\mathcal{AIPS} Memo 111

ATMCA: Phase Referencing using more than one Calibrator

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ABSTRACT

The VLBI astrometric accuracy and image quality of a target source can be improved if more than one reference calibrator is observed with the target. The improvement is obtained by determining the phase gradient in the sky in the region of the sources, mostly caused by an inaccuracy of the troposphere model. Even if the target is sufficiently strong to use self-calibration methods to determine the image, its precise location can be improved with phase referencing. This memo describes the scheduling strategy for multi-calibrator phase referencing, the reduction of the data, and the use of a new \mathcal{AIPS} task ATMCA, which combines the phase or multi-band delay information from the several calibrators.

1. Introduction

By using only one calibrator for a target source, it is assumed that the phase observed in the direction of the calibrator is equal to that of the target. However, even with a calibrator-target separation of only one degree, persistent (time-scale of one hour or more) phase gradients above each antenna commonly occur, and these produce target phase errors which degrade the position and image quality. By phase referencing with several calibrators, these phase gradient can be estimated, hence the phase in the direction of the target can be more accurately determined (Fomalont, 2004). This procedure not only removes the effect of the unmodeled component of the troposphere used in the correlator, but that from many other effects such as the ionospheric refraction, antenna-location errors, apriori astrometric errors—all effects that produce a phase gradient in the sky. The very short term phase fluctuations of less than about five minutes in time cover a small region of sky and cannot easily be removed using multi-calibrators. But these fluctuations tend to be random and average out with no appreciable effect to the position accuracy and image quality, apart from a slight loss of coherence.

Recently, a new \mathcal{AIPS} task, ATMCA, has been written to combine the data from several calibrators to improve the image quality and position accuracy of a target source. Another method in \mathcal{AIPS} , using the task DELZN for determining the unmodeled component of the troposphere, is described in AIPS memo 110 (Mioduszewski and Kogan 2004). The observing technique adds occasional all-sky observations of good quality calibrators, placed before and/or after the normal phase-referencing observations, to estimate the tropospheric model error that is then removed from observations. This method requires the all-sky measurements of wide spanned bandwidth in order to measure multi-band delays, and the technique assumes that the phase or delay error is dependent only on elevation. The task ATMCA only requires calibrator data in the vicinity of the target source; however, the selection of calibrators and some of the processing is somewhat different than normal phase referencing, and will be discussed in this memo.

This memo will be organized as follows: In §2 the appropriate calibrator properties, their relative location in the sky with respect to the target, and a cycling strategy among the sources, are described. In §3, the initial AIPS reductions will be briefly outlined, since these differ only slightly from normal phase reference observations which are discussed elsewhere (\mathcal{AIPS} Cookbook, Wrobel et al. 2000). In §4, the use of the new \mathcal{AIPS} task ATMCA will be described in some detail.

We will concentrate on the use the visibility phase in the memo. In principle, the multi-band delay can also be used, but this quantity is normally not accurately measured in VLBA observations, particularly those involved with phase referencing.

2. The Choice of Calibrators

If you wish to calibrate an experiment that has already been observed, then you are "stuck" with the calibrators already chosen, although ATMCA may still be useful. However, in designing a new experiment, you can improve the dynamic range and astrometric accuracy with the following guidelines.

Lists of phase referencing calibrators are given in the VLBA Calibrator web site *http://magnolia.nrao.edu/vlba_calib/index.html*. Generally, each target requires its own set of calibrators, unless several target sources are within about five degrees of each other. There are several criteria for determining acceptable calibrators:

•Proximity of the calibrator to the target is the most important property, other things being equal. The length of the calibrator scan must be sufficiently long to obtain reasonable signal to noise. As a general rule for the VLBA, if the correlated flux density of the calibrator is greater than 50 mJy at the longest VLBA baselines, it can be detected in a one-minute integration. Thus, it is better to choose a relatively weak, but close, calibrator than one which is much brighter but significantly farther from the target. This is because once you have more than 5:1 signal to noise ratio (SNR) on a relatively weak calibrator scan, the phase error will be smaller than the angular-dependent phase offsets caused between the target and calibrators.

- •It is marginally better to choose a calibrator without much angular structure and with an accurate position (Ma et al, 1998, Fey et al, 2004). However, as long as the candidate calibrator is detectable at all baselines and has a position error less than 5 mas, it is acceptable and should be considered if it is $< 3^{\circ}$ from the target.
- •If the target is strong and detectable in a single scan, then it can be used as a calibrator. This case is discussed in more detail below.

For many target locations, you will often find several calibrator candidates within 4° of the target, but search out to 7° if necessary. In order to obtain the robust solutions using ATMCA to determine the phase gradients, some calibrator-target configurations are better than others. Six examples of sky distributions of calibrators and a target are given in Fig. 1, and each will be treated somewhat differently using ATMCA.

The configurations that are recommended are: (a) Nearly linear disposition of two calibrators and target, or (c) Three calibrators surrounding the target. Configuration (d) s similar to (a) and (e) is over-kill but is useful if one or more of the calibrator properties is uncertain. Configuration (f) can only be used with ATMCA if the target is sufficient strong. More details concerning each calibrator configuration are given below.

- (a) Two calibrators in line with the target source: In order to determine the phase calibration for the target, only a simple extrapolation of the measured phase difference between calibrator 0 and 1 is needed. The phase gradient perpendicular to the line of sources is irrelevant. It is slightly better if the target is between the two calibrators. A deviation of 45° or less from a straight line (as measured at the middle source) is sufficiently close to linearity to use the straight-line interpolation between calibrators.
- (b) Non-linear distribution of two calibrators with target: The target directions to the two calibrators are nearly perpendicular, and this will be a common occurrence. Obviously, it is impossible to determine the phase contribution at the target using Calibrators 0 and 1. However, if we assume that the phase gradient above each antenna is in the elevation direction (likely if the tropospheric and ionospheric model errors are the most dominant), we can estimate the phase contribution at T from phase observations of 0 and 1, assuming the phase gradient is in the elevation direction as shown. If the separation of T from 0 and 1 is more than 3°, it is worth while adding another calibrator, even if 5° to 10° away from the target.
- (c) Three well-placed calibrators: The phase difference between calibrators 0 and 1 and the phase difference between 0 and 2 are sufficient to determine the two-dimensional phase gradient, from which an estimate of the phase at T can be obtained. This configuration will give a robust solution.



Fig. 1.— Calibrator-Target Configurations: Six different target and calibrator sky configurations are shown by each panel. The symbol designations are: $\mathbf{T} = \text{target}; \mathbf{0} = \text{main calibrator}; \mathbf{1,2,3} = \text{secondary calibrators}$. A tick-mark size of one degree roughly corresponds to the present calibrator density.

- (d) Three poorly-placed calibrators: Calibrators 1 and 2 are too close to give a good twodimensional sampling of the phase gradient. But, the 0-2 or the 0-1 line segments are sufficiently close to linear to revert to case a, used twice with the two phase slopes averaged.
- (e) Lots of Calibrators: Many calibrators and all of them can be used.
- (f) Only one calibrator: This situation corresponds to normal phase referencing. If, and only if, the target source is sufficiently strong to be detected in a few minutes of integration, it is possible to determine the phase slope in the elevation direction. In this case the time-scale of the elevation phase slope determination must be relatively long, since a short time-scale solution will remove all phase residuals of the target. For this scheme to work, the elevation phase slope among the antennas must not be strongly correlated. A similar method has been used by Brunthaler, Reid & Falcke (2003).

3. The Observational Scheme

The calibrator and subsequent reduction schemes depend on the strength of the target. If the target is weak, then all of the calibrators are used to determine the phase correction at the target source. On the other hand, if the target is strong (sometimes it is stronger than the other calibrators), then it should be used as the 'main calibrator', with all of the other calibrators as secondaries.

3.1. Weak Target

At first glance it seems that all of the observing time will be spent on calibrators, rather than the target which needs long integration time for good images and accurate positions. But, in most programs more than 50% of the time can be allocated to the target even with the use of several calibrators. This is because most of the observation time will be spent with simple phase referencing between **0** and **T**, with the suggesting switching cycle, described by Ulvestad (1999) and Wrobel etal. (2000).

Additional observations of calibrators 1, 2, 3 are needed at less often than the basic 0-T switching times since only the persistent phase errors (say, time scales of 30 minutes or longer) in the vicinity of the target are measured. However, because of short-term phase scatter, observation of a secondary calibrator every 15 minutes is recommended.

Suggested observing schedules for a target which is too weak to be detected are:

where one out of every four \mathbf{T} observations is replaced by the calibrator **1**. For several additional calibrators, consider

or

$$0-T-0-T-0-T-0-1-2-0-T-0-T-0-T-0-1-2-0-T-0-T \dots$$

$$0 - T - 0 - T - 0 - T - 0 - 1 - 2 - 0 - 3 - 4 - 0 - T - 0 - T - 0 - T - 0 - 1 - 2 - 0 - 3 - 4 - 0 - T - 0 - T \dots$$

If the target source is sufficiently strong to be detected—for observations where the accurate position of the target source is desired—, an observing scheme is

$$T-0-T-1-T-2-T-3-T-0-T-1-T-2-T-3-T-0\ldots$$

In this case the target is used as the main calibrator for connecting the phase, with residual phases obtained for all of the other calibrators.

There are two considerations in determining the length of each scan. First, the separation of consecutive scans of the main calibrator (usually $\mathbf{0}$, but \mathbf{T} in the last scheme above) must be less than the coherence time. This depends on frequency and observing conditions and is discussed in

Wrobel et al. For frequencies between 1.4 and 8 GHz, a maximum separation time of 5 min is recommended, especially for the longer spacings. At higher and lower frequencies, a shorter time span may be needed. Second, each scan must be sufficiently long in order to obtain at least 5:1 snr for each calibrator, and lengths are short as 20 sec are okay. Use the SCHED parameter DWELL, rather then DUR, to specify the precise integration time on a scan. For most observations, the target can be observed nearly 50% of the time, even when three calibrators are used.

Try to avoid low elevation observations as much as possible. Generally, observations below 20° elevation begin to show departures from a phase-gradient in the region of the sources, and observations below 15° elevation should be routinely excluded. Thus, MK and SC (BR and HN for southern sources) are not very useful for astrometric work during the first and last hour of long tracks.

Finally, whatever experimental frequency set-up is needed for the target scientific goals, it should be used for all sources. If the target is a narrow-lined maser source, then relatively strong calibrators may have to be chosen to be detectable in the limited bandwidth used for the maser IF set-up. It is also possible to choose a maser source as one of the calibrators with the same caveat about weak calibrators. Of course, an accurate bandpass across the frequency bandpass must be obtained, but this is a routine calibration. If there is extra time before and after the main observation period, all-sky observations of calibrators using a wide-spanned bandwidth can be added and analyzed using the \mathcal{AIPS} task DELZN, before using ATMCA.

3.2. Strong Target

If the target source is relatively strong, then it can be used as the main calibrator. In this case, a suggested observing scheme is

$$0 - T - 1 - T - 2 - T - 3 - T - 0 - T - 1 - T - 2 - T - 3 - T - 0 - T \dots$$

so that \mathbf{T} becomes the main calibrator, and 0, 1, 2, 3 are secondary calibrators. More comments on this case as given below.

4. VLBI Calibrations and Reductions

The reduction of the multi-calibrator data set is similar to that for conventional phase referencing, and see the following references for more detailed information (\mathcal{AIPS} Cookbook, Appendix C, Wrobel et. al. 2000)



Fig. 2.— Typical Phase Behavior During Experiment: The temporal phase behavior of the main calibrator observations for five VLBA antennas, with reference antenna at LA, are plotted. Each point comes from a one minute integration and are separated by 3 min. See the discussion in the text.

4.1. Normal Phase-referencing Calibrations

First, apply the usual apriori calibrations (ACCOR; APCAL; PCCOR; TECOR; CLCOR with OPCODE=PANG). Most of these can be conveniently done using the VLBACALA script. Then, run FRING only on the main calibrator ($\mathbf{0}$ or \mathbf{T}), with a solution interval over the scan. The output SN table will provides good indication of the data quality during the experiment. The delays should be stable within about 5 nsec per antenna/IF, the rates should not deviate by more the 3 mHz (except SC and MK at low elevations). Some editing may be needed to remove outlier observations.

A detailed inspection of the temporal phase properties of the solution are important, since further processing will interpret small corrections in the measured phase among several sources. An example of the antenna-based phase (LA was the reference antenna) for the main calibrator after the above processing is shown in Fig. 2. The phases for BR, OV and PT are relatively stable over the day, whereas the MK phases are very noisy (it was raining over most of the observing time). At the end of the time period when the SC elevation drops below 15° , the phase rate becomes large and it is difficult to interpolate the phase after time 1/00.7. Although it is possible to connect the phases, with the use of the rate, such a large temporal phase rate guarantees a large phase gradient in the sky which will be difficult to determine¹. Generally, little can be done to improve periods of poor phase behavior. Other indications of poor data quality is the SNR of a solution and relatively poor delay or rate stability of the solutions. After any appropriate editing and possible re-running of FRING, apply the final SN table to the data using CLCAL with your favorite interpolation option, INTERPOL='CUBE' or 'AMBG' is recommended.

As a precaution, now is a good time to run TASAV in order to save the calibration tables in order to repeat some of the following reductions if necessary. It is also convenient to run SPLAT in order to remove the intermediate SN and CL tables and average overall all channels to decrease the data volume and increase the speed of subsequent processing.

Since most calibrators are not point sources, it is wise to image the main calibrator)or the target source if used as the main calibrator) with this FRING'ed calibrated data. One phase self-calibration loop and then one amplitude calibration loop should be sufficient (usually done aside and not in the main data base). Then, with this source model, run CALIB on the main calibrator or target to determine the final phase and gain calibration of the entire data set. The resultant SN table phases should be near zero since the original FRINGwill have done a good job of calibration. However, there will be small, systematic residual phases associated with the deviation of the calibrator from the point source assumed in FRING. The SN table gain should be near 1.0, no more than 10% deviations in time, between antennas and IF's. Large excursions indicate data that should have been flagged or have poor phase stability within the scan.

4.2. Removing Calibrator Position Offsets

The following steps are needed for the successful interpretation of ATMCA. We want the phase residuals of the secondary calibrators, as calibrated by the main calibrator, to represent the phase gradients in the sky, but NOT THEIR RELATIVE POSITION OFFSET FROM THE MAIN CALIBRATOR. These offsets are associated with the slight errors in apriori positions used in the correlation of the data. Even when using high quality ICRF sources (Ma et. al. 1996; Fey et. al. 2004), the typical positional precision is 0.3 mas and the weaker VLBA calibrators may have position errors somewhat larger than 1 mas. In principle offsets as large as 50 mas are not a problem.

If the target is used as the main calibrator, its position may not be well-known. This position error will produce a large offset position of all of the calibrators. These offsets are, in fact, the estimate of the target position error.

The secondary calibrator offsets can be determined by imaging each with IMAGR with the

¹If the phase rate is constant over the observations, it is then a clock drift and is more safely removed since this is a temporal effect only

calibrated data². Measure the offset position of each secondary calibrator from the phase center; with x positive to the east and y positive to the north, in arcseconds. Then, run CLCOR with SOURCE = '1'; OPCODE = 'ANTC'; GAINVER = (last CL#), GAINUSE = (last CL#+1), CLCORPRM=0,0,0,0,x1,y1 (don't ask). Then run CLCOR again with SOURCE='2', GAINVER=(last CL#+1), GAINUSE=(last CL#+1), CLCORPRM=0,0,0,0,x2,y2; etc. For the hard-core astrometrists, these applications of CLCOR also update the SU table containing the source positions and the appropriate columns of the CL for the correlation model. Hence, the \mathcal{AIPS} astrometric tables are updated as if the correlator had actually processed the data at the revised positions. After making the CLCOR position corrections, re-image the calibrators to make sure that they are indeed at the phase center, generally within 0.1 mas.

Finally, run CALIB on the visibility data on *all* of the calibrators. This step generates the SN table which contains the residual antenna-based phases associated with the calibrators; zero for the main calibrator (of course) and the instrumental phase errors at the location of the secondary calibrator. This is the main input to ATMCA. If a clean model for any of the calibrators is used, then CALIB must be run on each source separately, but with the same output SN table.

4.3. Reduction Summary

You are now ready to use ATMCA. But first, a re-listing of the previous reduction steps.

- Apply the usual apriori gain and phase calibrations and use the VLBA log for appropriate editing.
- Run FRING on the main calibrator 0 or target T and check for reasonable solutions. Interpolate the calibrations in the SN table using CLCAL.
- •(optional) Run SPLAT to obtain a data base with the channels averaged. This will speed up subsequent reductions.
- •Self-calibrate on the main calibrator in the usual manner with the last CALIB producing the final gain and phase calibrations which are interpolated to the entire data set.
- •Image all of the calibrators (if necessary) and the target (if sufficiently strong)..
- •Measure the offsets of the secondary calibrators from the phase center, and use CLCOR to move them to the phase center.

 $^{^{2}}$ The calibrator images may be significantly distorted, especially if they are more distant than 3° from the main calibrator. For observations longer than about three hours, or observations over several sessions, the brightest point of the image is a good estimate of the correct position. With moderate image distortion, the centroid position is a better position estimate. For observations with less than three hours, there is a strong correlation between the source offset and atmospheric errors, so that the location of the image peak can be in error by more than 1 mas. In this case additional observations to tie the secondary calibrator positions to that of the main calibrator must be done with additional observations

•Run CALIB on *all* of the calibrators (with models if you believe them) to produce an SN table with the residual phases for all calibrators.

5. Using the AIPS task ATMCA

The input to ATMCA is, thus, an SN table containing the time-variable residual antenna-based phases, sampled in the direction to the calibrators. The phase zero is defined by the main calibrator or target, if strong. The goal of ATMCA is to determine the phase gradient in the sky which is consistent with the phases sampled by the secondary calibrators, and then to apply this gradient to the phase of all sources.

5.1. Running ATMCA

The most important ATMCA parameters are:

- •SNVER: The SN table version which contains the residual antenna-based phases for all calibrators. Look at the phases with SNPLT to make sure that for the additional calibrators the phases are reasonably continuous with time (the main calibrator should have zero phase). There will be a noise component, mainly produced by short term tropospheric effects, but it should not dominate over the systematic phase drifts with time. If it does, then ATMCA will not produce very useful results. Obvious outliers should be removed. ATMCA will be able to interpret the occasional phase ambiguities of integer multiples of 360°. Determine the appropriate solution interval time which seems realistic from the density of source observations and the random noise component. This period should include at least one and probably two scans of the additional calibrators, and is typically 30 to 60 minutes in length (see SOLINT below).
- •GAINVER: The CL table which was used by CALIB to generate the SNVER table of residual phases. ATMCA copies the GAINVER CL table to a version one larger than the highest CL table version and applies the correction for the phase gradient to it. The production of this new CL table is controlled by APARM(2).
- •APARM(1): This is most important control of ATMCA since it designates the algorithm to be used to correct the phase. The calibrator/target configuration will determine which of the options is most useful, although two options are sometimes possible.
 - -APARM(1) = 1, Linear Phase Interpolation Between Calibrators: The alignment of the calibrators and target are within 45° of linearity, so the phase difference between the calibrators can be interpolated to the target source. Cases (a) and (d) in Fig. 1 are configurations which should use this option.

- -APARM(1) = 2, Phase Slope and Orientation: The alignment of the calibrators and target has two-dimensional coverage so that the phase slope and orientation can be determined; cases (c) and (e) in Fig. 1.
- -APARM(1) = 5, Phase/Elevation: The phase slope is assumed to be in the elevation direction over each antenna. This algorithm must be used in case (b) (angle between calibrators is too large for APARM(1)=1), but can be used for cases (c) and (e) also.
- -Case f: If only one calibrator has been used, and the target source is detectable, then it is possible to determine a long-term phase/elevation dependent using APARM(1)=5. For example, use the calibrator as the phase reference source. Then, image the target source and move its position to the image peak in order to remove the target position-error phase terms. The remaining antenna-based phase errors are associated with the angulardependent phase error between the calibrator and the target. The use of ATMCA with APARM(1)=5 then determines a phase gradient in the elevation direction for each antenna that removes the phase residual for the target. The solution interval must be long so that there is sufficient parallactic angle rotation, so that the elevation difference between the target and calibrator changes significantly. It is recommended to use only one solution interval over the entire observation period.

The use of APARM(4) = 5 is not recommended at the present time.

- •APARM(2): APARM(2) = 1 will create the output CL table after correction of the phases for all sources designated in SOURCE, see below.
- •APARM(3): APARM(3) = 1 will create the output SN table which incorporates the corrections by ATMCA. It can be used with SNPLT to check on the residuals. Do not use this SN for further corrections: it is used for display only. Using the output CL table to re-image the calibrators and target is also useful for checking on the corrections.
- •APARM(4): APARM(4) = 0 for most cases
- •APARM(5): The phases for some calibrators in the SN table may go over the $\pm 180^{\circ}$ cut. It is crucial for ATMCA to recognize lobe ambiguities. Generally, there is a time for which there are no phase ambiguities for all of the sources (often near the center of the time range when the sources are at the highest elevation), and this is the time that should be inserted. APARM(5)=DD.DD where DD.DD is the time in days where the phases for all antennas and sources have no phase ambiguity.
- •APARM(6): It is sometimes useful to see the revised SN table after application of phase ambiguities. For APARM(6)=0, do not make this table. For APARM(6) = 1, two .SN tables are made. The first table contains the original input phases with the lobe ambiguities taken into consideration. These phases can be plotted using SNPLT with OPCODE = 'REAL'. The second SN table is the residual phase after correction by ATMCA. These should be near zero.
- •SOURCES: Which sources to apply the ATMCA solution. This can usually be kept blank which means that the ATMCA corrections are applied to all calibrators and targets.



Fig. 3.— Calibrator and Target Phases Before and After ATMCA: The left-hand plot shows the residual phases for the calibrators and target that was input to ATMCA. The right-hand plot show the ATMCA corrected phases for the calibrators and target. The source configuration was similar to case c (Fig. 1). The red points are for the main calibrator 0; the yellow crosses are for calibrator 1; the light blue crosses as for calibrator 2; the small blue points are for the target T which was strong enough to be detected, but not used in —ATMCA.

- •CALSOUR: The first CALSOUR must be the main phase calibrator. The other calibrators should then be listed. A strong target source which has been used as the main calibrator should be listed first. Do not use CALSOUR = '' since the main calibrator must be the first source in the list.
- •BIF; EIF: Only one IF is needed for ATMCA, but if $BIF \neq EIF$, then the SN table inputs are averaged over IF's before making a solution.
- •SOLINT: The solution time interval in minutes is a relatively important parameter. The minimum solution time interval should contain at least one observation scan of each calibrator. To average out the random phase fluctuations which are associated with short-term tropospheric and ionospheric fluctuations, we recommend a solution time interval which contains at least two observations of each calibrator. However, you can fiddle with this parameter and see what gives the best target image.
- •OPTYPE: This should be kept blank or set to PHAS. ATMCA can be used with the multi-band delay, but this quantity is not normally measured sufficiently accurately in most phase-referencing experiments.



Fig. 4.— Target Source Image Before and After ATMCA: (left) The image of the target source using normal phase referencing. The peak flux density is 25.9 mJy and the maximum side lobe is 1.7 mJy. (right) The image of the target source using a calibrator configuration similar to that in case c (see Fig. 1). The peak flux density is 27.5 mJy and the maximum side lobe level is 0.7 mJy. The contour levels are at 0.6, 1.2, 2.4, 4.8, 9.6 mJy/beam for both images.

5.2. Has ATMCA Worked?

The main way of determining if ATMCA has worked is to compare the antenna-based phases shown in Fig. 3. The plot on the left shown the calibrator antenna-based phases that were input to ATMCA. The plot on the right, obtained by running CALIB on the output CL table and using SNPLT should show that most of the phases are near zero. Look for periods of poor phase stability or problems with lobe ambiguities. Low elevation observations will often show deviations. The SOLINT parameter may need modification.

Next, image all of the calibrators and the target and see if the image quality has improved after using ATMCA. A good example of the target image improvement is shown in Fig. 4, associated with the same experiment associated in Fig. 3. The original persistent phase errors, while only slightly decreasing the peak flux density of the source, scatter emission into relatively large sidelobes near the image peak. After application of ATMCA the image quality has significantly improved, with the peak sidelobe level decreasing to 40% of its original value, and the peak flux density increasing by about 10%. The rms error of the position of the peak has decreased from 0.1 to 0.03 mas (Fomalont & Kopeikin 2003).

5.3. Summary

Phase referencing is now used for about half of the VLBA experiments. Uncertainties in the tropospheric and ionospheric delays over all of the antennas produce phase errors which erodes the quality of the images and the position accuracy of a target source. Until better modeling or measurement of these delay components are available, multi-source phase referencing can remove a major part of this error. The overhead of less than 25% in the observing time to include one

or more additional calibrators near the target can increase in the image quality and astrometric precision by more than a factor of two.

We thank Lorent Sjowerman and Amy Mioduszewski for many comments and improvement to this memo.

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