Memo to: 65-m design group
from: S. von Hoerner

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Dynamical Bracing of Declination Wheel

In order to prevent a rotation of the wheel about its long axis (43 - 58), we had originally introduced bar 41 - 51. As pointed out by J. Antebi of SGH, this is too soft since the extension of 41 - 51 beyond 51 almost crosses the axis 43 - 58, which would result in a very low dynamical frquency. We then did some experiments, and W.Y. Wong found that replacing 41 - 51 by some better bracing, like 40 - 51 or 51 - 56, would need major changes in the whole structure for regaining homology, which seems possible but would take too much time.

We thus decided to leave 41 - 51 as it is, for homology, but to add prestressed guy ropes or rods from point 54 to either 45 (basic square) or 59 (elevation bearing). In the following we derive first the axial stress of the rope needed for a good efficient stiffness, and second the cross-sectional area A needed for a given dynamical frequency ν.

1. The Effective Elasticity of a Sagging Rope

This problem has been treated already in Report 8 (May 2, 1966). The result is

$$E_s = E \left\{ 1 - \frac{1}{12} \frac{E \rho^2 L^2 A^3}{F^3} \right\} \tag{1}$$

where

$$\begin{aligned} E_s &= dF/dx = \text{effective modulus of elasticity} \\ E &= \text{material modulus} \\ \rho &= \text{material density} \\ L &= \text{length of rope} \\ A &= \text{area of rope (cross section of steel)} \\ F &= \text{external load, see Fig. 1.} \end{aligned} \tag{2}$$

The graph of Report 8 shows that a rope of high-strength steel (150 ksi yield) is better than a Corten rod (50 ksi) only if the length is more than 400 ft. But since the difference is not crucial while long ropes are easier for erection, we still decide on ropes.

Let us demand that the modulus of elasticity drops only by $\leq 13\%$, from $E = 23\,000$ ksi for Bethlehem steel ropes, to an effective modulus of

$$E_s \geq 20\,000 \text{ ksi.} \tag{3}$$

From (1) we then obtain, with stress $S = F/A$,

$$S^3 \geq 0.641\, E\, \rho^2\, L^2 \tag{4}$$

and with $\rho = 0.283$ lb/inch^3 we have

$$S \geq 10.1 \text{ ksi } (L/1000 \text{ inch})^{2/3} . \tag{5}$$

With L from Fig.2 we obtain for the stress needed for stiffness (3) finally

$$S \geq \begin{cases} 8.56 \text{ ksi, for pt. } 45, \\ 10.89 \text{ ksi, for pt. } 59. \end{cases} \tag{6}$$

2. <u>The Dynamical Frquency ν</u>

We use equation (6) of Report 20 (Dec. 16, 1968)

$$\nu = 3.13 \sqrt{K/W} \tag{7}$$

for a simple system with one mass of weight W (kip) and one spring of stiffness K (kip/inch), yielding ν in cps. The total weight of the declination wheel is 72 tons or 144 kip, and 1/4 of this is braced by each single guy rope:

$$W = 36 \text{ kip.} \tag{8}$$

The stiffness regarding axis 43 - 58 is, see Fig. 2,

$$K = E_s \frac{A}{L} \left(\frac{h}{r} \cos \alpha\right)^2 \tag{9}$$

or with (3) and the values of Fig. 2:

$$K = \begin{cases} 23.6 \frac{\text{kip}}{\text{inch}} A, \text{ for pt. } 45, \\ 14.2 \frac{\text{kip}}{\text{inch}} A, \text{ for pt. } 59. \end{cases} \tag{10}$$

The frequency ν then is, with (7) and (8),

$$\nu = \begin{cases} 2.53 \text{ cps } \sqrt{A}, \text{ for pt. } 45, \\ 1.97 \text{ cps } \sqrt{A}, \text{ for pt. } 59. \end{cases} \tag{11}$$

Table 1. Rope area A needed for dynamical frequency ν, and axial prestress load F needed for efficient stiffness of rope.

ν	pt. 45		pt. 59	
	A	F	A	F
cps	inch^2	kip	inch^2	kip
2.5	.97	8.3	1.63	17.8
3.0	1.40	12.0	2.34	25.5
3.5	1.91	16.3	3.19	34.7
4.0	2.49	21.3	4.16	45.3

Table 1 shows area A and load F needed for various ν. For single telescope members we have always demanded $\nu \geq 2.5$ cps. Since the wheel is rather heavy we should demand a higher limit, say

$$\nu \geq 3.5 \text{ cps.} \tag{12}$$

This leads to

$$A = \begin{cases} 1.9 \text{ inch}^2, \text{ for pt. } 45, \\ 3.2 \text{ inch}^2, \text{ for pt. } 59. \end{cases} \tag{13}$$

As to the actually needed prestress load F_a we must proceed in two steps. First, ropes of area A are added to the structure which then is analyzed for gravitational deformation with Ω = 1 (zenith) and Ω = 2 (horizon). The resulting forces in the guy ropes then are added quadratically,

$$F_g = \sqrt{F_1^2 + F_2^2}\,, \tag{14}$$

for the worst possible case. With F from Table 1, the actual prestress load then should be

$$F_a = F + F_g \tag{15}$$

Second, this load yields an additional stress in the telescope members which is to be found in a separate analysis, to be added to the survival stress in these members. The resulting sum must still fulfill the stability condition (stress ratio $Q \leq 1$). This should be checked, but problems are not expected.

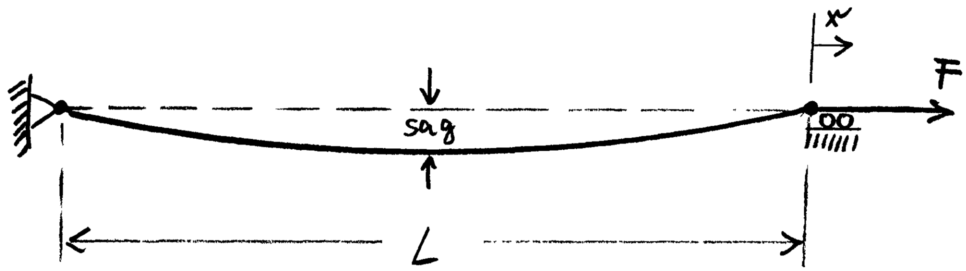

Fig. 1. Rope, sagging under its own weight.
The effective modulus of elesticity is defined as $E_s = dF/dx$.

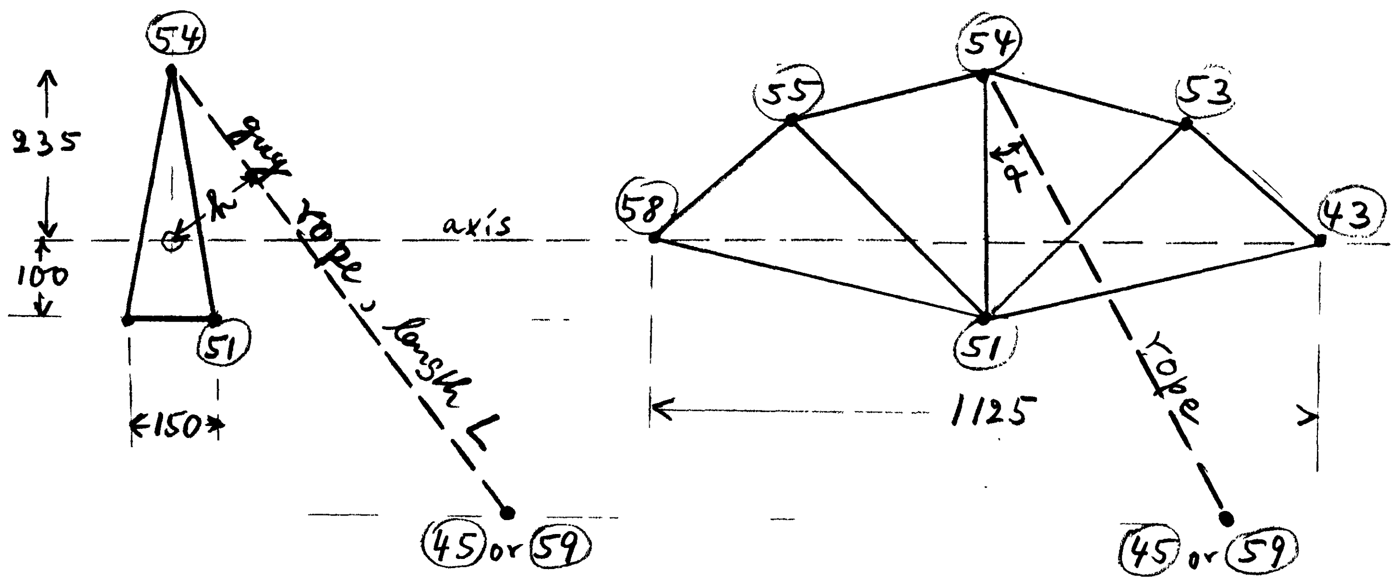

Fig. 2. The declination wheel (sizes in inch).
Radius of gyration, axis 43 - 58, r = 140 inch.

If point 54 is guyed to either point 45 or 59, then:

	L inch	h inch	α degree
pt. 45	780	190	45
pt. 59	1120	125	0