NATIONAL RADIO ASTRONOMY OBSERVATORY
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GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

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PROJECT: LFST

SUBJECT: Flat Antenna

The Flat Fixed-Elevation Transit Antenna

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In my first LFST-paper ("The design of large steerable antennas") I suggested an off-axis part of a parabolic mirror, sitting flat on the ground. The basic features of this concept are shown and described in Fig. 1. It is a fixed-elevation transit telescope; as compared to the other design of the same class, the tilted spherical mirror, it picks up much less wind force and it has much shorter connections to the ground, which should reduce the price, especially for shorter wavelengths and large diameters. The present investigation tries to find a good geometrical shape, to estimate the total weight of the structure, and to compare it with the other design.

1. Free Parameters

As an example, we have chosen an aperture diameter of 200 m, an elevation angle of 40° with a range of $\pm 5^\circ$, and a diameter of 15 m for the secondary mirror (for wavelengths up to 1.5 m). The secondary mirror was calculated as to give a symmetrical feed illumination (for polarization measurments) and a total feed illumination angle of 110° (for multy-frequency observations).

The first free parameter, then, is the height of the primary focus above the mirror. Too small a height gives a deep curvature for the mirror and not enough clearance for the feed tower. Too large a height gives a very high feed tower and long feed tracks. As a good compromise, we have adopted a height of 156 m. Once these values are chosen, many geometrical properties are fixed; they are shown in Fig. 2.

The second free parameter is the height H of axis M above the mirror; a third one is the distance between feed package and feed tracks, but usually this distance should be chosen as small as possible without having the tracks blocking the line of sight. If we take a small value for H, as shown in Fig. 3, we do not need any additional structure between feed package and feed tracks, but then the cylindrical trough becomes very high. If we take a large value for H, as shown in Fig. 4, the trough becomes much

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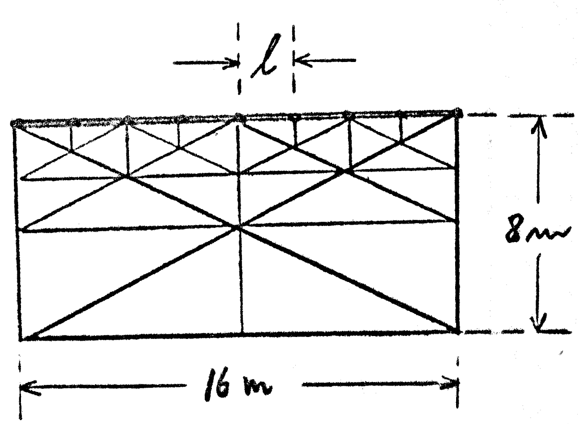
flatter, but then we need longer feed tracks, and we have quite a bit of structure right at the most delicate point of the telescope.

Since any additional feed structure would affect the pointing and driving accuracy, we finally chose the largest value for H which does not need additional feed structure. This turns out to be H = 300 m as shown in Fig.5. The cylindrical trough, then, would be too high; in order to decrese its height, we split the trough into five segments, each segment being 10° of a circle around axis M. The height of the trough is defined by length and tilt of the second segment from the left. The distance between the trough and the lower part of the mirror structure is defined by the clearance needed for the rotation by $\pm 5^{\circ}$ around M. The back-up structure of the mirror needs a certain thickness for rigidity, for which we have chosen 8 m.

The bottom of the mirror surface then is 26 m above ground, the highest point of the surface is 42 m above ground. The highest point of the trough is 32 m above ground. The height of the azimuth drive on the tower is 164 m, and the height of the upper end of the feed track is 210 m. The largest circular track, for the azimuth drive of the trough, has a radius of 389 m. The surface area is about 49 000 m^2.

2. The Surface

The back-up structure of the mirror in Fig.5 consists of rectangular boxes of size 16 by 16 by 8 m. Toward the surface, we divide into smaller and smaller boxes, for example as shown below, until we reach the surface with boxes of size ℓ:



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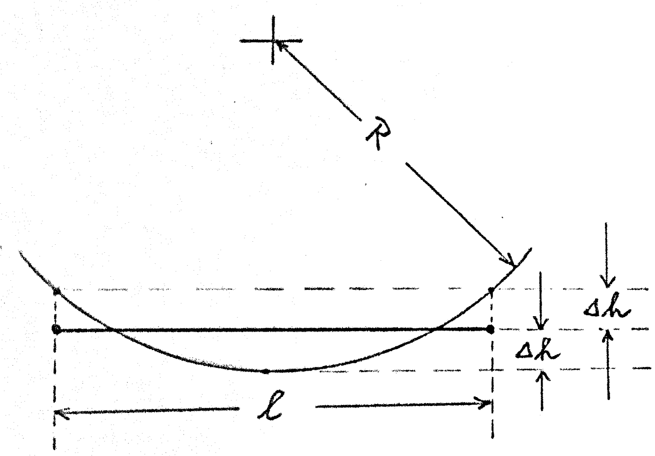
We want to take ℓ so small that the parabolic surface of the mirror can be replaced by straight lines of length ℓ. This means we want to use plane panels of size ℓ by ℓ. If we call R the radius of curvature of the surface, and Δh the maximum deviation of a straight line of length ℓ, we find



$$\Delta h = \frac{\ell^2}{16\ R} \tag{2}$$

We demand $\Delta h \leq \lambda/16$ and obtain for the largest size of plane panels

$$\ell = \sqrt{R\ \lambda}\ . \tag{3}$$

The radius of curvature depends on position and direction. The radius is smallest, R = 244 m, at the edge of the mirror closest to the tower and parallel to this edge. The largest panel size then is:

λ	2 cm	3 cm	5 cm	10 cm
ℓ	2.2 m	2.7 m	3.5 m	4.9 m

(4)

For the following, we adopt ℓ = 2 m which is good for any wavelength $\lambda \geq 1.7$ cm.

The surface itself might consist of aluminum sheet, 2 mm thick, which gives a weight of 270 tons. Each panel should be backed-up by some aluminum ribs, strong enough to allow walking on the panels if they are held at their edges. We adopt

500 tons for surface and ribs. (5)

The single panel then has a weight of 41 kg, including the ribs.

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3. Wind Forces

We adopt the following velocities and forces:

observation	25 mph;	12.5 kg/m^2	=	2.6 lb/ft^2	(6)
survival	110 "	242 "	=	50 "	

In the most unfavourable observing position, and at the edge of the mirror, the surface has an elevation angle of 16°. With $\sin 16^\circ = 0.276$, the uplifting wind force is $0.276 \times 12.5 = 3.45$ kg/m^2. In stow position, the angle is 10°, and the uplifting (or downward) force becomes 42 kg/m^2.

Each wheel of the mirror structure is at the joint of 9 members (4 from the box sides, 4 from the diagonals, and 1 vertical member). If we assume a number of 20 wheels, each wheel supports an area of 2450 m^2, and each member supports 273 m^2.

As the main structure we assume boxes of size 32 by 32 by 8 m, and we assume equal cross sections Q for all sides and diagonals of these boxes, and for the members joining at the wheels. The maximum deformation of the surface then turns out to be

$$\frac{\Delta h}{\text{cm}} = 4.3 \frac{\text{cm}^2}{Q}, \quad \text{for observation.} \tag{7}$$

Under survival condition, the maximum uplifting force at the edge of the mirror is 42 kg/m^2. In order not to fly away, the structure either must be held down in stow position, or it must have the following total weight (surface, structure, wheels)

$$2000 \text{ tons, against uplifting in survival wind.} \tag{8}$$

For a slenderness ration of $l/r = 100$ and for V45 steel, the cross section must be at least 2.2 inch^2 for survival winds. But if we have to allow for a snow load of, say, 30 lb/ft^2, we need at least 7.4 inch^2 cross sections. For A-36 steel, we obtain

$$Q \geq \begin{cases} 2.8 \text{ inch}^2 & \text{for winds of 110 mph} \\ 9.2 \text{ inch}^2 & \text{for 30 lb/ft}^2 \text{ of snow} \end{cases} \tag{9}$$

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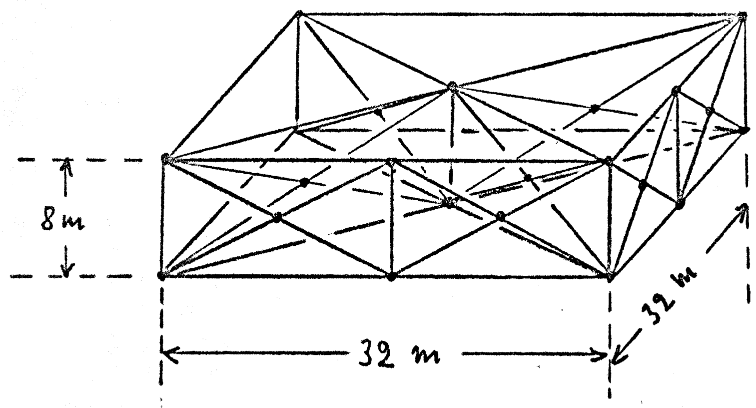
4. The Minimum Structure

First, we do not consider the purpose of the structure and just estimate the weight of the minimum stable structure, which could be manufactured and erected at low cost. This weight depends entirely on the choice of the longest unbraced length, ℓ, and the choice of the slenderness ratio, ℓ/r. For the boxes of the main structure, 32 by 32 by 8 m, as shown below,



(10)

we adopt an unbraced length of $\ell = 4\sqrt{2} = 5.7$ m for all diagonals and $\ell = 4$ m for all orthogonal members; and we adopt $\ell/r = 100$ for both (with respect to wind forces, the diagonals are main members, too). We further adopt members built from two unequal leg angles combined, and from the values adopted we find that we must use A74 angles for the diagonals, and A53 angles for the other members (with a wall thickness of 3/8 and 5/16 inch, resp.). The weight of the main back-up structure, including the wheel supports, then becomes 1310 tons.

We fill up the structure to obtain boxes 8 x 8 x 8 m, taking $\ell = 2.83$ m and $\ell/r = 142$ for diagonals, and $\ell = 2$ m and $\ell/r = 100$ for the other members. This additional structure turns out to be 460 tons. Next, we need the fine structure ending at the surface panels of 2 x 2 m; this will have about the same weight as the one calculated last, and we adopt 500 tons for it. The total back-up structure then is about 2300 tons.

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The weight of the trough shall only be estimated by comparison with the calculated back-up structure. We start with the 1310 tons of the main structure; since we do not have to divide into so small units as before, we add only 1/2 of the additional 460 tons for the 8 x 8 m boxes, and we omit the fine structure completely. This adds up to 1540 tons. Now, we compare the volumes. Since the volume of the trough is larger than that of the back-up structure, but not twice as large, we should be on the safe side if we multiply with 2, obtaining 3100 tons for the trough. In summary, we have:

500	tons	surface
2300	"	back-up structure
3100	"	trough
5900	tons	total

or about

$$6000 \text{ tons for minimum structure.} \qquad (11)$$

Second, we estimate the behaviour of this minimum structure under the wind forces. The diagonals of the main structure have a cross section of 7.96 inch^2, the other members have 4.80 inch^2, and the average is $Q = 6.38$ inch^2 = 41.2 cm^2. From equation (7) we then get $\Delta h = 0.104$ cm. We assume that the trough deforms by the same amount; this adds up to $\Delta h = 0.21$ cm, and with $\Delta h = \lambda/16$ we obtain $\lambda = 3.4$ cm. To be on the safe side, we increase slightly to

$$\lambda = 4 \text{ cm} \quad \text{for minimum structure.} \qquad (12)$$

The average cross section of 6.38 inch^2 should now be compared with equation (9) for survival conditions. It certainly is enough for the strongest wind, and it is enough for 21 lb/ft^2 of snow (or 4.0 inches of solid ice layer), which seems alright for regions with only moderate winters. The weight of back-up structure and surface is 2800 tons, which is more than needed for equation (8); it would prevent uplifting for winds up to 130 mph. In summary, we obtain

$$\text{survival conditions for minimum structure:} \left\{ \begin{array}{l} \text{wind} = 130 \text{ mph} \\ \text{snow} = 21 \text{ lb/ft}^2 \\ \text{ice} = 4 \text{ inches} \end{array} \right. \qquad (13)$$

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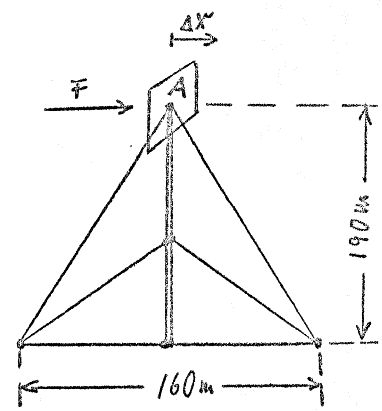
5. The Tower

Again, we first calculate a minimum structure. Including the azimuth-rotating top with the feed tracks, we calculate a tower of 190 m height, with 4 legs and with 3 horizontal connections 33 m long and as strong as the legs. The legs shall consist of 4 main chords, 5m apart and unbraced for $\ell = 7$ m. We choose $\ell/r = 90$ and obtain r = 2.9 inch. We combine again two angles with unequal legs, and arrive at A94 angles with a combined weight of 52.6 lb/ft and a combined cross section of 15.5 inch^2. The maximum width for any wind direction is 9 inches, while a rope of same cross section has a diameter of 4 inches. The total length of the main chords then is 4.7 km, and the weight is 307 tons. We add 60% for struts and diagonals and obtain 491 tons. As for the ropes, we think that it is the best solution if we have just as much steel in the tensioned ropes as we have in the compressed members, thus we add 307 tons for the ropes. The total weight then is 798 tons, or about

800 tons for minimum tower. (14)

Second, we ask for the wind forces. The whole tower, with chords, bracings and ropes, picks up the wind with an area of 2000 m^2. We replace the tower by the simplified structure shown. We place 1/4 of the total area at the top, A = 500 m^2, which gives a force F = 6.25 tons during observation, and F = 121 tons for survival wind. The stress in the guy ropes from survival wind then is only 0.39 tons/cm^2 = 5600 psi, which means that survival is no problem. During observation, the displacement amounts to $\Delta x = 0.50$ cm; we multiply by 16 and obtain $\lambda = 8$ cm for the minimum tower. Since the dish structure is the more expensive part and gives already $\lambda = 4$ cm, we should increase the tower weight until we get the same wavelength, which gives 1600 tons. To be completely on the safe side, we round up and have



$$\text{tower} \begin{cases} \lambda = 4 \text{ cm} \\ 2000 \text{ tons.} \end{cases} \qquad (15)$$

6. Comparison

We compare this type of transit telescope with the other type, the tilted sphere, as suggested by Findlay and worked out by Faelten. It is shown in Fig.6 in the same scale and for the same specifications as used in the previous figures. Most of the comparison can be obtained directly by looking at Fig.5 and Fig.6.

The feed support of Fig.6 and the tower of Fig.5 have the same height and need the same rigidity; also the curved track, the feed package and the elevation drive are almost the same. The tower has the disadvantage that the track on its top must rotate by 360° around a vertical axis, but then the feed support of Fig.6 must rotate as a whole, while the tower of Fig.5 is fixed to the ground. I think that both present about the same degree of difficulty and will cost about the same amount of money, for all possible values of D and λ.

The difference of both designs lies in the mounting of the primary mirror. In Fig.6 the large weight results from the wind force, picked up by a large area high above ground. In Fig.5 the area shown to the wind is about 5 times smaller, and the average height above ground is about 3 times smaller (which would give a factor of 15 for the survival weight, if we did not need a minimum structure). The flat structure needs much less steel, and even the minimum structure of 8000 tons can be used down to a wavelength of 4 cm. These features, together with the fact that the minimum weight increases only with D^2, makes this structure a good candidate for the "largest feasible" one.

On the other side, an antenna flat on the ground needs a slant illumination. It can be shown that a spherical mirror then would need an unreasonably large secondary mirror, and we are forced to use a paraboloid which then must be physically rotated by $\pm 5^{\circ}$ in elevation to give the observer enough integration time. This is the main disadvantage of the flat antenna. The second disadvantage is the large radius of azimuth rotation, which is 389 m in Fig.5 as compared to only 83 m in Fig.6. Some of this disadvantage will be balanced by the fact that the weight of Fig.6 is much higher and is more concentrated toward the edge, which needs much stronger foundations than are needed for the lower and evenly distributed weight in Fig.5. A final comparison, of course, needs an actual design with a cost estimate based on it.

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7. The Total Weight as a Function of D and λ

We call W_o the total weight of the minimum structure (dish, trough, tower), λ_o the shortest wavelength for this minimum structure, and W the total weight needed for other wavelengths. In my first LFST-paper I found that

$$W_o \sim D^2 \qquad \text{and} \qquad W \sim D^4/\lambda. \tag{16}$$

Since the cross sections must be $Q \sim W^{2/3}$, we have $Q \sim D^{4/3}$ for the minimum structure. And since wind deformations are proportional to (area x length)/Q, we find

$$\lambda_o \sim D^{5/3}. \tag{17}$$

We normalize with the numerical values found for D = 200 m and obtain:

$$\lambda_o = 4\text{ cm}\left\{\frac{D}{200\text{ m}}\right\}^{5/3} \tag{18}$$

and

$$W = \begin{cases} W_o = 8000\text{ tons}\left\{\frac{D}{200\text{ m}}\right\}^2 & \text{for } \lambda \geq \lambda_o \quad (19) \\ 8000\text{ tons}\left\{\frac{D}{200\text{ m}}\right\}^4 \frac{4\text{ cm}}{\lambda} & \text{for } \lambda \leq \lambda_o . \quad (20) \end{cases}$$

The result is shown in Fig.7. We see, for example, that an antenna with 200 m aperture can be built for λ = 2 cm with 16 000 tons of steel, while an antenna with 300 m aperture and a weight of 18 000 tons will be limited to λ = 8 cm. An antenna with 400 m aperture cannot be built below 32 000 tons, which then would give λ = 13 cm.

We need a cost estimate, including foundations, based on an actual design, before we can adopt a "largest feasible" limit for D and λ.

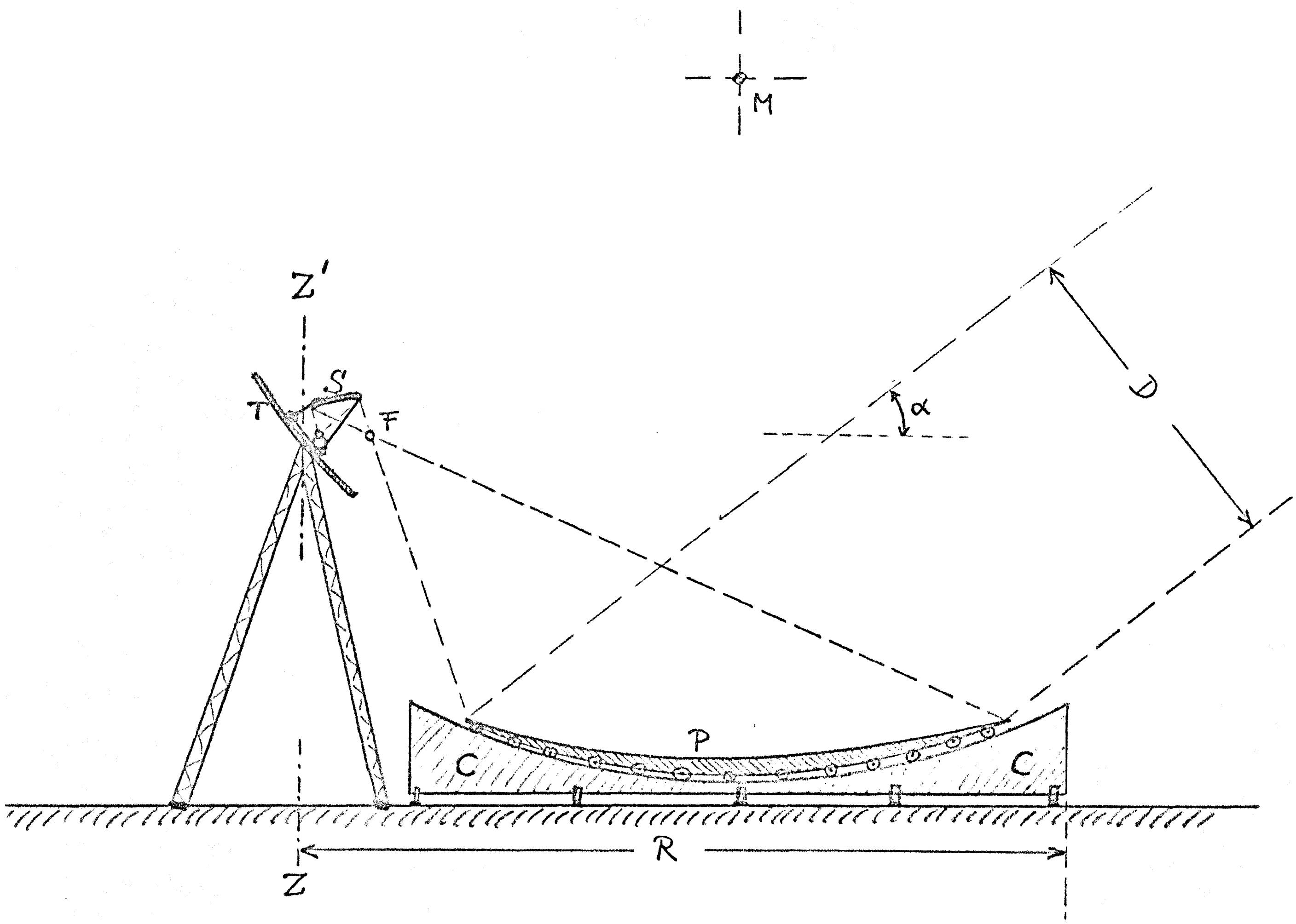


Fig.1: The flat antenna concept.

The primary mirror P is an oval, off-axis part of a paraboloid of revolution, having its primary focus in F. The secondary Gregorian mirror S is needed for obtaining a symmetrically illuminated beam with a symmetrically illuminating feed (for polarization measurments). The antenna beam has a round aperture of diameter D and elevation angle $\alpha \pm 5^{\circ}$.

The primary mirror moves by $\pm 5^{\circ}$ around axis M in a flat, cylindrical trough C. The trough rotates on horizontal circular tracks by 360° around axis Z-Z'. Secondary mirror, feed and observing cabin are moved as one package along a circular track T by $\pm 5^{\circ}$ around M. The track T is mounted on a fixed tower and rotates by 360° around axis Z-Z'.

Fig. 2: Fixed dimensions for flat mirror, once the following is chosen:

200 m aperture diameter

$40^{\circ} \pm 5^{\circ}$ elevation angle

156 m height of focus

15 m diameter second. mirror

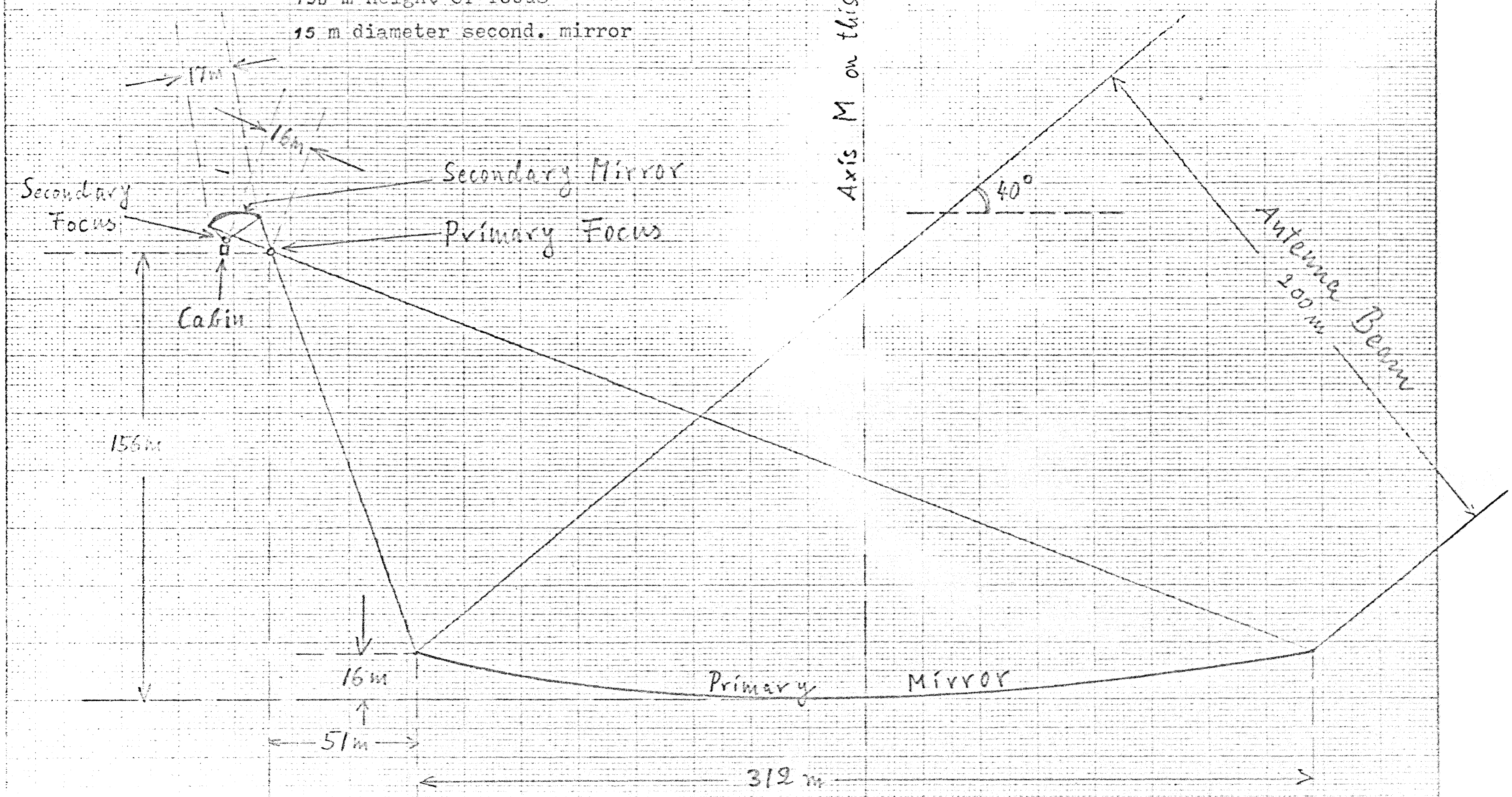


Fig.3: Too low a value for H (260 m) makes the cylindrical trough too high (61 m).

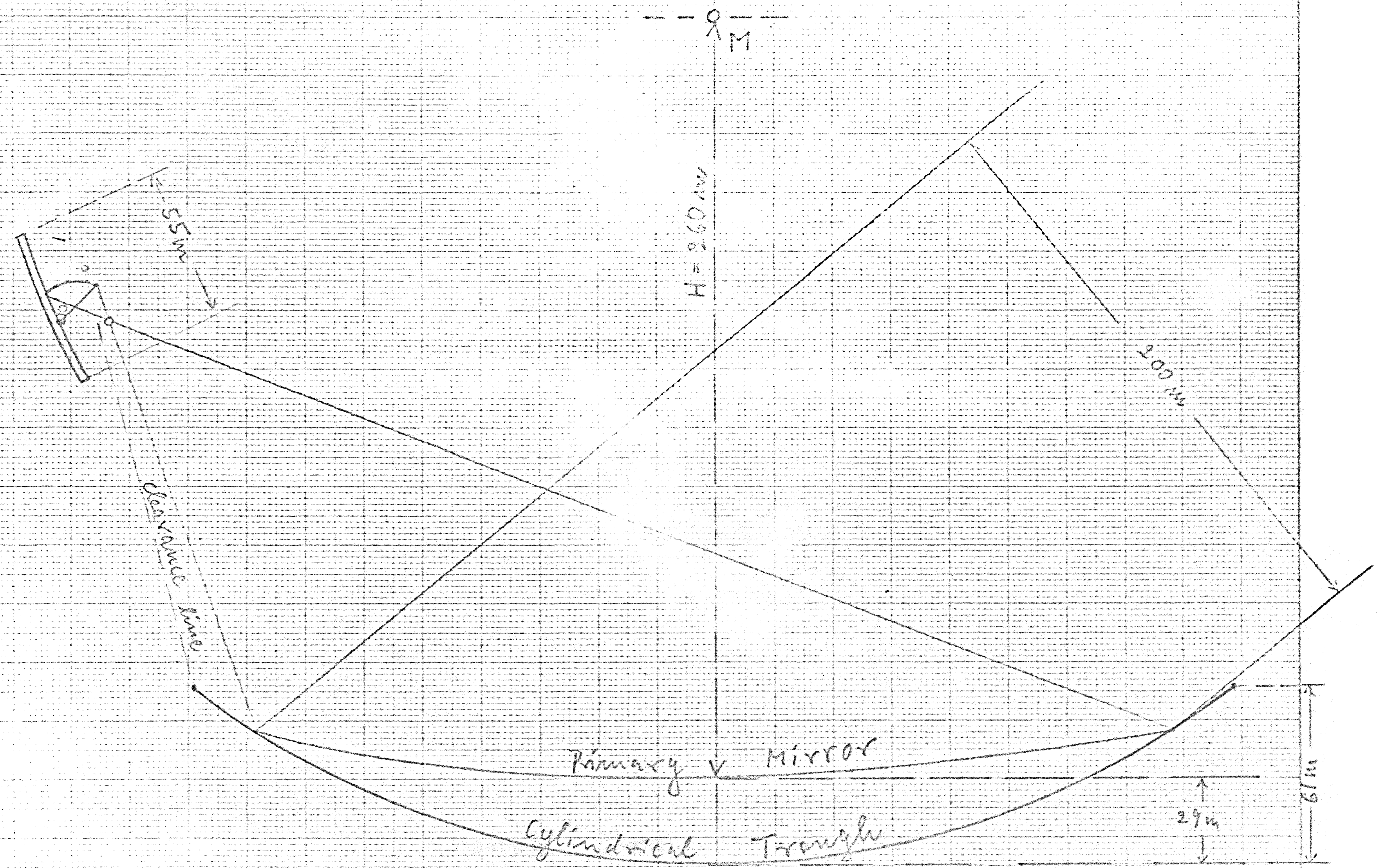


Fig.4: A large value for H (340 m) makes the trough flatter (50 m), but it needs additional feed structure and longer feed tracks (68 m).

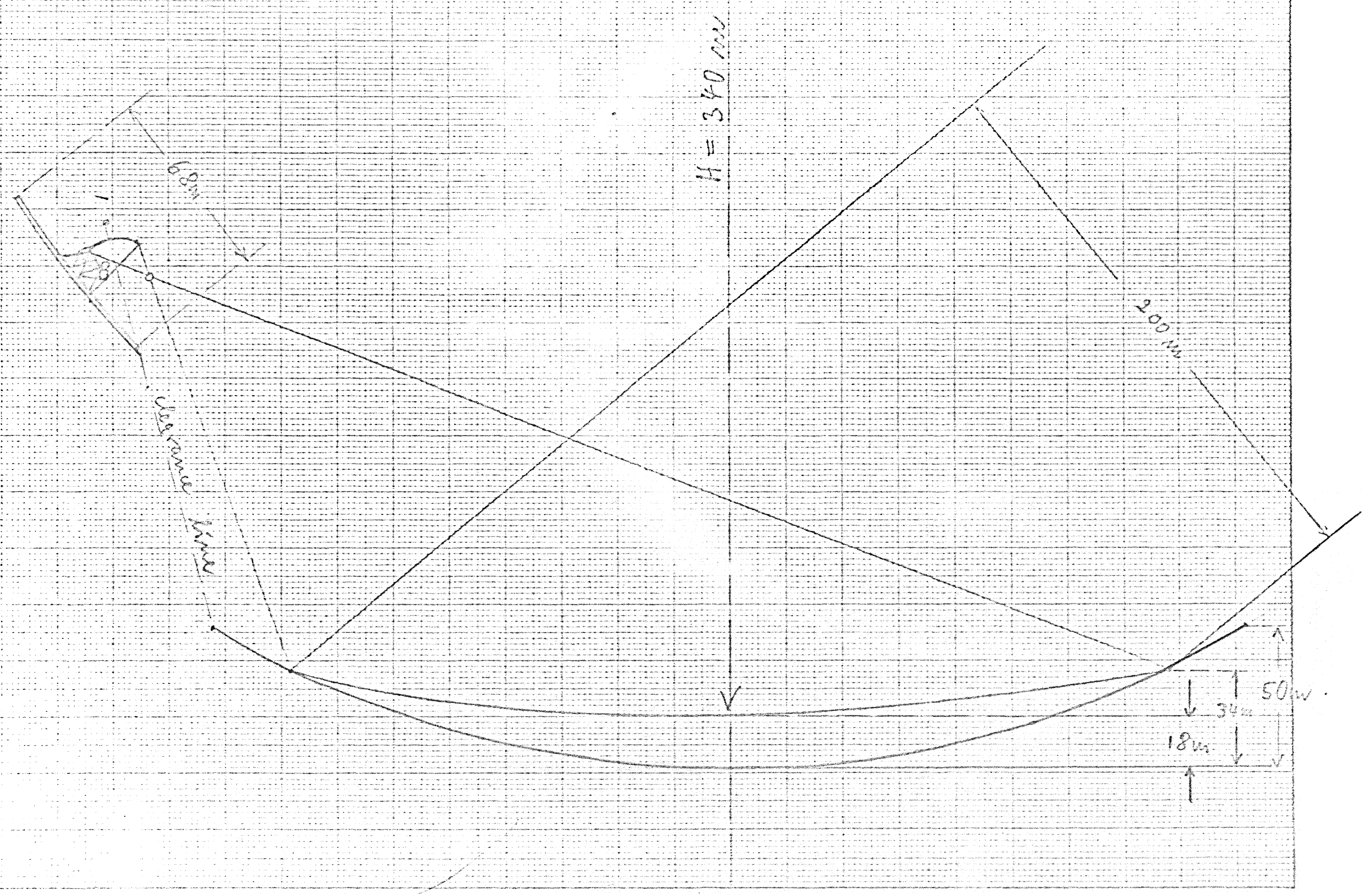


Fig.5: H = 300 m is adopted. This is the largest value without additional feed structure.

The trough is split into 5 segments for decreasing its height.

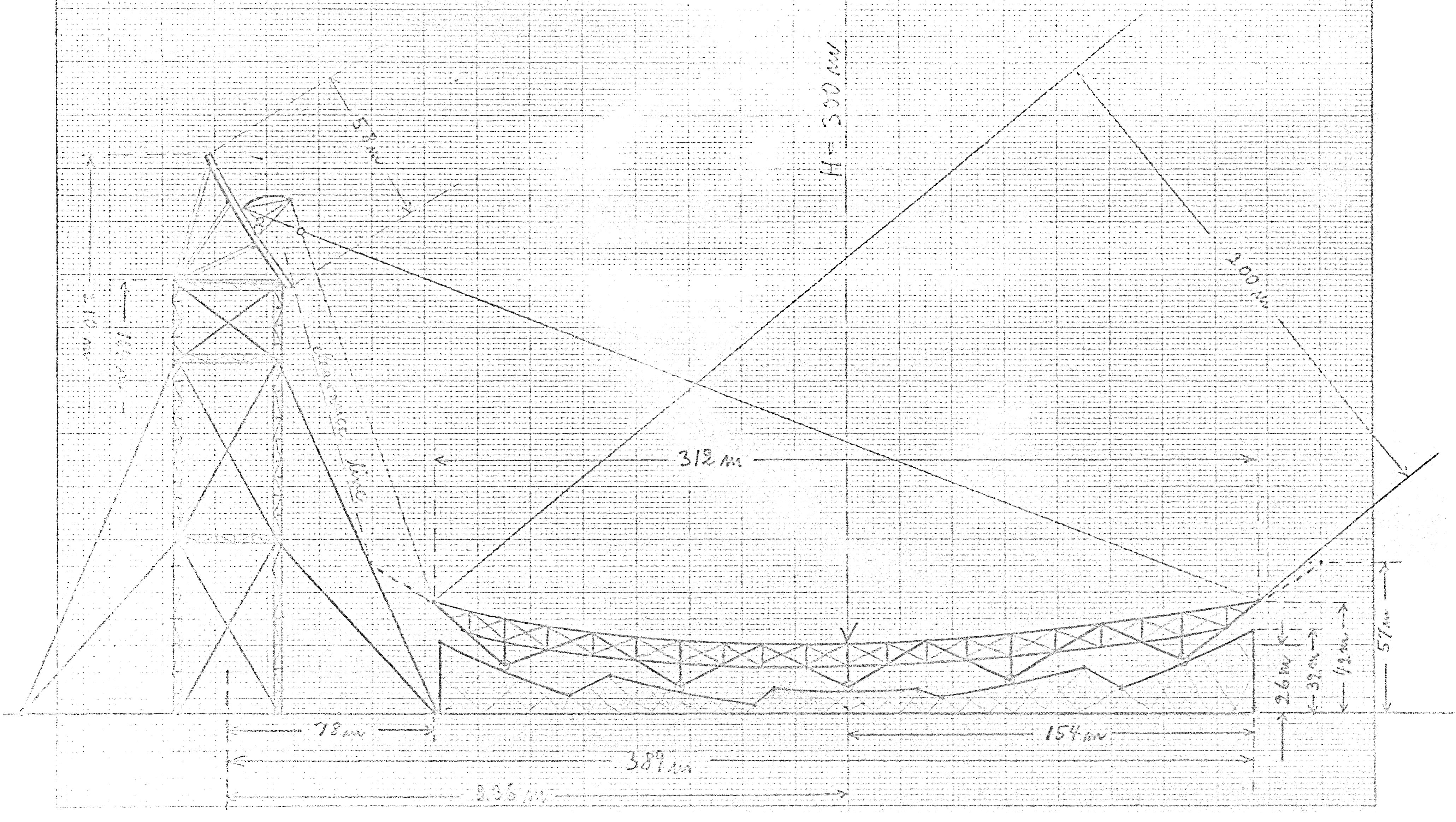
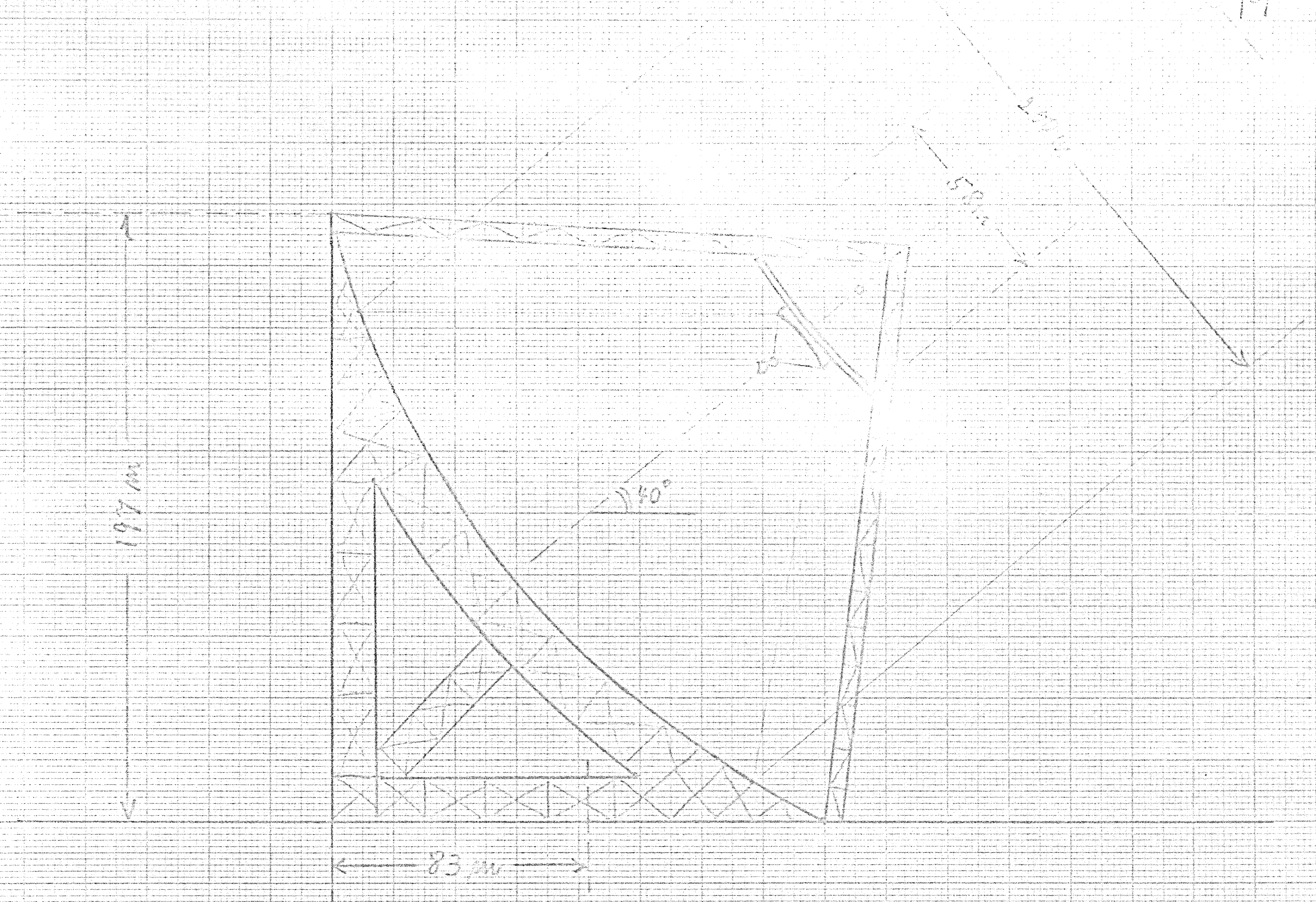
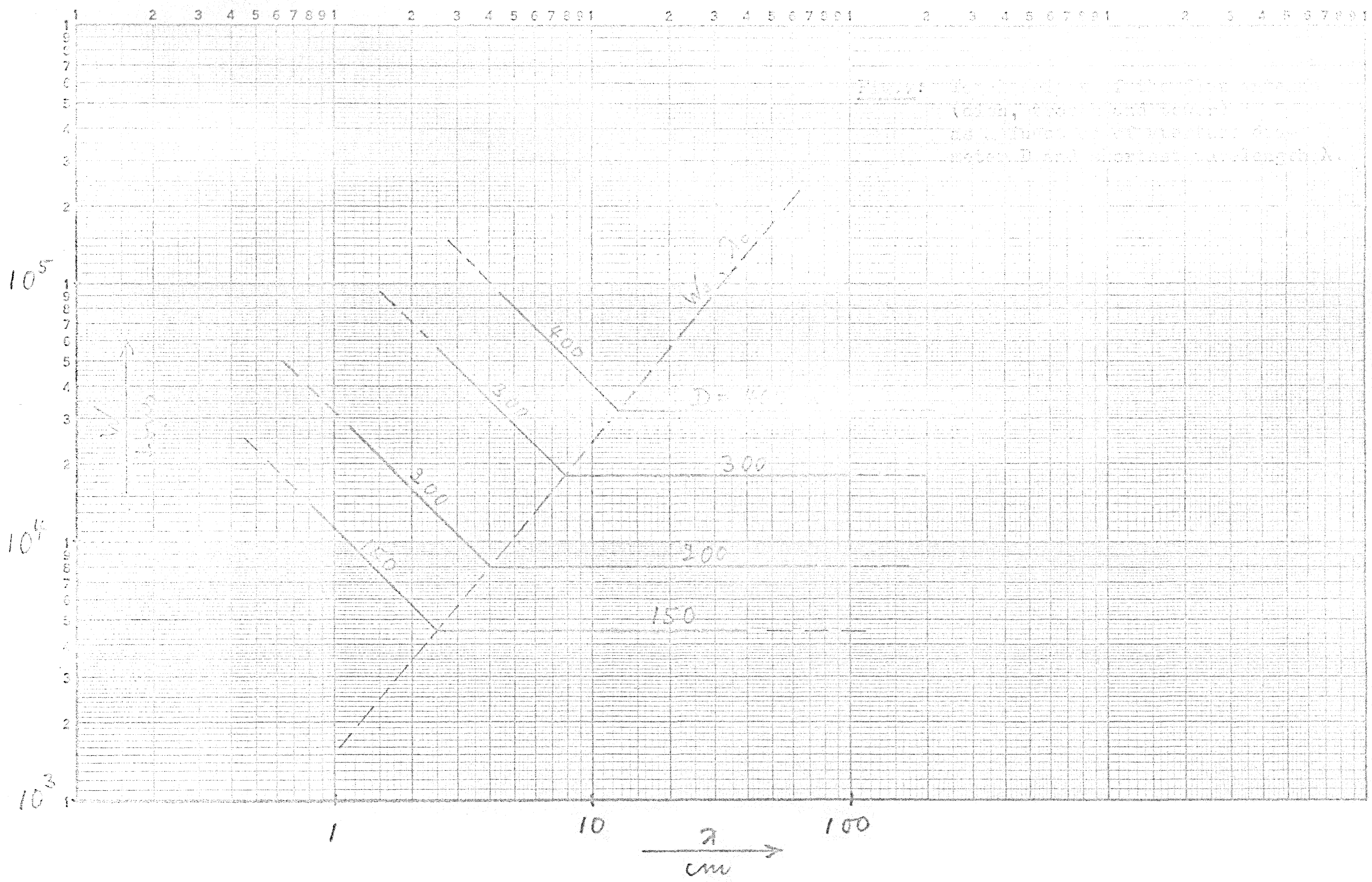


Fig. 6: For comparison: The tilted spherical antenna, for the same specifications (200 m aperture, 40° ± 5° elevation).




10^5
10^4
10^3
1
10
100
λ
cm
400
300
200
150
300
200
150