Small Surface Plates for Large Radio Telescopes

S. von Hoerner; NRAO, Green Bank, W.Va.

Summary

Four triangular plates (side length $\ell = 3.12$ ft = 95 cm) are cut from 1/8 inch aluminum sheet, their sides are rivetet on ribs of aluminum channels. Internal adjustment at four points is provided by a simple second system of ribs underneath; the plate center and the middle of each side can be pulled down by given amounts by four adjustment screws. The plate shape is measured at 21 equally spaced surface points with an rms accuracy of .0005 inch = .013 mm. The results then are scaled to $\ell = 2.22$ ft = 67.5 cm for the new design of the homologous telescope with D = 213 ft = 65 m diameter. This type of surface then costs 9.5 $/ft^2 including the internal adjustments, and 7.3 $/ft^2 without it.

The gravitational sag under dead load gives an rms deviation from the average of .0008 inch = .021 mm without internal adjustments, and .0006 inch = .015 mm with it. The rms deviation between the average plate shape and the telescope paraboloid is .0067 inch = .169 mm without adjustement, and only .0011 inch = .027 mm with it, varying as ℓ^2/D. The internal bumpiness of the plates (rms deviation of single point from average shape) is .0054 inch = .137 mm without adjustments, and .0036 inch = .092 mm with it, varying as $\ell^{1.3}$.

If the corners of all plates are exactly on the design paraboloid of the telescope, the total rms deviation of the surface from the best-fit paraboloid is .0089 inch = .225 mm without internal adjustments, about as good as honeycomb panels; and it is .0039 inch = .100 mm with adjustments, almost as good as a milled surface of extremely high cost.

For finding the performance of the 65-m telescope, 16 contributions to the total rms surface deviation are added up. Calling $\lambda = 16$ rms(Δz), the shortest wavelength of observation is $\lambda = 2.3$ mm at zenith without wind or temperature differences. At 60° from zenith, $\lambda \leq 3.4$ mm for 55% of all nights (disregarding clouds and snow); $\lambda \leq 4.0$ mm for 67% of all nights; $\lambda \leq 6.0$ mm for 68% of all time; and $\lambda \leq 9.2$ mm for 93% of all time.

I. Introduction

At present there are three methods for manufacturing the surface of a radio telescope to be used at short wavelengths. The following data are from discussions with Rohr Company:

1. Curved skin with ribs. Rohr offers plates 4 x 10 ft, ribs 10 inch apart, 1 mm skin. Yielding an rms accuracy of .013 inch = .33 mm at best. At a cost of 10 $/ft^2$ plus 200 k$ for tooling, totaling 15 $/ft^2$ for a telescope of D = 65 m = 213 ft diameter.

2. Honeycomb. Yields an rms of .008 inch = .20 mm. At 25 $/ft^2$ plus 250 k$ for tooling, totaling 32 $/ft^2$ for D = 65 m.

3. Milled surface (like the Kitt Peak 36-ft telescope). May yield an rms of .003 inch = .076 mm at best, at a cost of up to 200 $/ft^2$. The 36-ft was milled in one piece, but that is already too large with respect to thermal deformations. It seems to be somewhat more difficult to mill smaller, separate pieces.

For observing at a wavelength λ, the total rms deviation of the surface from the best-fit paraboloid of revolution should not be more than λ/16. And about 1/2 of that, λ/32, may be allowed for the manufacturing accuracy of the surface plates (assuming that this is one out of four uncorrelated and equal major contributions to the total deviation). Values for λ = 32 rms(Δz) are shown in Table 1, together with the cost.

Table 1. Accuracy, cost, and shortest wavelength for three available surface types.

	rms(Δz) mm	cost $/ft^2	λ mm
Curved skin and ribs	.33	15	10.6
Honeycomb	.20	32	6.4
Milled surface	.076	200	2.4

For the proposed homologous telescope, a different approach has been suggested: making the surface plates so small that they can be flat. For a 300-ft telescope and λ = 1 cm, an estimate yielded a side length ℓ of 3.12 ft = 95 cm for triangular plates. A cost estimate for material and manufacturing from ALCOA resulted in 4.4 $/ft^2$ for somewhat larger plates (ℓ = 114 cm), including tooling, which goes up to 5.3 $/ft^2$ for ℓ = 95 cm. The plates are cut from 1/8 inch aluminum sheets; their three sides are riveted on ribs of aluminum channel.

A 300-ft telescope then needs 18 000 triangular plates of ℓ = 95 cm side length; and 9 000 corner adjusters are needed on the telescope, each carrying six triangle corners. For the new design, with a smaller diameter of D = 65 m = 213 ft, it is suggested to keep the number of plates and corner adjusters the same, yielding ℓ = 2.22 ft = 67.5 cm. These smaller plates then will be 7.3 $/ft^2, but will almost reach the accuracy of honeycomb.

If a wavelength $\lambda < 6$ mm is wanted, and since the step from honeycomb to a milled surface is extremely expensive, a further improvement of the small plates, by internal adjustments, should be considered. First, a central adjustment screw pulls the plate center down toward a light channel fastened underneath two of the ribs. An experiment gave some but not much improvement. Second, three additional screws, one on the middle of each triangle side, pull the middle of each rib down toward a light angle beam fastened at both its ends a little below the ends of the rib, the spacing being provided by little blocks. This experiment improved the accuracy considerably.

The final version then has four internal adjustment screws per plate: one at its center, and three at the middle of the sides; this is the maximum possible number of independent adjustments for a triangle. It needs three additional angles under the ribs, and one light channel across and below the plate center. The manufacturer should adjust all four screws before delivery, to a given height below the plane defined by the three plate corners, with an accuracy of .002 inch = .05 mm maximum. This can be done within a few minutes per plate, with a proper template having four dial indicators and resting with three pins on the plate corners. It is estimated that the additional material, the labour and the adjustment will inrease the cost from 7.3 to 9.5 $/ft^2, yielding almost the accuracy of a milled surface.

This approach has the advantage of high accuracy at low costs, and the disadvantage of needing the large number of 9000 corner adjusters on the telescope. But this disadvantage will not be crucial if a new measuring technique (now in preparation) can be applied, which will allow measuring 9000 points or more within 1/2 hour, and probably yielding an rms accuracy of .003 inch = .08 mm. The mechanical adjustment of the corner adjusters then can be done by 10 men within 4 weeks (assuming 2 men and 5 minutes per point).

The present investigation wants to determine the actual shape of such plates, with and without adjustment srews, and their rms deviation from the telescope parabola. The results then are combined with all other surface errors for finding the shortest wavelength λ under various observing conditions.

II. Experimental Method

1. The Plates

A sample of four triangular plates are cut from 1/8 inch aluminum sheet, and their sides are riveted on ribs of aluminum channel (1 1/2 x 3/4; 1/8 inch thick). No straightening or trimming was done. The rivets are 3/16 thick, with 11/32 heads, and spaced by 1 5/8 inch. Each plate has a weight of 11.4 lb. The plates are designed for carrying a man of 200 lb on any point, without any permanent deformation.

For the central adjustment screw, a thin channel is fastened to the lower side of two of the ribs. For the adjustment screws at each side, an angle beam is put under each rib and parallel to it; spacing is provided by two small blocks between rib and angle at both ends, and the centers of rib and angle are pulled together by the screw. For later application on the telescope, some more practical sulution might be found.

Fig.1 shows a sketch of a plate and its side adjustment. For measuring the shape of the surface, it was decided to use 21 equally spaced points as shown; this number turned out to be large enough since no appreciable short-scale bumpiness was found.

2. Measuring Technique

For measuring the shape of the plate surface, one would like to have a dial indicator moving in a plane; or the deviations from a plane should at least be small, measurable and repeating. After various tries with other methods, it was decided to use the large milling machine of the Green Bank workshop (span of 5.5 feet). A rod replaces the milling bit, and a dial indicator (with .0005 inch divisions) is fastened vertically to the rod, such that it can be moved out of the way and back to working condition without any measurable change of its position. The dial indicator then is moved manually, by rotating the arm of the mill about its pillar, and by cranking the motor block along the arm.

The plates rest horizontally with their three corners on adjustable support screws on a triangular jig fastened on the working platform of the mill. The plates can be measured in normal position (skin pointing up) as well as upside-down, for separating the plate shape proper from the gravitational sag. For this separation one then needs also the skin thickness at all 21 measuring points; this is measured by moving the plate, from one point to the next, over a rigid pin fastened on the working platform, with the dial indicator exactly above the pin.

Since the motor block of the milling machine does not move exactly in a plane, this measuring technique needs a calibration. A template was made out of four newly bought straight edges, forming the three sides of a triangle and one center line. This template rests on the same adjustable support screws as used for the plates. But the middle of each straight edge is supported by a spring carrying half the weight; the unsupported length then is 1.5 ft, and the gravitational sag of the edge is only 3×10^{-5} inch max. The quality of this template was checked by rotating it twice by 60° and measuring again; no measurable difference was found.

The calibration yielded (1) a strong effect of .0055 inch = .14 mm maximum, regarding the position of the motor box along the arm; (2) a hysteresis of .0006 inch = .015 mm maximum, explained by one-sided piling-up of lubricant. Both effects (1) and (2) are shown in Fig.2 and are used for correcting the plate measurments. The following smaller effects are not used for corrections: (3) an effect of .0003 inch = .008 mm rms, regarding the rotation of the arm about the pillar; (4) a temperature effect of .00015 inch = .004 mm rms; and (5) a short-term repeatability of the same amount.

For obtaining the combined measuring error, a total of 12 single contributions was added up, regarding the template, the mill, the positioning of the plates, and the reading error. The result is, averaged over the whole plate:

$$\text{rms measuring error} = .0005 \text{ inch} = .013 \text{ mm}. \qquad (1)$$

Setting up a plate on the jig, adjusting the support screws, and measuring the 21 points is done by one man in 20 minutes. Adjusting the center and three sides to given amounts is done by one man in an additional 6 minutes per plate.

3. Definitions and Formulas

All four plates, with various amounts of internal adjustment, were measured in "up" position, with the skin pointing upward. Plates 1 and 2, once without adjustment and once with it, were also measured in "down" position; also the skin thickness was measured for these two plates at all 21 points. We call

u = dial indicator reading in "up" position;

d = dial indicator reading in "down" position;

t = measured skin thickness (1/8 inch nominal);

z = plate shape in "up" position (deviation from plane defined by 3 corners);

z_g = gravitational sag under dead load;

z_p = plate shape proper (no gravity);

c = calibration as shown in Fig.2.

We define z as positive if pointing down. Since a positive dial reading points up, we have

$$u = -z_p - z_g + c, \tag{2}$$

$$d = +z_p - z_g + c + t; \tag{3}$$

and

$$z = z_p + z_g . \tag{4}$$

In most cases, only z is wanted and is found from

$$z = c - u . \tag{5}$$

In some cases, the influence of gravity shall be separated according to

$$z_g = c - \frac{d+u-t}{2} , \tag{6}$$

and

$$z_p = \frac{d-u-t}{2} = z - z_g . \tag{7}$$

We define the following running indices, where "distance" refers to the plate center:

$1 \leq i \leq 21$ point number in plate, see Fig.1;
$1 \leq j \leq n_k$ point number in distance group k;
$1 \leq k \leq 5$ distance group;
$1 \leq m \leq N$ plate number (N=4 in our case);

and the following quantities:

b = distance of corner point from plate center;
r_k = (distance of group k from plate center) / b;
n_k = number/plate of points at distance r_k;
N = total number of plates investigated;

and finally call

$$q_k = \overline{r^2} - r_k^2 . \tag{8}$$

Then, for example,

z_{jkm} = deviation (from plane defined by 3 corners) of point j, within distance group k, of plate m.

The arrangements of the 21 points is shown in Fig.1, and the values of r and q are given in Table 2.

Table 2. The 21 points of Fig. 1, divided in distance groups k, according to distance r from plate center.

i	k	n_k	$.9253\ r_k$	r_k	q_k	$n_k\ q_k$	p_k
9, 13, 14	1	3	2/13	.1663	.2487	.7464	1.5
8, 10, 17	2	3	4/13	.3325	.1658	.4976	1.0
3, 4, 12, 15, 16, 18	3	6	$2\sqrt{7}/13$	.4399	.0829	.4976	1.0
2, 5, 7, 11, 19, 20	4	6	$2/\sqrt{13}$	.5995	-.0829	- .4976	-1.0
1, 6, 21	5	3	10/13	.8313	-.4177	-1.2440	-2.5

The .9253 in Table 2 results from the corner points being shifted inward by 4.1 cm, see Fig. 1. The values p_k are defined by

$$p_k = n_k q_k / .4976 \qquad (9)$$

and one finds

$$\overline{r^2} = .2764 \qquad (10)$$

and

$$Q = \overline{q^2} = .04126 = 1/24.23 , \qquad (11)$$

while, by definition,

$$\overline{q} = \sum{}' p_k = 0 . \qquad (12)$$

The following averages are used:

$$z_{km} = \frac{1}{n_k} \sum_{j=1}^{n_k}{}' z_{jkm} = \text{average z in distance group k, of plate m;} \qquad (13)$$

$$z_k = \frac{1}{N} \sum_{m=1}^{N}{}' z_{km} = \text{average z in distance group k, all plates;} \qquad (14)$$

$$Z_m^2 = \frac{1}{21} \sum_{i=1}^{21}{}' z_{im}^2 = \text{average } z^2 \text{ of plate m;} \qquad (15)$$

$$z_m = \frac{1}{21} \sum_{i=1}^{21}{}' z_{im} = \text{average z of plate m;} \qquad (16)$$

$$z_o = \frac{1}{N} \sum_{m=1}^{N}{}' z_m = \text{average z of all plates.} \qquad (17)$$

4. The best-fitting Adjustments

The paraboloid of the telescope shall have a given focal length and direction, but it shall be best-fitted in height (parallel translation up or down). The focal ratio is always used as

$$f/D = .427 \;. \tag{18}$$

The corner points defining the plane of reference are shifted inward by 4.1 cm, which reduces b by a factor of .9253, and we assume in general the same factor; thus

$$b = (.9253/\sqrt{3})\,\ell. \tag{19}$$

Consider a single plate at the apex. A paraboloid of revolution, having its apex at the center of the plane of reference of the plate, and its axis perpendicular to it, has the equation

$$z_{par,o} = -\frac{r^2}{4f} \;, \tag{20}$$

and a parallel translation to the best fit in height gives, with (8),

$$z_{par} = s\,(\overline{r^2} - r^2) + z_m = s\,q_k + z_m \;, \tag{21}$$

where

$$s = b^2/4f \;; \tag{22}$$

or, with (18) and (19),

$$s = .1674\,\ell^2/D \;. \tag{23}$$

For example, with D = 65 m and ℓ = 67.5 cm, we have

$$s = .0462 \text{ inch} = 1.173 \text{ mm}. \tag{24}$$

Equations (20) to (24) apply directly to the plate right at the telescope apex, and in good approximation to plates nearby. At larger distances x from the axis, one should replace 2f by R, the local average radius of curvature (average of radial and tangential direction). Using the geometrical mean instead of the average, one can derive an easy formula

$$R = 2f\left[1 + (x/2f)^2\right] , \tag{25}$$

and equation (22) then reads

$$s = b^2/2R = (b^2/4f) \Big/ \left[1 + (x/2f)^2\right]. \tag{26}$$

At the telescope rim, (26) is 25 % smaller than (22), and 14 % in the average. But in the following estimates, we use (22) instead of (26), considering the worst case only.

For determining the best amount of internal adjustment, one needs to know R_m, the best-fitting radius of curvature, for plate m and with various amounts of adjustment. In addition to the averages of (13) to (17), we define a curvature term P for the single plate and for all plates, as $P = \overline{zq}$:

$$P_m = \frac{1}{21} \sum_{k=1}^{5} n_k q_k z_{km} = .02370 \sum_{k=1}^{5} p_k z_{km} , \qquad (27)$$

$$P_o = .02370 \sum_{k=1}^{5} p_k z_k . \qquad (28)$$

It can be shown that the best-fitting radius then is

$$R_m = \frac{b^2 Q}{2 P_m} , \qquad (29)$$

and the adjustments should be varied until $R_m \approx R = 2f$, or

$$P_m \approx Q s = \frac{Q b^2}{4f} . \qquad (30)$$

Regarding the rms deviation, we consider plate m as being located at the apex, all three corners exactly level. The rms deviation between the plate surface, z, and a paraboloid of focal length f and best-fitting height, z_{par} from (21), is

$$\Delta_m = \text{rms} (z_{im} - z_{par}). \qquad (31)$$

It can be shown that Δ_m can be split up into two contributions,

$$\Delta_m = \sqrt{\Delta_{mo}^2 + \Delta_{mc}^2} \qquad (32)$$

with

$$\Delta_{mo} = \sqrt{Z_m^2 - z_m^2 - P_m^2/Q} \qquad (33)$$

= rms deviation of plate from paraboloid with best-fitting curvature R_m ;

and

$$\Delta_{mc} = \sqrt{Q (s - P_m/Q)^2} \qquad (34)$$

= rms deviation between paraboloid of curvature R_m and paraboloid of curvature $R = 2f$.

The task, then, is finding

z_c = height of center adjustment, below plane of reference, (35)

z_s = height of side adjustement, below plane of reference, such that Δ_m = minimum. Instead of using this demand, it was found easier and almost correct to demand condition (30) which yields

$$\Delta_{mc} \approx 0 , \tag{36}$$

and, simultaneously, demanding

$$\Delta_{mo} = \text{minimum.} \tag{37}$$

In this way the best adjustments were determined, for "up" position, as

$$\begin{aligned} z_c &= .060 \text{ inch} = 1.53 \text{ mm}, \\ z_s &= .052 \text{ inch} = 1.32 \text{ mm}, \end{aligned} \tag{38}$$

for D = 65 m and ℓ = 67.5 cm. For other values, both adjustments vary as

$$z_c, z_s \sim \ell^2/D. \tag{39}$$

All four plates then were adjusted to the same values, given by (38), and were measured again. The following refers to these measurments after adjustment.

5. Application to Telescope

Consider the telescope surface consisting of a large number of such plates, similar to the ones measured, and assume that all plate corners are exactly adjusted on a paraboloid of revolution given by (20). The plates have internal adjustments according to (38) for the final version; or center adjustment only, or none, for comparison.

The best-fit paraboloid then is shifted down by z_o from (17). It also shows a small change of focal length, $df = -.71\, z_o$ as can be shown, which is disregarded in the following. The rms deviation Δ_p between the plates and the best-fit paraboloid is split up into two contributions, regarding the internal "bumpiness" of the plate surfaces, and the quality of the fit achieved by the internal adjustment. In addition, we need a correction regarding the small number N of plates measured as compared to the large number used; we call

$$\Delta_b^2 = \overline{(z - z_k)^2} = \frac{1}{21N} \sum_{m=1}^{N} \sum_{k=1}^{5} \sum_{j=1}^{n_k} (z_{jkm} - z_k)^2 \quad = \text{bumpiness} \tag{40}$$

$$\Delta_a^2 = \overline{(z_k - z_{par})^2} = \frac{1}{21} \sum_{k=1}^{5} n_k (z_k - z_{par})^2 \quad = \text{quality of inter.adjustm.} \tag{41}$$

$$\Delta_N^2 = \frac{1}{N-1}\overline{(z_m-z_o)^2} = \frac{1}{N(N-1)}\sum_{m=1}^{N}(z_m-z_o)^2 = \text{correction.} \quad (42)$$

The total rms deviation of all plate surfaces then is

$$\Delta_p = \sqrt{\Delta_b^2 + \Delta_a^2 + \Delta_N^2} \, . \quad (43)$$

For scaling to various values of D and ℓ, we have

$$\Delta_b, \Delta_N \sim \ell^\beta \, , \quad (44)$$

$$\Delta_a \sim \ell^2/D, \quad (45)$$

where the exponent β of the bumpiness must be found experimentally.

6. Error Contributions from Plate Corners

In addition to (43), we have three contributions regarding the plate corners. First, the plate corner rests on the support of a corner adjuster, the plate surface thus being lifted by the amount t_c of the corner thickness (thickness of skin, of rib flange, and of a little plate, each one being 1/8 inch nominal) with an average of about 3/8 inch = 9.6 mm. We call

$$\Delta_{ct} = \text{rms difference between single } t_c \text{ and the average.} \quad (46)$$

Second, the corner adjusters will have manufacturing inaccuracies. We call Δ_{ca} the rms deviation in height between a single corner support and the average of six supports on one adjuster. It was estimated for our present design that the following specification could be met without difficulties:

$$\Delta_{ca} = .002 \text{ inch} = .05 \text{ mm.} \quad (47)$$

Third, the corner adjusters should be welded on the telescope perpendicular to the surface; but the surface position is not known exactly at this time, and maybe one should consider a flexible mount instead of a rigid welding. Menwhile we assume an adjuster scaled down to 1/2 of the present design (which was done for D = 300 ft, and is too clumsy, anyway), and we assume that the coordinates of the surface structure (of the panels) have been measured with an rms accuracy of .080 inch = 2 mm. It can be shown that the resulting corner error then is

$$\Delta_{c\alpha} = .0022 \text{ inch} = .056 \text{ mm.} \tag{48}$$

The total error of a single corner then is

$$\mathcal{E}_c = \sqrt{\Delta_{ct}^2 + \Delta_{ca}^2 + \Delta_{c\alpha}^2} = \sqrt{\Delta_{ct}^2 + (.075 \text{ mm})^2} \;. \tag{49}$$

With respect to scaling, smaller plates can be thinner, and

$$\Delta_{ct} \sim \ell \tag{50}$$

is adopted; whereas Δ_{ca} and $\Delta_{c\alpha}$ are considered to be constant.

If the corners of a triangular plate are vertically shifted by random amounts of rms $\mathcal{E}_c$, it can be shown that the rms surface shift then is

$$\Delta_c = \mathcal{E}_c / \sqrt{2} \text{; independent of } \ell \text{ and D;} \tag{51}$$

which means that a large number of corner adjusters contributes the same surface error as a small number would. With (49) we then have

$$\Delta_c = \sqrt{\Delta_{ct}^2/2 + (.053 \text{ mm})^2} \;. \tag{52}$$

Finally, we call

$$\Delta_{pc} = \sqrt{\Delta_p^2 + \Delta_c^2} \;, \tag{53}$$

which is the total rms surface deviation arising from the use of small triangular plates and their corner adjusters (but not including the final telescope adjustment to be discussed later on).

III. Numerical Results

1. Surface distortion by Rivets

Measurments of z have been taken along one side of a plate, on a line going through the centers of the rivets, with 8 measurments per inch; 3 measurments on each rivet, and 10 in between rivets.

The result is shown in Fig. 3. The riveting depresses the surface next to the rivets by about 2/1000 inch, and the rivet heads stick out by about the same amount. Along this line centered on the rivets, the average distortion is only

$$\text{rms } z = .0012 \text{ inch} = .047 \text{ mm.} \tag{54}$$

Since this concerns less than 1/10 of the plate surface, it will be neglected further on.

2. Skin Thickness and Gravitational Sag

The skin thickness, needed for equation (6), has been measured for Plates 1 and 2 at 21 points each. The average thickness of these 42 points is found as

$$\bar{t} = .1244 \text{ inch} = 3.16 \text{ mm,} \tag{55}$$

to be compared with 1/8 = .125 inch nominal. The rms deviation of the single point from the average is only

$$\text{rms } (t - \bar{t}) = .00037 \text{ inch} = .0094 \text{ mm,} \tag{56}$$

which means that the thickness of the aluminum sheets is amazingly constant. Even the maximum deviation from the average is only

$$\max |t - \bar{t}| = .0011 \text{ inch} = .028 \text{ mm.} \tag{57}$$

The sag under dead load then is calculated from equation (6) for these two plates, once without and once with internal adjustments of center and sides. The results are shown in Fig.4. The difference, of course, is not due to the adjustment itself, but to the additional beams fastened underneath the plates, yielding much additional stiffness/weight for the center point and some for the sides.

Without adjustment, the center sag is found as z_{gc} = .0062 inch = .158 mm, which compares favourably with .0088 inch predicted and used in Report 25 (March 12, 1969), where the torsional stiffness of the ribs was neglected. The average sag over the plate is $\overline{z_g}$ = .00289 inch = .0735 mm, and the rms deviation from the average,

$$\Delta_g = \text{rms}\ (z_g - \overline{z_g}) \ , \qquad (58)$$

is found as .00164 inch = .0417 mm, as compared to .00171 inch predicted in Report 25. The best-fitting radius of curvature is found from (27) and (29) as R_g = 2330 ft = 710 m.

The above-mentioned values are determined with ℓ = 95 cm plate size. They vary as ℓ^2 if the thickness of skin and ribs is scaled in proportion to ℓ. With this assumption, the values of Table 3 are calculated from the measured values (except for R which stays constant).

Table 3. The gravitational sag of triangular plates of ℓ = 67.5 cm side length, without and with internal adjustments.

	center	average	rms dev.	curvature
	z_{gc} mm	$\overline{z_g}$ mm	Δ_g mm	R_g m
No adjustm.	.0793	.0369	.0209	710
With adjustm.	.0404	.0280	.0151	977

If the plates at the apex are internally adjusted with values (38) for "up" position, then the sag does not contribute explicitly to the total surface deviation if the telescope looks at zenith. If the telescope then is tilted to horizon, the contribution from the sag to be added to the total deviation is Δ_g as defined in (58) and as given in Table 3. The fact, that plates toward the rim behave differently from those at the apex, has been already treated ("A 300-ft High Precision Radio Telescope"; NRAO, May 1969; Vol I, page 5-4) with the result that this effect is negligible except for a small change in focal length. Thus,

$$\Delta_g = .000596 \text{ inch} = .0151 \text{ mm} \qquad (59)$$

is to be used for the new design with D = 65 m.

3. Bumpiness of the Surface

Fig. 5 shows the shape of the surface along a center line, for the bumpiest plate and the most even one; both in "up" position without internal adjustments. We see the dominating large-scale waves, but also some shorter waves of smaller amplitude.

The main task is finding the bumpiness as a function of the plate size. Instead of cutting and riveting many plates of various size, the four plates with $\ell = 95$ cm have been measured along all three center lines up to various distances x from the plate corner, as sketched in Fig. 6. A straight line then is put through the plate corner and the surface point at distance x, and the deviation of the surface from this line is called z_b. The rms value of z_b is shown in Fig. 6 as a function of x. The best-fitting ~~straight~~ exponential line has a slope of $\beta = 1.30 \pm .15$, to be used in (44) as the exponent of the bumpiness. Thus

$$\Delta_b \, , \, \Delta_N \sim \ell^{1.30} \, . \qquad (60)$$

The bumpiness depends on the plate size, as well as on the internal adjustment which in effect reduces the "unadjusted" size. All four plates have been measured in "up" position at all 21 points of Fig. 1, with and without adjustments. The bumpiness as defined in equation (40) is shown in Table 4, scaled from $\ell = 95$ cm to 67.5 cm according to (60). Table 4 also gives the correction Δ_N as defined in equation (42), with N = 4. The last line of Table 4 then gives the values of Δ_b and Δ_N to be used for the new design with D = 65 m diameter.

Table 4. Bumpiness Δ_b and correction Δ_N, for $\ell = 67.5$ cm and various internal adjustments.

	Δ_b mm	Δ_N mm
No adjustment	.137	.057
Center only	.099	.029
Sides and center	.092	.029

We see in Table 4 that the bumpiness is considerably reduced by adjusting the soft plate center, whereas adjusting the stiff sides does not yield much decrease.

4. Internal Adjustment, Prabolic Fit, and Corner Error

After the best-fitting values (38) for the internal adjustment were determined, all four plates are adjusted with these values and are measured at all 21 points. For each distance r_k from the plate center, the average z_k according to (14) then is calculated; from (40) we find the bumpiness Δ_b, and from (41) the rms deviation Δ_a between the average shape z_k and the telescope parabola z_{par}. This is done in two steps: first with center adjustment only, and second with adjustments of sides and center.

Fig. 7 shows the result with center adjustment only. Since the plate regions toward the corners are not affected, a rather odd shape results. The fit to the parabola is slightly improved as compared to the unadjusted plate (straight line).

The fit is considerably improved by adding the side adjustments, as shown in Fig. 8. The fit actually is now so good that the total surface deviation is mainly determined by the bumpiness and not any more by the fit of the average shape.

The measured values are scaled according to (45) and (60) to ℓ = 67.5 cm, and the resulting Δ_a is given in Table 5, together with the total surface deviation Δ_p according to (43) which combines fit, bumpiness and correction.

Comparing Δ_p from Table 5 with rms(Δz) from Table 1, we find that the plain triangular plates (no adjustment) are slightly worse than honeycomb, while the center adjustment makes them slightly better. The triangular plates with both side and center adjustment come close to the quality of a milled surface, in spite of their low cost of only 9.5 $/ft^2, see Introduction.

Table 5. The quality of the adjustment fit Δ_a, the total of the plate error Δ_p, and the total of plate and corner error Δ_{pc}. For D = 65 m = 213 ft, and ℓ = 67.5 cm = 2.22 ft.

	Δ_a mm	Δ_p mm	Δ_{pc} mm
No adjustment	.169	.225	.235
Center only	.134	.169	.182
Sides and center	.027	.100	.120

Next, the corner errors must be added. The corner thickness has been measured at all 12 corners, and the rms deviation from the average according to (46) is .00323 inch

= .0820 mm. This then is scaled with (50) to ℓ = 67.5 cm, yielding

$$\Delta_{ct} = .00229 \text{ inch} = .0582 \text{ mm.} \tag{61}$$

Together with the two other corner errors, from manufacturing (47) and from tilt (48), we find from (52) the rms contribution to the surface deviation as

$$\Delta_c = .00264 \text{ inch} = .0671 \text{ mm.} \tag{62}$$

This is added quadratically to Δ_p according to (53), and the resulting total rms deviation Δ_{pc} is given in the last column of Table 5. Thus, the value to be used for the new design, for the combined rms deviation resulting from the use of small triangular plates and their corner adjusters, is

$$\Delta_{pc} = .00473 \text{ inch} = .120 \text{ mm.} \tag{63}$$

Table 6. Values of the rms deviation Δ_{pc} for several D and ℓ.

	D = 300 ft ℓ = 95 cm N = 18000	213 ft 95 cm 9000	213 ft 67.5 cm 18000
No adjustment	.415 mm	.532 mm	.235 mm
Center only	.322	.416	.182
Sides and center	.179	.187	.120

Table 6 shows Δ_{pc} for three combinations of D and ℓ: the old 300-ft design, the new diameter of 65 m but the old plate size, and the new design with smaller plates. The last two cases show that one should not decrease the number N of plates, because going to N = 9000 increases Δ_{pc} already by 56 %. The first and third case show that Δ_{pc}, with complete adjustment, varies roughly as $\Delta_{pc} \sim D$, if we scale $\ell \sim D$.

Finally, surface deviations and number of plates are shown in Fig. 9 as functions of ℓ for D = 65 m. An accuracy considerably better than that of the present design could indeed be achieved by a larger number of smaller plates, but then it becomes more and more crucial that means be found for decreasing the corner errors, too. Without corner error, N = 30000 plates of size ℓ = 52 cm could enable observations even at λ = 2 mm, for D = 65 m.

IV. The Performance of the 65 m Telescope

1. The Surface Deviation

The new design of D = 65 m = 213 ft is mainly scaled down from the previous design of 300 ft ("A 300 Foot High-Precision Radio Telescope"; NRAO, May 1959; Vol. I, II, and III. Scaling to various diameters is described in Vol. I, Chapter 7).

Table 7 gives 16 items of the surface deviation from the best-fit paraboloid of revolution, for D = 65 m and a plate size of ℓ = 67.5 cm. For scaling to other D and ℓ, the single items vary as

$$\Delta z \sim D^{\gamma} \ell^{\beta} \qquad (64)$$

with γ and β given in Table 7. The single items come from the following sources:

<u>Items 1 through 6</u> from the present measurments;

<u>Item 7</u> assumes that a new measuring technique (to be described later) can be successfully applied, measuring 9000 points or more within 1/2 hour, with an rms accuracy of .003 inch = .08 mm. Or it assumes that some such technique will be developed within the next 4 years.

<u>Item 8</u> assumes an rms accuracy of .06 mm for the mechanical adjustment, which means turning a corner adjuster by a given angle. With a thread of 32/inch, this demands an angular accuracy of $\pm 27^{\circ}$, or about 1/4 of a right angle.

<u>Item 9</u> is scaled from the 300-ft design.

<u>Item 10</u> results from the present measurments, equation (59).

<u>Items 11 to 16</u> are scaled from the 300-ft design.

As to the thermal deformations, a good protective white paint is assumed. Our own measurments of $\Delta T = 5\ ^{\circ}C$, between sunshine and shadow on clear summer days, has also been confirmed by measurments of Rohr Co. Furthermore, it should be mentioned that calm nights give small values of ΔT, while fast changes of ambient air temperature (larger ΔT) are connected with higher winds.

Items 4 through 8 give $1/\sqrt{2}$ of the actual corner error, according to (51).

Table 7. Single contributions to the rms surface deviation, for D = 65 m and $\ell = 67.5$ cm. Scaling to other D and ℓ according to $D^{\gamma}\ell^{\beta}$.

Items	γ	β	rms(Δz) mm	Combinations (mm) and remarks
Plates (ℓ = 67.5 cm)				
1. Bumpiness	0	1.3	.092	
2. Average shape	-1	2	.027	.100
3. Number correction	0	1.3	.029	
Corners ($\varepsilon/\sqrt{2}$)				
4. Corner thickness	0	1	.041	
5. Adjuster tolerance	0	0	.035	.067 — (1.–8.) .140 Telescope at zenith, no wind, $\Delta T = 0$.
6. Adjuster tilt	0	0	.040	
Telescope adjustment ($\varepsilon/\sqrt{2}$)				(λ = 2.24 mm)
7. Measuring	1	0	.057	.071
8. Adjusting	0	0	.042	
Gravity				
9. Use of standard pipes	2	0	.122	
10. Sag of plate and ribs	0	2	.015	.148 Tilt of 90°;
11. Sag of large panels	2	0	.080	otherwise $\sim (1 - \cos \zeta)$.
12. External load on panels	2	0	.019	
Wind				
13. Plate and ribs	0	1	.013	
14. Panels	1.2	0	.027	.229 Wind of 18 mph (3/4 all time) otherwise $\sim v^2$.
15. Back-up structure	1.2	0	.227	
Temperature				
16. Thermal def. $\Delta T = 1^{\circ}C$	1	0	.112	most of all nights,
16. Thermal def. $\Delta T = 5^{\circ}C$	1	0	.560	full sunshine and calm.

2. The Shortest Wavelength of Observation

Figs. 10 and 11 show the shortest wavelength, defined as

$$\lambda = 16 \text{ rms}(\Delta z), \quad (65)$$

for various observing conditions. We see that the influence of gravity is very small; the thermal deformations in sunshine are rather dominant, while the wind deformations are mostly in between. The results are summarized in Table 8.

Table 8. Shortest wavelength λ for various conditions.

	Conditions	Fraction of time (disregarding clouds, snow)
$\lambda \leq 9.2$ mm	Sunshine and calm; or wind ≤ 28 mph	93 % of all time
$\lambda \leq 6.0$ mm	Sunshine and $5.5 \leq v \leq 21$ mph; or night, and wind ≤ 21 mph	54 % of all days, 82 % of all nights = 68 % of all time
$\lambda \leq 4.0$ mm	Night and wind ≤ 15 mph	67 % of all nights
$\lambda \leq 3.4$ mm	Night and wind ≤ 11 mph	55 % of all nights
$\lambda = 2.24$ mm	Zenith, no wind, $\Delta T = 0$	0

3. The Pointing Error

A final estimate of the pointing error has to wait for a finished re-design of the pointing system, and for an experiment (in preparation) with a servoed platform and a laser beacon. Preliminary estimates of O. Heine lead to an rms pointing error of

$$\Delta\varphi = 2.3 \text{ arcsec}. \quad (66)$$

For some observations it helps if at least the pointing knowledge is more accurate. Preliminary estimates show that the pointing at any given time will be known within

$$\Delta\varphi_0 = 1.5 \text{ arcsec}. \quad (67)$$

As compared to a beamwidth of

$$\beta = 11 \text{ arcsec (for } \lambda = 3 \text{ mm)}, \quad (68)$$

the pointing errors then are

$$\Delta\varphi = .210\ \beta, \qquad (69)$$
$$\Delta\varphi_o = .137\ \beta,$$

which may be compared to $\Delta\varphi = .22\ \beta$ for our 140-ft at $\lambda = 2$ cm.

A final improvement of the pointing error, from 2.3 down to 1,5 arcsec, seems possible with a Cassegrain mirror oriented by fast servo motors, which decreases the dynamical lag of the pointing.

It also should be mentioned that a preliminary cost estimate, scaled from the previous 300-ft design, yields 4 - 5 M$ for the total cost.

Finally, Fig. 12 shows the attenuation of the atmosphere as a function of wavelength. Several known or suspected molecular lines are added (from L. Snyder).

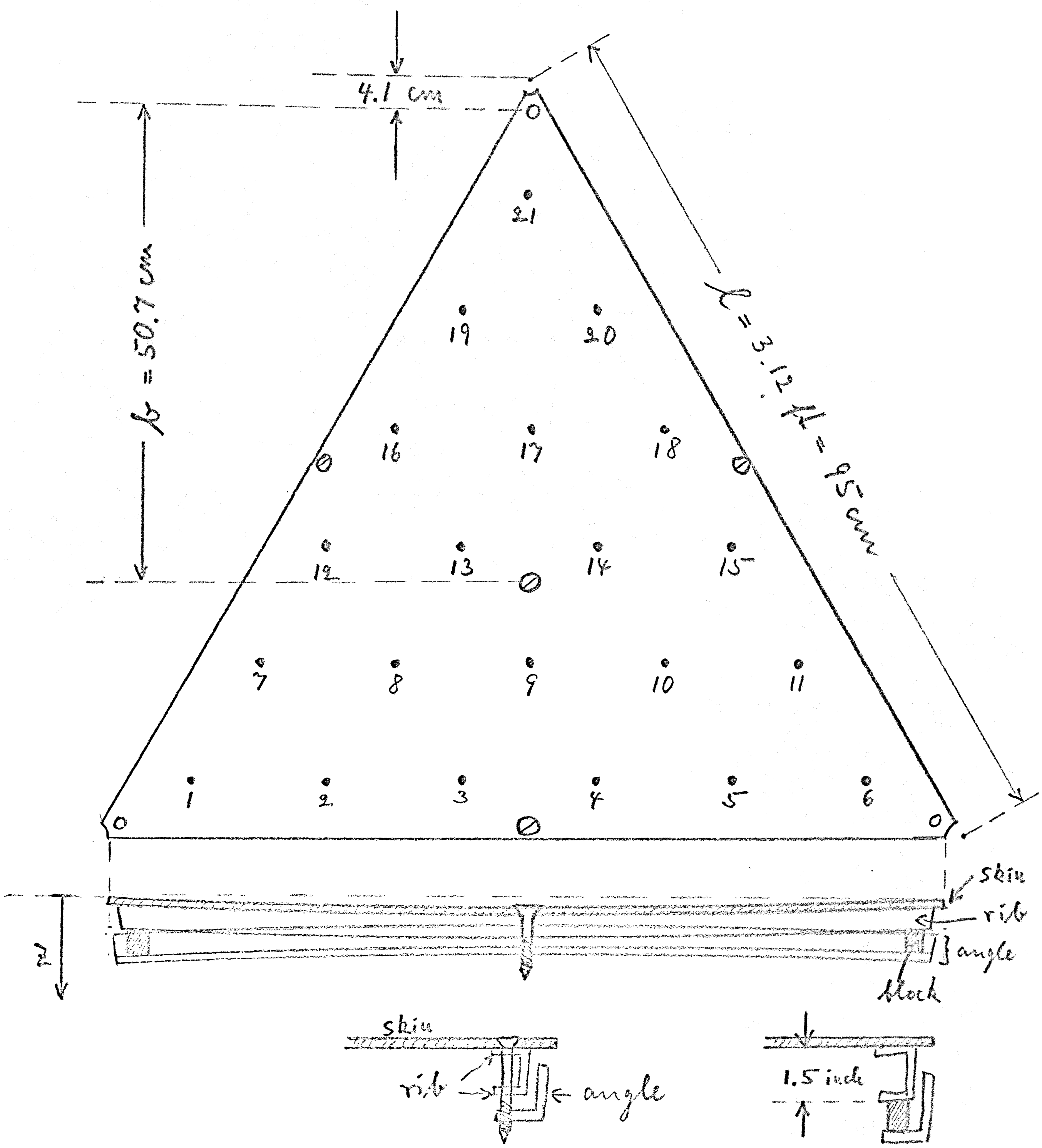


Fig. 1. The triangular plate and its side adjustment.

o 3 corner points defining the plane of reference;

∅ 4 srews for the internal adjustment, pulling down;

• 21 points where deviation z from plane is measured.

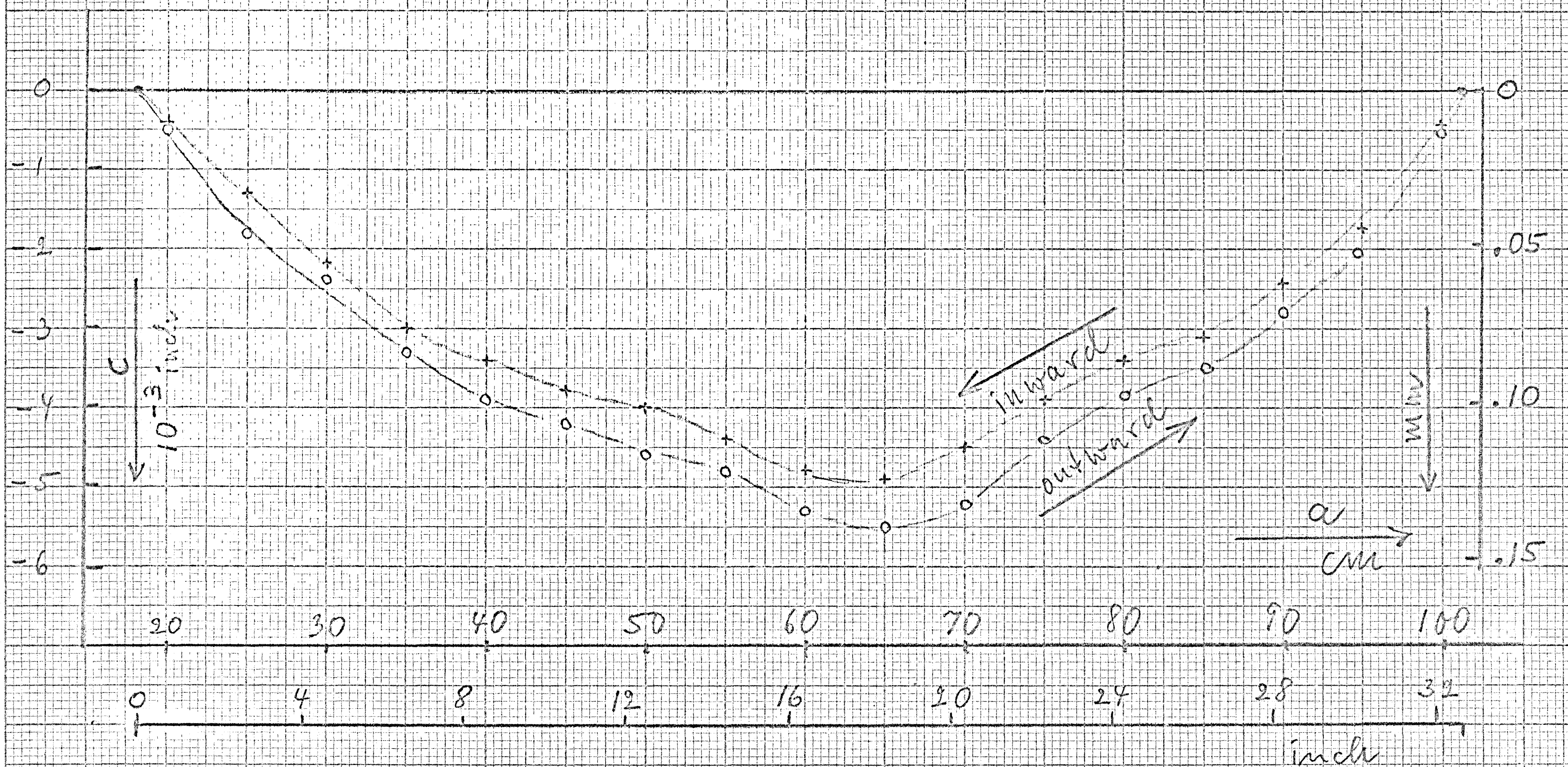


Fig. 2. Calibration curve of the milling machine used for the measurments.

a = distance along arm of machine (arbitrary zero);

c = reading of dial indicator on straight template.

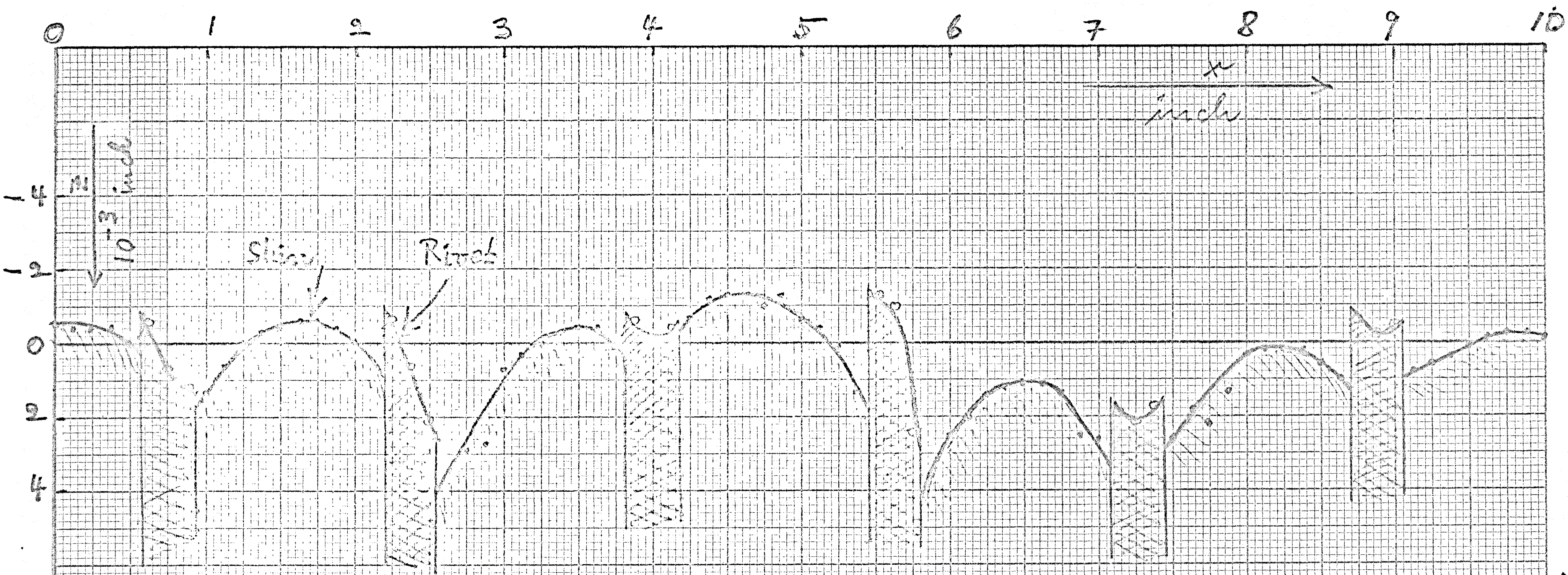


Fig. 3. Surface distortion, along a line centered on the rivets at the side of a plate.

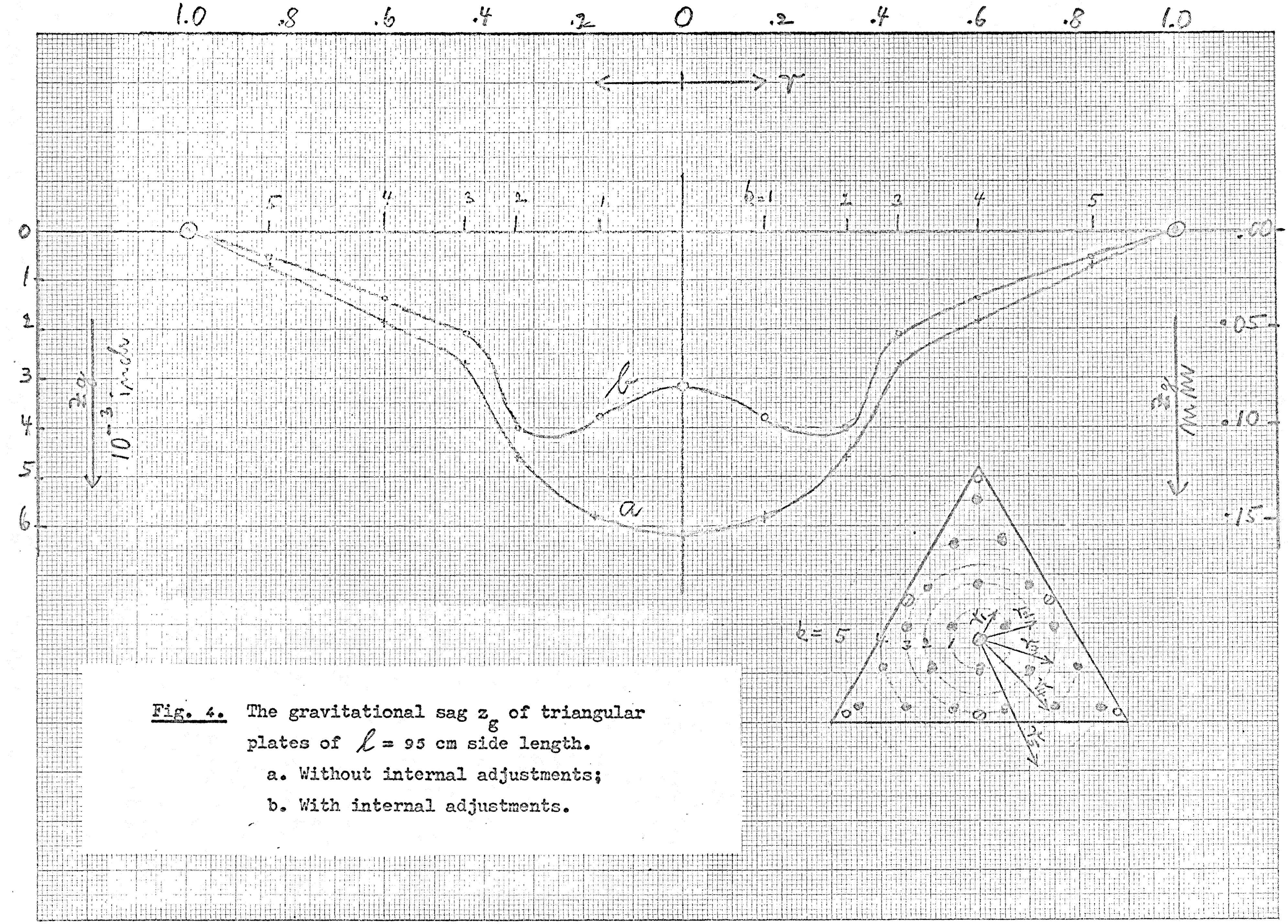


Fig. 4. The gravitational sag z_g of triangular plates of ℓ = 95 cm side length.
a. Without internal adjustments;
b. With internal adjustments.

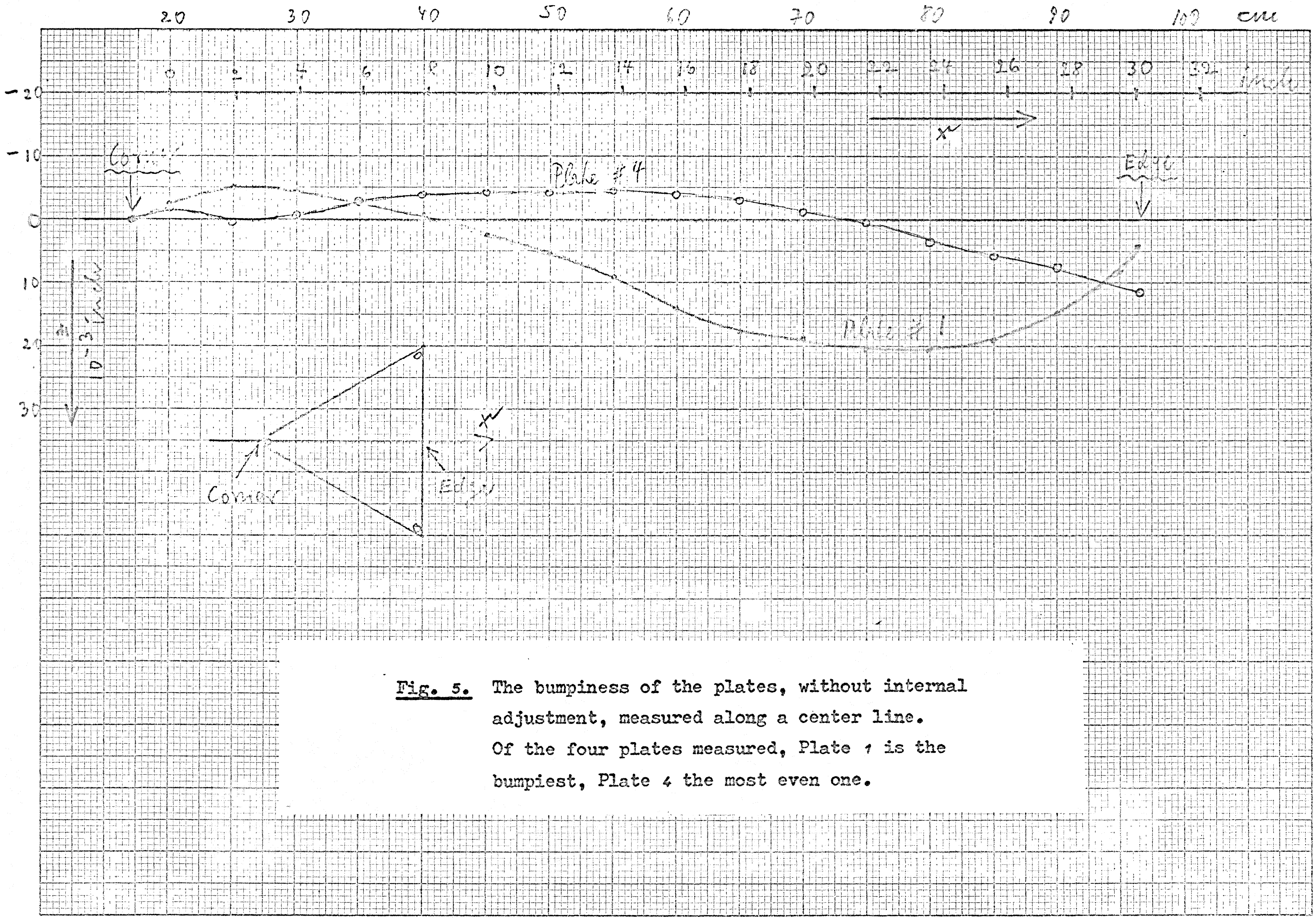


Fig. 5. The bumpiness of the plates, without internal adjustment, measured along a center line. Of the four plates measured, Plate 1 is the bumpiest, Plate 4 the most even one.

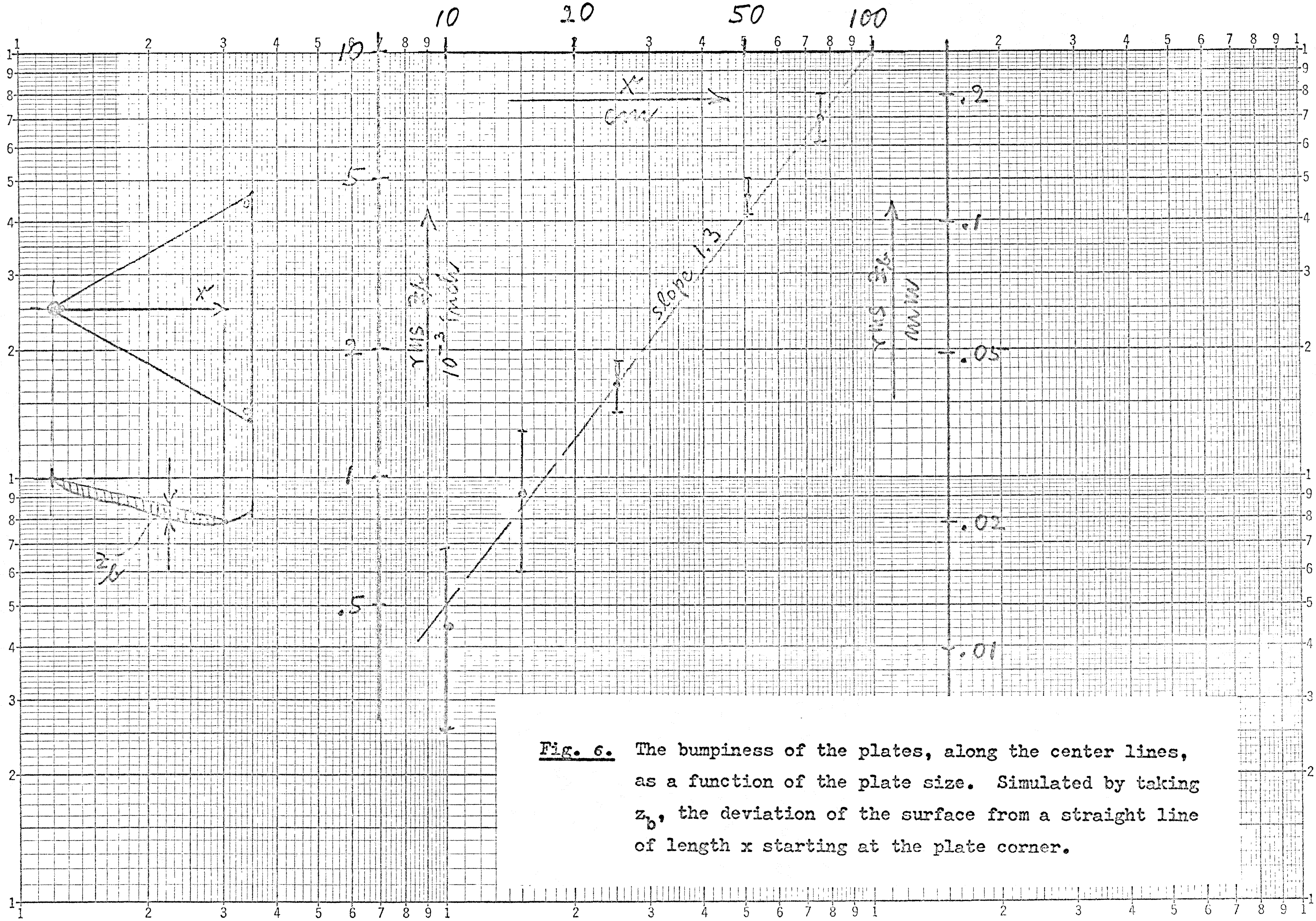


Fig. 6. The bumpiness of the plates, along the center lines, as a function of the plate size. Simulated by taking z_b, the deviation of the surface from a straight line of length x starting at the plate corner.

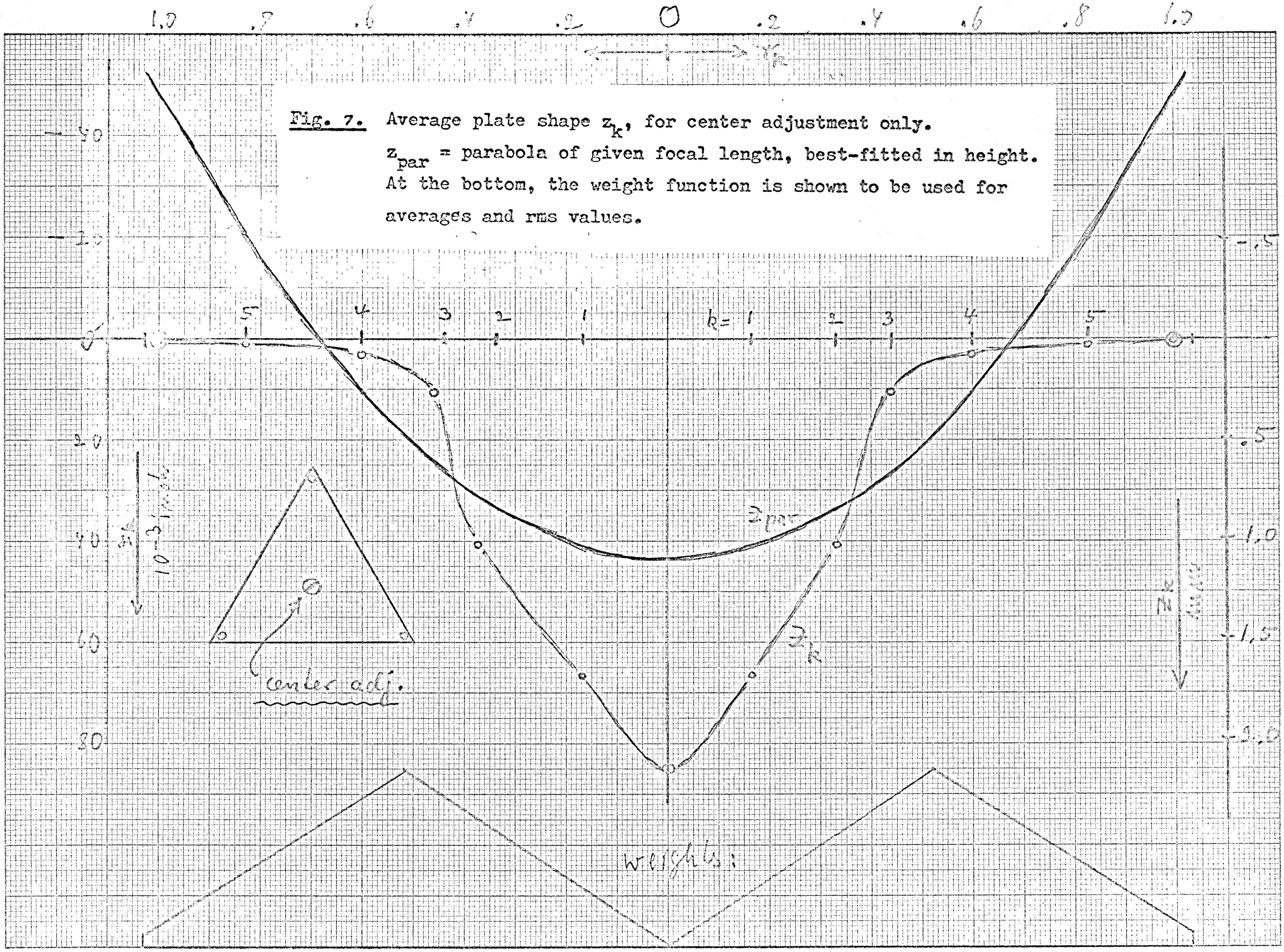


Fig. 7. Average plate shape z_k, for center adjustment only.
z_{par} = parabola of given focal length, best-fitted in height.
At the bottom, the weight function is shown to be used for averages and rms values.

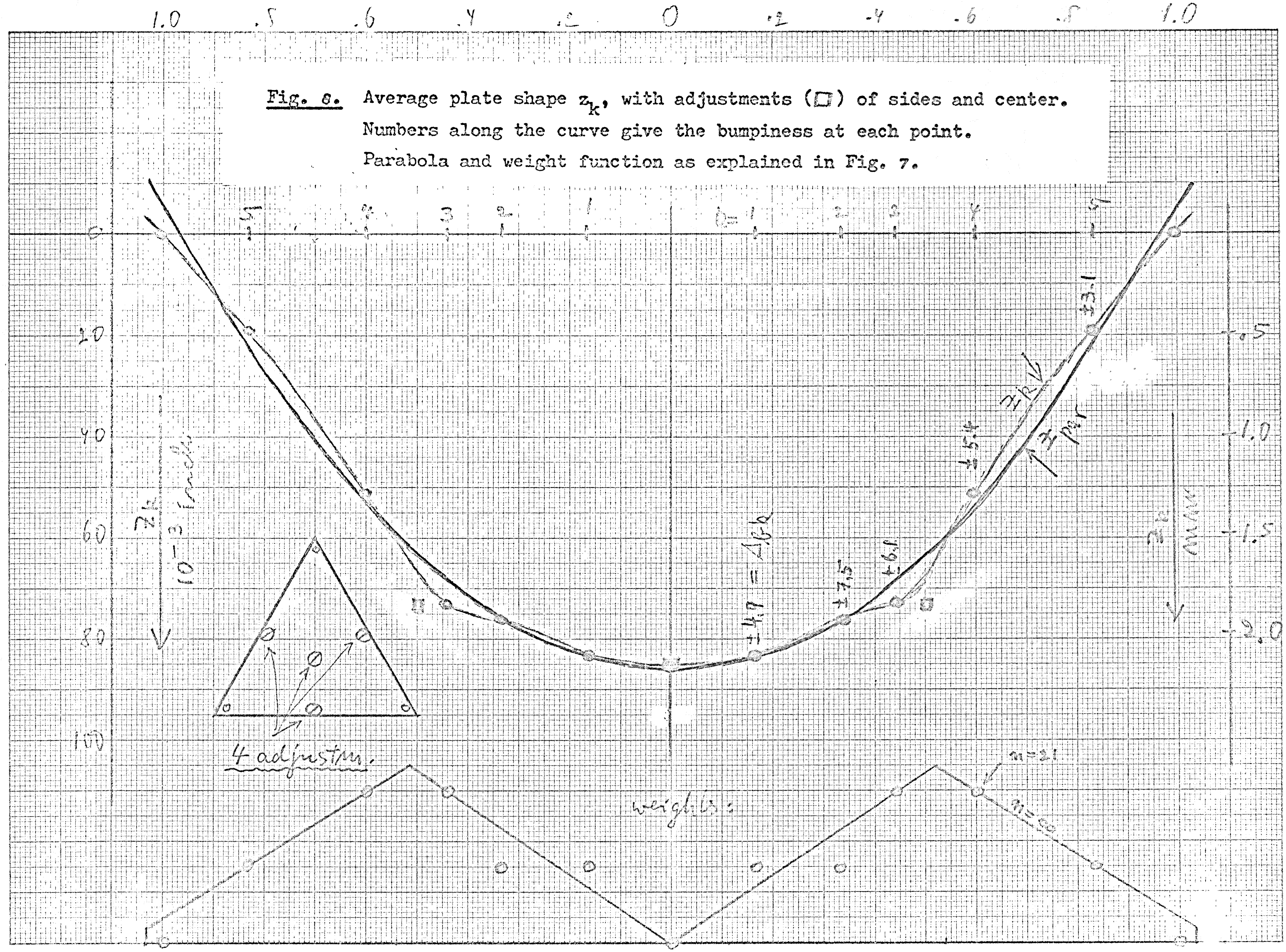
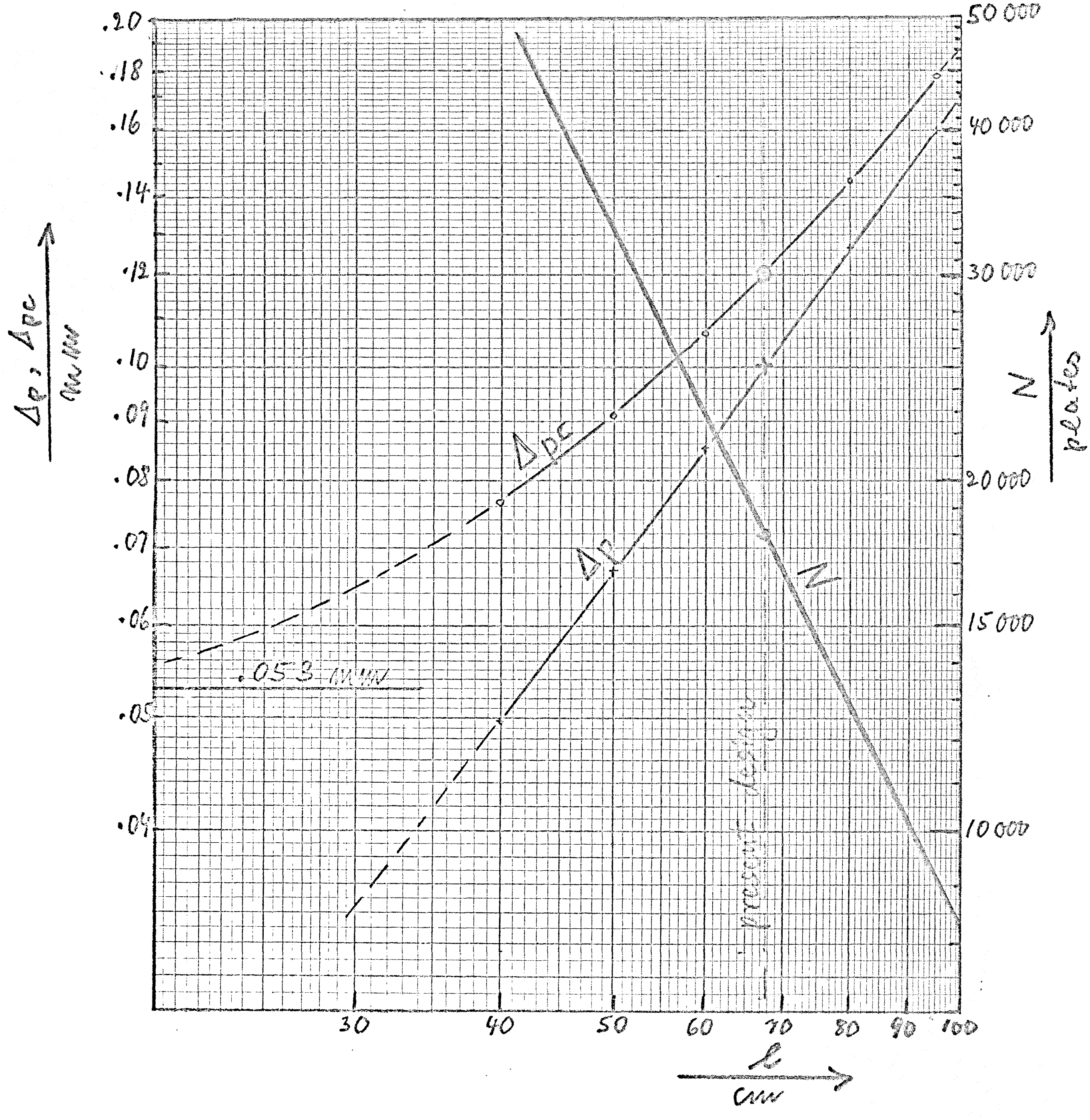


Fig. 8. Average plate shape z_k, with adjustments (□) of sides and center. Numbers along the curve give the bumpiness at each point. Parabola and weight function as explained in Fig. 7.



<u>Fig. 9.</u> The rms surface deviation of the plates, as a function of plate size ℓ, for D = 65 m telescope diameter.

Δ_p = plate deviation only (if corners are exactly on paraboloid);
Δ_{pc} = combination of plate deviation and corner errors;
N = number of plates needed for D = 65 m.

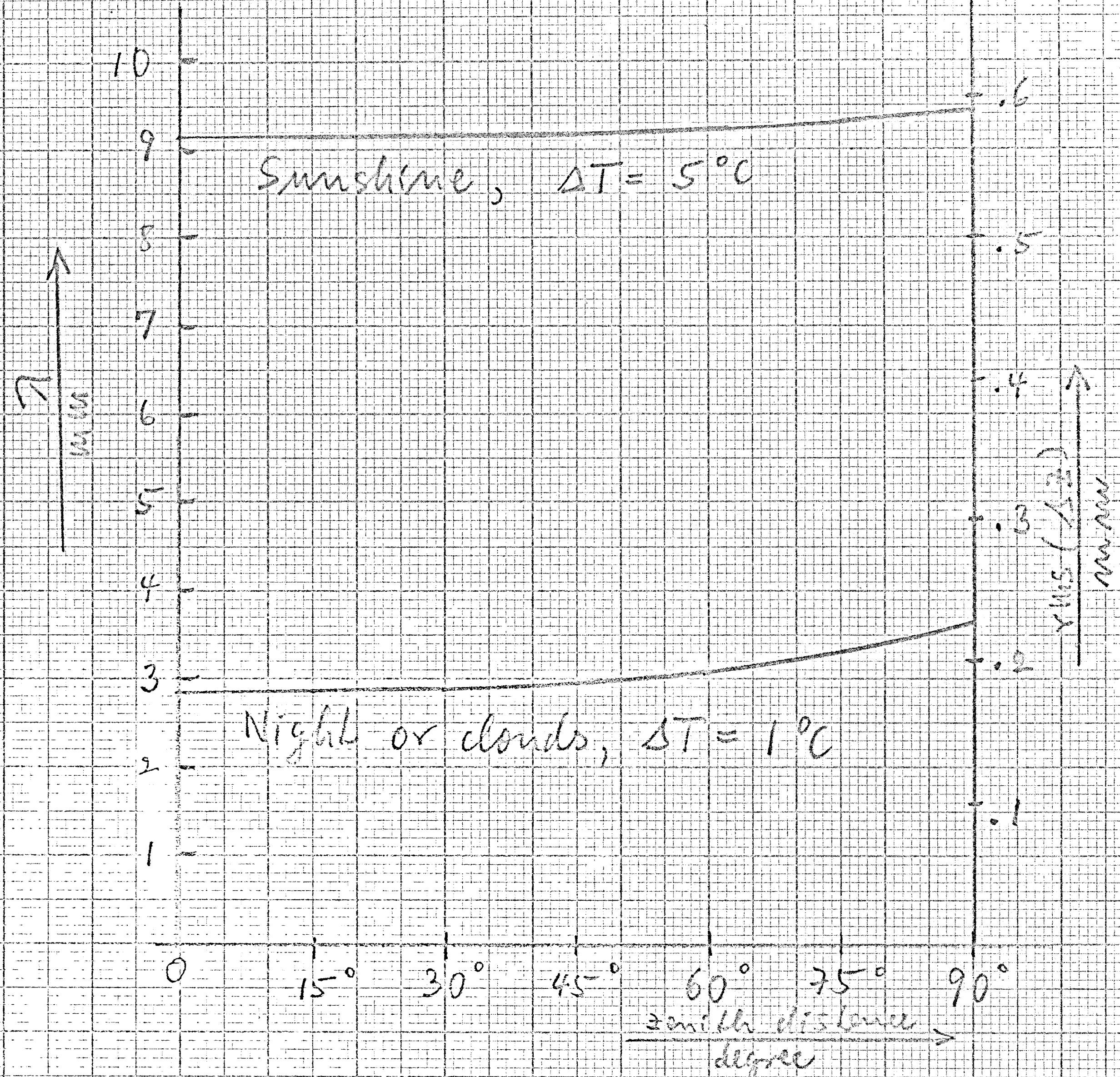
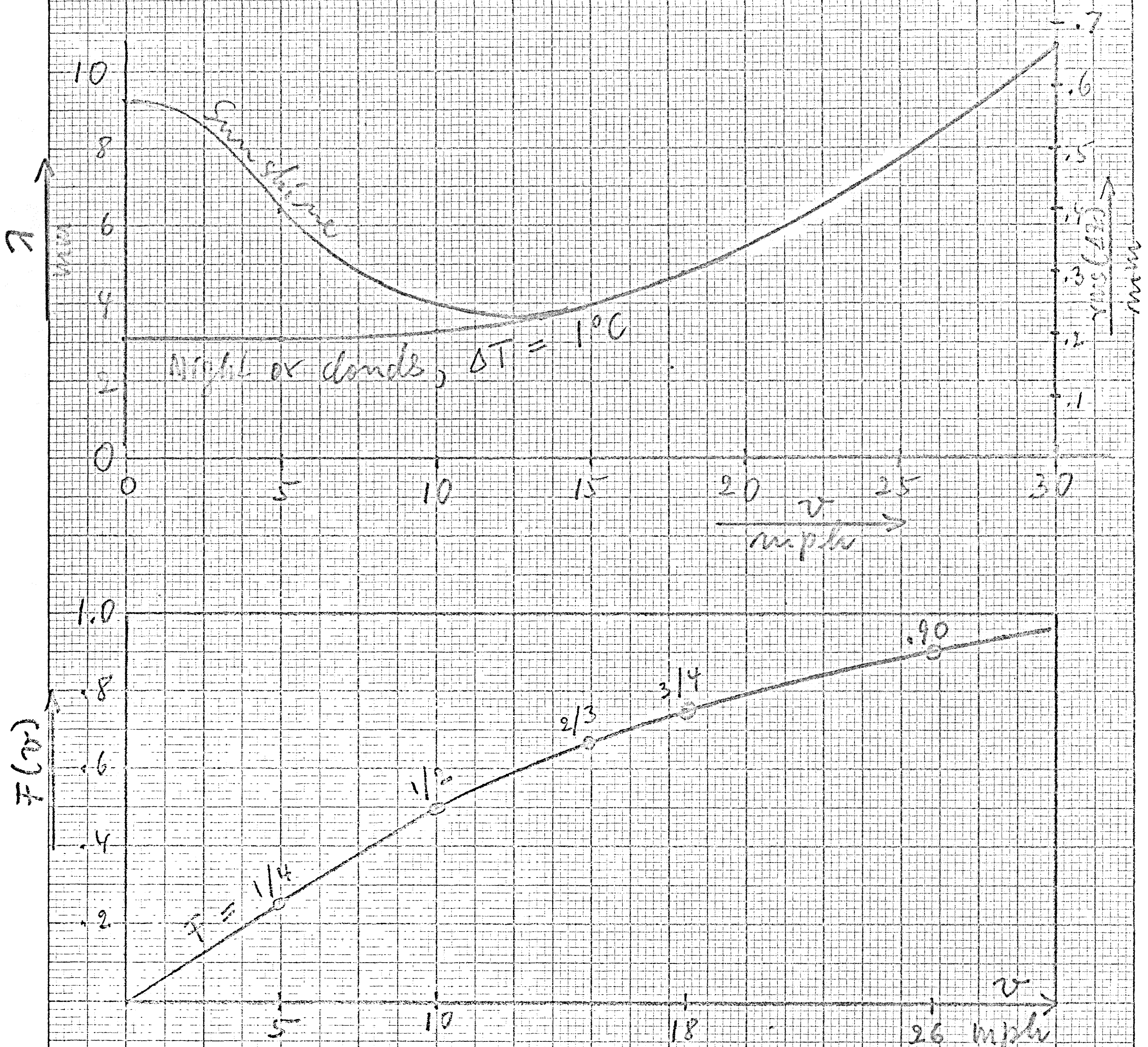


Fig. 10. The shortest wavelength $\lambda = 16$ rms(Δz) of the 65 m telescope, as a function of the zenith distance; for low winds ($<$ 10 mph).



<u>Fig. 11.</u> The shortest wavelength $\lambda = 16$ rms(Δz) of the 65 m telescope, as a function of the wind velocity; for a zenith distance of 60°.

F(v) = cumulative velocity distribution at Green Bank.

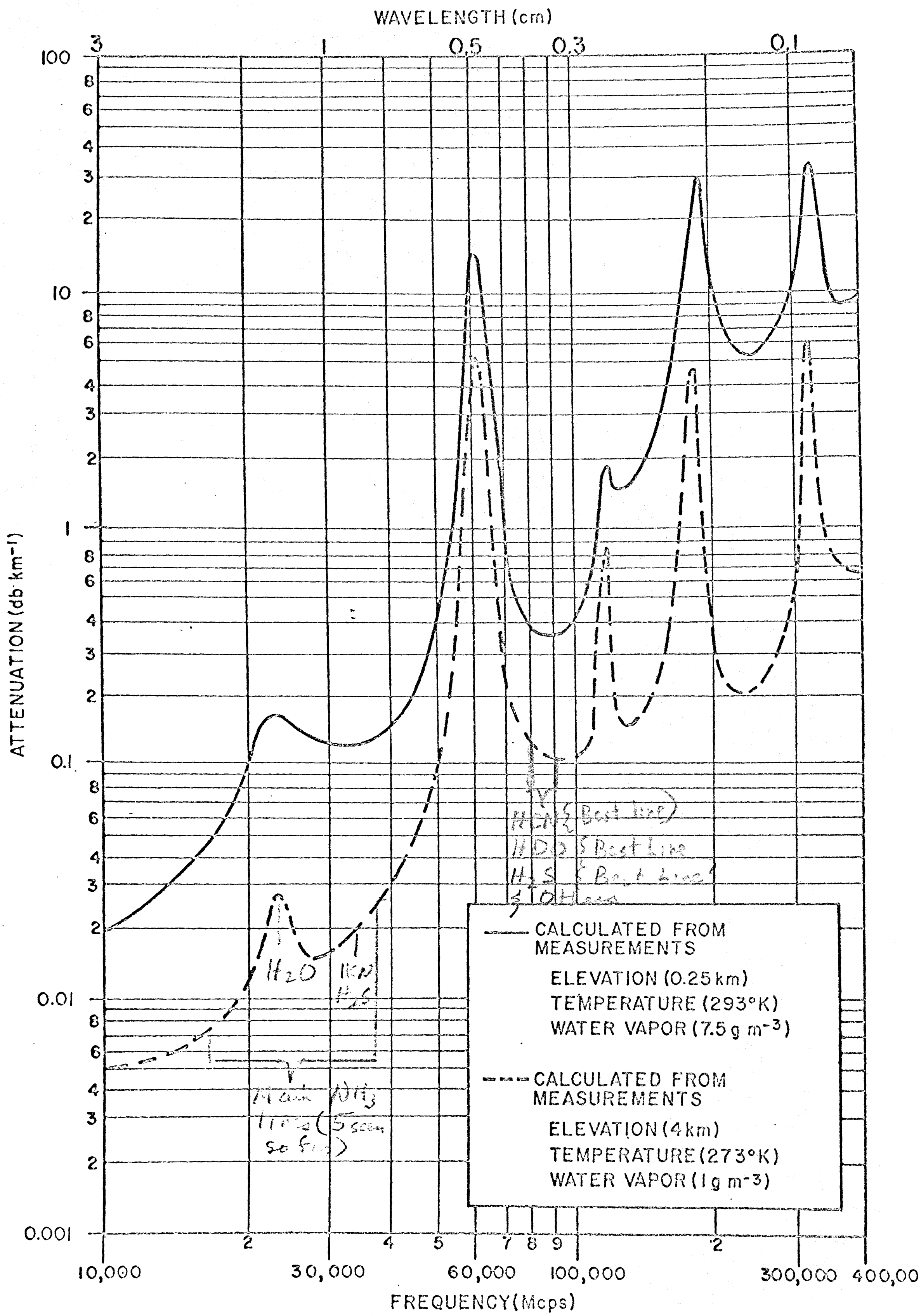


Fig. 22-1. Attenuation, calculated from measurements, by combined water vapor and oxygen. (After C. W. Tolbert, A. W. Straiton, J. H. Douglas, *Electrical Engineering Lab. Rept. No. 104,* University of Texas, 1958.)

Fig. 12. Attenuation from the atmosphere.