it the other way and mount the round part on a third tower on wheels, guiding from there the lower end of the hub, as shown in Figure 9. One possibility might be to have a slit along the center line of the round part and a chain gliding behind this slit; a pin at the end of the hub sticks through the slit into the chain.

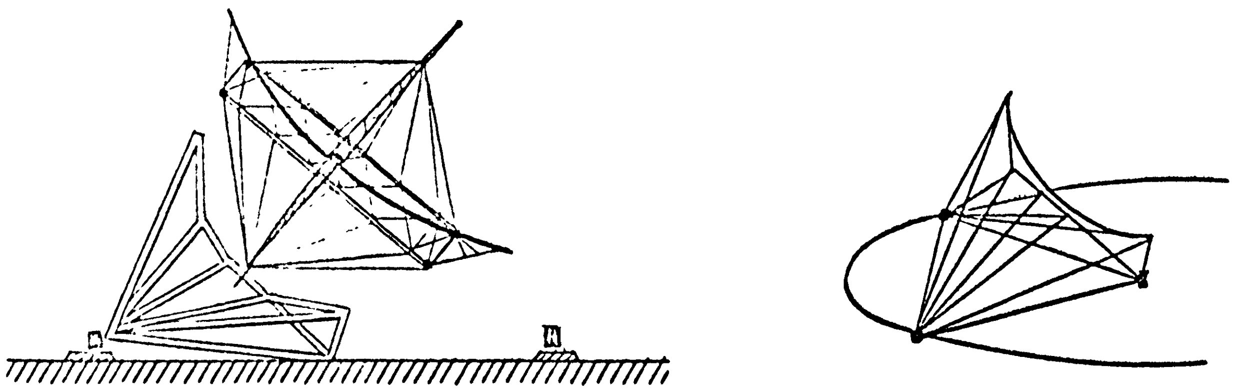

Figure 9. Third tower for elevation drive

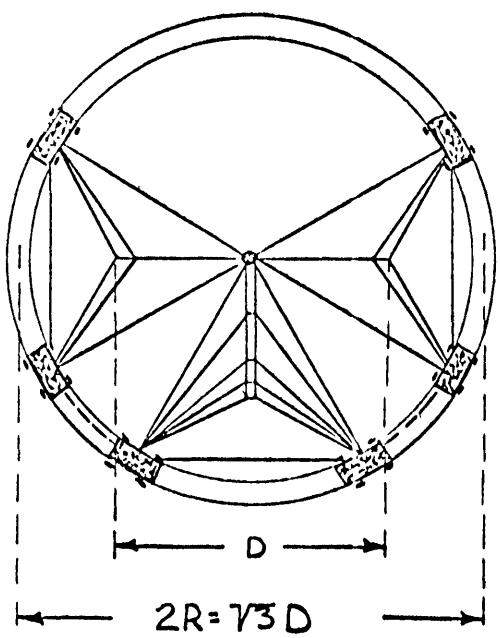

Figure 10. All three towers as seen from above. Each leg mounted on a freight car filled with rocks. Dashed lines are connections between towers.

Our last question is whether or not we should provide additional foundations in six points for a special stow position, plus some equipment for holding the tower legs fixed to these points. In Table 3 we have calculated for our model the forces (including counterweights) for stow position and the strongest wind, as compared to forces in observing conditions at 25 mph. The result reads: up to about 300 feet diameter (antenna surface) one might as well just make the whole track a little stronger. But for larger dishes it becomes definitely more economical to provide a specially fortified stow position.

Table 3

Forces for Stow and Observation

diameter feet	$\frac{F_{stow}}{F_{obs}}$
300	1.45
400	2.17
500	3.17
600	3.12

III. Diameter, Wavelength, and Price

1. General Picture

For any antenna of given diameter, the connection between price and shortest wavelength will show three characteristic points. The first point is the price of the minimum structure for $\lambda \to \infty$, just for a stable self-support. If we now insert surfaces of decreasing mesh size for observing at decreasing wavelengths, we take up increasing wind forces in the surface, and thus have to build stronger structures for safety; the result is a steady but slow increase in price with decreasing wavelength. During this process we always calculate the gravitational deflection of the resulting structure and find it smaller than $\lambda/16$ (which can be tolerated for observation), until we reach the second characteristic point where the deflection just equals $\lambda/16$. From there on to still shorter wavelengths, the weight of the structure is no longer defined by the wind force, but by the demand that the deflection be smaller than $\lambda/16$. This results in a steep increase of the price. Finally, we approach the shortest possible wavelength, as given in Table 2, which could be reached only with infinite mass (because of the passive weight of the surface). This limit is our third characteristic point. The position of each characteristic point, of course, is itself a function of the diameter chosen.

> It is obvious from these considerations that an economical antenna should be somewhere close to the second characteristic point, where the strength needed for safety just yields the rigidity needed for observation. At any other point, we have a waste of either strength or rigidity.

For practical application we need numerical values. We thus go to some more details and try to find the price as function of size and wavelength. For reasons of simplicity we confine this investigation to the dish and the three towers (omitting drives and foundations, which are estimated later on for one example). And as a convenient measure of price we just take the weight of the material needed. Altogether, we ask for the weight of dish plus towers as function of diameter and wavelength.

2. Balance of Forces

In order to obtain the weight, all cross sections must be determined. We do this in three steps. First, we want the structure to be balanced in the sense that all forces (pre-stressing, gravitation, wind) in the average create the same stress in all members; this yields the maximum forces in all members relative to each other. Second, we need one of these forces determined in an absolute way, which is done by requiring safety in stow conditions. Third, we calculate the gravitational deflection; if it is larger than $\lambda/16$ we fortify the active weight until we reach this limit. In the following we skip most of the details and just give all assumptions used and the results derived.

First, balance for the wheel of Figure 5, for example, is obtained if

$$\begin{aligned} F(\text{rim}) &= F_o/2\pi \qquad & F(\text{all outer cables}) &= F_o/\sqrt{2} \\ F(\text{hub}) &= F_o/4 \qquad & F(\text{all center cables}) &= F_o/2 \end{aligned} \tag{20}$$

where

$$F_o = \text{sum of all radial forces projected into the plane of the rim.}$$

For the additions of Figure 6 we demand that its square of heavy cables take up as much force as half of all outer cables of the wheel; this also defines the forces in the second diagonal and the additional force in the hub.

Since the transformation from force to weight follows equation (11), we need the non-guyed length ℓ of compression members. Ideally, this would be D/2 for hub and diagonal, and the distance between cables for the rim. Such a structure would be perfectly stable after being completely finished, but it never could be erected. In order to be on the safe side, we take ℓ = D for hub and diagonal, and ℓ = D/2 for the rim. From equations (11), (13) and (20) we obtain for the total weight of the dish structure (rim, hub, cables and strengthening, plus the surface)

$$W_{dish} = 0.140\ F_o\ D + (30.3 + 8\ \lambda^{-2/3})\ D^2 \tag{21}$$

the active part of which is (averaged over all dish elevations)

$$W_{act} = 0.110\ F_o\ D \tag{22}$$

Second, we relate this result to the wind force in stow position, and I think we are on the safe side if we demand $F_o = 2\ F_{w,st}$, which gives with equation (14) finally

$$W_{dish} = (30 + 8\ \lambda^{-2/3})\ D^2 \begin{cases} + 87\ D^3 & \text{for } \lambda \leq 0.2 \text{ m} \\ + 30\ \lambda^{-2/3}\ D^3 & \text{for } \lambda \geq 0.2 \text{ m} \end{cases} \tag{23}$$

Third, the weight factor, averaged over all dish elevations, turns out to be

$$K = 1.27 \begin{cases} + (0.444 + 0.117\ \lambda^{-2/3})/D & \text{for } \lambda \leq 0.2 \text{ m} \\ + (1.30\ \lambda^{2/3} + 0.343)/D & \text{for } \lambda \geq 0.2 \text{ m} \end{cases} \tag{24}$$

3. Increased Rigidity

The structure, up to here, is entirely defined by the strength needed for safety. If the value of K obtained from equation (24) is smaller than the one demanded by equation (10), the rigidity of the structure is good enough for observation, and the final weight is given by equation (23). But if K(24) is larger than K(10), we must increase the active weight. Both values of K are equal at the second characteristic point mentioned earlier. Table 4 gives some examples, compared with the shortest possible wavelength λ_3 at the third characteristic point, from equation (7), where $K = 1$.

Table 4

Wavelength λ_2 at the second characteristic point, for various surface diameters a. At λ_2, the strength needed for safety just yields the rigidity needed for observation. For comparison: λ_3 is the shortest possible wavelength.

a feet	D 100 meter	λ_2 meter	λ_3 meter
100	0.242	0.036	0.005
200	.484	.069	.019
300	.726	.111	.043
400	.968	.162	.076
500	1.22	.243	.122
600	1.45	.337	.172

The second characteristic point is also the most economical point, but perhaps we have to go to shorter wavelengths. The structure then is defined entirely by the rigidity needed for observation. For the weight of the complete dish, in the range $\lambda_2 \geq \lambda \geq \lambda_3$, we find after some calculation:

$$W_{dish} = \begin{cases} \dfrac{\lambda}{\lambda - \lambda_3} \left(18.6\ D + 30 + 8\ \lambda^{-2/3}\right) D^2 & \text{for } \lambda \leq 0.2 \text{ m} \\[2ex] \dfrac{\lambda}{\lambda - \lambda_3} \left[(6.4\ D + 8)\ \lambda^{-2/3} + 30\right] D^2 & \text{for } \lambda \geq 0.2 \text{ m} \end{cases} \qquad (25)$$

From equations (23) and (25) we have calculated the values given in Table 5. As compared to conventional antennas, the weights are lower by almost a factor of ten. The step line in the table represents $\lambda = \lambda_2$.

Table 5.

The weight of the complete dish (all structure plus surface) for various surface diameters a. The step line is the most economical point, the last row ($\lambda = \infty$) is the minimum weight for self support.

λ in meter	W_{dish} in tons			
	a = 300 feet	400 feet	500 feet	600 feet
0.06	143	-	-	-
0.08	99	-	-	-
0.10	75	340	-	-
0.15	63	155	650	-
0.20	60	127	293	1210
0.30	49	103	190	340
0.50	39	81	147	231
1.00	30	60	107	166
2.00	24	47	82	126
∞	14	25	40	57

4. The Towers

We adopt two supporting towers as shown in Figure 8, and one elevation tower as in Figure 9. For simplicity we calculate the weight assuming all towers to be regular tetrahedrons, all three of the same strength. Actually, the towers have additional members and thus more weight, but the elevation tower does not need so much strength. These deviations go the opposite way and may about cancel each other. The horizontal connections between any two legs of each tower are given the same strength as the legs. The strength of a leg is calculated according to the weight of the complete dish, plus the wind force in most unfavourable stow conditions, both acting only on the legs of the two supporting towers. The result is given in Table 6.

Table 6

The Combined Weight of all Three Towers (without drives, wheels, counterweights)

λ	W_{3tow} in tons			
in meter	a = 300 feet	400 feet	500 feet	600 feet
0.06	104	-	-	-
0.08	99	-	-	-
0.10	96	239	-	-
0.15	94	209	474	-
0.20	93	204	400	842
0.30	81	172	323	533
0.50	69	143	263	419
1.00	58	116	207	323
2.00	51	99	172	263
	39	70	112	161

5. The Total Weight

Figure 11 shows the total weight of dish plus towers for various surface diameters as function of the shortest wavelength. This figure should be used for decisions about diameter and wavelength for large antennas. The three characteristic points are marked, and we see that the best choice is always somewhat to the left of the second point, before the steep increase toward the third point begins. For smaller dishes, there is a flat part, left of $\lambda = 20$ cm, because for smaller wavelengths we have assumed a closed surface in stow position, thus the wind force cannot increase any more by having smaller meshes for smaller wavelengths. From Figure 11 we get a first rough estimate for the total price of such an antenna, if we assume $1000/ton of steel plus erection, and if we then multiply by about a factor of two in order to allow for drives, bearings, foundations, tracks and cars.

6. Foundations and Total Price

The foundations will not be included in the general investigation. We just select a few "economical" antenna models, and calculate the forces acting on a single leg of a tower under unfavorable conditions.

In Figure 11 we select for each diameter a wavelength somewhat left of λ_2, as given in Table 7. The next column shows the weight of dish plus towers, from which we obtain a rough estimate of total price (including foundations and drives) given in the fourth column. The next six columns show the maximum forces on a single leg, for stow and observation: downward force, upward force, and the sum of both. If we fight uplifting forces by counterweights, as we propose to do for observation, this sum is the force the rail has to withstand.

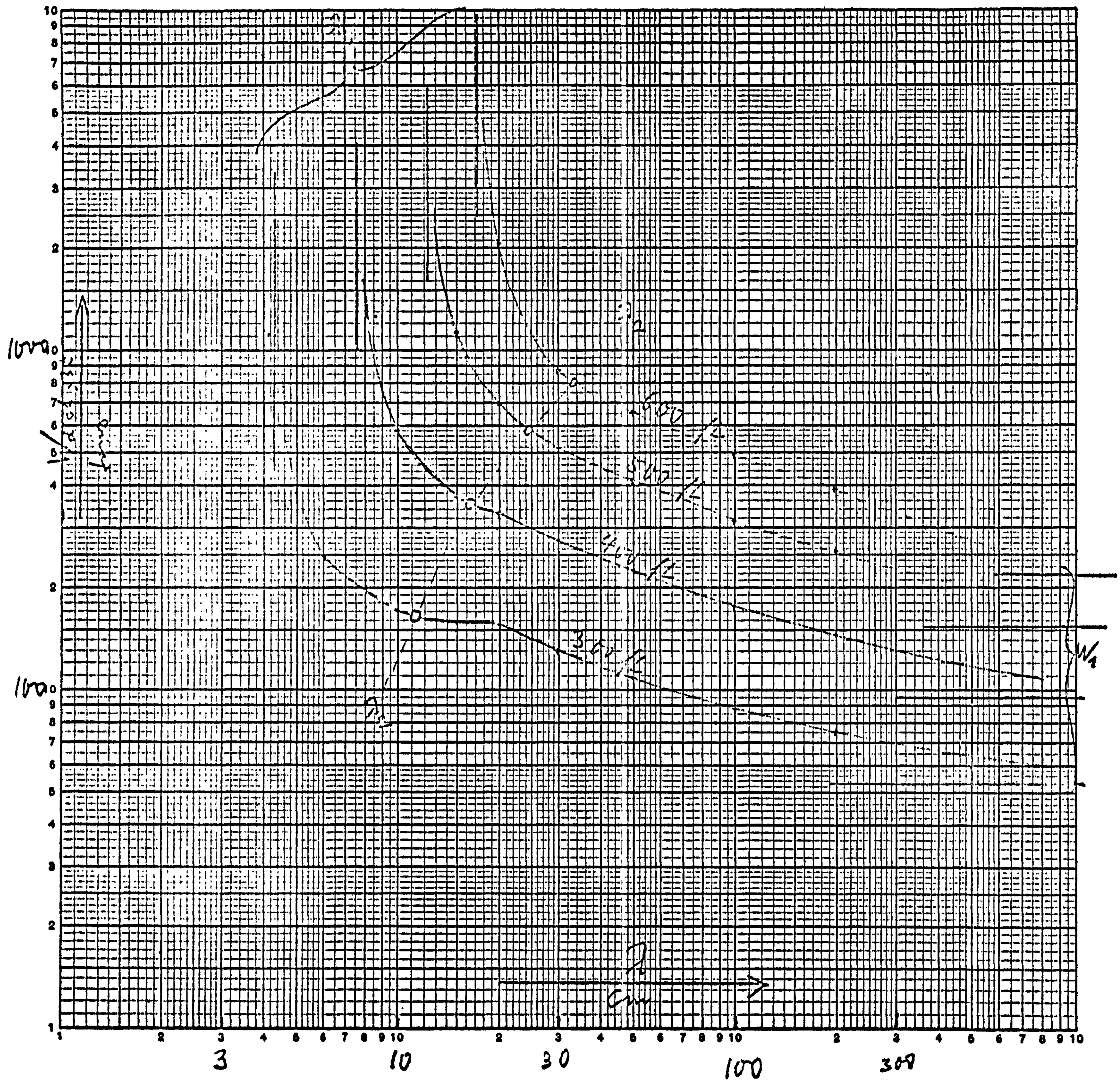

Fig. 11. The total weight of dish plus three towers.

W is the weight of a minimum structure for self-support
The most economical point is at λ_2(waste of neither strength nor rigidity)
λ_3 is the shortest possible wavelength

Table 7

Some Selected "Economical" Antennas. See text.

a	λ	$W_{dish,3t}$	Total price	Max. forces on single tower-leg					
				Stow position			Observing posit.		
				down	up	sum	down	up	sum
Feet	cm	tons	10^6 \$	tons			tons		
300	5	380	0.76	114	59	173	79	40	119
400	10	580	1.16	193	111	304	104	38	142
500	20	690	1.38	284	185	469	115	33	148
600	30	870	1.74	317	192	509	135	28	163

The forces given in Table 7 should now be compared with the limits given for normal railway tracks and cars:

Railway track

Railway track	Price	\$80,000/mile
	max. load on single axle	30 tons
	max. load per 100 feet	450 tons
	accuracy after 1 year	± 1/2 inch

Freight cars

	steel hopper	largest gondola
price per car	\$17,000	\$35,000
empty weight	20 tons	40 tons
total weight loaded	70 tons	160 tons

In order to be on the safe side, we ought to multiply all forces from Table 7 by a factor of, say, 1.5; but on the other side, the load limits given above might easily be surpassed by 50% with respect to the slow motion of a telescope as compared to a train. We thus conclude that even the largest antenna from Table 7 does not need more than one gondola per leg of the supporting towers for observation. A steel hopper will probably do for the legs of the elevation tower; the capacity per 100 feet is nowhere surpassed for observation. The stow positions will need extra support from concrete, and some means for tying the legs down.

IV. Total Price for 500 Feet Diameter and 20 cm Wavelength.

For the special purpose of observing lunar occultations, a shortest wavelength of about 50 cm would be sufficient, but a lower limit would be preferred if possible; since the instrument will be free for other users half of the time, we choose 20 cm. From Figure 11 we then select a surface diameter of 500 feet. This represents a large, general purpose instrument for many uses.

Dish 300 tons; Towers 400 tons	700 tons;	\$1000/ton	\$700,000	
additional for more difficult erection			100,000	
Bearings and pintle bearing			60,000	
Drives			150,000	
Dish surface	2×10^5 ft^2;	10 cent/ft^2	20,000	
Controls			80,000	
Miscellaneous			10,000	
		Complete dish, towers, drives		\$1,120,000
Railroad	0.834 miles;	\$80,000/mile	70,000	
Cars (4 gondolas, 2 hoppers)			170,000	
Stow supports			100,000	
Pintle foundation			40,000	
		Complete foundations and cars		\$ 380,000
		Total =	\$1,500,000	