

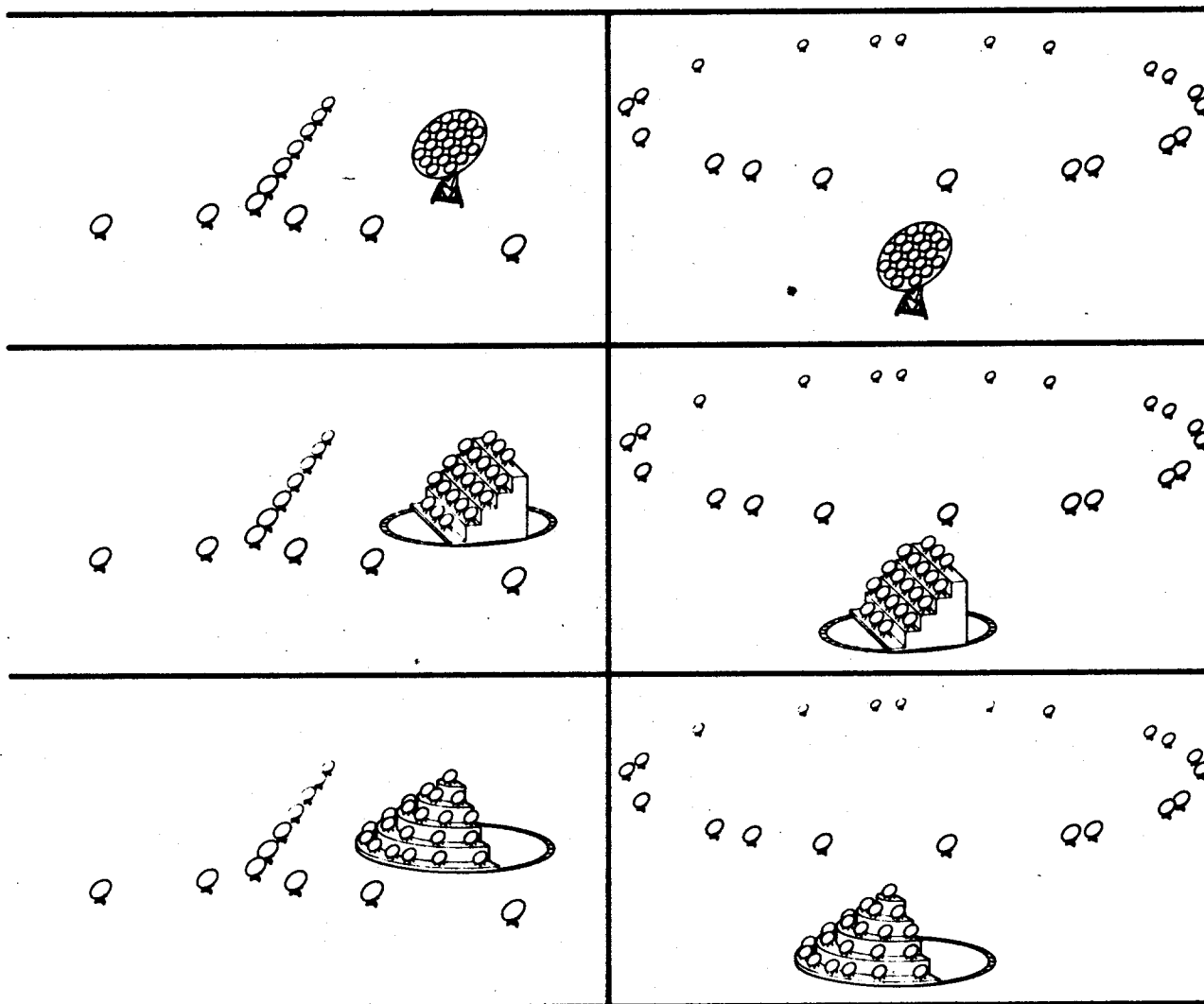
The Summer 1985 Concept of the Proposed NRAO Millimeter Array

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The purpose of this memo is to summarize the current state of thinking on the paradigm Millimeter Array system being discussed just before the Millimeter Array Science Workshop in Sept.-Oct. 1985. The memo will consist of updated versions of the summary material provided for the public discussions in Tucson/Charlottesville in May/June 1985, with some explanatory text.

Two arrays are proposed in order to cover the range of desired resolutions and fields of view. The current paradigm consists of a large array of ~21 movable antennas of ~10m diameter and a small array of ~21 antennas of ~4m diameter which are mounted on a structure ~29m in size in a multi-telescope (M-T) configuration with ~50% filling factor and 25m resolution. Figure 1 schematically illustrates the major options being discussed for configurations for the two arrays. The antennas in both arrays will operate in both aperture synthesis mode and single antenna mode, and Figure 2 summarizes the observing modes that are used to cover all resolutions from the $56'' \lambda_{\text{mm}}$ beams size of the 4m antennas down to the resolution of the largest arrays of the 10m antennas. Single dish observing in total power/beam switching mode covers the largest two size scales, the M-T in aperture synthesis mode covers the next step of a factor of 2.5 in resolution, and the large array configurations $\geq 90\text{m}$ which cover all higher resolutions. Figure 3 illustrates the fundamental mosaicing problem involving the combination of 4m and 10m data to observe the field of view of the 4m antennas with the resolution of the 10m antennas or array of 10m antennas. It can be shown that to have the mosaiced beams in Figure 2 give the same sensitivity in the overlap regions of two (or four) beams, the ratio of antenna sizes should be 2.414 (or 3). A ratio of 2.5 is used for the current paradigm. The text and schematic equations in Figure 3 summarize the basic data processing problem in combining the data to provide images of the desired resolution. The main reason for two arrays with antennas of two different sizes is to allow imaging of a wide range of spatial scales. Figure 4 is a one page summary of array/mosaic parameters.

Millimeter Array



Possible 300 m and Multi-Telescope Configurations

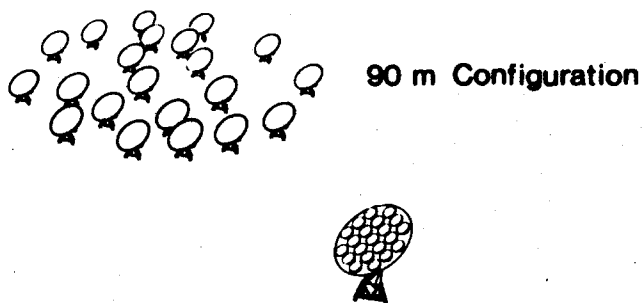


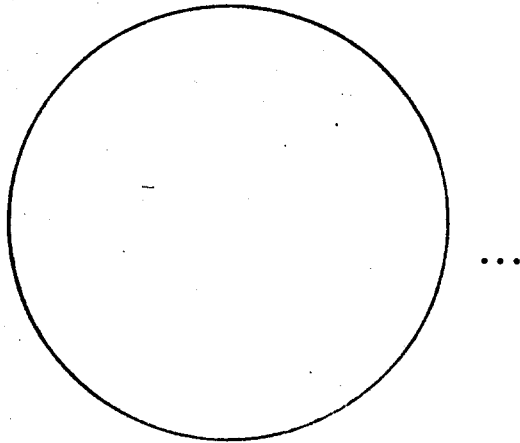
Figure 1

Millimeter Array Imaging/Mosiacing Problem

Resolution

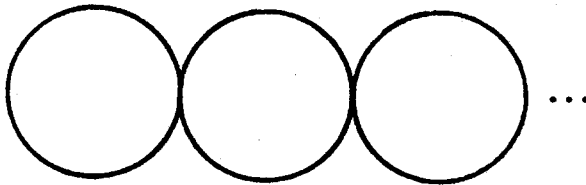
Instrument/Observing Mode

$56'' \lambda_{\text{mm}}$



4 m. Antennas / In Total
Power/Beam Switching Mode

$22'' \lambda_{\text{mm}}$



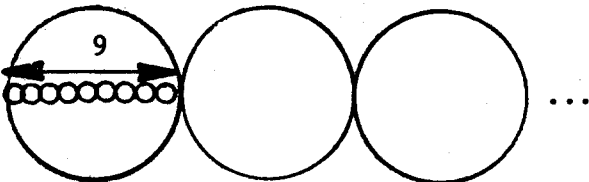
10 m. Antennas / In Total
Power/Beam Switching Mode

$7.6'' \lambda_{\text{mm}}$



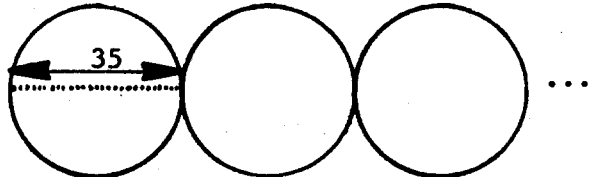
25 m. Multi-Telescope Array
of 21 4 m. Antennas In
Aperture Synthesis Mode

$2.2'' \lambda_{\text{mm}}$



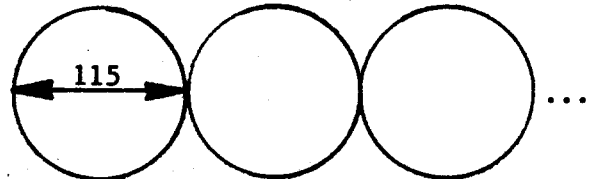
90 m. Configuration of
of 21 10 m. Antennas In
Aperture Synthesis Mode

$0.65'' \lambda_{\text{mm}}$



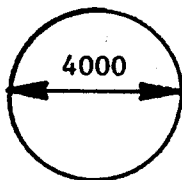
300 m. Configuration of
of 21 10 m. Antennas In
Aperture Synthesis Mode

$0.19'' \lambda_{\text{mm}}$



1000 m. Configuration of
of 21 10 m. Antennas In
Aperture Synthesis Mode

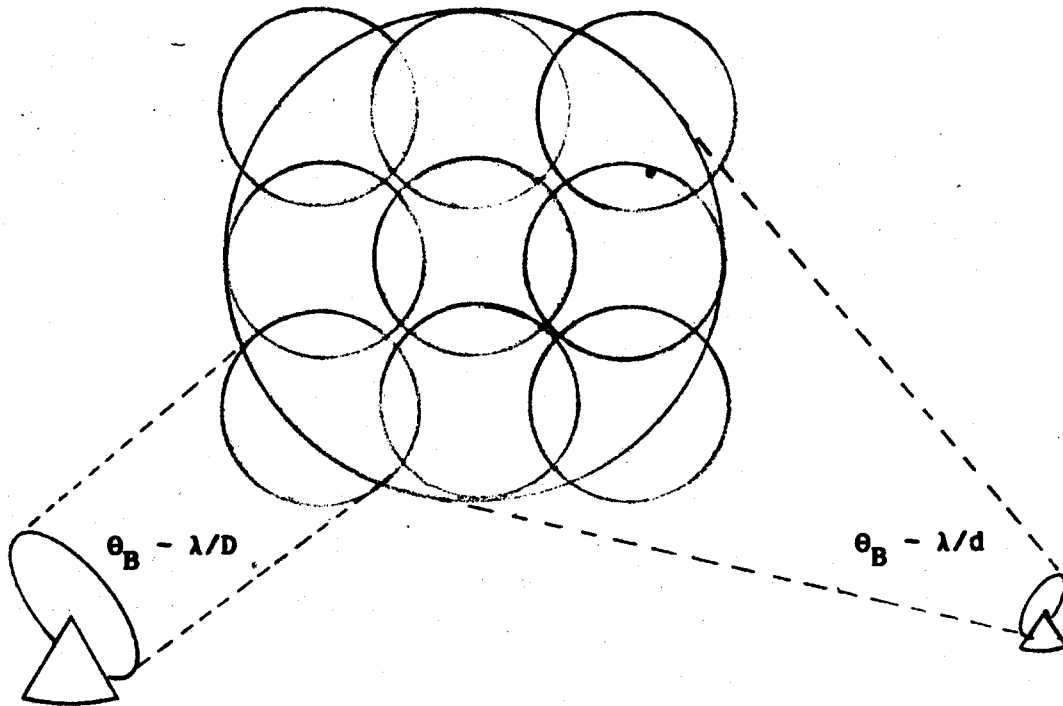
$0.006'' \lambda_{\text{mm}}$



35 km. Configuration of
of 21 10 m. Antennas In
Phase Closure
Calibration Mode

Figure 2

The Mosaic Problem



D = 10 m. Antennas
Imaging λ/d field in
Total Power/Beam
Switching Mode

Array of D = 10 m. Antennas
Observing 9 λ/D Fields ($-\lambda/d$)
In Aperture Synthesis Mode

Multi-Telescope Array
of d = 4 m. Antennas
Observing λ/d Field in
Aperture Synthesis Mode



$\{I_{TP}(\alpha, \delta)\}_{\text{Image}}$

$9 \{V_{JK}(u, v)\}_{\text{Grid}}^{B/2D}$

$\{V_{jk}(u, v)\}_{\text{Grid}}^{B/2d}$ or $\{V_{jk}(u, v)\}_{\text{Grid}}^{B/2d}$

$$I_{\text{Complete Image}}(\alpha, \delta) = FT^* [FT^* \{I_{TP}(\alpha, \delta)\}_{\text{Image}} + \{V_{jk}(u, v)\}_{\text{Grid}}^{B/2d} + 9\{V_{JK}(u, v)\}_{\text{Grid}}^{B/2D}]$$

* FT = Fourier Transform

Figure 3

MILLIMETER ARRAY

Purpose:

Spectral Line and Continuum Imaging in the 9, 3, 2, and 1 mm. bands with a wide range of spatial sampling

Electronics Parameters:

Receiver temperatures: 100 ($\nu_{\text{obs, GHz}}/100$) K Bandwidths: 1 GHz (Cont.) to 100 kHz (Sp. Line)
Hybrid Correlator (512 Channels)

Antennas: 21 Antennas with diameter of 10 m. (1600 m² Collecting area),
movable on transporters to 90, 300, 1000, ... m. configurations

21 Antennas with diameter of 4 m., mounted upon a Multi-Telescope (M-T) structure ~25 m. in size with M-T structure controlling azimuth and (perhaps) elevation pointing

Summary of Imaging Parameters¹ of Millimeter Array:

Instrument	Instrument Size	Antenna Size	Antenna Beam Size	Synthesis Beam Size	Map Size	Mosaicing Map Size
4m Antenna ²	4m	4m	56" λ_{mm}		n	
10m Antenna ²	10m	10m	22" λ_{mm}		3	
25m M-T ³	25m	4m	56" λ_{mm}	7.6" λ_{mm}	15	15n
90m Config. ³	90m	10m	22" λ_{mm}	2.2" λ_{mm}	20	50n
300m Config. ³	300m	10m	22" λ_{mm}	0.65" λ_{mm}	70	172n
1km Config. ³	1km	10m	22" λ_{mm}	0.19" λ_{mm}	230	590n
Nkm Config. ⁴	Nkm	10m	22" λ_{mm}	(0.19"/N) λ_{mm}	230N	

- ¹ Boldface entries identify the resolution of the instrumental configuration
² Total Power/Beam Switching Observing Mode
³ Aperture Synthesis Observing Mode
⁴ Phase Closure Calibration Mode

Figure 4

The previous material deals basically with the resolutions and fields of view that can be obtained with the paradigm Millimeter Array. The next major category of array parameters deals with sensitivity to detection of point sources or extended sources.

The principal sensitivity parameters for an aperture synthesis array are: system temperature (T_{sys} , in units of °K); antenna diameter (D_m , in units of meters); bandwidth ($\Delta\nu_{\text{GHz}}$, in units of GHz); integration time (Δt_{min} , in units of minutes); and the number of antenna pairs ($N_B = N(N-1)/2$, where N is the number of antennas in the array). Assuming a three-level correlator, the rms noise for each image pixel (or point source detection sensitivity) is given by (cf. MMA Memo 33):

$$\sigma = 1.71 (T_{\text{sys}}/100) / \{ (D_m/10)^2 [\Delta\nu_{\text{GHz}} \Delta t_{\text{min}} (N_B/210) (n_a/n_M)]^{1/2} \} \text{ mJy} \quad (1)$$

adopting scaling parameters appropriate to snapshot observations with 21 antennas of 10 m diameter, a 1 GHz bandwidth (appropriate for continuum problems), and a system temperature (100 K) appropriate to the 3 mm region. In Equation (1) n_a is the average number of data points per occupied cell in the gridded u - v plane, a quantity which is dependent upon the adopted weighting and tapering. With no tapering and "natural" weighting, $n_a = n_M$, the mean number of data points in each occupied cell, whereas for no tapering and "uniform" weighting, $n_a = n_{\text{HM}}$, the harmonic mean number of data points per occupied cell. These two types of means are indicators of the distribution of data points in the u - v plane, and hence are important array parameters.

Surface brightness sensitivity is derived from equation (1), the scaling constant between Jy/beam and brightness temperature, and the synthesized beam solid angle. Although the synthesized beam solid angle ($\Omega_b = 1.1331\theta_b^2$) is dependent upon array geometry, one predictable case is that of the beam for a uniformly filled (or weighted) array (with maximum diameter or baseline, B_{km} , in units of km). The rms surface brightness sensitivity is then given by

$$\Delta T_b = 0.64 (T_{\text{sys}}/100) [B_{\text{km}} / (D_m/10)]^2 / \{ f_{\text{geom}} [\Delta\nu_{\text{GHz}} \Delta t_{\text{min}} (N_B/210) (n_a/n_M)]^{1/2} \} \text{ K} \quad (2)$$

where we have defined f_{geom} to be the ratio of the true beam solid angle (Ω_b) to that for a uniformly filled aperture ($\Omega_{b,\text{un}}$). T_{sys} is determined by

$$T_{\text{sys}} = T_{\text{rcvr}} + T_{\text{atmo}} [1 - \exp(-\tau_1 \sec \zeta)] \quad (3)$$

where τ_1 is the optical depth for unit air mass (at the zenith), ζ is the zenith angle, and for simplicity we approximate the air mass by $\sec \zeta$. While $T_{\text{atmo}} = 280$ K is a reasonable estimate, the value of T_{rcvr} varies as a function of frequency and τ_1 varies with both frequency and atmospheric conditions. Based on discussions of probable receiver sensitivities in the future, we will adopt $T_{\text{rcvr}} = 100(v_{\text{GHz}}/100)$ K (single side band).

The following Table 1 is an updated version of the sensitivity parameters listed in the table in MMA Memo 29, but with 21 antenna configurations for the arrays of 10 m antennas (Y21, a VLA-like configuration; R5CIR21, a circular array with randomized antenna locations; and FCIR90M, a filled circle array which is 90 m in diameter) and one possible 21 antenna configuration of 4 m antennas in a Multi-Telescope array (TRACKM21).

Figure 5 is a plot of ΔT_b as a function of array size (B_{km}), for the smaller MMA configurations and observing situations with eight hour of integration time, 21 antennas with diameters of 10m or 4m, and an average system temperature of 100 K. The left abscissa of Figure 5 is for the 1 GHz bandwidth appropriate to continuum problems and the right abscissa is for spectral line problems with 100 kHz channel width. The line with alternating long and short dashes is Equation (2) with $f_{\text{geom}} = 1$ and $n_a = n_M$. The other lines and symbols are obtained from correct computations (cf. Table 1) of f_{geom} and n_a , for both uniform and natural weighting, for the Y21, R5CIR21, FCIR90m, and TrackM21 configurations.

In addition to quantities already defined, Table 1 contains entries for σ_{sid} , a fractional estimate of the beam sidelobe level (for natural and uniform weighting) as defined by Cornwell in MMA memo 18. The numbers in Table 1, the plots in Figure 5, and the coefficients in the above equations reflect system temperatures of 100 K. For other system temperatures one multiplies σ and ΔT_b by $(T_{\text{sys}}/100)$. When both polarizations are combined these quantities are divided by 1.414 because the coefficients in Equations (1) and (2) are then 1.21 and 0.45, respectively.

Table 1

Summary of Parameters for $\delta = 60^\circ$ Obs. with Various Arrays and $T_{\text{sys}} = 100^\circ$

Config.	Y21		R5CIR21		FCIR90M		TRACKM21	
Config. Diam.	300 m.		300 m.		90 m.		25 m.	
Antenna Diam.	10 m.		10 m.		10 m.		4 m.	
Gr. u-v Plane	71 X 71		71 X 71		17 X 17		15 X 15	
Obs. Time	8 ^h	2 ^m	8 ^h	2 ^m	8 ^h	2 ^m	8 ^h	2 ^m
$n_{\text{HM}}/n_{\text{M}}$	0.26	0.93	0.67	0.99	0.18	0.77	0.13	0.60
f_{geom}	5.8	2.1	1.1	1.0	1.9	1.3	2.0	1.6
$\sigma_{\text{sid,nat}}/\lambda_{\text{mm}}$.0325	.0565	.0200	.0499	.0717	.0798	.1006	.1064
$\sigma_{\text{sid,un}}/\lambda_{\text{mm}}$.0189	.0526	.0171	.0494	.0570	.0700	.0754	.0857
$\theta_{\text{b,nat}}/\lambda_{\text{mm}}$	1.30"	1.20"	0.51"	0.49"	2.09"	1.99"	6.95"	6.90"
$\theta_{\text{b,un}}/\lambda_{\text{mm}}$	0.54"	0.82"	0.48"	0.49"	1.53"	1.75"	4.88"	5.44"
σ_{nat} (mJy)	0.079	1.22	0.079	1.22	0.079	1.22	0.49	7.6
σ_{un} (mJy)	0.154	1.28	0.096	1.23	0.187	1.40	1.38	9.9
$\Delta T_{\text{b,nat}}$ (mK)	0.64	11.5	4.07	68.8	0.25	4.2	0.14	2.2
$\Delta T_{\text{b,un}}$ (mk)	7.08	25.8	5.62	69.1	1.12	6.2	0.79	4.5

Divide by 1.414 when both polarizations are combined

The sensitivity numbers in Table 1 are reasonable estimates for 100 GHz observations under good atmospheric conditions, however they are always too small at least a factor of 3 for observations above 200 GHz because of atmospheric absorption and emission. In Equation (3) we expressed the system temperature as a composite of receiver noise and the effects of observing emission from a relatively "hot" atmosphere. However, absorption of the source signal, so that observation of a source with brightness temperature T_{b} through an atmosphere with temperature T_{atmo} , zenith optical depth of τ_1 , and zenith angle ζ will give an observed brightness temperature which is

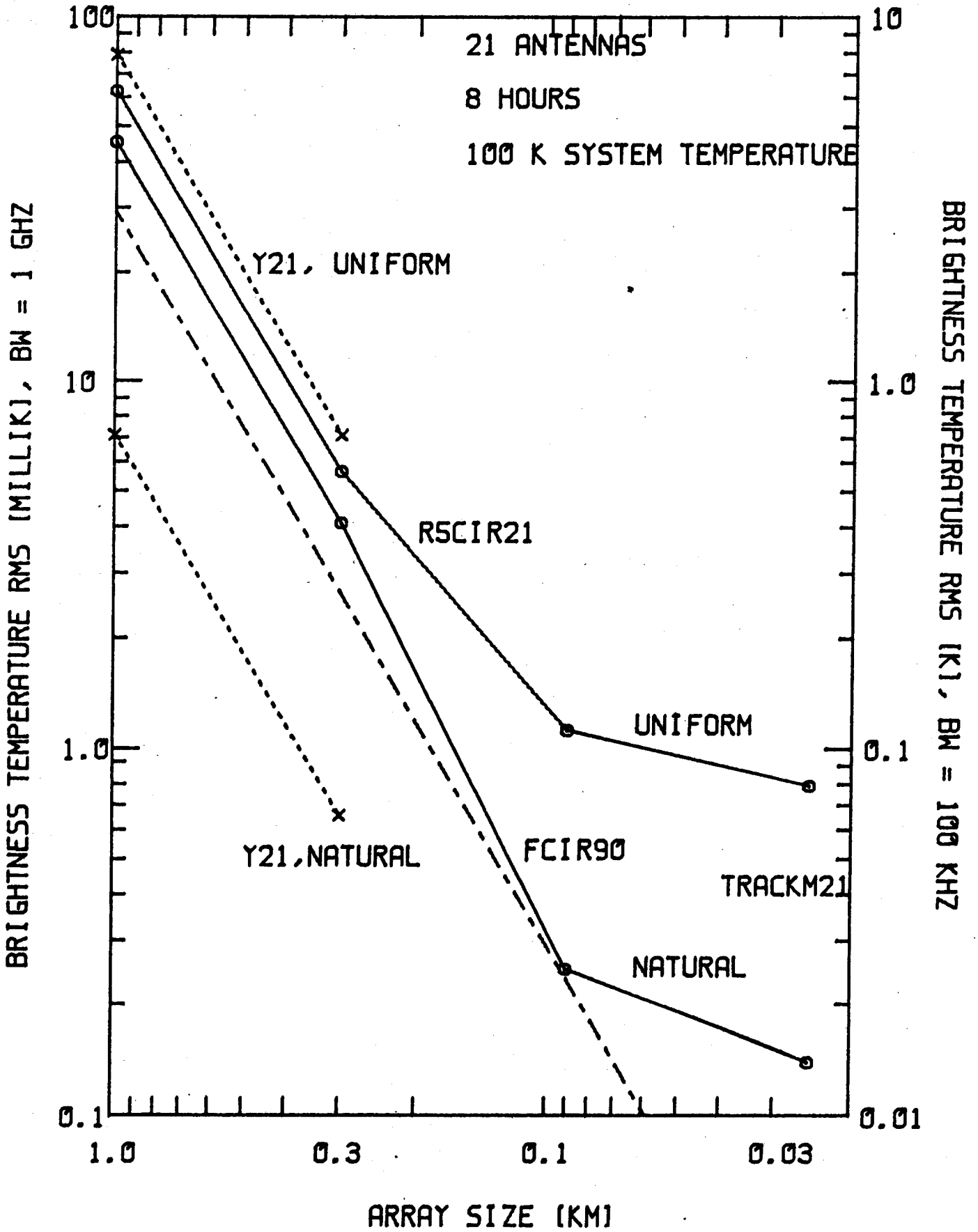


Figure 5

$$T_{b,obs} = T_b \exp(-\tau_1 \sec \zeta) \quad (5)$$

The purpose of sensitivity Equations is to evaluate the signal to noise for observations of sources of flux density S_ν and brightness temperature T_b . For this purpose the dependence on receiver temperature and atmosphere is

$$S_\nu/\sigma = T_b/\Delta T_b \propto \exp(-\tau_1 \sec \zeta)/\{T_{rcvr} + T_{atmo}[1 - \exp(-\tau_1 \sec \zeta)]\} \quad (6)$$

so one can define an effective system temperature,

$$T_{sys,eff} = \{T_{rcvr} \exp[\tau_1 \sec \zeta] + T_{atmo} [\exp(\tau_1 \sec \zeta) - 1]\} \quad (7)$$

that includes the atmospheric effects of both emission and absorption. Equation (7) should be used with Equations (1) and (2) to determine the effective signal/noise, and for $\nu \geq 200$ GHz, the atmosphere can dominate $T_{sys,eff}$.

Unfortunately, it is necessary to adopt typical values for τ_1 in order to estimate properly signal/noise. Table 2 gives values of $T_{sys,eff}$ for the latitude of the VLA site, $T_{atmo} = 280$ K, frequencies of 100 and 230 GHz, declinations of 30° , 0° , and -30° , a range of values of τ_1 , and the range of hour angles that can be accessed for each declination. Note that the observing time needed to obtain a particular level of signal to noise is proportional to $(T_{sys,eff})^2$.

Table 2 illustrates how strongly atmospheric absorption and emission affect the system temperature for the larger values of τ_1 and air mass. The top of Mauna Kea is known to frequently have τ_1 at 230 GHz in the range 0.1-0.15, while the NRAO 12m site has best days with $\tau_1 \sim 0.25$. It is restating the obvious to note that: (1) the lowest possible τ_1 at 230 GHz, for a reasonable amount of time, is an extremely important parameter of a site; (2) observing to large air masses is counter-productive in signal/noise-limited observations; and (3) since observations near meridian transit are most productive, one has another reason for requiring excellent synthesized beam characteristics for short observations.

Table 2

Values of $T_{\text{sys,eff}}$ for 100 and 230 GHz and a Range of Declinations, Zenith Optical Depths, and Hour Angles

Latitude, Declination: 34.0 30.0 freq(GHz): 100 Trcvr: 100 K										Latitude, Declination: 34.0 30.0 freq(GHz): 230 Trcvr: 230 K									
HA =	0.0	1.0	2.0	3.0	4.0	5.0	6.0	hrs		HA =	0.0	1.0	2.0	3.0	4.0	5.0	6.0	hrs	
z =	4.0	13.3	25.7	38.1	50.3	62.3	73.8	degrees		z =	4.0	13.3	25.7	38.1	50.3	62.3	73.8	degrees	
tau	Tsys = Trcvr*EXP(tau*sec z) + Tatmo*[EXP(tau*sec z) - 1]									tau	Tsys = Trcvr*EXP(tau*sec z) + Tatmo*[EXP(tau*sec z) - 1]								
0.00	100	100	100	100	100	100	100	degrees K		0.00	230	230	230	230	230	230	230	degrees K	
0.05	120	120	122	125	131	143	174	degrees K		0.05	256	257	259	263	272	288	330	degrees K	
0.10	140	141	145	151	164	191	263	degrees K		0.10	284	285	290	299	316	352	449	degrees K	
0.15	162	163	169	180	201	245	370	degrees K		0.15	313	315	322	337	365	424	592	degrees K	
0.20	184	187	194	210	240	304	497	degrees K		0.20	343	346	357	378	418	504	763	degrees K	
0.25	208	211	221	242	282	370	649	degrees K		0.25	375	379	393	421	474	593	967	degrees K	
0.30	233	237	250	276	328	444	831	degrees K		0.30	409	414	431	467	536	692	1211	degrees K	
0.35	260	264	280	313	377	526	1049	degrees K		0.35	444	451	472	516	602	802	1503	degrees K	
0.40	287	293	312	352	431	617	1309	degrees K		0.40	482	489	515	568	674	925	1852	degrees K	
0.45	317	323	346	393	489	719	1620	degrees K		0.45	521	530	560	623	752	1061	2270	degrees K	
0.50	347	355	382	437	551	833	1992	degrees K		0.50	562	573	608	682	836	1213	2769	degrees K	
0.55	380	389	419	484	619	959	2437	degrees K		0.55	605	618	659	746	927	1383	3367	degrees K	
0.60	413	424	459	534	692	1099	2969	degrees K		0.60	651	665	712	813	1025	1571	4081	degrees K	
0.65	449	461	502	588	772	1256	3605	degrees K		0.65	698	715	769	884	1131	1781	4934	degrees K	
0.70	487	500	546	645	857	1430	4366	degrees K		0.70	749	767	829	961	1246	2015	5956	degrees K	
0.75	526	541	593	705	950	1624	5276	degrees K		0.75	802	822	892	1042	1371	2275	7177	degrees K	
0.80	567	585	643	770	1050	1840	6364	degrees K		0.80	857	880	959	1129	1505	2565	8637	degrees K	
0.85	611	630	696	839	1158	2080	7665	degrees K		0.85	916	942	1030	1221	1650	2888		degrees K	
0.90	657	678	751	912	1275	2348	9220	degrees K		0.90	977	1006	1104	1320	1808	3247		degrees K	
0.95	705	729	810	990	1402	2646		degrees K		0.95	1042	1074	1183	1425	1978	3647		degrees K	
1.00	755	782	872	1073	1539	2978		degrees K		1.00	1110	1145	1267	1536	2162	4092		degrees K	

Latitude, Declination: 34.0 0.0 freq(GHz): 100 Trcvr: 100 K										Latitude, Declination: 34.0 0.0 freq(GHz): 230 Trcvr: 230 K									
HA =	0.0	1.0	2.0	3.0	4.0	5.0	hrs		HA =	0.0	1.0	2.0	3.0	4.0	5.0	hrs			
z =	34.0	36.8	44.1	54.1	65.5	77.6	degrees		z =	34.0	36.8	44.1	54.1	65.5	77.6	degrees			
tau	Tsys = Trcvr*EXP(tau*sec z) + Tatmo*[EXP(tau*sec z) - 1]									tau	Tsys = Trcvr*EXP(tau*sec z) + Tatmo*[EXP(tau*sec z) - 1]								
0.00	100	100	100	100	100	100	degrees K		0.00	230	230	230	230	230	230	degrees K			
0.05	124	124	127	134	149	200	degrees K		0.05	262	263	267	275	295	364	degrees K			
0.10	149	151	157	171	204	326	degrees K		0.10	295	298	306	325	369	533	degrees K			
0.15	175	178	188	211	266	485	degrees K		0.15	331	335	348	379	452	746	degrees K			
0.20	204	208	222	255	336	685	degrees K		0.20	369	375	394	437	546	1015	degrees K			
0.25	234	239	258	302	415	938	degrees K		0.25	409	417	442	501	652	1355	degrees K			
0.30	266	273	297	354	504	1258	degrees K		0.30	452	462	495	571	772	1784	degrees K			
0.35	300	308	339	410	604	1662	degrees K		0.35	498	510	550	647	907	2326	degrees K			
0.40	336	346	383	472	717	2171	degrees K		0.40	546	560	610	729	1059	3010	degrees K			
0.45	374	387	431	539	845	2815	degrees K		0.45	598	615	674	819	1230	3873	degrees K			
0.50	415	429	482	612	990	3627	degrees K		0.50	652	672	743	917	1424	4963	degrees K			
0.55	458	475	537	691	1152	4652	degrees K		0.55	710	734	817	1023	1642	6339	degrees K			
0.60	504	524	596	778	1336	5946	degrees K		0.60	772	799	896	1139	1889	8076	degrees K			
0.65	552	576	660	872	1543	7579	degrees K		0.65	837	868	981	1266	2167		degrees K			
0.70	604	631	727	974	1777	9642	degrees K		0.70	907	942	1072	1403	2480		degrees K			
0.75	659	689	800	1086	2040		degrees K		0.75	980	1021	1170	1553	2834		degrees K			
0.80	717	752	878	1207	2338		degrees K		0.80	1059	1105	1274	1716	3233		degrees K			
0.85	779	818	961	1340	2673		degrees K		0.85	1142	1194	1386	1894	3684		degrees K			
0.90	845	889	1051	1484	3052		degrees K		0.90	1230	1289	1506	2088	4192		degrees K			
0.95	915	965	1147	1641	3479		degrees K		0.95	1324	1390	1635	2299	4765		degrees K			
1.00	990	1045	1250	1812	3961		degrees K		1.00	1424	1498	1773	2528	5412		degrees K			

Latitude, Declination: 34.0 -30.0 freq(GHz): 100 Trcvr: 100 K										Latitude, Declination: 34.0 -30.0 freq(GHz): 230 Trcvr: 230 K									
HA =	0.0	1.0	2.0	3.0	hrs		HA =	0.0	1.0	2.0	3.0	hrs							
z =	64.0	65.5	70.0	76.8	degrees		z =	64.0	65.5	70.0	76.8	degrees							
tau	Tsys = Trcvr*EXP(tau*sec z) + Tatmo*[EXP(tau*sec z) - 1]									tau	Tsys = Trcvr*EXP(tau*sec z) + Tatmo*[EXP(tau*sec z) - 1]								
0.00	100	100	100	100	degrees K		0.00	230	230	230	230	degrees K							
0.05	146	149	160	193	degrees K		0.05	292	295	310	355	degrees K							
0.10	197	204	229	309	degrees K		0.10	361	369	403	511	degrees K							
0.15	255	266	309	453	degrees K		0.15	438	453	511	704	degrees K							
0.20	320	336	402	633	degrees K		0.20	525	547	635	946	degrees K							
0.25	392	415	509	857	degrees K		0.25	622	653	779	1246	degrees K							
0.30	473	504	633	1136	degrees K		0.30	731	773	946	1620	degrees K							
0.35	564	605	777	1483	degrees K		0.35	853	908	1138	2086	degrees K							
0.40	666	719	943	1915	degrees K		0.40	990	1061	1362	2666	degrees K							
0.45	781	847	1136	2453	degrees K		0.45	1144	1233	1620	3388	degrees K							
0.50	909	992	1358	3123	degrees K		0.50	1316	1427	1919	4287	degrees K							
0.55	1053	1155	1616	3957	degrees K		0.55	1508	1646	2265	5406	degrees K							
0.60	1213	1339	1914	4995	degrees K		0.60	1724	1893	2665	6800	degrees K							
0.65	1394	1547	2260	6288	degrees K		0.65	1967	2172	3128	8535	degrees K							
0.70	1596	1782	2659	7898	degrees K		0.70	2238	2487	3665		degrees K							
0.75	1823	2047	3122	9903	degrees K		0.75	2542	2843	4285		degrees K							
0.80	2077	2345	3657		degrees K		0.80	2883	3244	5004		degrees K							
0.85	2362	2682	4276		degrees K		0.85	3265	3696	5835		degrees K							
0.90	2681	3063	4993		degrees K		0.90	3694	4206	6797		degrees K							
0.95	3039	3492	5823		degrees K		0.95	4174	4782	7910		degrees K							
1.00	3439	3976	6783		degrees K		1.00	4712	5433	9199		degrees K							