MILLIMETER ARRAY
MEMO NO. 50

Report of the Central Element working group

T.J. Cornwell
R. Braun
D. Emerson
J.M. Uson
National Radio Astronomy Observatory *

February 12, 1988

1 Introduction

The working group on the Central Element was formed to look into the question of the type of Central Element needed to provide short-spacing information. In this memo, we will report on our current understanding of this question, and upon what further work needs to be done.

One very important attribute of the Millimeter Array is that it measure "all" Fourier components relevant to a region of the sky, up to some cutoff in maximum spacing of a few kilometers. Particular weight has been attached to the measurement of the short-spacing information crucial for the imaging of large objects. By short, we mean spacings less than or in the neighbourhood of the antenna diameter 7.5 meters, say 0-10 meters. In conventional interferometers, these and shorter spacings are often missing completely. However, since the MMA will regularly image objects much larger than an individual primary beam, these spacings are vital. There are three possible schemes for obtaining these measurements:

Large Single-Dish: This is the most conventional of the schemes. A large single-dish is used to image the sky in total power mode. The required spacings could be obtained by inverse Fourier transformation of the image, followed by correction of the sensitivity function of the dish (i.e. the inverse Fourier transform of the primary beam). According to the conventional wisdom, the size of the single-dish should be 2-3 times larger than the size of the required spacings.

*The National Radio Astronomy Observatory (NRAO) is operated by Associated Universities, Inc., under contract with the National Science Foundation.

Multi-Telescope Array: An array of *smaller* telescopes is used as an interferometric array, and a subset of the required spacings are obtained directly. The size of the elements should be 2-3 times *smaller* than the size of the large array elements. To avoid shadowing between the densely packed elements, the elements should be mounted on a backing structure which can point at and track the object. Structures on the largest scale-size are derived from total-power measurements using the larger interferometer dishes.

Homogeneous Array: This is the most radical of the schemes. There would not be a Central Element: each antenna in the interferometer array would be equipped to observe in a total-power mode, preferably simultaneously with interferometer measurements. As the array is scanned for mosaicing, the total-power observations can be used to form an image, which could be inverse-transformed to obtain spacings up to some fraction of the dish-diameter. Ekers and Rots [8] showed that this can be generalized for the interferometer elements, and that it provides information on scales ranging down from the shortest spacing by some fraction of the dish diameter. Therefore, if the shortest spacing is about a dish diameter, and if the extrapolated measurements are good at offsets of up to half a dish diameter, then there is no gap to be filled.

The processing of the collected data would probably be performed via a Maximum Entropy "mosaicing" algorithm [4,5] rather than the Ekers-Rots scheme discussed above. The advantages of the mosaicing algorithm are:

- The constraints on data collection are less severe since a regular grid of pointings is not required. Data from single-dishes, and from interferometric arrays with elements of different sizes, may be combined easily.
- The deconvolution is performed at the same time as the estimation of the short-spacings, leading to superior imaging.
- Correct weighting by SNR is simple.

From the earliest days of work on the design of the MMA, it was recognised that the Large Single-Dish option requires the use of an array of feeds to provide a balance between the sensitivity of the interferometer part of the array and the Central Element. For this reason, the technologically most-conservative path was thought to be based around use of the Multi-Telescope Array. More recently, it was realized that a Homogeneous Array also could, in principle, measure all the required spacings.

Although all three option are theoretically feasible, there may be practical difficulties with each. The task of the working group is to investigate these obstacles and reassess the viability of each options. The main questions can be summarized:

1. Given perfect observations, will mosaicing allow recovery of all spacings:
   (a) down to about half the dish diameter from the interferometric measurements?
   (b) up to about half the dish diameter from the total-power measurements?
2. Will the various instrumental errors corrupt the mosaicing? For example, will pointing errors be a limiting factor? Or reversing the question, what are the necessary specifications on pointing, illumination stability, cross-talk, etc?
3. How do observational details affect the design? For example, can we design a homogeneous array which allows sufficient non-shadowed integration time? Beam-switching will be required for total-power measurements–how far a throw is necessary? Can the optics accomodate it?
4. Are the total power measurements significantly more difficult than the interferometric measurements? Is a single feed array with N elements easier to maintain than the feeds on N separate telescopes?

In this memo we will discuss both some partial answers to these questions, and how further answers may be obtained. Most of the information we now have comes either from experiments using the VLA, or from simple analytical investigations. In the future, we expect to augment this with further experiments, using the VLA and other telescopes, with more theoretical work, and with an extensive computer simulation program.

In section 2, we discuss mosaicing experiments with the VLA, and we indicate future tests which should be performed. In section 3, we discuss results of our theoretical investigations. Then in section 4, we present a plan for computer simulation which should provide more-definitive answers.

2 Experiments with the VLA

Mosaicing is now used widely at the VLA for imaging objects spanning more than one primary beam. Many large experiments have been performed to date. The main conclusions are:

- Mosaicing using the **VTESS** algorithm [4,5] is feasible and can produce low dynamic range images of extended objects. The spacings recovered faithfully reach down to about half a dish diameter.
- For high dynamic range imaging, our poor knowledge of the VLA primary beam is the limiting factor in the deconvolution. A modified scheme is then preferred in which only the low spatial frequency data for all fields are processed simultaneously in **VTESS**, while the high frequency data are

processed independently and combined linearly into an image [1]. To produce a final image, the high- and low-resolution images are combined together linearly into one image.

- The disadvantages of MEM, namely bias and super-resolution, can be overcome by convolving the final image with a CLEAN-beam and adding the residuals. (A mosaicing algorithm similar to **VTESS** but without the restriction to positive-definite brightness is being tested in AIPS as task **UTESS**.)

Three detailed investigations of the use of mosaicing with the VLA have been made. These focus upon the estimation of short-spacings, the effect of pointing errors, and the limitations on dynamic range.

2.1 Estimation of Short-Spacings

In this experiment, discussed in detail elsewhere [3], a comparison was made between a mosaiced VLA image and a conventional Penticton image of a very extended object, Simeis 57. Comparison with another interferometric array rather than a single-dish is preferred because of the lower level of systematic errors present in interferometric images. Twenty-two pointings of the VLA were made to cover the same area as one pointing of the Penticton array. At the observing wavelength of λ20cm, this corresponded to a total field of view of about 2°. The conclusions from this experiment are:

- For the VLA at λ20cm, it is possible to estimate spacings about 10m shorter than the smallest measured. Given the poor model of the VLA primary beam used, truncated at the 7% level, this is gratifyingly good.

- The mosaicing algorithm, **VTESS**, produces results superior to those from linear combination of separately deconvolved images. The extra computational cost in the joint deconvolution is about a factor of two for 22 pointings.

2.2 The Effect of Pointing Errors

The r.m.s. pointing errors of the VLA are about 10–15 arcseconds in normal conditions. At λ2cm, this is about $\frac{1}{18}$th – $\frac{1}{12}$th of the full-width-half-maximum of the primary beam. For the 7.5m diameter dishes proposed for the MMA, observing at 345GHz, this corresponds to a pointing error in the range 1.2–1.8 arcseconds, or about twice as good as the current specification. A similar test at λ1.3cm would translate into a MMA pointing error in the range 1.9–2.8 arcseconds. Therefore a VLA mosaicing experiment at either of these wavelengths should tell us whether the current pointing specification is adequate, at least for estimation of the longer spacings. Using data kindly provided by Mike Bietenholz, we have performed such an experiment at λ2cm on the Crab Nebula. These tests are discussed in full elsewhere [7]. The main results are:

- After defining the support of the object, a mosaiced image was obtained which compared favourably with the λ6cm image.
- The final dynamic range of the image is only about 60 or so, but it is difficult to estimate since the mosaiced region is filled with emission. The misfit for each pointing was suprisingly large, with little obvious connection to the peak brightness. The dynamic range per field (defined as the ratio of the peak brightness to the misfit) varied from 7 up to 25. Some possible causes for this would be pointing errors, and very large errors in the primary beam pattern. The variations in dynamic range would seem to make the latter less plausible.

2.3 Limitations on Dynamic Range

This experiment [1] dealt with some of the limitations on the final dynamic range of a compact 3 pointing VLA mosaic of the Cygnus Loop at λ6cm. These limitations were due to a combination of inconsistencies amongst the pointings arising from a poor model of the primary beam, and interpolation errors in reducing the dirty images to a common geometry. The conclusions were:

- Direct, joint deconvolution of the three pointings using `VTESS` yielded a final image with dynamic range of only 50.
- Interpolation errors of well-sampled dirty images impose a dynamic range limit of about 200.
- Subtraction of compact, bright background sources from the visibility data of each pointing eliminates both problems: inconsistencies and interpolation errors.
- A effective general solution is a hybrid scheme, in which the large-scale structure is found from a joint deconvolution of the short spacings, and the fine-scale structure is found from separate deconvolution of the longer spacings from each pointing.

This test stresses the importance of good knowledge of the primary beam, and of accurate interpolation schemes for geometrical transformations.

2.4 Future tests

There remain a large number of tests which will provide useful information. We suggest the following:

1. A repeat of the Crab observations after holography of the high-frequency beam pattern. Re-focusing of the feeds might be worthwhile. Also, a pointing run should be performed during the observations. [Cornwell, Uson]

2. A comparison of a mosaiced VLA image of the Rosette Nebula with data from NRAO 140' observations. Of interest here is the level of agreement between the visibilities derived from the two different observations. [Braun, Lizst]

3. A VLA mosaicing observation of the quiet Sun. This will be supplemented by Arecibo observations, to which it can be compared on the overlapping spacings. [Cornwell, Bastian]

4. Hatcreek interferometer mosaicing observations of the Crab Nebula, and of the Sunyaev-Zeldovich decrement towards two clusters. The goals are to test mosaicing at millimeter wavelengths, and also on very weak objects. In addition it may be possible to measure the total power from the interferometer measurements. The short-spacing information on the Crab will be compared to observations made by one of us (DE) with the Pico de Veleta 30m telescope. [Uson, Cornwell, Emerson]

5. VLA spectral-line mosaicing observations of a sufficiently large object, perhaps M33. In spectral-line mode, the total-power in a channel can be derived from the measured autocorrelation function. [Braun, Cornwell, Uson]

The relevance of these tests is as follows: tests 1, 4 and 5 relate to the Homogeneous Array option, tests 2, 3 and 4 to the Large-Single Dish, and no tests are directly relevant to the Multi-Telescope Array.

2.5 Other Developments

In addition to these explicit tests, we are interested in various technological developments now being pursued by various groups.

Arrays of feeds: Success in building high-quality systems will be very important for the Large-Single Dish option.

Simultaneous total power and interferometry: One of the attractive features of the Homogeneous Array is the possibility of coupling the total power and the interferometric measurements. It would then be easier to maintain internal consistency of the measurements by, for example, some form of self-calibration.

Shaping of illumination patterns: Stable, high-efficiency illumination patterns are important for all mosaicing observations.

3 Theoretical Investigations

Mosaicing is a particularly thorny procedure to analyse theoretically since in part it relies upon a non-linear algorithm, MEM, for a crucial step. Consequently, we do not believe that a full theoretical analysis of mosaicing is possible. We have, however, investigated the related Ekers-Rots scheme of short-spacing estimation [8]. The E-R scheme obtains short-spacing information from the Fourier transform of the visibility function sampled by a single, scanned interferometer; the transform being with respect to the scan-position. Since it is a purely linear procedure, analysis is fairly simple [2]. A summary of the conclusions of this analysis follow:

1. Scanning of the image-plane is not strictly necessary. Instead, a grid of discrete samples will suffice, provided that the separation is not greater than $\frac{\lambda}{2D}$, where λ is the observing wavelength, and D is the antenna diameter.

2. All errors are boosted by a gain factor $\frac{a(0)}{a(\xi)}$ where ξ is the desired offset in spatial frequency from the nominal baseline, and $a(\xi)$ is the Fourier transform of the primary beam of the interferometer.

3. For small numbers of pointings, the error in the derived visibility diminishes as the inverse-square-root of the number of pointings.

Another memo used this analysis to investigate the importance of pointing errors. The most-crucial aspect of the pointing errors is the scaling with antenna diameter. It was shown that provided these errors do not increase faster than the fourth power of the diameter, then the homogeneous array option introduces amplification of pointing errors by at most a factor or two. For more reasonable dependencies, the excess error is negligible [6].

4 Computer Simulations

Since theoretical investigations do not seem to be promising, and since experimental tests are sometimes ambiguous, we suggest that many of the issues concerning the central element are best addressed by computer simulations. We also believe that simulations are necessary to settle more-general questions about the millimeter array design. Since the MMA will be the first radio-interferometric array designed for mosaicing, we believe that a demonstration of the array via computer simulation is vital. We suggest that a simulation package be developed to test mosaicing of complex objects in realistic conditions. The package should produce pseudo-data with the following attributes:

1. Sampling of the u, v-plane appropriate to the various arrays.

2. Realistic primary beams.

3. Appropriate levels of receiver noise.

4. Realistic levels of antenna-based calibration errors, both in amplitude and phase.

5. Pointing errors with appropriate amplitude and temporal correlation.

6. Aperture illumination errors.

Not all of these need be tested simultaneously, but all of these must be explored. The AIPS mosaicing tasks can be used for the actual imaging.

In addition to a comparison of the three main design choices for the Central Elements, a number of other, more-general design choices should be addressed with this package. For example, it has been suggested that since the sidelobes of the primary beams are likely to be a limiting factor, offset-paraboloid antennas are to be preferred to the current on-axis paraboloid with its associated large blockage by the Cassegrain system.

The work in such a package is substantial: we estimate that coding of the package will require about 2 man-years. Subsequent testing will probably consume 0.5–1 man-year per year until the design stage is complete. This manpower is not currently available.

5 Summary

So far, our investigations have centered mainly around measuring spacings interferometrically with the Homogeneous Array option, which, on the evidence of the tests performed so far, is likely to work satisfactorily. However, as yet, we have not demonstrated that acceptable total power measurements can be obtained with any of the three options. We have presented plans for future work which will explore all three options, in both interferometric and total power modes. We have described a number of experimental tests which are important, and we have described how computer simulation can be used to provide some answers.

References

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