MILLIMETER ARRAY
MEMO NO. 38

# Crystalline antenna arrays

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## INTRODUCTION

During the early days of the design of the mm array configuration, we erroneously believed that the snapshot coverage and the surface brightness sensitivity curve were crucial. As part of an attempt to address problem of the snapshot coverage, I developed a method to optimise the coverage with respect to various measures of uniformity. In this note, I will summarise both the procedure used and show the beautiful, symmetric configurations which were obtained for one u,v plane coverage measure.

## SUMMARY OF THE PROBLEM

We wish to find positions for $N$ antennas within some area such that a measure of the u,v coverage is optimised. The form of the measure could be very complicated e.g. r.m.s. sidelobes, mean square separation of u,v points, geometric mean separation of u,v points, number of unfilled cells in a grid of appropriate size, etc.

Let $\mathbf{r}_i$ be the vector position of the i'th antenna, and let $K$ be the set of allowed positions for antennas : it may consist of a grid, a compact region or a number of disconnected regions. Let the measure of snapshot coverage be $m(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N)$. The problem is then to maximise or minimise globally the value of $m(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N)$ such that $(\mathbf{r}_i \in K : i = 1, N)$.

Although this seems a simple optimization problem, derivatives may thus be difficult to obtain, or simply not defined, and there may also be multiple extrema which could trap a gradient-based optimization algorithm. An alternative to conventional methods, simulated annealing, which has been recently developed (see e.g. Kirkpatrick et al. 1983), can be used to attack this problem.

## OPTIMIZATION STRATEGY

Simulated annealing is essentially a statistical approach : configurations are tried at random and accepted according to the following rules. Let $E$ be the function to be minimised, and let $T$ be a user-controlled "temperature" (the meaning will become clear).

(1) If $E_r < E_{r-1}$ then the new, $r$'th configuration is accepted.

(2) Compute $p(E_r) = e^{-E_r/T}$, and $X_r$, a random number drawn from a uniform distribution ranging from 0 to 1. If $p(E_r) < X_r$ then accept the new, $r$'th configuration.

Thus the algorithm only goes down-hill on average. Some fraction of the time, dependent upon the temperature, the algorithm goes up-hill, and it can therefore escape

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from local minima if the temperature is varied sufficiently slowly. The art of this algorithm consists in choosing the appropriate "annealing schedule"; as Kirkpatrick et al. (1983) indicate, many of the usual statistical mechanics tricks can be used to aid in this choice. For example, the specific heat can be monitored for signs of the onset of freezing. I have not found such sophistication to be required and resorted to a simple cooling law : multiply $T$ by some factor $g$, e.g. $g = 0.9$, after a given number of new configurations have been accepted at any given temperature.

## Application to the optimization of snapshot coverage

One can argue for any number of choices for the measure function. I decided to maximise the distance between the u,v points from a snapshot, with the justification that the resulting coverage will be unique, and will have low sidelobes. To concentrate more on closer points, I decided to use the logarithm of the distance between u,v points rather than the square. This choice is somewhat arbitrary but the results obtained are interesting.

The u,v coverage is given by the set of difference vectors :

$$(\mathbf{u}_{i,j} = \mathbf{r}_i - \mathbf{r}_j : i,j = 1, N)$$

The measure is thus :

$$m(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N) = \sum_{i,j,k,l} \log(|\mathbf{u}_{i,j} - \mathbf{u}_{k,l}|)$$

where redundant spacings are ignored in the sum.

The antennas were constrained to lie with a circle of arbitrary radius. Trial configurations may be constructed randomly, but I have found that it is best to change one antenna location at a time, and by an amount $\delta\mathbf{r}_i$ such that the corresponding $\delta E$ is comparable to $T$. This does not affect the result obtained but merely the rate of convergence. To verify that true consistent minima had been obtained, most of the annealings were performed a few times with different, random initial conditions.

## Discussion

The resulting arrays are listed in Appendix A and shown in Figure 1. The arrays have beautiful crystalline structure, which always has bi-lateral symmetry. These arrays provide very good snapshot coverage of the u,v plane and may, therefore, be of some use in interferometric arrays for which the instantaneous coverage must be very good. The principles involved could be extended to non-instantaneous coverage only if analytic forms for the measure can be calculated. The arrays are very redundant in rotation when seen face-on, and may therefore be unsuitable for some uses such as space-borne optical arrays.

## Appendix A: Listings of the antenna positions

Number of Antennas = 3

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.4320858 | -0.2515986 |
| -0.4328533 | -0.2502755 |

Number of Antennas = 4

| x | y |
|---|---|
| -0.4189747 | -0.2728738 |
| 0.2398397 | 0.4387220 |
| -0.2289764 | 0.4444882 |
| 0.4152750 | -0.2784720 |

Number of Antennas = 5

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.4755248 | 0.1545189 |
| -0.2941984 | -0.4042856 |
| -0.4751981 | 0.1555202 |
| 0.2935914 | -0.4047270 |

Number of Antennas = 6

| x | y |
|---|---|
| 0.3151123 | 0.3882064 |
| -0.4936455 | 7.9460695E-02 |
| -0.3241841 | 0.3806634 |
| 0.1795142 | -0.4666631 |
| -0.1666702 | -0.4714030 |
| 0.4915545 | 9.1509834E-02 |

Number of Antennas = 7

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.4948349 | -7.1682535E-02 |
| 0.3989847 | 0.3013487 |
| 0.1810795 | -0.4660578 |

| | |
|---|---|
| -0.3999324 | 0.3000896 |
| -0.1796049 | -0.4666281 |
| -0.4946291 | -7.3088318E-02 |

Number of Antennas = 8

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.2965855 | 0.4025382 |
| -1.3281007E-03 | -0.4999981 |
| -0.4938155 | -7.8396916E-02 |
| -0.4024266 | -0.2967358 |
| 0.4935313 | -8.0164373E-02 |
| 0.4014871 | -0.2980067 |
| -0.2949823 | 0.4037142 |

Number of Antennas = 9

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.2119921 | -0.4528345 |
| 0.4330290 | -0.2499709 |
| -0.4332467 | -0.2495928 |
| 0.2867274 | 0.4096187 |
| -0.4982555 | 4.1731380E-02 |
| 0.4976544 | 4.8371203E-02 |
| -0.2064091 | -0.4554068 |
| -0.2910378 | 0.4065673 |

Number of Antennas = 10

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.4888961 | -0.1047875 |
| -0.2445619 | -0.4361070 |
| 0.2425469 | -0.4372308 |
| 0.4496760 | 0.2186115 |
| 0.3462886 | 0.3606716 |
| -0.3447906 | 0.3621033 |

| | |
|---|---|
| -3.2835179E-03 | -0.4999890 |
| -0.4520736 | 0.2136100 |
| -0.4904117 | -9.7448446E-02 |

Number of Antennas = 11

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.2912835 | 0.4063911 |
| -0.4568465 | 0.2032023 |
| -0.3457744 | -0.3611643 |
| 0.3457167 | -0.3612197 |
| 0.4572277 | 0.2023420 |
| 0.4989131 | -3.2948442E-02 |
| -0.4990395 | -3.0969921E-02 |
| -0.1671764 | -0.4712238 |
| -0.2899925 | 0.4073131 |
| 0.1667254 | -0.4713836 |

Number of Antennas = 12

| x | y |
|---|---|
| 0.0000000 | 0.5000000 |
| 0.4527406 | 0.2121913 |
| -0.4983997 | 3.9967656E-02 |
| 0.4033182 | -0.2955227 |
| -0.2082298 | 0.4545767 |
| -0.4024793 | -0.2966645 |
| -0.4542729 | 0.2088913 |
| 5.5871205E-04 | -0.4999995 |
| 0.2072576 | 0.4550205 |
| 0.4979849 | 4.4839311E-02 |
| 0.2959012 | -0.4030411 |
| -0.2927386 | -0.4053441 |

## References

Kirkpatrick, S., Gelatt Jr, C.D., and Vecchi, M.P., *Optimization by Simulated Annealing*, Science **220** (1983), 671-679.

Figure 1

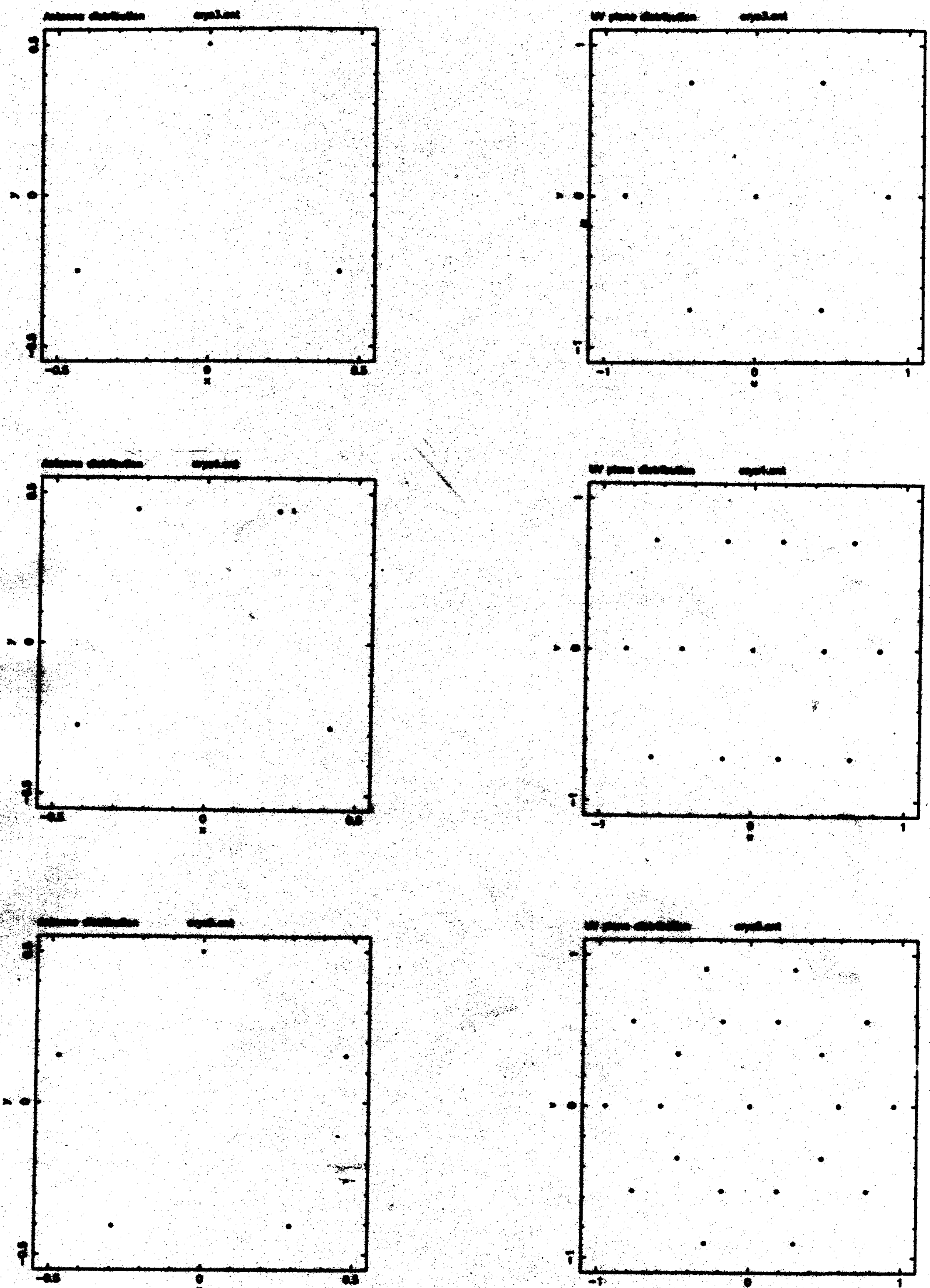

Figure 1 (cont)

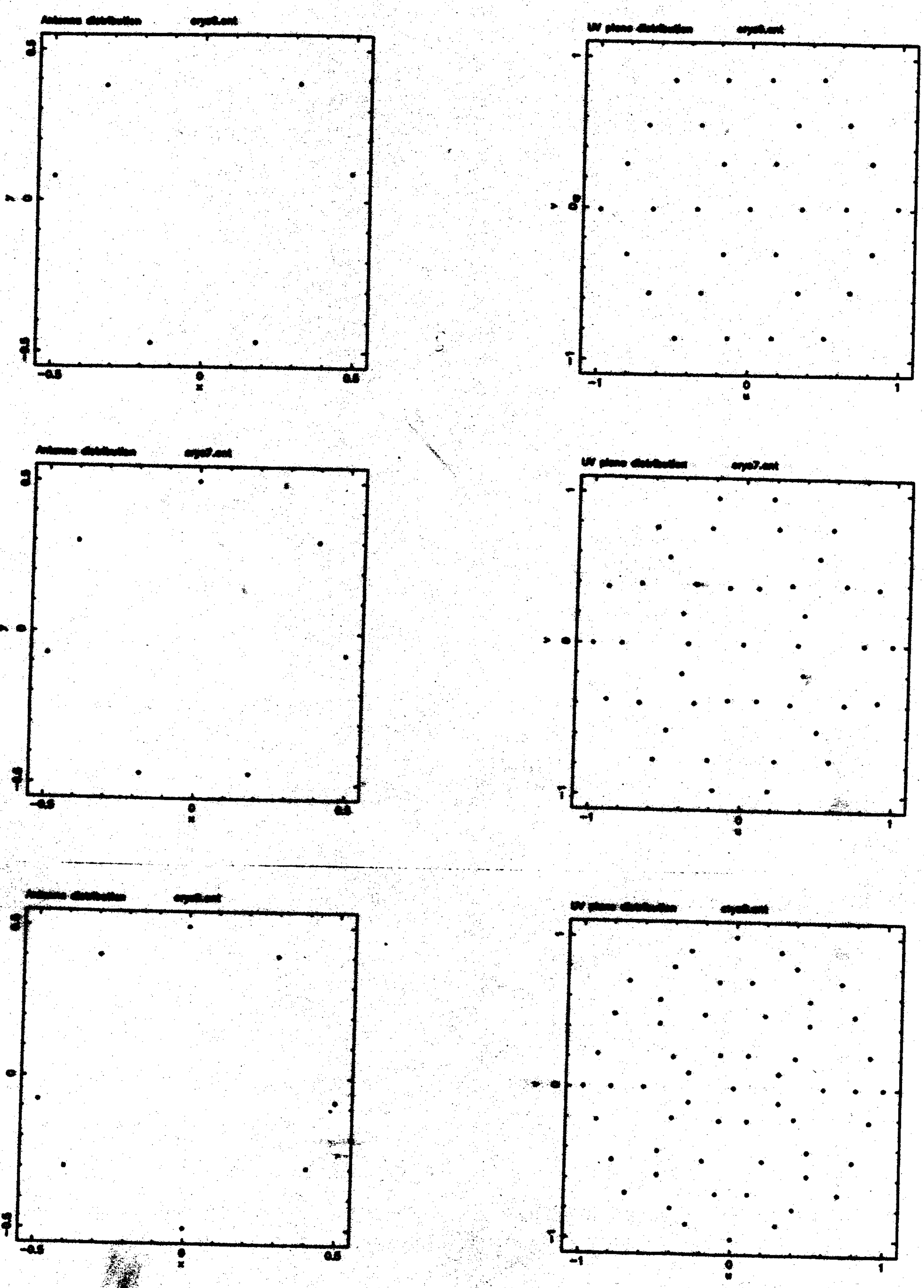

Figure 1 (cont).

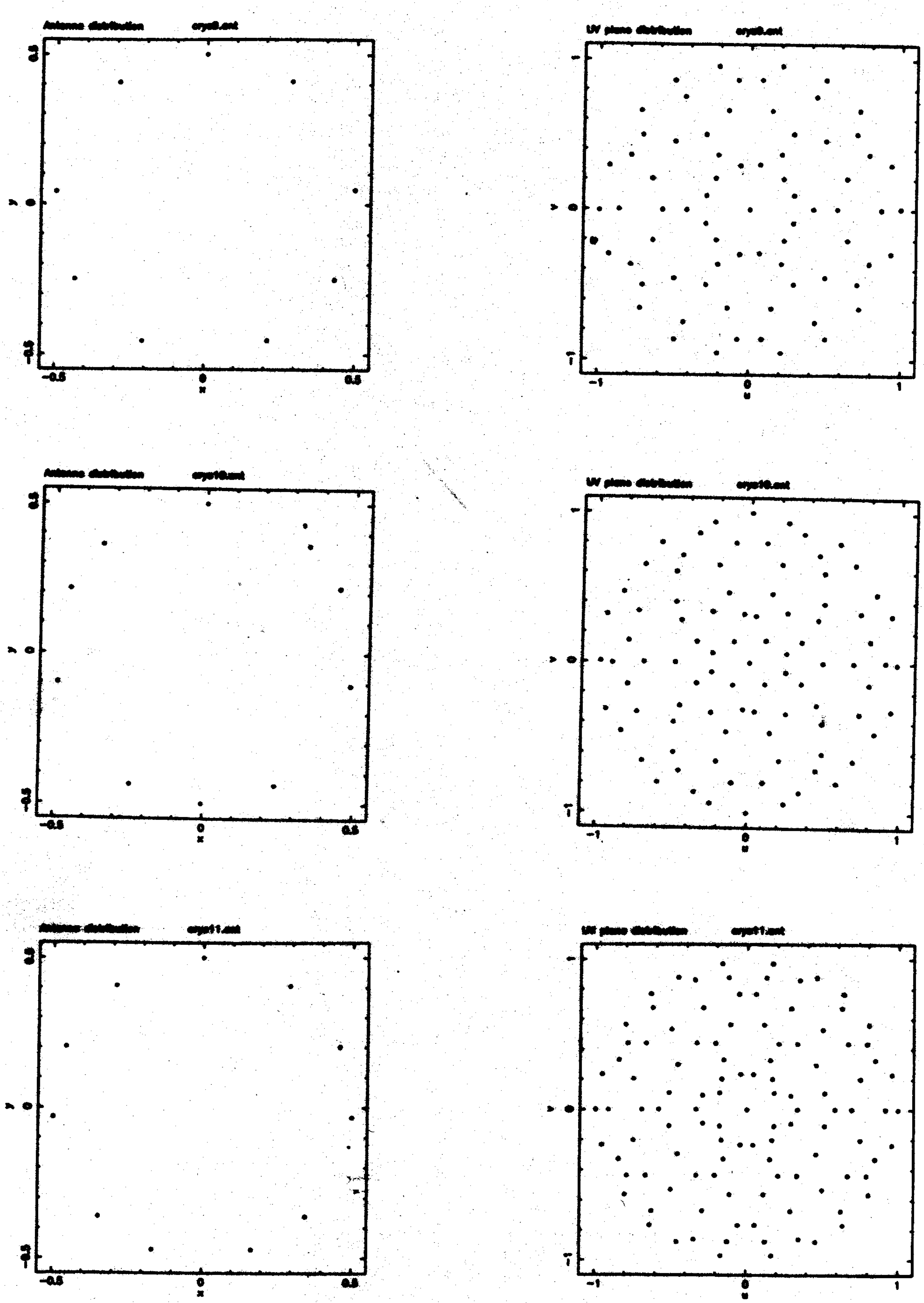

Figure 1 (cont)