\mathcal{AIPS} Memo 108

Weights for VLA Data

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Abstract

A method for calculating the properly calibrated weights for VLA data in AIPS (or AIPS++, or any other package) is presented, along with some related information on the "nominal sensitivity" quantity stored in the VLA archive data. A method of determining the quantity T_{sys}/η_a for each antenna using the properly calibrated weights is also presented.

1 Introduction

In AIPS Memo 103 (Desai 2000), a nice scheme for calculating weights for VLA data in AIPS is outlined. This scheme allows for proper relative weighting of data based on the different surface and receiver characteristics for each antenna, and has been an important part of VLA data reduction since implemented into FILLM by Eric Greisen. Unfortunately, there is a scaling error in that memo, so the recommended weights aren't truly calibrated. In addition, the implementation in FILLM is not strictly as recommended in AIPS Memo 103, resulting in a different scaling factor, which is also in error. Since the error is only a scaling factor in the weights, as long as only VLA data which has all gone through this weighting scheme is used (including combining together different data sets), the error should not have any affect on the end-result. The exception is that continuum data from before and after the change to full complex correlation should not be put together after using the current scheme (see discussion below). In addition, it should not be assumed that these weights are calibrated correctly because of this error, i.e., one should not expect to be able to examine the weights at the end of the calibration process to deduce information about true visibility variance or rms, or antenna and receiver system characteristics (e.g., the antenna G/T).

This memo outlines the proper way to do weighting of VLA data, which should result in true calibrated weights. It is not recommended that a change be made to the way that FILLM calculates weights by default, since in most cases this scaling factor is transparent (the ones that come to mind where it is important are when combining VLA data with data from another telescope, or, again, when combining VLA continuum data from before and after the change to full complex correlation), but rather use this note as a guide to how the weighting should be done if true weights are desired. It is recommended, however, that a user selectable option in FILLM to get this behavior

be added (e.g., have the default DOWEIGHT=1 imply that the current scheme is used, but allow for DOWEIGHT=2 to specify that the scheme described herein be used, or something similar).

2 Deriving the AIPS weight

AIPS defines the "weight" on the visibility for the baseline between antennas i and j, w_{ij} as the inverse variance (i.e., in the standard way):

$$w_{ij} = \frac{1}{\sigma_{ij}^2} \quad , \tag{1}$$

where σ_{ij} is the standard deviation. The standard deviation can be written, in units of Wm⁻²Hz⁻¹:

$$\sigma_{ij} = \frac{\sqrt{2} k \sqrt{T_{sys_i} T_{sys_j}}}{\eta_c \sqrt{\eta_{a_i} \eta_{a_j}} A \sqrt{\Delta \nu \Delta t}} \quad , \tag{2}$$

where k is Boltzmann's constant, A is the physical antenna area, T_{sys_i} and η_{a_i} are the system temperature and aperture efficiency for antenna i, η_c is the correlator efficiency, $\Delta \nu$ is the bandwidth, and Δt is the integration time. I've ignored other system loss terms, assuming they are small. Substituting equation 2 into equation 1 yields:

$$w_{ij} = \frac{\eta_c^2 A^2}{2 k^2} \Delta \nu \Delta t \; \frac{\eta_{a_i}}{T_{sys_i}} \frac{\eta_{a_j}}{T_{sys_j}} \; \times \; 10^{-52} \quad , \tag{3}$$

where the factor of 10^{-52} converts the weight into units of inverse Janskys squared (Jy⁻²).

To calculate the weight, therefore, it is necessary to know the quantity η_a/T_{sys} for the two antennas forming the baseline. That quantity can be determined from the so-called "nominal sensitivity", S_i which is written on the archive tape for each antenna at each integration. That quantity is defined as (Butler 1998):

$$S_i = \frac{3}{V_{sd_i}} \left(\frac{1}{\kappa} \frac{T'_{cal_i} g_i}{\eta'_{a_i}} \right) \quad , \tag{4}$$

where V_{sd_i} is the sync-detector voltage, T'_{cal_i} and η'_{a_i} are the assumed values for the noise tube temperature and aperture efficiency, g_i is the peculiar gain, and κ is a value which combines the area of the dish, Boltzmann's constant, the front end gain, and other constants. In the current online system, $\kappa = 21.59$, but prior to May 1, 1990, the on-line system used $\kappa = 24.32$ (see Appendix A for comments on this). Assuming that the total power voltage is constant at 3 V, then (Butler 1998):

$$T_{sys_i} = \frac{45 T_{cal_i}}{V_{sd_i}} \quad , \tag{5}$$

where T_{cal_i} is the *true* noise tube temperature (as opposed to that assumed in the on-line system). Substituting equation 5 into equation 4 yields:

$$S_i = \frac{T_{sys_i}}{15 T_{cal_i}} \left(\frac{1}{\kappa} \frac{T'_{cal_i} g_i}{\eta'_{a_i}} \right) \quad . \tag{6}$$

The VLA on-line system calculates visibilities in dekaJanskys as:

$$\hat{V}_{ij} = 256 \ \sqrt{S_i S_j} \ \hat{r}_{ij} \quad , \tag{7}$$

where \hat{r}_{ij} is the normalized correlation coefficient. The relationship between ρ_{ij} and \hat{r}_{ij} is (Butler 1998):

$$\rho_{ij} = 1.236 \, \hat{r}_{ij} \quad , \tag{8}$$

 $\mathbf{so},$

$$\hat{V}_{ij} = \frac{256}{1.236} \rho_{ij} \sqrt{S_i S_j} \quad . \tag{9}$$

Substituting equation 6 into equation 9 yields:

$$\hat{V}_{ij} = \frac{13.81}{\kappa} \rho_{ij} \sqrt{\frac{T_{sys_i} T'_{cal_i} g_i}{T_{cal_i} \eta'_{a_i}}} \frac{T_{sys_j} T'_{cal_j} g_j}{T_{cal_j} \eta'_{a_j}} \quad .$$
(10)

During calibration, complex antenna gain factors are determined which multiply the visibilities to put them on a properly calibrated flux density scale (in Jy). If we refer to the amplitude of this complex calibration gain for antenna i as G_i , then the calibrated visibilities are:

$$V'_{ij} = \frac{13.81}{\kappa} \rho_{ij} \ G_i \ G_j \ \sqrt{\frac{T_{sys_i} T'_{cal_i} g_i}{T_{cal_i} \eta'_{a_i}}} \ \frac{T_{sys_j} T'_{cal_j} g_j}{T_{cal_j} \eta'_{a_j}} \quad .$$
(11)

From theory, the conversion from correlation coefficient ρ_{ij} to true visibility amplitude V_{ij} in Jy is:

$$V_{ij} = \frac{2k}{A} \sqrt{\frac{T_{sys_i} T_{sys_j}}{\eta_{a_i} \eta_{a_j}}} \rho_{ij} \times 10^{26} \quad .$$
(12)

For the VLA, $A = 491 \text{ m}^2$, so

$$V_{ij} = 5.625 \ \rho_{ij} \ \sqrt{\frac{T_{sys_i} T_{sys_j}}{\eta_{a_i} \eta_{a_j}}} \quad .$$
(13)

Set equation 11 and equation 13 equal, call $\kappa' = 10 \times 256/(1.236 \times 15 \times 5.625) = 24.55$, and solve for the true aperture efficiency:

$$\eta_{a_i} = \frac{\kappa}{\kappa'} \frac{T_{cal_i} \ \eta'_{a_i} \ 10}{T'_{cal_i} \ g_i \ G_i^2} \quad .$$
(14)

From equation 6, we know that

$$T_{sys_{i}} = \frac{15 \ T_{cal_{i}} \ S_{i} \ \kappa \ \eta'_{a_{i}}}{T'_{cal_{i}} \ g_{i}} \quad .$$
(15)

Combining equation 14 and equation 15 yields:

$$\frac{\eta_{a_i}}{T_{sys_i}} = \frac{10}{15 \kappa' S_i G_i^2} \quad . \tag{16}$$

As an aside, note that Rick Perley's "K-term" for calculating sensitivity on the VLA (see Taylor et al. 2002, section 3.2 and equations 1-3) can be calculated for each antenna via:

$$K_i \sim 0.1186 \; \frac{T_{sys_i}}{\eta_{a_i}} \sim 0.178 \; \kappa' \; S_i \; G_i^2 \quad .$$
 (17)

Substituting equation 16 into equation 1 yields:

$$w_{ij} = \frac{\eta_c^2 A^2}{2 k^2} \Delta \nu \Delta t \; \frac{10^2}{15^2 \kappa'^2 S_i S_j G_i^2 G_j^2} \times \; 10^{-52} \quad , \tag{18}$$

Desai (2000) claimed that there was no need to worry about the correlator efficiency, since it was accounted for in the nominal sensitivity. This is not true, it is necessary to account for it, as has been shown above. This is because correlator efficiency as defined here does not affect the amplitude scale (the scaling from correlation coefficient to Janskys), but does result in a decrease in SNR, or an effective increase in the noise (or decrease in the weight). So, it *is* necessary to know what the correlator efficiency is for the VLA, and to include it when calculating the weight. Unfortunately, it is not a single number. There are separate values when using the correlator for spectral line and continuum, and the continuum case is further complicated by the fact that the correlator was modified several years ago for full complex correlation, changing the efficiency. Before the full complex correlation improvement, the value when using the correlator in spectral line mode was $\eta_c \sim 0.77$, while that for continuum mode was $\eta_c \sim 0.79$ (Crane & Napier 1994). After the improvement, the continuum mode value increased to $\eta_c \sim 0.87$ (Bagri 1997; Bagri 1998). Ignoring the difference between 0.77 and 0.79 (use $\eta_c = 0.78$ for both spectral line and continuum modes before the full complex correlation improvement), using A = 491 m², and putting in the other constant numerical terms yields:

$$w_{ij} = \begin{cases} 2.84 \times 10^{-5} \ \frac{\Delta\nu \ \Delta t}{S_i \ S_j} \ \frac{1}{G_i^2 \ G_j^2} & \text{Case 1,} \\ \\ 3.53 \times 10^{-5} \ \frac{\Delta\nu \ \Delta t}{S_i \ S_j} \ \frac{1}{G_i^2 \ G_j^2} & \text{Case 2,} \end{cases}$$
(19)

where Case 1 is spectral line data taken at any time, or continuum data taken before July 30, 1998, and Case 2 is continuum data taken after July 30, 1998 (that is the date when, as accurately as can be reconstructed by Ken Sowinski, the change to full complex correlation was made in the on-line system).

Does this make sense? Invert equation 19 for the standard deviation, and use $S_j \equiv S_i$ and $G_j \equiv G_i$:

$$\sigma_{ij} = \frac{1}{\sqrt{w_{ij}}} \sim \frac{S_i \ G_i^2}{\sqrt{3 \times 10^{-5} \ \Delta\nu \ \Delta t}} \quad .$$

Experience with the VLA at X- and C-bands is that the nominal sensitivity is of the order of 0.2, and the squared gain factors are roughly 10. Plug in numbers for continuum ($\Delta \nu \sim 45$ MHz) and 10 second integrations, and this gives $\sigma_{ij} \sim 18$ mJy. This is a perfectly reasonable value for the rms per visibility.

So, the weight that should be attached to each visibility before calibration (at the FILLM stage) is:

$$\hat{w}_{ij} = \begin{cases} 2.84 \times 10^{-5} \ \frac{\Delta\nu \ \Delta t}{S_i \ S_j} & \text{Case 1,} \\ 3.53 \times 10^{-5} \ \frac{\Delta\nu \ \Delta t}{S_i \ S_j} & \text{Case 2.} \end{cases}$$
(21)

After calibration, this should be adjusted by the gain amplitudes:

$$w'_{ij} = \hat{w}_{ij} \ \frac{1}{G_i^2 \ G_j^2} \quad . \tag{22}$$

3 Comparison with AIPS Memo 103

AIPS Memo 103 recommended the following weight calculation, using the notation used herein:

$$\hat{w}_{ij_{103}} = \frac{\Delta\nu\,\Delta t}{S_i\,S_j} \quad . \tag{23}$$

Again, this has the right functional form, but is missing the scaling factor.

4 Comparison with current FILLM implementation

FILLM calculates the weights in subroutine MCWAIT. This subroutine is passed weights which are in 10's of seconds (i.e., a 10 second integration has an associated weight of 1.0), and modifies them. In detail, the bit of code that does this (taking out loops, special cases, and condensing the code) is currently (in all 3 of OLD, NEW, and TST [31DEC00, 31DEC01, and 31DEC02]):

 $\begin{array}{l} \text{XBW} = \text{SQRT} \ (0.12 \ \ \text{RBW}) \ / \ \text{SQRT} \ (1000.) \\ \text{CORFAC}(\text{IS}) = \ \text{XBW} \ / \ \text{MCANNS}(\text{IS},\text{IA1}) \\ \text{CORFAC}(\text{IS}+4) = \ \text{XBW} \ / \ \text{MCANNS}(\text{IS},\text{IA2}) \\ \text{CFACT} = \ \text{CORFAC}(\text{IP1}) \ \ \ \text{CORFAC}(\text{IP2}) \\ \text{VIS}(\text{INDEX}+2) = \ \text{VIS}(\text{INDEX}+2) \ \ \ \text{CFACT} \end{array}$

where RBW is the bandwidth, and MCANNS(IS, IAi) is the nominal sensitivity for antenna i and polarization IS as contained in the archive. Writing this in the notation used herein:

$$\hat{w}_{ij_{\text{FILLM}}} = 1.20 \times 10^{-5} \ \frac{\Delta \nu \,\Delta t}{S_i \, S_j} \quad .$$
 (24)

Again, this has the right functional form, and at least it *has* a scaling factor, but that factor is not right. It is a factor of 2.4 or 2.9 too low when compared to the correct value. The scaling factor in FILLM was determined by simply adjusting the numerical factors until the observed and expected weights agreed crudely ("chi-by-eye", if you will) for a particular L-band data set being reduced at the time that the weighting scheme was being implemented (as explained by Eric Greisen).

Note that AIPS++ currently calculates the visibility weights in exactly the same way as the current AIPS FILLM does (with the same scaling factor), and hence suffers from the same problem (see: http://aips2.nrao.edu/released/docs/user/NRAO/node74.html).

5 Other issues

The on-line system only calculates the S_i at every 10 second tick, so fluctuations in system temperature on shorter timescales are not reflected in the S_i . In fact, the system temperature is smoothed to roughly 6 seconds (the sync-detector voltage values are smoothed to that timescale - see description in Butler 1998) anyway for normal integration times (all > 1.667 sec), so there shouldn't be substantial variations on timescales less than 10 seconds. But users should be aware of the possibility.

For sources which need the full van Vleck quantization correction (those which are very strong), there will be an error in the weights, since the equivalent of the nominal sensitivity will have a different value in that case, which is not accounted for above.

It is unclear if this affects the solar-mode observing, and how those data are handled in FILLM. It probably makes no difference, but that has not been checked.

6 Why bother?

If the scaling error in the current FILLM is not important for most cases, then why bother with the correct scaling? The reasons are twofold. First, if it is desired to combine data from the VLA taken before and after the change to full complex correlation, then if the current scheme is used, the weights will not be right between the datasets. The scheme proposed in this memo provides a solution to this problem. A similar argument may be made for the case when VLA data is combined with data from another telescope. Secondly, if the weights were really properly calibrated, then they could be used to deduce information about the system which is hard to determine by other means. If the weights are really properly calibrated, then it should be possible to calculate the value of T_{sys}/η_a for each antenna (see Appendix B). One other possible use is if VLA data is combined with data from other telescopes which have realistic weights attached to them (on a calibrated flux density scale), then there will be no required mucking about with reweighting the data.

7 Conclusion

A method has been presented that calculates properly calibrated weights for VLA data. The proper calibration is obtained by assuring that the initial raw visibilities and the weights assigned to them are on the same (uncalibrated) flux density scale. After proper calibration, assuming that any calibrations that are applied to the raw visibilities are also applied to the weights, the weights will be properly calibrated, i.e., in units of Jy^{-2} . The current weights assigned in AIPS via FILLM are nearly right, but off by a scaling factor. It is not recommended that the current calculation of the weights in FILLM be replaced by the one presented here, since for most cases this scaling error is unimportant. However, it *is* recommended that an option be added to do the proper scaling in FILLM (e.g., have the default DOWEIGHT=1 imply that the current scheme is used, but allow for DOWEIGHT=2 to specify that the scheme presented herein be used). This could also be obtained by using the task WTMOD, but that seems less attractive. Having these properly calibrated weights would allow for straightforward combination of all VLA data, as well as examination of the weights to determine actual system parameters.

Appendix A. Derivation of κ

This appendix describes the calculation of the correct value for the quantity κ used in the on-line system "nominal sensitivity".

Assume that the peculiar gain is adjusted by monitoring so that:

$$\frac{T_{cal_i}' g_i}{\eta_{a_i}'} = \frac{T_{cal_i}}{\eta_{a_i}} \quad .$$
(25)

This peculiar gain adjustment is done at all VLA bands except Q-band via the MODCAL procedure. Now substitute this into equation 10:

$$\hat{V}_{ij} = \frac{256}{15 \times 1.236 \times \kappa} \rho_{ij} \sqrt{\frac{T_{sys_i}}{\eta_{a_i}}} \frac{T_{sys_j}}{\eta_{a_j}} \quad .$$
(26)

Set this equal to equation 12, (but note that it needs to be in DJy, so the scaling factor is 10^{25} instead of 10^{26}) and solve for κ :

$$\kappa = \frac{256}{1.236 \times 15 \times 10^{25}} \frac{A}{2k} \quad . \tag{27}$$

For the VLA, $A = 491 \text{ m}^2$, so

$$\kappa = 24.55 \quad . \tag{28}$$

This is exactly the κ' term above, which is no coincidence.

The value for κ used in the on-line system until May 1, 1990 (this date is a best estimate from Ken Sowinski based on perusal of old change logs and module listings) was $\kappa = 24.32$. This agrees well with the value of 24.55 derived above (to better than 1%). The value was changed in the May 1, 1990 on-line code upgrade to $\kappa = 21.59$. This change was made in the midst of an overhaul of the solar observing code in the on-line system. It is likely that the new value of κ was simply calculated incorrectly (and Ken does not disagree with this assessment). The difference is probably manifested in a bias in the on-line values of the peculiar gain, and has not been noticed before because of the other various scaling factors which can be in error in the on-line system (e.g., the antenna efficiency, which is assumed to be the same for all antennas).

Appendix B. Deriving T_{sys}/η_a from weights

This appendix describes the use of properly calibrated weights to determine the interesting quantity T_{sys}/η_a for each antenna.

Define for each antenna:

$$\alpha_i \equiv \sqrt{\frac{T_{sys_i}}{\eta_{a_i}}} \quad , \tag{29}$$

then

$$\frac{1}{\sqrt{w_{ij}}} = \sigma_{ij} = \beta \ \alpha_i \ \alpha_j \quad , \tag{30}$$

where β combines all the known quantities:

$$\beta = \frac{\sqrt{2}k}{\eta_c A \sqrt{\Delta\nu \,\Delta t}} \quad . \tag{31}$$

Given the values of $\sigma_{ij} = 1/\sqrt{w_{ij}}$ for all of the baselines, it should then be possible to back out the values of α_i and hence the quantity T_{sys_i}/η_{a_i} for all of the antennas.

First, given N antennas, form an upper diagonal $N \times N$ matrix **A** where $A_{ij} = \sigma_{ij}/\beta$ for i < j, and $A_{ij} = 0$ for i = j:

$$\mathbf{A} = \begin{pmatrix} 0 & \alpha_{1}\alpha_{2} & \alpha_{1}\alpha_{3} & \alpha_{1}\alpha_{4} & \alpha_{1}\alpha_{5} & \dots & \alpha_{1}\alpha_{N} \\ 0 & 0 & \alpha_{2}\alpha_{3} & \alpha_{2}\alpha_{4} & \alpha_{2}\alpha_{5} & \dots & \alpha_{2}\alpha_{N} \\ & & \alpha_{3}\alpha_{4} & \alpha_{3}\alpha_{5} & \dots & \alpha_{3}\alpha_{N} \\ & & & \alpha_{4}\alpha_{5} & \dots & \alpha_{4}\alpha_{N} \\ \vdots & & \ddots & & \vdots \\ & & & & \alpha_{N-2}\alpha_{N-1} & \alpha_{N-2}\alpha_{N} \\ & & & & & \alpha_{N-1}\alpha_{N} \\ 0 & & \dots & & 0 \end{pmatrix}$$
(32)

Then, form a chi-squared quantity:

$$\chi^{2} = \sum_{i=1}^{N-1} \sum_{j>i}^{N} \left(A_{i,j} - \alpha'_{i} \alpha'_{j} \right)^{2} \quad , \tag{33}$$

where α'_i is some estimate of α_i (the true value). The derivatives of this chi-squared quantity with respect to the values of the α'_k are:

$$\frac{\partial \chi^2}{\partial \alpha'_k} = -2 \left[\sum_{i=1}^{k-1} \left(A_{i,k} - \alpha'_i \alpha'_k \right) \, \alpha'_i + \sum_{j=k+1}^N \left(A_{k,j} - \alpha'_k \alpha'_j \right) \, \alpha'_j \right] \quad , \tag{34}$$

for each k = 1, 2, ..., N. Setting this equal to 0, to minimize chi-squared, implies:

$$\alpha'_{k} = \frac{\sum_{i=1}^{k-1} A_{i,k} \, \alpha'_{i} + \sum_{j=k+1}^{N} A_{k,j} \, \alpha'_{j}}{\sum_{l \neq k} \left(\alpha'_{l}\right)^{2}} \quad .$$
(35)

A method for finding the best estimate of the true values for the α_k is to come up with some initial estimate, then iterate using the above relation and the given current estimates of the α'_k , i.e.:

estimate the initial α'_k do until some tolerance is reached do for each antenna k use equation 35 to find the new estimate of α'_k od od

A reasonable initial estimate is:

$$\alpha'_{k} = \frac{1}{\sqrt{N-1}} \sqrt{\sum_{i=1}^{k-1} A_{i,k} + \sum_{j=k+1}^{N} A_{k,j}} \quad .$$
(36)

For the tolerance criteria, check both the maximum relative change of any of the α'_k , and the relative change in χ^2 from iteration to iteration. Also, as a practical matter, reverse the order of evaluation of the antennas each time through the loop.

This method works very well on simulated data. It has been implemented in AIPS by modifying FIXWT (into a task called FIXW2 - *not* in standard AIPS), which does the calculation of the σ_{ij} and then the α_k . When tested on real VLA data, however, the best fit solutions leave what seem to be excessively large residuals (the final χ^2 seems too big). It is unclear whether this is related to baseline-based errors or some other effect.

A note:

The chi-squared equation (equation 33) can be re-cast as:

$$\chi^{2} = \operatorname{tr}\left(\left[\mathbf{A} - \mathbf{A}'\right] \left[\mathbf{A} - \mathbf{A}'\right]^{\mathrm{T}}\right) \quad , \qquad (37)$$

where $tr(\mathbf{M})$ is the *trace* of matrix \mathbf{M} (the sum of the diagonal elements), and \mathbf{A}' is given by:

This probably has some snazzy matrix solution (minimizing the χ^2 in equation 37), but it is beyond my skill.

Acknowledgements

Ken Sowinski provided extremely helpful input, checked the calculation in Appendix A, and reminded me about the change in correlator efficiency resulting from the full complex correlation improvement. Eric Greisen provided helpful input on the history of the weight scheme implementation in FILLM.

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