MILLIMETER ARRAY
MEMO NO. 18

Quality indicators for the MM array

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Following the various discussions recently about quality indicators for the mm array design, I would like to suggest two measures which may be easily evaluated using UVMAP, FFT and one special task WEIGH (written by Bob Sault) in AIPS. I also intend to use something like these two measures in my optimisation of the random array.* They are designed to minimise the number of Fourier transforms needed.

(1) To quantify the sidelobe level of a configuration we can use Parseval's theorem to estimate the rms sidelobe from the distribution of weights in the uv plane.

$$\sigma_b^2 = \Sigma_{i,j}\, b_{i,j}^2 / N_{pixels}$$

where $b_{i,j}$ is the beam. This is equal to :

$$\sigma_b^2 = \Sigma_{k,l}\, w_{k,l}^2 / (\Sigma_{k,l}\, w_{k,l})^2$$

which may be easily evaluated. The change of this with taper is also of interest since good surface brightness sensitivity may only come at the expense of sidelobe level. For a Gaussian taper the sum is :

$$\sigma_b^2(\theta) = (\Sigma_{k,l}\, w_{k,l}^2 . \exp(-2.\alpha.r_{k,l}^2.\theta^2)) / (\Sigma_{k,l}\, w_{k,l} . \exp(-\alpha.r_{k,l}^2.\theta^2))^2$$

where θ is the width (FWHM) of the Gaussian, α is a scaling factor = $Pi^2/(4.\ln(2))$, $w_{k,l}$ is the weight in the (k,l)th cell, and :

$$r_{k,l}^2 = (u_{k,l}^2 + v_{k,l}^2).$$

The run of $\sigma_b(\theta)$ with θ should communicate information about the distribution of holes in the u,v plane.

(2) To evaluate the surface brightness sensitivity I suggest that we calculate the expected noise level for a variety of tapers and plot surface brightness ((noise level)/(area of Gaussian)) as a function of the size of a Gaussian. For a naturally weighted image, the rms noise level is :

$$\sigma_{map}(\theta) = \sigma_v / (\Sigma_{k,l}\, w_{k,l} \exp(-2.r_{k,l}^2.\alpha.\theta^2))^{1/2} \quad \text{Jy/beam}$$

and the surface brightness sensitivity is :

$$S(\theta) = \sigma_{map}(\theta)/((1.1331).\theta^2) \quad \text{Jy/(unit area)}$$

where σ_V is the rms noise for one sample. Note the factor of 2 in the Gaussian term. The special task in AIPS, WEIGH, evaluates this sum from the Fourier transformed beam for a number of values of θ and also $\sigma_b(\theta)$ from (1).

One can regard the formula for $\sigma_{map}(\theta)$ as giving the error in a least-squares fit of a Gaussian of specified size to the data. The taper should then be viewed as producing an optimum matched filter for the detection of the Gaussian. Therefore the size of the corresponding clean beam, which one might think should be involved in some way, is immaterial.

$\sigma_b(\theta)$ and $S(\theta)$ measure different aspects of the array and should be complementary : to minimise the rms sidelobe level we should spread the samples as evenly as possible, but then the run of surface brightness sensitivity will reflect a preponderance of long spacings and will be small for very extended structures. Conversely, optimising the surface brightness sensitivity for large sources will increase the sidelobe level. These two descriptions of the u,v coverage will probably help weed out the poor configurations, but it is possible that to discriminate between good configurations we have to use more sophisticated methods.

Both are dependent upon the gridding used so we should be careful to maintain the field of view and number of pixels in the images for all corresponding cases.

To summarise, I suggest that for each trial configuration we calculate $\sigma_b(\theta)$ and $S(\theta)$ for a range of θ, probably at intervals over something like a range 1 to 100 times the natural beam size. There will thus be 4*(number of trial declinations) tables or graphs for each configuration if we use natural and uniform weighting.

To evaluate these sums in AIPS use the following procedure :

(1) Run UVMAP to produce a beam using natural weighting, cell-summing and no grid correction. Note the sum of weights for later use.

(2) Run FFT on the beam to produce the gridded weights.

(3) Run WEIGH to find $\sigma_b(\theta)$ and $S(\theta)$ for a range of θ. The sum of weights from step (1) must be inputted since all information about the number of valid data points is otherwise lost.