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Homologous Deformations of Tiltable Telescopes

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Summary

A telescope tilted in elevation angle must deform under its dead load, and this sets a lower limit to the shortest wavelength of observation once the diameter is chosen. A most natural way of passing this limit is by designing a structure which deforms unhindered, but which deforms one paraboloid of revolution into another, thus yielding a perfect mirror for any angle of tilt. Focal length and axial direction are permitted to change (to be servo-corrected by focal adjustments).

First, a proof is given that any stable structure has homologous solutions, at least in a mathematical sense (whereas physical solutions must have all bar areas positive, and practical solutions must fulfill all specifications with a low total weight). Second, a mathematical method is described for obtaining such solutions on a computer, it is linearized and iterative, changing all bar areas simultaneously in each iteration step. Calling ΔH the rms deviation between the deformed surface and a best-fit paraboloid, the goal is to obtain a set of bar areas such that $\Delta H = 0$, for a fixed but arbitrary number N of equally-spaced surface points. Third, exact solutions (physical and practical) are obtained for a variety of structures, yielding $\Delta H \leq 10^{-4}$ inch after only 2 - 4 iterations. Fourth, the total weight is obtained as defined by wind deformations during observations and by stresses during survival conditions. Fifth, the sensitivity of ΔH to manufacturing tolerances is investigated.

Two designs for 300 feet diameter are given. The first is defined by survival, yielding a shortest wavelength of $\lambda = 4.4$ cm with a total weight of 476 tons (same weight as our 300-foot at NRAO, but more than three times shorter λ). The second is defined by wind deformations, yielding $\lambda = 2.1$ cm with 748 tons. Both can be built from off-the-shelf pipes and can observe in sunshine and shadow. Focal adjustments are about one inch. It seems that gravity can be omitted without paying any price.

I. The Gravitational Limit

There are three natural limits (as opposed to financial ones) for diameter D and shortest wavelength λ, for tiltable conventional telescopes [1]:

$$\text{stress limit} \qquad D \leq D_{st} \approx 600 \text{ m}; \tag{1}$$

$$\text{thermal limit} \qquad \lambda \geq \lambda_{th} \approx 2.4 \text{ cm} \frac{D}{100 \text{ m}}; \tag{2}$$

$$\text{gravitational limit} \qquad \lambda \geq \lambda_{gr} \approx 8.0 \text{ cm} \left\{\frac{D}{100 \text{ m}}\right\}^2. \tag{3}$$

The stress limit is reached when the dead load of the structure produces at the bearings the maximum allowed stress of the material; at present we are far below this limit. The thermal limit applies to a telescope with good protective paint in full sunshine ($\Delta T = 5^\circ$C); this limit can be passed by a factor 2 - 5 in a radome, or an open dome (36-foot telescope at Kitt Peak), or at night. The gravitational limit arises from the deformations under dead load if the telescope is tilted from zenith to horizon; the value given, 8.0 cm, applies to an economical structure and is the same for steel and aluminum, it can be brought down to 5.3 cm with an uneconomically high total weight. We see in Figure 1 that the gravitational limit is the essential one for large telescopes ($D \geq 100$ ft); some telescopes come very close to it, but not a single existing tiltable telescope passes it.

II. Homologous Deformations

There are several ways of passing this gravitational limit: (1) Fixed elevation transit telescopes (the 1000-foot dish in Arecibo does not move at all; the LFST group has worked out three 600-foot telescopes moving 360° in azimuth);

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[1] S. von Hoerner: "Design of Large Steerable Antennas", Astron. J. 72, 35, 1967. (First published as an LFST-REPORT, June 1965. The LFST group, headed by Dr. Findlay of NRAO, investigates the possibilities for a Largest Feasible Steerable Telescope; its reports and summaries can be obtained from Green Bank on request.)

(2) Motors in the structure or at the surface panels, correcting the deformations; (3) Levers and counterweights as in large optical telescopes. But the most natural way seems to be: (4) Designing a structure which deforms completely unhindered, but which deforms one paraboloid of revolution into another one, thus yielding a perfect mirror for any angle of tilt. Since this deformation transforms one member of a given family of surfaces into another member of the same family, we suggest to call it a "homologous deformation" [1], and the permitted changes of focal length and axial direction we call "homology parameters".

Homologous deformations can be demanded for an arbitrary number N of equally spaced structural points, holding the surface or the panels, where N must be chosen so large that any deformation between neighboring points can be neglected ($\leq \lambda/16$). The minimum N then is proportional to λ^{-1}, and we must have $N = 2$ for $\lambda = \lambda_{gr}$ (two bearings of the conventional telescope). But the design of the panels is somewhat eased if we demand at least:

$$N = 3 \frac{\lambda_{gr}}{\lambda} . \tag{4}$$

Since small deformations can be superimposed, homology holds for all angles of tilt if it holds for two; a paraboloid of revolution is defined by 6 points, and since deformations parallel to the surface do not matter, we obtain homology if a set of 2(N-6) conditions is fulfilled. On the other hand, a structure of N unconstrained points needs at least 3(N-2) members just for stability, and even for a fixed geometry we still have 3(N-2) degrees of freedom just for the bar areas. Since $3(N-2) - 2(N-6) = N+6 > 0$, the problem is solvable and there is a family of solutions with at least N+6 free parameters.

But this existence proof holds for "mathematical solutions", whereas a "physical solution" demands all bar areas to be positive and finite, and a "practical solution" must fulfill all specifications with a small total weight. There certainly are structures which have only mathematical solutions but no physical ones, for example, if we mount the two elevation bearings at surface points.

As a first approach to homology, the concept of an "equal softness structure" was introduced in [1], which is explained in Figure 2. A conventional structure mostly has hard and soft surface points, which clearly is illustrated by the measured deformation patterns of several telescopes. Since we cannot make the soft points hard, we have to make the hard points soft, although this might hurt our feelings. We obtain about equal softness when the structural ways to the nearest bearings are about equal for all surface points. A structure like Figure 2c comes already close to a homologous deformation, while a structure like Figure 2a mostly has no practical solution at all.

III. The Homology Method

In 1965, a mathematical method was developed [2] for obtaining homology solutions on a computer. The homology problem would lead to a set of 2N highly non-linear equations, but the method used is linearized and iterative. With the input data we give a complete structure, its geometry (coordinates) as well as a "first guess" of all bar areas A_γ. The method then keeps the geometry unchanged, but it changes all A_γ simultaneously in each iteration step, demanding a zero rms deviation ΔH between the surface points and a best-fit

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[2] S. von Hoerner: "Homologous Deformations of Tiltable Telescopes", Journal of the Structural Division, ASCE, 93, 461, 1967.
(First published as LFST-REPORT No. 4; Nov. 1965).

paraboloid of revolution. We define

$$\Delta H = \left\{ \frac{1}{2N} \left[\sum_{i=1}^{N} (\delta z_i)^2_{\text{zenith}} + \sum_{i=1}^{N} (\delta z_i)^2_{\text{horizon}} \right] \right\}^{1/2} \tag{5}$$

as the "deviation from homology", where $\delta z_i = \Delta z_i - b(\Delta x_i, \Delta y_i)$ is the difference in z-direction (parallel to the optical axis) between the deformed surface point i and the best-fit paraboloid of revolution; Δx_i, Δy_i, Δz_i are the deformations of point i. The task is to find a set of bar areas such that $\Delta H = 0$. This task is represented by a set of 2N linear equations, which we call "homology equations".

From all possible homology solutions (we have at least N+6 free parameters), the method selects that solution which is most similar to the first guess. With this demand we try to avoid impractical solutions. The first guess should be made such that the structure withstands the survival conditions and has only small wind deformations, both with a minimum total weight, and the homology iterations should stay close to this condition. Finally, keeping all changes as small as possible gives the best hope for a good convergence. The method is described in full detail in paper [2], and here we give only a brief outline. The present method neglects the bending stiffness, regarding each joint as a pin-joint.

The method used is a generalization of Newton's method for finding the zero point of a function. If x is wanted such that $y(x) = 0$, Newton's method starts with some initial value x_o (first guess), and iterates according to $x_{i+1} = x_i - y_i \,/\, (dy/dx)_i$. This is generalized to n variables, where $n = m+4$ = number of members plus number of homology parameters. The quantity whose zero is wanted is ΔH from (5), and we see that we now need $\partial\Delta H/\partial A_\gamma$, the derivatives of ΔH with respect to all bar areas. In (5), ΔH goes back to the deformations Δz of the surface points which are given as

$$\Delta z = K^{-1} F \tag{6}$$

where K^{-1} is the inverse of the stiffness matrix K, and F is the force vector given by dead loads and surface weight. We then need the derivatives of all elements of K^{-1} with respect to all bar areas, $T_{ij\gamma} = K^{-1}_{ij} / \partial A_\gamma$ (a tensor of three dimensions), and we also need all $\partial F_j / \partial A_\gamma$. Two facts make the method easy. First, the derivatives of K^{-1} can be obtained from those of K with a formula derived in the appendix of [2], which can be written as

$$T_{ij\gamma} = - \sum_p \sum_q K^{-1}_{ip} \frac{\partial K_{pq}}{\partial A_\gamma} K^{-1}_{qj} \quad , \tag{7}$$

(which simply is the matrix equivalent to $(1/y)' = - y'/y^2$). Second, the elements of K are always linear in the A_γ, and thus $\partial K_{pq} / \partial A_\gamma$ = constant throughout all iterations for a given and unchanged geometry, and the same is true for the $\partial F_j / \partial A_\gamma$. In this way, the wanted change of ΔH (for obtaining $\Delta H = 0$) is finally traced back to the unknowns, the needed changes of the bar areas.

The combined task of (a) achieving homology, and (b) selecting that solution which is most similar to the first guess, is treated by the method of Lagrangean multipliers, but we have also included the possibility of making the homology parameters as small as wanted if they should turn out too large; we thus minimize

$$\sum_{\gamma=1}^{m} (dA_\gamma/A_\gamma)^2 + \omega \sum_{k=1}^{4} h_k^2 = \text{Min.} \tag{8}$$

where dA_γ is the needed change of bar area A_γ, and $h_1 \ldots h_4$ are the homology parameters; ω is a factor given with the input data which tells how important we consider small homology parameters. One iteration step then runs as follows. We build up the stiffness matrix, its inverse, and the force vectors. After several matrix operations we arrive at (8) where we finally have to solve a set of n linear equations, yielding the changes dA_γ which then are added to the old values of A_γ. With these improved bar areas we again build up a <u>new</u> stiffness matrix and force vectors, and we repeat the procedure until a given number of iterations is finished.

The present method solves an optimum condition, equation (8), together with a set of 2N constraint equations (homology equations) for obtaining $\Delta H = 0$; the external specifications (survival stress, wind deformation) are checked separately later on. This way was chosen for obtaining best convergence in case that convergence is a problem. But from our present experience, convergence is no severe problem, and the best method then would be a different and more direct one. In addition to the homology equations, one should set up the external specifications in a linearized way. The demand on wind deformation would be represented by a set of w inequalities (if w different wind directions are specified), and the demand on maximum stress would yield a set of m inequalities. The optimum condition then should minimize the total weight. In the present method we have constraint equations only, whereas in the new method we would have equalities as well as inequalities. This task can be solved by a combination of Lagrange multiplies and Fritz John multipliers, as shown by Mangasarian and Fromovitz [3]. I would like to add that this method seems to be the best and most direct one for a large variety of optimization tasks in engineering. The only remaining problem is again the convergence;

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[3] O. L. Mangasarian and S. Fromovitz: "The Fritz John Necessary Optimality Conditions in the Presence of Equality and Inequality Constraints", J. Math Anal. Applic. 17, 37, 1967.

we found that the linearization of the homology equations gives fast convergence for a wide range of first guesses, but we cannot tell without trying whether this also holds for the linearization of the external specifications.

IV. The Total Weight

Homology has nothing to do with the total weight. If we multiply all bar areas (and the surface weight) by a factor q, then the weight increases by a factor q, but so does the stiffness, and all gravitational deformations stay the same. Since the weight must be defined somehow, the present program keeps it constant. It also keeps constant the counterweight needed for balance. Mostly we make our structures completely balanced (counterweight zero) before applying the homology iterations, but we also can choose any given amount of counterweight. The iterations then keep the counterweight constant, zero or not. This means we have two more constraint equations in addition to our 2N homology equations.

For an actual design, the total weight is defined by either one of three conditions, see [1]. (a) Stable self-support under dead loads, for a telescope inside a radome; (b) Stability under specified survival forces, for an exposed telescope and medium wavelengths; (c) Specified wind deformations during observation, for an exposed telescope and very short wavelengths. In the present program, this final total weight is not obtained automatically. We start with a first guess fulfilling all conditions, then we iterate until homology is reached, and thereafter we check with a separate program whether all conditions still are fulfilled. If not, we make an improved first guess and try again. (Sometimes, we also changed the geometry slightly.) Although this procedure does not look very elegant, it still seems to be the best one for gaining experience and understanding.

It turned out that this method is very easy for case (c), mostly not too difficult for case (b), but a little troublesome for case (c) if one really wants a low total weight. This difference is easily understood. If the wind deformation is too large by a factor q, we simply have to multiply each bar area and the surface weight by q, and we obtain an exact solution even without repeating any iteration. Survival stresses again go down (although by a smaller factor) if we multiply all bar areas by the same factor, but the dead load stresses stay the same. Thus in case (a) we have to try an essentially different first guess. It seems that one really should solve case (a) with the method suggested at the end of the last section.

The additional program calculates the stress in each bar in zenith position, S_z, and in horizon position, S_h, where the maximum stress for any elevation then is

$$S_m^2 = (S_z^2 + S_h^2)^{1/2}. \tag{9}$$

This is done either for dead loads in case of a radome, or for survival loads in case of an exposed telescope. The program then calculates the maximum allowed stress, S_o, according to the ℓ/r ratio of this member (more exactly: of its chords); each member is actually a built-up member according to Figure 3, where the influence of the lacings on weight and stiffness has been taken into account. The stress factor

$$Q = S_m/S_o \tag{10}$$

then is printed for each member, and the stability condition is fulfilled if all $Q_\gamma \leq 1$. As to the numerical value of the survival condition, we have adopted a time of, say, 30 years after which a telescope becomes obsolete, and a chance of, say, 1 per cent for losing the telescope in a storm before it becomes obsolete. We then obtain from wind statistics at Green Bank [4] a survival wind of 110 mph at 200 feet height (= 90 mph as measured at 40 ft height, gusts included). Using this value, the telescope then is also stable against a snow load of 20 lb/ft^2, or 4 inch of solid ice. And since we should be able to dump the snow by tilting, we specify this load for any elevation angle according to (9).

The wind deformation is, at present, calculated only for a wind face-on, assuming that this is the worst condition (different angles are planned for future checks). Since all gravitational deformations are omitted by homology, we omit the dead loads and regard the wind loads only. We calculate the rms surface deformation in z-direction (no best fit this time) and call it $\Delta\zeta$. If the telescope is held and guided in an economical way (Section VI), the better part of $\Delta\zeta$ is just a parallel translation which does not matter. The remaining part which matters is due to (a) gusts of any size but faster than the servo loop of the drive, giving rise to pointing errors, and (b) gusts of any speed but smaller than the telescope radius, giving rise to surface deformations. An estimate of this remaining fraction of $\Delta\zeta$ is in preparation, but at present we just adopted $0.5\Delta\zeta$. Furthermore, the calculations of Simpson, Gumpertz and Hager (Section VII, 4) showed an rms difference in half the path length of only $0.777\ \Delta\zeta$, and both effects combined give $0.389\ \Delta\zeta$. On the other hand, we should add the deformations of the towers holding the telescope, where an estimate showed that we should add about 60 per cent (for an economical tower design),

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[4] S. von Hoerner: "Statistics of Wind Velocities at Green Bank"; LFST REPORT No. 16; Dec. 1966.

thus obtaining 0.632 $\Delta\zeta$. Finally, this value should be 1/16 of λ, and we arrive at a shortest wavelength λ for an rms deformation $\Delta\zeta$:

$$\lambda = 10\ \Delta\zeta. \tag{11}$$

How do we specify the highest wind during observation? I have asked several observers, what fraction of their observing time they would be willing to lose at the shortest wavelength, due to high winds, in order to get the largest possible telescope for a given amount of money; the answer was "about one quarter". According to our wind statistics [4], we then arrive at a wind speed of 22 mph at 300 feet height (= 17 mph as measured at 40 feet height, gusts included); the wind is higher than that for 15 per cent of all time during summer, 30 per cent during winter, and 24 per cent all year. This speed of 22 mph, together with (11), then gives the specification for the deformation $\Delta\zeta$.

V. Miscellaneous

1. Sensitivity. The homology program delivers the final bar areas with 6 digits, but what accuracy do we actually need? What is the sensitivity of a homology solution to manufacturing deviations? This is answered by a small auxiliary program called "Sensitivity". We start with perfect homology, and then assume that each bar area is changed according to $A_\gamma(1 + \varepsilon_\gamma)$, where the ε_γ are uncorrelated random numbers with mean zero and variance $\varepsilon^2 \ll 1$. The resulting deviation from homology, ΔH, then can be obtained analytically (without using actual random numbers); and demanding $\Delta H \leq \lambda/16$ yields the maximum tolerated ε. Fortunately, it turned out that rms deviations of the bar areas of about 12 per cent can be tolerated for most practical purposes

(see Table 4). This means we can use off-the-shelf shapes or pipes which give 10 per cent, if we always choose from the steel manual that size which comes closest to our A_γ from the computer output.

2. Thermal Deformations. If the gravitational limit is passed by homology or else, the next natural limit then is the thermal limit. We do not calculate the actual deformation of a given structure under a given temperature distribution. Instead, we use with some confidence estimate (2) which assumes a temperature difference of $\Delta T = 5^{o}C$ in unfavorable places; it further assumes that all bars are made from steel and that the surface, if made from aluminum, is allowed to "float" on the back-up structure of the panels. As to the expected values of ΔT, we use some experiments described in [5]. For good protective white paint, we found $\Delta T = 5^{o}C$ as the difference between sunshine and shadow on the average for clear, sunny summer days at noon. A second effect is given by the time-lag of heavy members in case of rapidly changing ambient air temperature (mostly around sunrise and sunset). For hollow members with white paint in winds below 5 mph, the time scale τ of thermal adaption was found by experiments as 1.73 hours per inch of wall thickness for steel, and 1.14 hours per inch of wall thickness for aluminum (length and diameter do not matter within wide ranges). The time scales are half these values for T and L shapes and solid rods; the time scales of unpainted aluminum and galvanized steel are 1.8 times longer. If the air changes by $\dot{T}$ (^{o}C/hour), a member lags behind with $\Delta T = -\tau\dot{T}$. At Green Bank, on 1/4 of all days, the measured maximum change of the day is $\dot{T} \geq 3.5^{o}C$/hour. Thus, if $\Delta T \leq 5\ ^{o}C$ is demanded for 3/4 of all days, the heaviest members then must have

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[5] S. von Hoerner: "Thermal Deformations of Telescopes", LFST-REPORT No. 17; Jan. 1967.

a wall thickness below 0.83 inch for steel pipes, or below 1.66 inch for open shapes or rods. This, of course, can always be met by splitting up heavy members into several thinner ones; but then the wind resistance increases, and a good compromise is needed. One also could blow ambient air through hollow members at 15 - 20 mph, which would reduce thermal deformations by about a factor 3.

I would like to add that the effect of thermal deformations (just as in case of wind deformations) will be reduced by at least a factor 2, if the telescope pointing is done as suggested in Section VI.

3. Non-Parabolic Panels. It might be of advantage to make the surface panels of a shape which is different from a parabolic one, but easier to produce and to measure. For any given shape, a maximum size can be calculated. Formulas are given in [6] for flat plates, spherical panels (to be measured and adjusted with a pendulum), and for toroidal panels (two-axes pendulum). These formulas are derived such that no deviation from the true paraboloid is more than $\lambda/16$, which means that the rms deviation from the best-fit paraboloid is about $\lambda/40$. In this way the values of Table 4 are calculated.

VI. Telescope Pointing by Optical Means

The following is connected with homologous deformations only insofar as, in the present situation, we must find much cheaper ways of building telescopes, otherwise we do not get the money for any large telescope, homologous or not.

Many of the older radio telescopes used a polar mount, which is the only way of driving a telescope without a computer (or analog coordinate converter).

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[6] S. von Hoerner: "Non-Parabolid Panels, and Surface Adjustment" LFST-REPORT No. 18; June 1967.

Since nowadays large telescopes always will have an on-line computer anyway, they mostly use an alt-azimuth mount which has many obvious advantages structurally and even some advantages observationally. But in both types of mount, the pointing of the telescope mostly is measured at the axes or drive rings (too far away from the telescope surface), and with respect to some structural elements or rails (stressed by heavy loads). The most logical way seems to be measuring the pointing where it matters (right at the apex), and with respect to something unstressed and unmovable (fixed points on the ground), which can be done by optical means. The JPL antenna at Goldstone, California, comes close to this demand, measuring the pointing at the apex and with respect to an internal unstressed pillar reaching close to the apex. But this internal pillar gives some structural (and financial) disadvantages. One should go one step further and use a sufficient number of light beacons right on the ground, as suggested in [1].

Some satellites, rockets, and balloon telescopes already use optical pointing devices, "locked-in" to the bright rim of the Earth or Sun, or to some brighter stars. An investigation is planned into the availability, accuracy and cost of such devices, and into their application to radio telescopes. The basic idea is to have a rotatable platform mounted behind the apex, with several small optical systems (theodolites) equipped with photocells, looking at as many light beacons (flashlights) mounted on concrete blocks on the ground, see Figure 4. A servo-system keeps this platform "locked-in" to the beacons. At the joints between platform and telescope structure, we measure two angles and thus obtain the pointing direction of the telescope. Measuring these two angles, and guiding the telescope into the desired direction with a second servo-system, can be done by normal techniques already in use; new is only the reference system defined by the platform, locked-in optically to the ground.

In principle, we need three beacons, and only two if the direction of gravity is measured independently by some kind of pendulum. Actually, we should have about twice as many beacons, because in some telescope positions one or the other light path will be blocked by the surface or some structural members. In a first approach, the pointing direction of the telescope can be defined by the structural element where the platform is mounted, and maybe this is already all we need in the next future. In a more sophisticated version, a second platform, mounted in front of the apex, can look at the feed and three or more points on the dish surface, from which the computer can find the (best-fitting) pointing direction even of a slightly deformed dish.

The disadvantages of this method are, first, that it does not work in heavy fog or cloudburst, but then we cannot observe at very short wavelengths anyway; and since we do not need high accuracies for long wavelengths, the telescope could be equipped with an additional pointing system of conventional type for those cases. Second, there is the usual human inertia against any new method; large optical telescopes are <u>still</u> polar-mounted, and nobody can explain why.

There are two major advantages. First, this method keeps the pointing accuracy completely independent of the accuracy of elevation rings and azimuth rails. As far as pointing is concerned, we could as well drive the telescope on a circular dirt road, and pull it into the right elevation by chains or ropes. Actually, one would use plain, normal railroad equipment for the azimuth ring, with $100,000 per mile for erection, $400 per mile and year for maintenance, and an accuracy of 1/4 to 1/2 inch vertical and lateral, see [7]. Second, with respect to thermal deformations, constant wind loads and all gusts slower than the servo-loops, we completely omit all deformations occurring between the apex and the ground (telescope suspension, bearings, elevation ring, towers, rails, and foundation). This will cut down the remaining effective deformation by at least a factor 2, even for the first approach, and the more sophisticated version should yield at least another factor 2. It may cut down the costs of foundations and rails by almost a factor 10, see [7].

VII. Numerical Results

The homology method was programmed at the Department of Civil Engineering of the University of Virginia, Charlottesville, Va. It was run with a Burroughs 5500 on several structures with good success. The results will be published somewhere else [8]; the following is only a short summary.

We began with very simple structures, just for gaining experience, and then proceeded to more complicated ones, looking more and more like telescopes. The last one successfully run has N = 13 homologous surface points, a total of p = 26 points (pin joints), and m = 112 members. The next one with N = 21, p = 34, m = 128 could not be run because of memory limitations (200,000 word disc, magnetic tape is too slow). A second version of the program is now in

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[7] S. von Hoerner: "Discussions with Railroad Engineers"; LFST=REPORT No. 14, Sept. 1966.

[8] M. Biswas, R. Jennings and S. von Hoerner; in preparation.

preparation and half finished; it calculates only one quadrant of a symmetrical structure. We then hope to reach about $N = 90$, $p = 190$, $m = 600$. For a structure of this size, a full treatment (3 iterations, stress and wind analysis) should take about 3 - 4 hours on our IBM 360/50.

1. Floating Sphere Telescope (Structure 3a). The first application to an actual telescope design was done for one of the LFST proposals: a complete sphere of 750 ft diameter, floating on water or pressurized air; one segment is cut off and replaced by a radome, and a stiffener ring at the opening holds the rim of a 656-foot parabolic reflector, see Figure 5. First, it was shown that a non-deforming ring can be obtained with no extra cost, just by a proper distribution of the stiffener ropes and counterweights needed anyway. Second, supported at this ring, a parabolic surface structure was designed as a 2-dimensional network of triangles, replacing a membrane by discrete members, with $m = 66$ and $N = 19$ (sufficient for $\lambda = 5$ cm). This structure ran successfully on first try; after only 4 iterations the calculating accuracy of the machine was reached, giving an rms deviation of only $\Delta H = 7 \times 10^{-6}$ inch. As to the homology parameters, the change of focal length was 3.9 inch, and the change of axial direction only 0.72 minutes of arc. For the first guess, we took all bar areas equal, and the largest change obtained in the final solution was 42 per cent, the average change 8 per cent. The result is not a shell which could be replaced by a membrane; it must be a framework since the radial members of the final solution are lighter than the ring members. Each iteration decreased ΔH by more than a factor of ten, and this quick convergence showed that the floating sphere telescope could easily be supplied with a homologous mirror of any accuracy wanted. Furthermore, the convergence and the final value of $\Delta H = 7 \times 10^{-6}$ inch showed that the homology problem actually has exact

solutions (as we have claimed with the existence proof), and that some structures also have practical solutions. Not only can we minimize ΔH, we can make it zero and thus omit gravity completely, in a natural and elegant way.

2. Octahedron and Suspension (Structures 2e). Within the memory limit of the present program, we tried to obtain a sufficient number N of surface points, with a minimum of total points p and members m, for a telescope to be held at two elevation bearings on top of two towers. The best basic principle seemed to be an octahedron held with two suspensions (Figure 6a) which we called Structure 2. First, we experimented with N = 9 (Structures 2a, b and c) in many variations, 3/4 of which converged to physical solutions. A try to obtain N = 20 with only p = 29 (Str. 2d) failed and was given up. We finally settled on Structure 2e as shown in Figure 6, with N = 13, p = 29, varying m from 102 to 116, and also slightly varying the geometry and the bearing restraints.

In all these experiments, we found that a good structure, close to equal softness and with a first guess well thought of, will mostly give already for the first guess a ΔH small enough for practical purposes and then converge nicely, see Table 1; a wrong geometry or a bad first guess will mostly give large ΔH and then converge to some negative bar areas. But one of the major advantages of the homology program is that it tells you what is wrong; it teaches its user after a few trials how to make a good first guess and how to choose a proper geometry. Furthermore, the numerical proof that ΔH can be made zero should encourage all designers who try to minimize ΔH by other methods, like trial and error in the simplest case.

The best of our trial structures seemed to be Structure 2e/4, see Table 1, with m = 112. Starting at two elevation bearings (points 25 and 26), two suspensions of three members each hold an octahedron, thus including the feed supports in the basic structure as suggested in [1]. The basic square of the octahedron then is used for obtaining an octagon. From the octagon and its center, 9 points, we reach the 13 surface points with a layer of 45 bars. The surface structure is represented by 28 surface bars, and the surface itself by an additional load of 15,000 poinds per surface point (2.76 lb/ft^2). The focus is at point 23. Each bar of this structure, actually, is a built-up member as shown in Figure 3. The two bearings should be held on top of two towers moving on wheels on an azimuth ring. Since the telescope will have more stiffness in x-direction than the towers, we neglect their stiffness and let the bearings move freely along the x-axis, making up for it by restraining point 24 in x-direction. Thus, the restraints of all three points 24, 25 and 26 are represented by gliding cylinder bearings. The actual tower stiffness will be introduced later on in the new program. Because of the present memory limit, we could not attach an elevation ring to the telescope.

After m and the geometry was settled, we started in earnest and tried to make a real good first guess such that the bar areas would meet the survival condition by only a small margin, while the ratios of the areas would minimize the wind deformation. This is the Structure 2e/16 of Tables 1, 2, 3. The diameter was chosen as D = 300 ft, for comparison with our 300-foot telescope at Green Bank. Structure 2e/16 started with ΔH = 0.025 inch and converged fast (Table 1) to a physical solution. Table 2 shows the original bar areas and those after three iterations; the largest increase is a factor 1.75, and the largest decrease a factor 1.64, while the average change is only 18 per cent. But we obtained survival stresses slightly larger than the allowed ones for three members ($Q = 1.27$ for member 2-15; 1.22 for 3-16; 1.12 for 8-16).

Improvement then was aimed in two directions; first, stability ($Q \leq 1$) with minimum weight just for survival, resulting in Structure 2e/21; second, stability and minimum weight for obtaining about $\lambda = 2$ cm for wind deformations, resulting in Structure 2e/18. Table 2 shows the final values of bar area A, survival stress S_m from (9), the ℓ/r ratio Λ, and the maximum allowed stress for this Λ and for steel of 33,000 psi yield. We see that $S_m < S_o$ for all members of both structures, thus both are stable in survival conditions. From Table 3 we see that $\lambda = 4.4$ cm, which still is a good wavelength for a 300-foot telescope, if achieved with only 476 tons total weight (the Green Bank 300-foot has 450 tons, the surface allows only $\lambda = 20$ cm, but the structure would allow $\lambda = 15$ cm). And Structure 2e/18 meets the goal closely enough with $\lambda = 2.12$ cm for calling it a final solution. Its weight is still fairly low with only 748 tons. Against survival, it is overdesigned by 21 per cent. For both structures, the change of focal length is about 1 inch, and the direction changes by only 3 minutes of arc. The mechanical adjustment of the feed, with respect to point 23, is only about 1 inch.

P. Weidlinger [9] has pointed out that the ideal minimum-weight structure should have the same stress S_m in all of its members; and for any given structure with different S_m, the ratio of the smallest S_m over the largest one thus is an easy and elegant estimate of how efficiently the material is used. I would like to modify this in two ways. First, with respect to members of different length, we use the stress factor Q instead of the stress S_m; second, it really does not matter much if one or the other member even has $Q = 0$, but what counts is the

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[9] P. Weidlinger, private communication.

Table 1. Convergence of the homology iterations.

ΔH = rms deviations of deformed surface from best-fit paraboloid;

$\delta A/A$ = rms relative change of bar areas between iterations;

df = change of focal length between zenith and horizon.

i = iteration (o = first guess)

structure	i	ΔH inch	$\delta A/A$ per cent	df inch
3a	0	0.0856	-	- 2.99
	1	.0149	15	- 3.71
	2	.00074	3.2	- 3.96
	3	.000016	.4	- 3.92
	4	.000007	.3	- 3.87
2e/4	0	0.202	-	+ 0.26
	1	.0240	21	- 0.46
	2	.00089	4	- 0.52
	3	.00024	3	- 0.49
2e/16	0	0.0248	-	+ 0.77
	1	.0043	15	.71
	2	.0006	5	.70
	3	.0004	3	.72
2e/18	0	0.0120	-	+ 0.80
	1	.0045	7	.84
	2	.00006	1.4	.85
2e/21	0	0.019	-	+ 0.98
	1	.0015	6.7	1.00
	2	.00011	2.4	1.03

Table 2. Members of Structures 2e/16, 2e/18 and 2e/21.
m = number of identical members;
A = bar area, in square inch;
S_m = survival stress according to equation (9);
Λ = ℓ/r ratio (of main chords of built-up members);
S_o = maximum allowed stress, for 33,000 yield steel and Λ.

structure		2e/16		2e/18				2e/21			
iteration		0	3	2	2	2	2	2	2	2	2
member	m	A_o	A_3	A_2	S_m	Λ	S_o	A_2	S_m	Λ	S_o
1 - 2	2	12	10.6	9.1	7.2	65	15.8	5.9	9.5	87	14.7
3	2	12	12.0	10.1	1.4	61	16.1	6.9	3.8	79	15.6
14	1	12	9.9	13.8	9.5	36	18.0	8.8	16.2	49	18.5
16	4	10	10.6	13.1	5.5	68	15.6	8.3	8.1	93	14.0
2 - 3	4	15	14.8	13.0	4.9	73	15.2	11.3	6.6	80	15.5
6	2	12	12.8	13.8	8.0	53	16.8	8.4	12.4	73	16.2
7	4	18	18.0	16.3	1.5	67	15.6	9.1	2.8	99	13.3
14	2	10	13.8	11.8	3.6	72	15.2	5.4	5.8	121	10.4
15	2	10	6.1	11.8	10.9	29	18.3	5.8	14.5	47	18.7
16	4	10	13.3	16.9	10.3	44	17.4	8.3	12.6	71	16.5
3 - 7	4	18	16.1	13.7	7.1	75	14.9	8.6	9.6	103	12.8
8	2	12	11.8	9.9	5.6	65	15.8	6.2	6.7	89	14.4
14	2	10	8.7	8.0	6.4	93	13.1	7.6	10.4	97	13.6
16	4	10	11.5	15.2	13.9	47	17.2	13.6	16.3	51	18.3
17	2	10	8.2	14.6	8.6	26	18.6	9.5	16.8	34	19.7
6 - 7	4	22	19.7	16.9	8.2	66	15.7	10.5	11.9	90	14.3
15	2	12	18.9	30.6	7.6	31	18.0	32.6	7.7	30	20.0
16	4	10	7.6	10.8	9.9	95	12.7	10.6	11.9	96	13.6
7 - 8	4	22	18.9	16.9	8.1	66	15.7	12.0	9.6	83	15.2
15	4	10	9.7	14.1	4.2	80	14.4	6.4	6.1	135	8.2
16	4	12	14.6	16.8	11.8	47	17.3	12.3	15.3	58	17.7
17	4	10	14.2	13.2	9.4	84	14.1	13.5	9.5	82	15.3
8 -16	4	10	10.9	14.2	12.1	79	14.6	15.3	12.5	75	16.0
17	2	12	15.2	18.9	11.6	43	17.5	14.1	16.4	53	18.2
14 -15	2	40	70.1	55.7	2.9	26	18.5	50.0	3.8	29	20.1
16	4	40	37.1	30.9	2.5	39	17.8	7.1	4.2	104	12.7
17	2	40	44.4	35.0	2.6	36	18.0	10.4	5.4	81	15.4
24	1	40	32.6	29.9	8.5	37	18.0	14.3	15.3	61	17.5
15 -16	4	40	25.4	19.2	14.1	39	17.8	12.4	16.1	52	18.2
24	2	50	53.6	53.0	9.1	43	17.5	35.9	12.5	55	18.0
16 -17	4	40	39.2	41.2	8.7	24	18.7	29.5	12.6	29	20.0
18	2	60	56.9	48.1	7.5	39	17.8	15.6	12.8	83	15.2
22	2	60	63.7	49.2	10.7	39	17.8	32.8	12.1	51	18.4
23	4	15	15.4	13.5	1.1	133	8.4	7.4	1.1	198	3.8
24	4	60	60.1	51.4	7.8	43	17.4	24.5	12.6	71	16.4
26	4	120	115.2	148.1	10.7	13	19.3	76.9	17.5	20	20.7
17 -24	2	50	56.6	55.7	7.1	41	17.6	39.3	11.7	52	18.3
24 -26	2	200	190.0	229.5	8.1	18	19.0	111.9	15.5	30	20.0

Table 3. Some final values for Structures 2e/16, 18, and 21.

	structure iterations	2e/16 3	2e/18 2	2e/21 2
rms	deviation from homology, ΔH (inch)	0.00040	0.00006	0.00011
	max. surface deformation, ΔP_m (inch)	1.29	1.04	1.52
	change of focal length, df (inch)	.72	.85	1.03
	change of direction, $d\phi$ (min. of arc)	4.20	2.37	3.06
	focal adjustment, rel. to point 23 ΔF_z (inch)	.90	.48	1.13
	focal adjustment, rel. to point 23 ΔF_x (inch)	1.89	.84	.89
	wind deformation, $\Delta\zeta$ (inch)	.102	.0835	.172
	shortest wavelength, λ (cm)	2.59	2.12	4.40
	max. stress factor, $Q = S_m/S_o$	1.27	.83	.90
	total weight on elev. axis (tons)	745	748	476

average. We thus define a weight efficiency η by

$$\eta = \frac{\text{maximum } Q}{\text{average } Q} . \tag{12}$$

This counts most for Structure 2e/18 where we just want to fulfill the survival condition. Figure 7 shows the distribution of the stress factors Q for 80 bars only, since 32 bars are defined by different criteria (28 surface bars which must be widely split-up and must resist bending forces, and 4 feed supports which should not go beyond $\ell/r = 200$). We see that 10 bars have rather low Q, but the remaining 70 bars form a nice, dense group. For all 80 bars, we obtain $\eta = 0.786$, which we consider close enough to 1 for calling it a final solution. The maximum is $Q = 0.90$, leaving a margin of 10 per cent for nuts and bolts and other things neglected.

3. Comparison with Theoretical Estimates. In 1965 several formulas were developed on purely theoretical grounds, for estimating the weight of tiltable telescopes of a "near-to-ideal design", and it is interesting to compare these old theoretical estimates with the present actual designs. For a structure defined by survival, and for $\lambda \leq 5$ cm, formula (24) of paper [1] reads (W measured in tons, D in 100 m):

$$W = 432\ D^3 + 160\ D^2 \tag{13}$$

which yields W = 467 tons as compared to W = 476 tons of Structure 2e/21. For a structure defined by wind deformations, $\lambda = 16\ \Delta\zeta$ was used in [1] instead of $10\ \Delta\zeta$ used now in (11) where we neglect parallel translations; if corrected for this difference, formula (30) of [1], for $\lambda \leq 5$ cm, reads (W in tons, D in 100 m, λ in cm):

$$W = (2025\ D^4 + 181\ D^3)\ /\ \lambda + 122\ D^2 \tag{14}$$

which yields W = 842 tons for $\lambda = 2.12$ cm, as compared to W = 748 tons of Structure 2e/18. This is a difference of 12 per cent, while the old estimates did "not ask for more accuracy than, say, $\pm$ 30 per cent". Finally, if a structure is defined by survival, formula (18) of [1] gives the shortest wavelength from wind deformations, corrected for (11), then as (λ in cm, D in 100 m):

$$\lambda = 4.7\ D \tag{15}$$

which yields $\lambda = 4.3$ cm as compared to $\lambda = 4.4$ cm of Structure 2e/21. All three results give a nice mutual confirmation of both estimate and design.

The fact that our homology solutions agree completely with the old estimates based on a near-to-ideal design, and that Structure 2e/21 gives the same weight as the 300-foot at Green Bank while beating its wavelength by more than a factor 3, shows that we have omitted gravity without paying any price for it, and I would like to emphasize this point.

4. Check Calculations. Usually, a computer program is checked, before its application, by some hand calculation of a simple case. This is impossible with our homology program. Since a paraboloid of revolution is defined by 6 points, we need at least 7 surface points to make the method work, which means at least a total of, say, 10 points and 30 members, which is far beyond the scope of a hand calculation. We thus have taken one of our final results, Structure 2e/18 after two iterations from Table 2, and have sent its coordinates and bar areas to Simpson, Gumpertz and Heger (Cambridge, Mass.), asking for a complete stress and deformation analysis under all of our load conditions including survival. For dead loads only, a best-fit paraboloid of revolution and the deviations ΔH from it should be calculated for elevation angles 0°, 45° and 90°, thus also checking at 45° our statement that homology holds in all angles if it holds in two.

The results are just as good as can be expected for the finite calculating accuracy of the computers. All stresses and deformations agree with our results within 5 decimals. For the rms deviations ΔH from the best-fit paraboloid, Simpson, Gumpertz and Heger obtain:

position	ΔH
horizon, 0°	0.000052 inch
45°	0.000051 inch
zenith, 90°	0.000049 inch

(16)

This agrees within the calculating accuracy with our value of $H = 6 \times 10^{-5}$ inch from Table 3 for the average of horizon and zenith according to (5), and it also shows that our statement is correct at 45° elevation angle.

VIII. Further Possibilities and Plans

1. Radome. We also tried to work out a telescope of $D = 300$ feet to be enclosed in a radome (dead loads only). But since this turned out more difficult, as explained in Section IV, and since we think that exposed telescopes can be made less expensive than those in a radome, we have not tried as hard as for the exposed ones. A series of first guesses was run, called Structure 2f/1 to 2f/8, using about the same geometry as Figure 6 and varying m between 104 and 112.

A final result (all $Q \leq 1$ with low total weight) has not yet been obtained, but the best solution comes already close to it, giving still $Q > 1$ for 6 members but with a maximum of only $Q = 1.24$, similar to Structure 2e/16. This solution, Structure 2f/5, has $m = 104$ bars, and a total weight of $W = 237$ tons. From our previous experience we feel confident that a final solution with about $W = 200$ tons can be reached after some more trials, especially with the new program, and we have dared to enter this expected value in Table 4 for comparison.

The new version of the program will be much more flexible than the present one, and it can be run in a mode which already comes close to the method suggested at the end of Section III, which is better suited to this task.

2. Telescopes of other D and λ. Although we have calculated our structures only with $D = 300$ feet, we can predict a selection of other telescopes as shown in Table 4; N is found from equations (4) and (3); ΔT from (2); $\Delta A/A$ is based on the calculated sensitivity of Structures 2e/18 and 2e/21, and then is scaled according to $\Delta A/A \sim \lambda/D^2$; and formulas for n and ℓ are given in [6]. The total weight for exposed telescopes is calculated from equation (14) which gave good

Table 4. Some Telescopes with Homologous Deformations.

D	λ	β	N	ΔT	ΔA/A	n	ℓ	W radome	W exposed
feet	cm	arc sec		°C	per cent		m	tons	tons
30	.0025	.7	80	.06	1.7	770	.01	3	-
85	.1	10	17	.8	9	165	.11	25	131
300	2	54	10	4.5	14	60	.96	200	888
400	3	61	12	5.2	12	56	1.35	410	1790
450	4	72	11	5.4	12	51	1.65	550	2150
500	5	81	11	6.7	12	47	1.95	750	2570
600	6	81	13	6.7	12	42	2.34	1300	4370

Free choice:

$$\left.\begin{array}{l} D = \text{telescope diameter} \\ \lambda = \text{shortest wavelength} \end{array}\right\} \quad \beta = 1.2\ \lambda/D = \text{half power beam width.}$$

Requirements:

N = minimum number of homologous surface points (the present program is memory-limited and gives N = 13; the planned version should reach N = 80 or more);

ΔT = maximum tolerated temperature differences in the structure (good protective paint gives about 5°C in sunshine);

ΔA/A = maximum tolerated rms deviation of bar areas from computed values (off-the-shelf structural pipes give 10 per cent.)

Surface:

n = minimum number of toroidal panels;

ℓ = maximum size of flat plates.

Weight (of elevation-moving structure: dish, surface, feed-legs) = W

radome: minimum structure for stable self-support, inside radome;

exposed: a) wind deformation $\leq \lambda/10$ for 17 mph on ground (22 mph at 300 ft height);

b) survival = 20 lb/ft^2 of snow, or 4 inch solid ice, or 90 mph on ground (110 mph at 200 ft height);

comparison: the NRAO 300-ft telescope (λ = 15 cm) has W = 450 tons.

agreement with Structures 2e/18 and 2e/21. The weight of telescopes in a radome is based on the expected value for 300 feet, and then scaled according to $W \sim D^{2.5}$.

The first line of Table 4 is an "infrared telescope" and needs some explanation. Several colleagues have suggested to build a small model of a homologous telescope, about 30 feet diameter. Its gravitational deformations ($\sim D^2$) then will be only 0.3 mm = 300 micron, and in order to show that they are homologous, they must be measured with an accuracy of only 10 micron; this can be done with an optical Michelson interferometer, as an experiment by J. Hungerbuhler of NRAO has shown. But a good model should do more than just demonstrate what a computer already has calculated; it should make itself useful as a telescope. The trouble then is that our atmosphere is opaque from 1 mm all the way down to 25 micron wavelength. A homologous telescope for that wavelength could be built, with N = 80 surface points, although it needs a temperature stability of 0.06 $^{\circ}$C and a structural accuracy of 1.6 per cent. The most severe problem, however, is how to obtain a large surface of almost optical quality within low costs, and this has not yet been solved. The second line of Table 4 is an 85-foot telescope for 1 mm wavelength which certainly can be built.

For the larger telescopes ($D \geq 300$ ft) we have always chosen the smallest λ such that N, ΔT and $\Delta A/A$ can easily be met. We see that all large telescopes can be designed from Structure 2e/18 (all $N < 13$), they can be built from off-the-shelf pipes (all $\Delta A/A > 10$ per cent), and they can observe in sunshine (all $\Delta T \approx 5\ ^{\circ}C$).

3. <u>Final Designs</u>. Our plan is, first, to finish the new program, allowing more complicated structures and more flexibility. Second, we will develop some good structures with 60 - 80 surface points; this is not necessary for observation, but it reduces the size of surface panels and eases the erection. Third, we will consider more details of survival stresses and wind deformations, like various angles of the wind and the actual resistance and bending of the members. Fourth, a dynamical analysis of the more promising structures will be done somewhere else.

Finally, we plan to work out three complete designs (D = 85 ft, 300 ft, 500 ft from Table 4) in all details, no matter what the financial hope for building them happens to be. They will be published, and anyone interested is welcome.

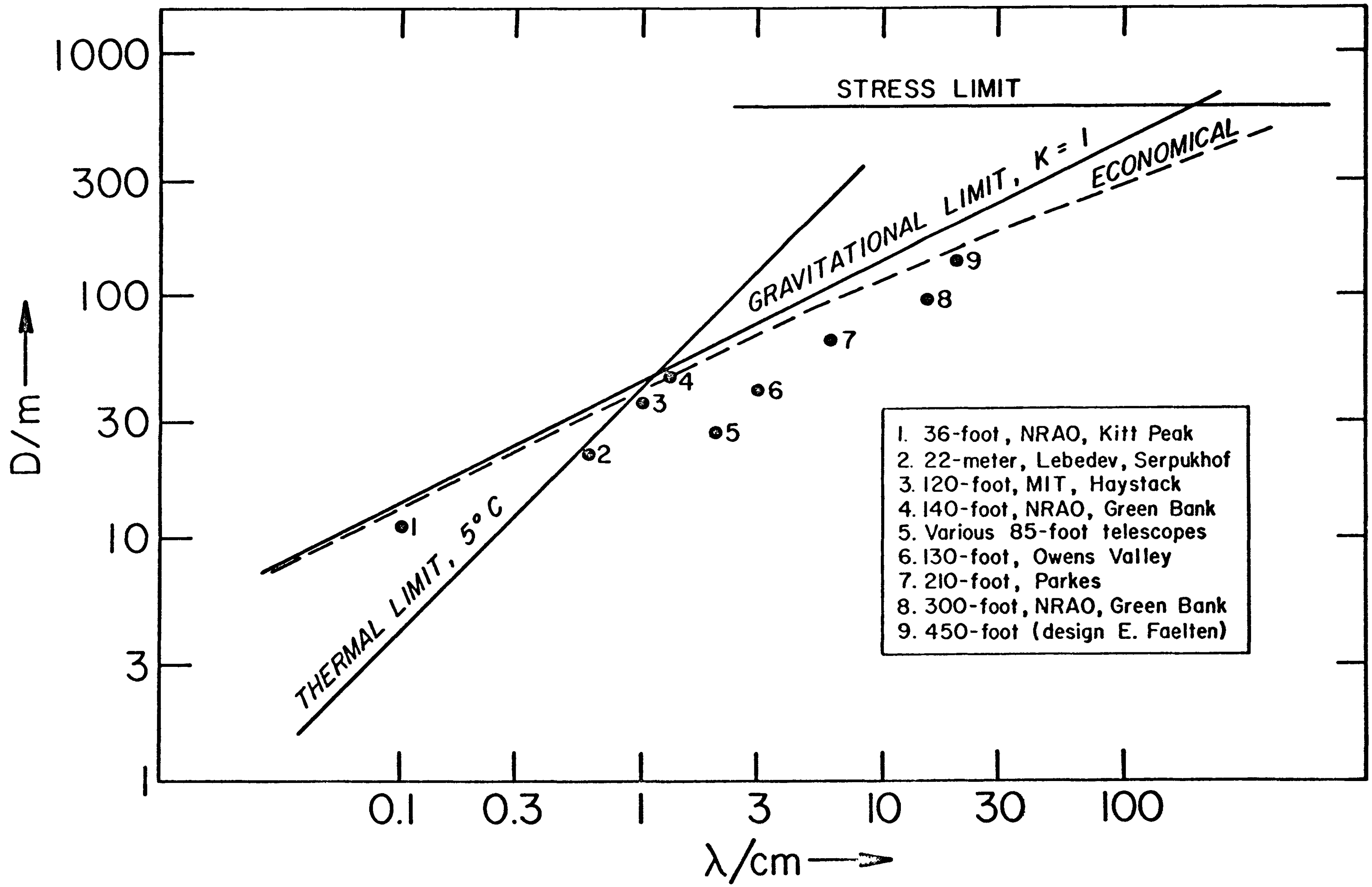


Figure 1. Three natural limits for conventional, tiltable telescopes, with nine actual examples for comparison. The Kitt Peak telescope is inside an open dome, passing the thermal limit set by sunshine and shadow.

D = telescope diameter, in meter; λ = shortest wavelength to be observed, in centimeter.

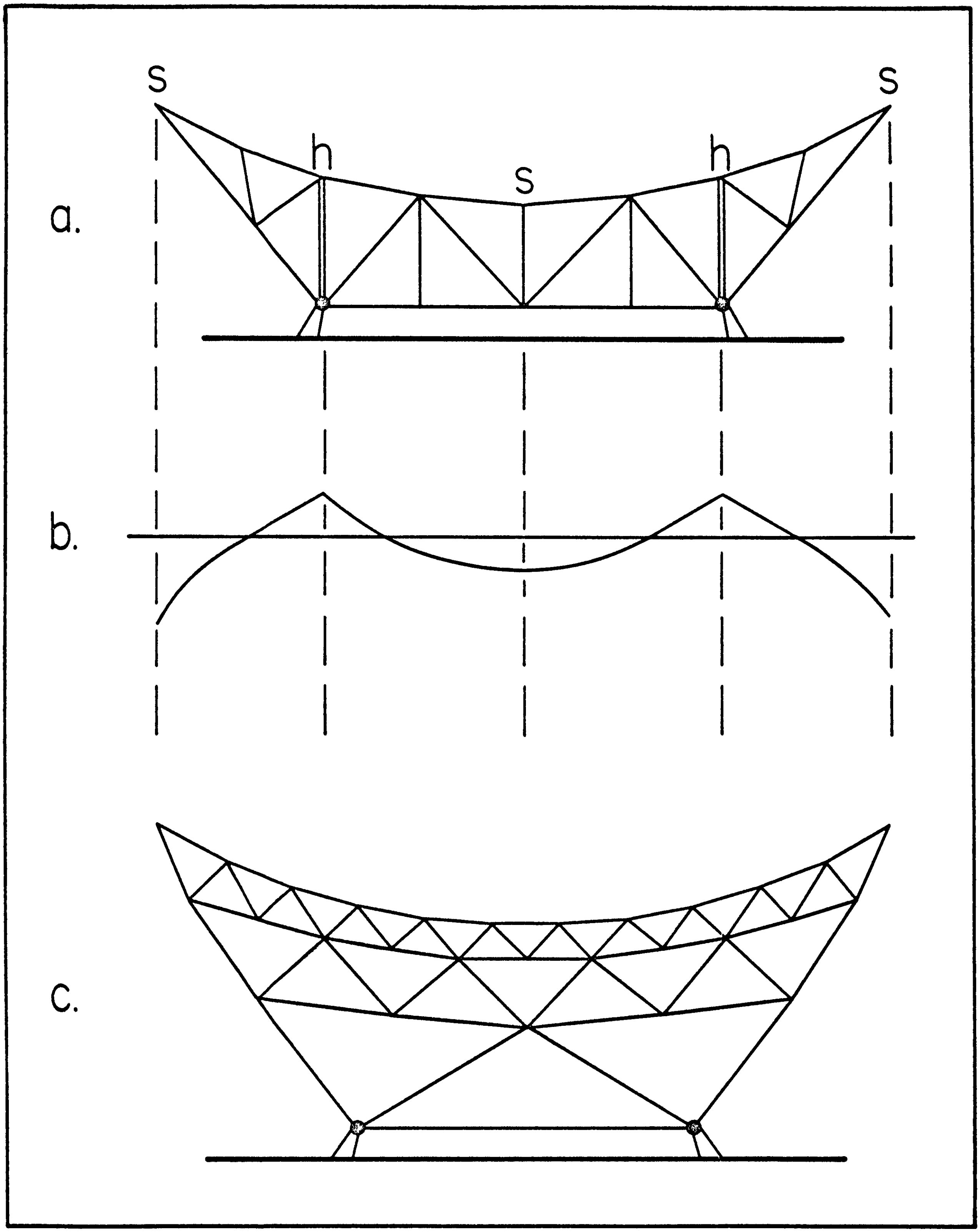


Figure 2. Equal-softness Structures.

(a) Conventional design, with hard (h) and soft (s) surface points.

(b) Deformation of this telescope, looking at zenith; the best-fit paraboloid is represented by a straight line.

(c) Structure where all surface points have about equal softness.

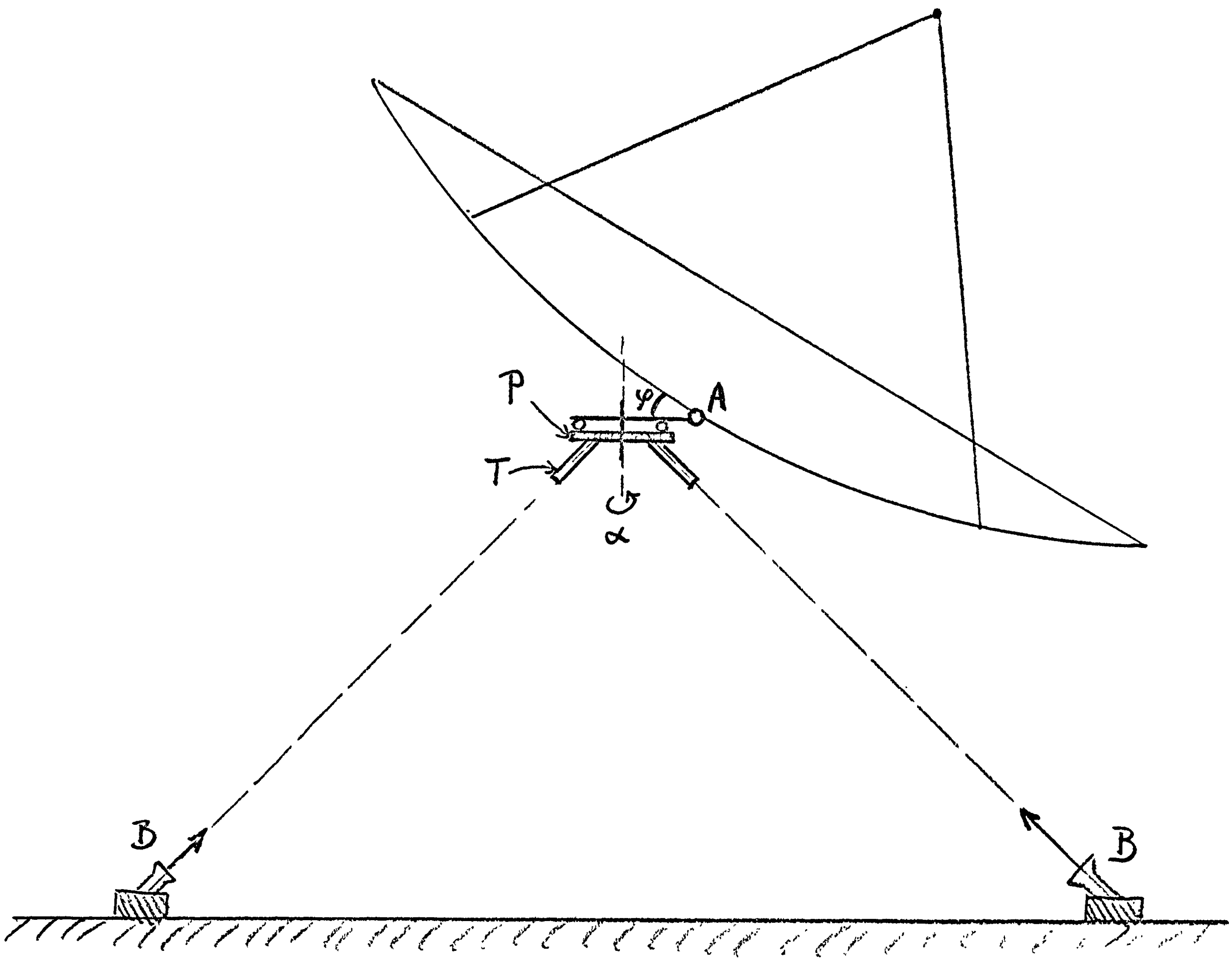
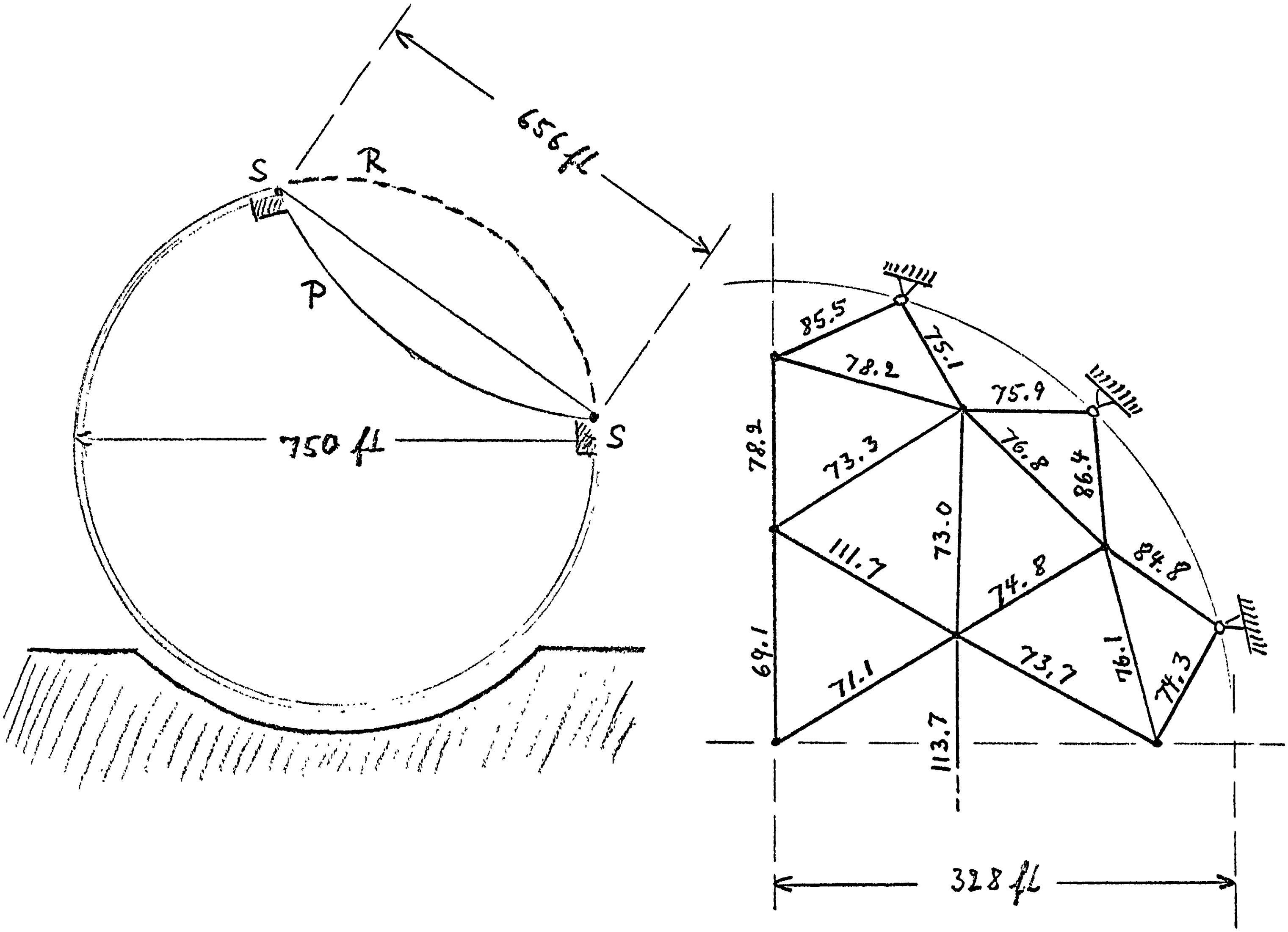


Figure 4. Position measurments by optical means.

A small tiltable and rotatable platform P is mounted behind the apex A and looks with about six theodolites T to as many optical beacons B fixed at the ground. Three servo motors keep the platform "locked-in" to the beacons; elevation φ and azimuth α then are measured between structure and platform. In this way, the position is measured where it matters and with respect to something unstressed and unmovable. No high accuracy is required for foundations, azimuth rails and elevation ring; also, all deformations between apex and ground are omitted.



<u>Figure 5</u>. Parabolic reflector for floating-sphere telescope.

The reflector structure P is a two-dimensional network, suspended at the stiffener ring S inside the radome R. First, a non-deforming ring could be obtained. Second, all 66 members of the reflector were given the same area of 80 square inch for the first guess; after four iterations, the rms deviation between the deformed surface and a best-fit paraboloid was only $\Delta H = 7 \times 10^{-6}$ inch. The final bar areas are written at each bar of one quadrant (all four quadrants are identical).

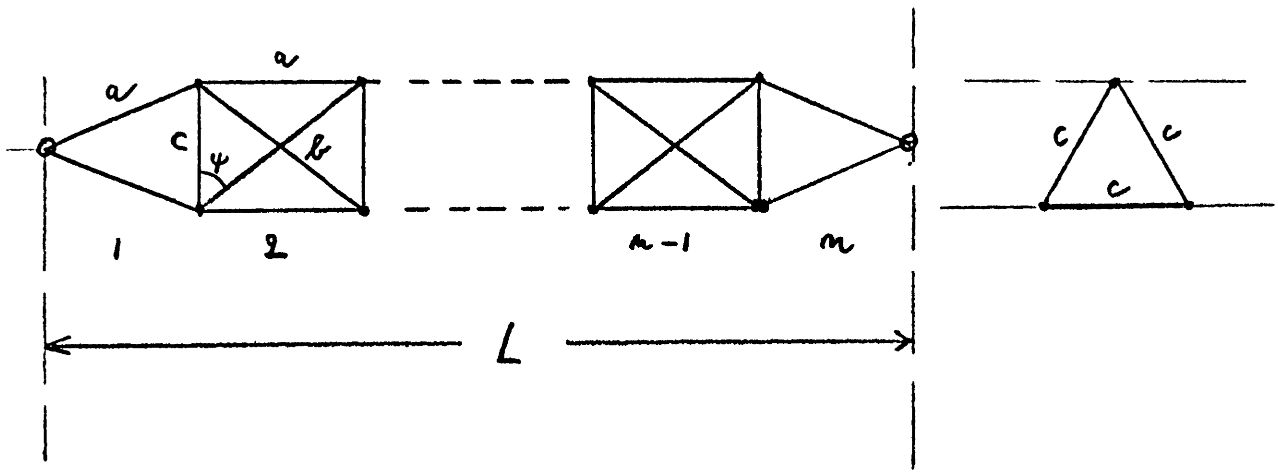
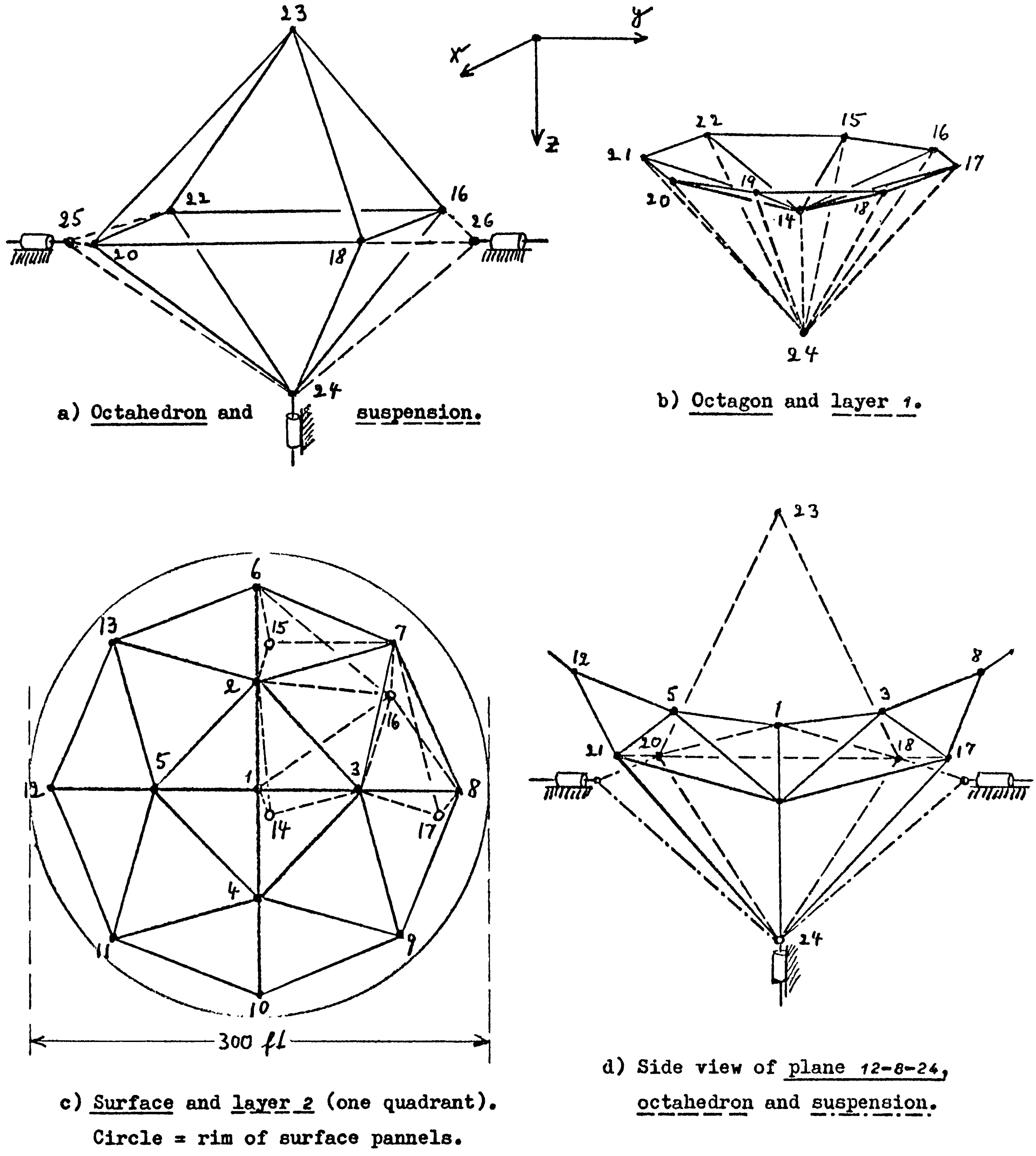


Figure 3. Built-up structural members.

In principle all members could be different; but at present the same values are adopted for all members: $n = 10$, $\psi = 55^{\circ}$, $A_b = A_c = 0.3\ A_a$. This built-up member then is represented in our program by a single shape or pipe of area $A = 3.84\ A_a$, density $\rho = 1.19\ \rho_o$, and unchanged elasticity E. With respect to stability, we call Λ the ℓ/r ratio of the single chord, use standard pipes of the Steel Construction Manual, and obtain $\Lambda = L\ /\ (2.88\ A^{2/3})$.



a) Octahedron and suspension.

b) Octagon and layer 1.

c) Surface and layer 2 (one quadrant). Circle = rim of surface pannels.

d) Side view of plane 12-8-24, octahedron and suspension.

Figure 8. Geometry of Structures 2e/4 to 2e/21. The basic structure is an octahedron, held by a suspension from two elevation bearings mounted on top of two towers. ~~Bar areas are given in Table 2,~~ final data in Table 3. This structure has 13 homologous surface points, a total of 26 points (pin joints) and 112 members.

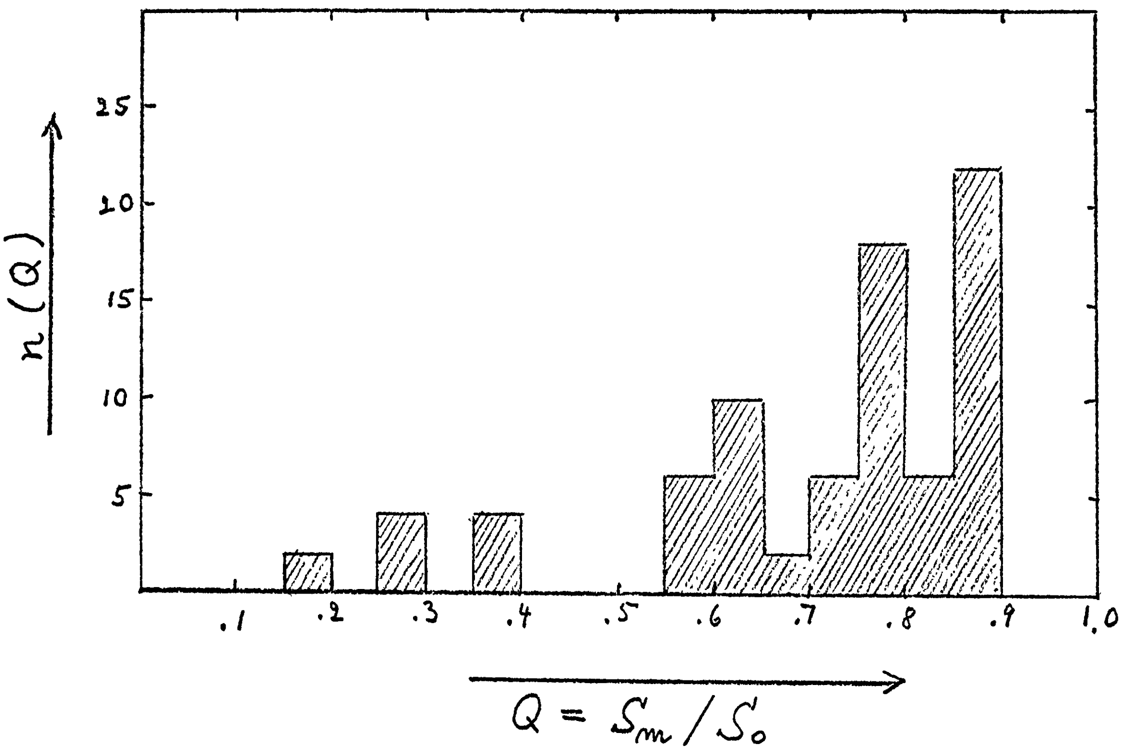


Figure 7. Distribution of stress factors Q from equations (9) and (10), for Structure 2e/21.