The relation between optical seeing and radio phase stability

T.J. Cornwell NRAO/VLA

Of particular interest in the design of the millimeter array is the origin and nature of the path length variations introduced by the troposphere. Since this subject has been well studied in the optical regime I felt it was worthwhile to see if the seeing at optical wavelengths can be reconciled with that at radio wavelengths. This may seem to be a foolish endeavour in view of the disparity in both wavelength of observation and scale size in the troposphere. Therefore, we should first justify this exercise. It is easily shown that the seeing disk appropiate for optical telescopes is the Fourier transform of the exponential of (minus one halfth) the structure function $D(\| \underline{x}_1 - \underline{x}_2 \|)$. Furthermore, there is evidence to suggest that a Kolmolgorov form :

$$D(\| \underline{x}_1 - \underline{x}_2 \|) \propto \| \underline{x}_1 - \underline{x}_2 \|^{5/3}$$

for the structure function provides a very good fit to observed seeing disks (see e.g. figure 1 of Woolf 1982). Thus, the spectrum of scale sizes in the troposphere must be well represented by the Kolmolgorov spectrum over a range in scale sizes from 10cm (the smallest scale size) to several meters (the size of the largest telescopes). Physically, the Kolmolgorov spectrum is formed by the steady state cascade of turbulent energy from an upper scale size down to a lower scale size determined by the effective viscosity. The upper scale size is probably determined by sources of sheared flow in the vicinity of the telescope(e.g. mountains). Hence if this scale size is larger than the size of a radio array then the phase structure function at radio wavelengths as measured by the array should obey the same Kolmolgorov form and hence the phase stability at radio wavelengths due to dry air fluctuations should be predictable from the seeing at optical wavelengths. Of course, if the seeing is mainly man-made as is now suspected in many cases (see e.g. Woolf 1982) then this extrapolation will be invalid. Hence for comparison, we should use as our input data figures for the best optical seeing available at the Plains of San Agustin. Failing this, I took the figures quoted by Woolf (1982) for the best seeing at a number of observatories. The conventional normalisation for the structure function is :

$$D(\| \underline{x}_1 - \underline{x}_2 \|) = 6.88(\| \underline{x}_1 - \underline{x}_2 \| / r_0)^{5/3}$$

where 6.88 is Fried's number. At spacings of r_0 the rms phase difference is about 2.6 rad. Table I shows an amalgamation of Woolf's numbers and the numbers measured by Dick Sramek (1983). These latter numbers correspond to the 1Km intercept in Sramek's results since we expect this to be more reliable than the numbers at shorter spacings.

Table I

λ50% (m)	r_0 (m)	Comments
5E-7	0.14	
1E-6	0.32	
1E-5	5.1	
1E-3	1300	300 GHz
3E-3	5400	90 GHz
1.3E-2	27000	22 GHz, Predicted
1.3E-2	17000	22 GHz, Measured by Sramek on a baseline of 1Km.

The 50% r_0 is the value observed 50% of the time. The degree of agreement between the observed and predicted r_0 is suprising, especially in view of the fact that the structure function measured by Sramek (1983) is hardly ever as steep as the Kolmolgorov form. We must conclude that either the agreement shown in Table I is entirely fortuitous or that at short spacings Sramek's numbers are too high. Although the latter seems more likely, since for example LO noise will add a constant term, there is no sign in the plots of two components for the structure function. Another curious point is that we know that at radio wavelengths the flucuations in the water vapour content dominate the path length changes whereas the Kolmolgorov model includes only the dry air flucuations. Either this is purely coincidental and the dry air fluctuations are un-important for radio arrays or they represent a lower bound for the phase errors below water vapour radiometers will be ineffective in correction.

If seeing and poor radio phase stability have a common origin then it may be possible to test for good sites for the millimeter array using an optical telescope equipped with e.g. a knife-edge. Before examining this possibility further more observations of the radio phase structure function are required so that we can determine whether it can be well represented by the Kolmolgorov form.

Note : Some of the points made in this memo were also made by B. Ulich at the March 1-2, 1984 mm. array advisory group meeting. I have therefore revised it to include remarks made by those present.

References :

Sramek, R., "VLA phase stability at 22GHz on baselines of 100m to 3Km", Millimeter memo 8, 1983.

Woolf, N., "High resolution imaging from the ground", Ann.Rev. Astron. and Astrophys. 20, 367-398, 1982.