MILLIMETER ARRAY
MEMO NO. 59

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Update of MMA Sensitivity Estimates

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This document is an update of sensitivity estimates for the NRAO Millimeter Array (MMA) as it will be discussed at the Nov. 15-18 Workshop in Socorro. It is being written in MathCAD 2.5 so text, mathematical computations, and plots can be mixed together, documenting the assumptions and equations leading to numbers and plots. The definitions and equations correspond to those in Chapter I in Volume I of the MMA Design Study.

First we define some (cgs) constants:

$k := 1.38062 \cdot 10^{-16}$ Boltzmann

$h := 6.6262 \cdot 10^{-27}$ and Planck constants

then we specify some of the assumed parameters. Some are fixed and others set the scale for scaling constants we will compute.

$N := 40 \quad D := 8 \cdot 100$ cm — for 40 antennas with diameters of 8 meters

$\epsilon_a := 0.7$ — for aperture efficiency at 230 GHz

$\epsilon_q := 0.82$ — for correlator quantization efficiency ~~(3-bit)~~ LEVEL

$g := 1$ — g = gamma, an array design constant

$\Gamma := 1$ — another array design constant

$BW := 2.0 \cdot 10^{9}$ Hz — (scaling) effective DSB continuum bandwidth

$T_{sys} := 200$ K — (scaling) 'good" atmosphere Tsys at 230 GHz

$\delta t := 60$ sec — (scaling) integration time

$B_{cm} := 10^{5}$ cm — (scaling) array diameter

$N_p := 2$ — (scaling) number of independent polarizations

$N_B := N \cdot \frac{N-1}{2} \quad N_B = 780$ — (scaling) number of antenna pairs or baselines

and we now calculate a (scaling) value for sensitivity for a single pair

$$\sigma_c := \frac{4 \cdot \sqrt{2} \cdot k \cdot T_{sys}}{\epsilon_a \cdot \epsilon_q \cdot \pi \cdot D^2 \cdot \sqrt{BW \cdot \delta t}} \cdot 10^{23} \quad \text{Jy} \qquad \sigma_c = 0.039 \quad \text{Jy}$$

Now we compute the (scaling) point source sensitivity for 2 polarizations and the array of 40 antennas.

$$\delta S := \frac{\sigma_c}{g \cdot \sqrt{N_p \cdot N_B}} \qquad \delta S = 0.000989 \quad \text{Jy} \qquad \delta S \cdot 1000 = 0.989 \quad \text{mJy}$$

**********> 1 mJy per minute <***********

which is an interesting number to remember.

The associated sensitivity for B.cm = 1 km, and Γ = 1, is

$$\delta T := \frac{\delta S \cdot B_{cm}^{2} \cdot 10^{-23}}{2 \cdot k \cdot \Gamma} \qquad \delta T = 0.358 \quad \text{K for 1 minute}$$

Now we define the functions, σ and δTb, for point source and surface brightness sensitivity for different parameters:

$$\sigma\left[T_{sys}, D_m, BW_{GHz}, \delta t_{min}, N_p, N_B\right] := \frac{\delta S \cdot \left[\frac{T_{sys}}{200}\right]}{g \cdot \left[\frac{D_m}{8}\right]^2 \cdot \sqrt{\frac{BW_{GHz}}{2} \cdot \delta t_{min} \cdot \frac{N_p}{2} \cdot \left[\frac{N_B}{780}\right]}}$$

and

$$\delta T_b\left[B_{km}, \Gamma, T_{sys}, D_m, BW_{GHz}, t_{min}, N_p, N_B\right] := \frac{\left[\frac{B_{km}}{10^{-5}}\right]^2 \cdot 10^{-23}}{2 \cdot k \cdot \Gamma} \cdot \sigma\left[T_{sys}, D_m, BW_{GHz}\right.$$

Checking to make sure they are correct we compute

$\sigma(200,8,2,1,2,780) \cdot 10^{3} = 0.989$ mJy and

$\delta T_b(1,1,200,8,2,1,2,780) = 0.358$ K

Results for other interesting integration times, 1, 8, and 24 hours:

$\sigma(200,8,2,60,2,780) \cdot 10^{6} = 127.705$ μJy per hour

$\sigma(200,8,2,8 \cdot 60,2,780) \cdot 10^{6} = 45.151$ μJy per 8 hours

$\sigma(200,8,2,24 \cdot 60,2,780) \cdot 10^{6} = 26.068$ μJy per 24 hours and

$\delta T_b(0.072,1,200,8,2,60,2,780)\cdot 10^3 = 0.24$ mK per hour for 72 m. array (most compact, 0.5 filling fac.)

$\delta T_b(0.072,1,200,8,2,8\cdot 60,2,780)\cdot 10^3 = 0.085$ mK per 8 hours for 72 m. array

$\delta T_b(0.072,1,200,8,2,24\cdot 60,2,780)\cdot 10^3 = 0.049$ mK per 24 hours for 72 m. array

Let us now set up for calculations for each of four configurations ranging from a compact configuration with 50% filling factor out to 3 km:

$i := 0 ..3$ define subscripts for array-dependent vectors

$B_{km} := \begin{bmatrix} 0.072 \\ 0.25 \\ 1 \\ 3 \end{bmatrix}$

Array	Synth. Beam	Nx=Ny
compact 72 meter configuraton	2.6"*λmm	18 pixels
250 meter configuration	0.76"*λmm	32
1000 meter configuration	0.19"*λmm	125
3 km configuration	0.06"*λmm	396

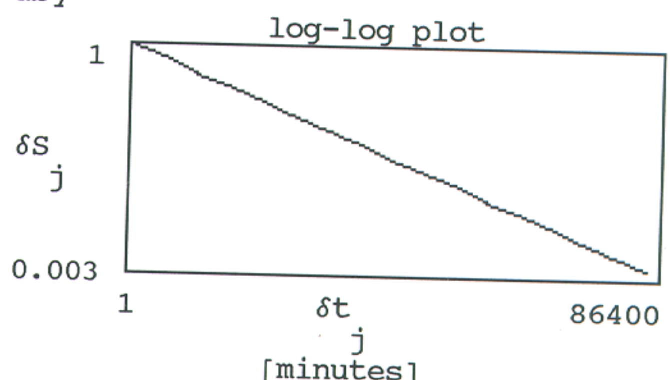
and for a range of integration times (δt) from 1 minute to 24 hours

$n := 17$ $\quad j := 0 ..n$ $\quad \delta t_j := 2^j$ minutes

so we can compute and plot the point source sensitivities for this range of integration times.

$\delta S_j := \sigma\left[200,8,2,\delta t_j,2,780\right]\cdot 1000$ mJy

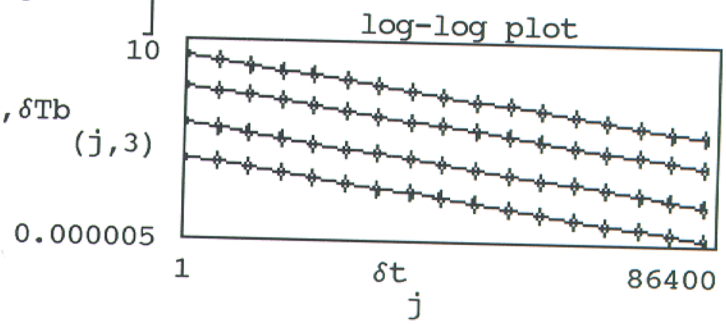
Figure 1 ============>
Figure 1 at the end of memo shows this 2 GHz continumm plot, plus lines for galactic spectroscopy with BW = 100 kHz, extra-galactic spectroscopy with BW = 2 MHz, and a "potential" curve if one can achieve BW = 20 GHz Tsys = 50.



and a matrix of surface brightness sensitivities for each configuration and the same range of integration times.

$\delta Tb_{j,i} := \delta T_b\left[B_{km_i},1,200,8,2,\delta t_j,2,780\right]$ K

Figure 2 $\quad \delta Tb_{(j,0)}, \delta Tb_{(j,1)}, \delta Tb_{(j,2)}, \delta Tb_{(j,3)}$



Better quality versions of Figures 1 and 2, for 230 GHz and Tsys = 200 K, can be found at the end of this document. In addition Figures 3 and 4 are equivalents for the special case of 35 GHz whre Tsys = 35K.

Estimation of System Temperature

Estimation of system temperatures for the MMA appropriate to the end of the 1990's is based upon extrapolation of receiver temperatures from current technology and assumptions about the properties of the atmosphere. At the first MMA Workshop in Green Bank we assumed receiver temperatures of 1 K per GHz of frequency. It is now believed that smaller receiver temperatures than this will be obtainable at the end of the 1990's, so we now take:

$$\mathrm{Trcvr}\left[\mathrm{nu}_{\mathrm{GHz}}\right] := 0.435\cdot \mathrm{nu}_{\mathrm{GHz}} + 9\cdot\left[\frac{\mathrm{nu}_{\mathrm{GHz}}}{115}\right]^{0.75}$$

for contributions of receivers and optics.

appropriate for 115, 230, and 345 GHz.

Defining the appropriate functions to go between temperature and radiation temperature

$$C := h\cdot\frac{10^{9}}{k} \qquad C = 0.048 \qquad \mathrm{Tprime}\left[\mathrm{nu}_{\mathrm{GHz}}, T\right] := \frac{C\cdot \mathrm{nu}_{\mathrm{GHz}}}{e^{C\cdot\frac{\mathrm{nu}_{\mathrm{GHz}}}{T}} - 1}$$

and assuming a 'warm' spillover efficiency of $\epsilon_{l} := 0.85$

and a 'cold' spillover efficiency of $\epsilon_{fss} := 0.85$

$$\mathrm{Tsystem}\left[\mathrm{nu}_{\mathrm{GHz}}, A, \tau\right] := \mathrm{Trcvr}\left[\mathrm{nu}_{\mathrm{GHz}}\right]\cdot e^{\tau\cdot A} \ldots$$
$$+ \epsilon_{l}\cdot \mathrm{Tprime}\left[\mathrm{nu}_{\mathrm{GHz}}, 280\right]\cdot\left[e^{\tau\cdot A} - 1\right] \ldots$$
$$+ \mathrm{Tprime}\left[\mathrm{nu}_{\mathrm{GHz}}, 2.7\right] \ldots$$
$$+ \left[1 - \epsilon_{l}\right]\cdot \mathrm{Tprime}\left[\mathrm{nu}_{\mathrm{GHz}}, 280\right]\cdot e^{\tau\cdot A}$$

where A is the air mass and and τ is the atmospheric optical depth at the zenith. We then have for 230 GHz, A = 1, and τ = 0.065*2 for 2 mm PWV:

$$\mathrm{Tsystem}(230,1,0.065\cdot 2) = 210.654$$

which is the basis for using a (scaling) Tsys = 200 K in the previous calculations of sensitivity. Now let us compute curves of Tsys as a function of zenith angle (Z) at 115, 230, and 345 GHz.

First we set up some vectors for optical depth τ and air mass A.

$$q := 0 ..20 \qquad m := 0 ..10$$

$$Z_q := q\cdot 3.5 \qquad A_q := \frac{1}{\cos\left[q\cdot 3.5\cdot \frac{\pi}{180}\right]} \qquad \tau := \begin{bmatrix} 0.05 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1.0 \end{bmatrix}$$

so A = 1 to 3 for Z (zenith angle) = 0 to 70 degrees

$$T_{m,q} := \mathrm{Tsystem}\left[115, A_q, \tau_m\right]$$

Tsys(115 GHz), τ = 0.05, 0.1, 0.2, 0.3 ...

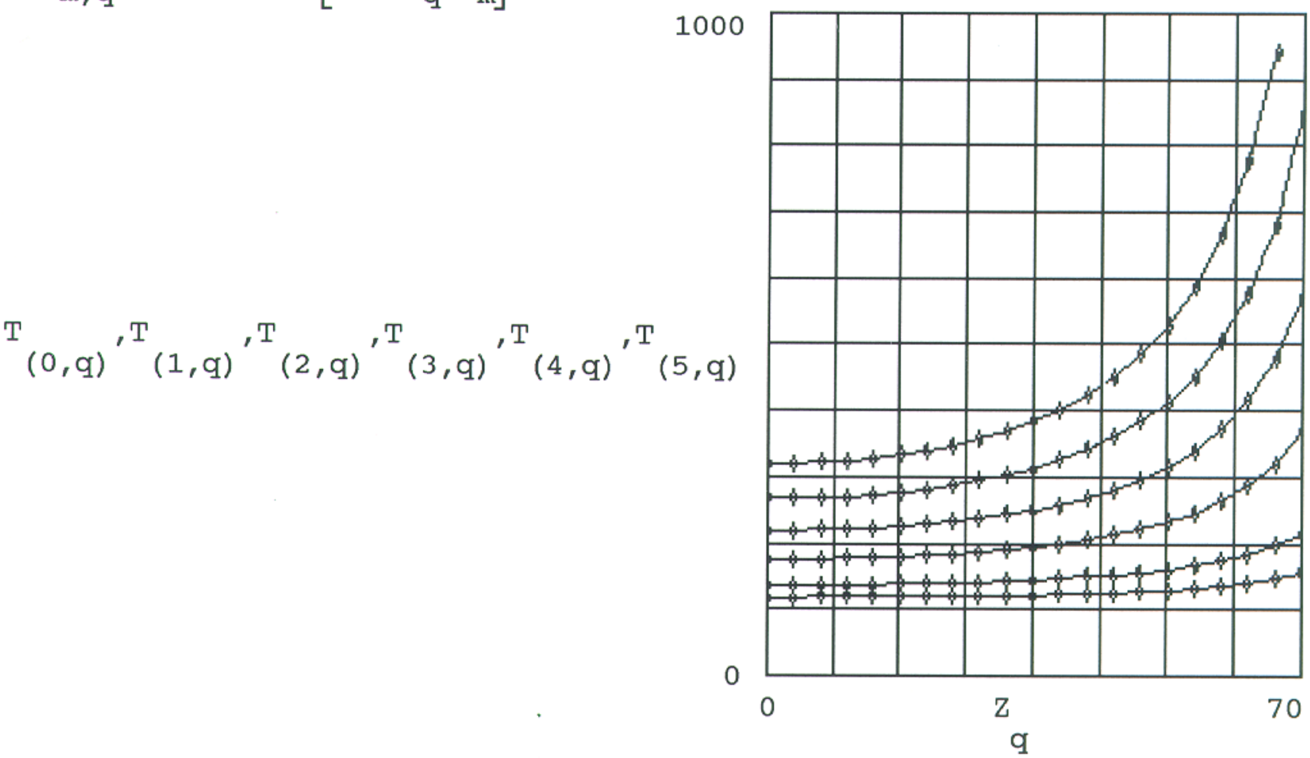


Figure 5

$T_{m,q} := \mathrm{Tsystem}\left[230, A_q, \tau_m\right]$

Tsys(230 GHz), τ = 0.05, 0.1, 0.2, 0.3 ...

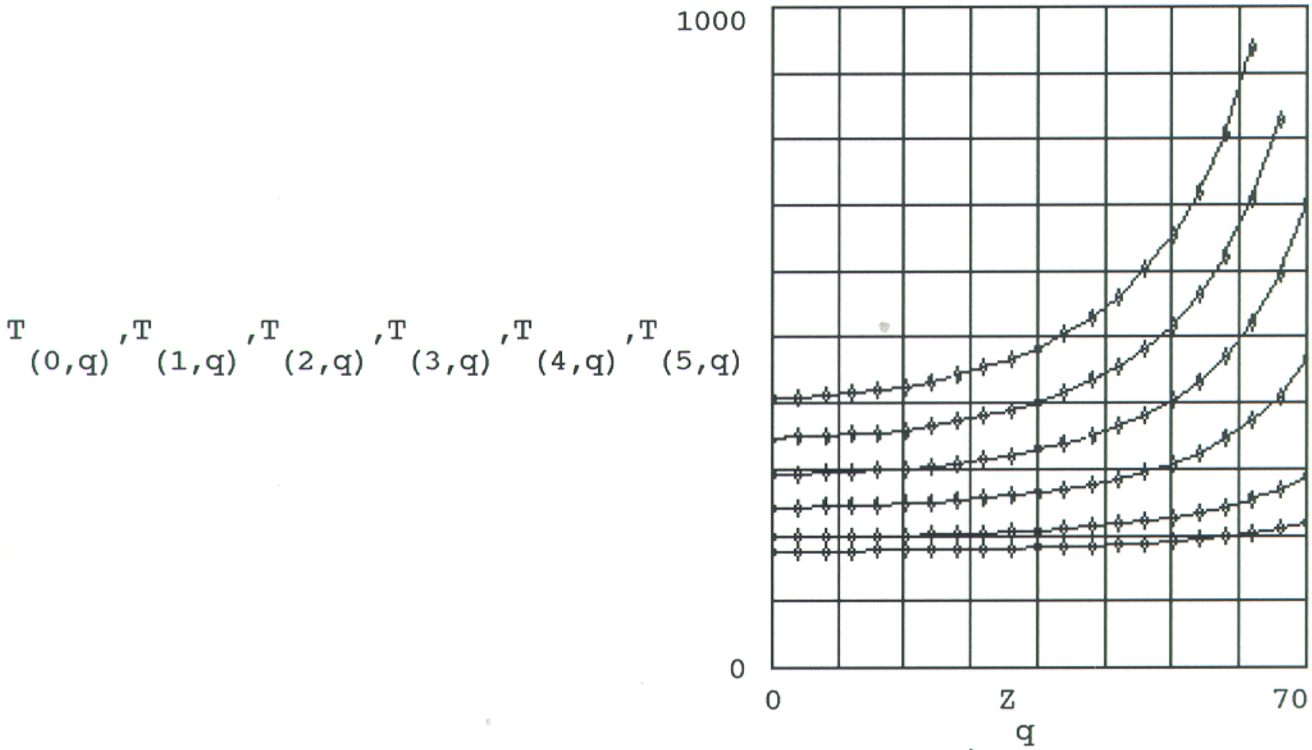


Figure 6

$T_{m,q} := \mathrm{Tsystem}\left[345, A_q, \tau_m\right]$

Tsys(345 GHz), τ = 0.05, 0.1, 0.2, 0.3 ...

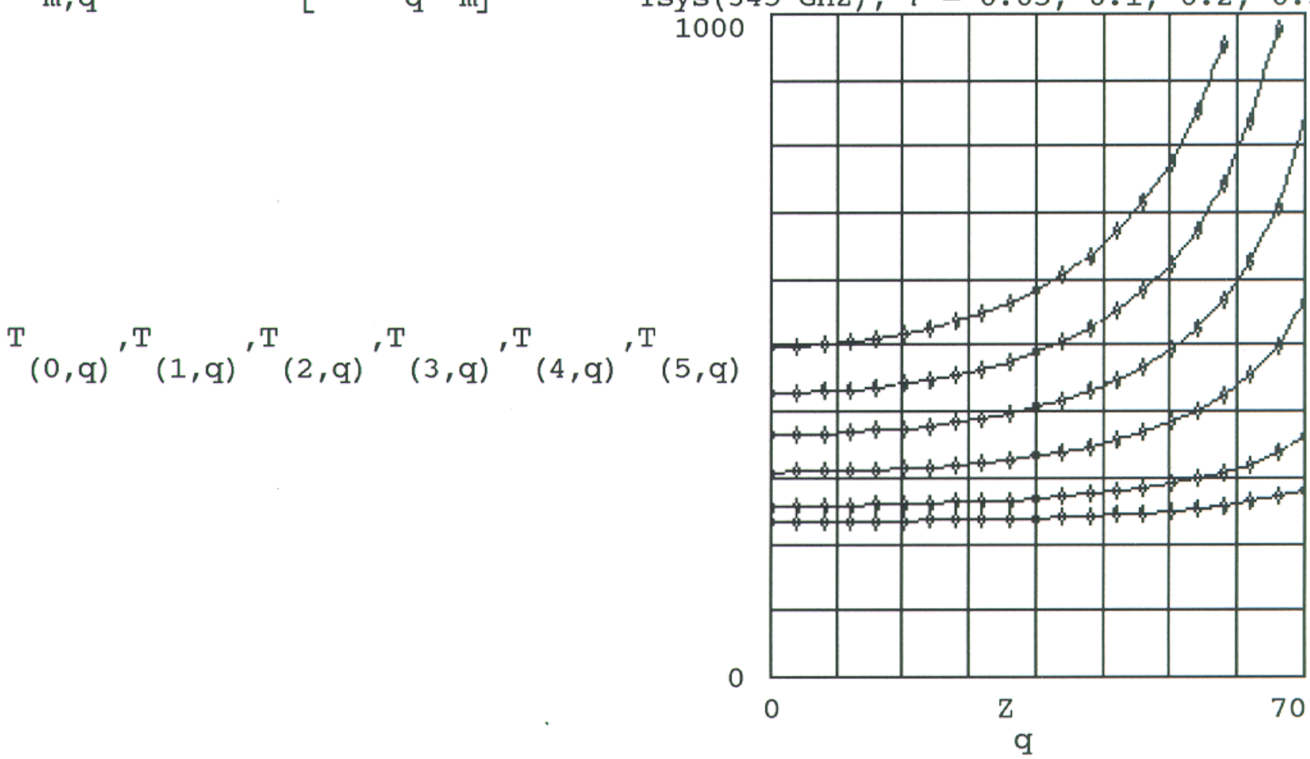
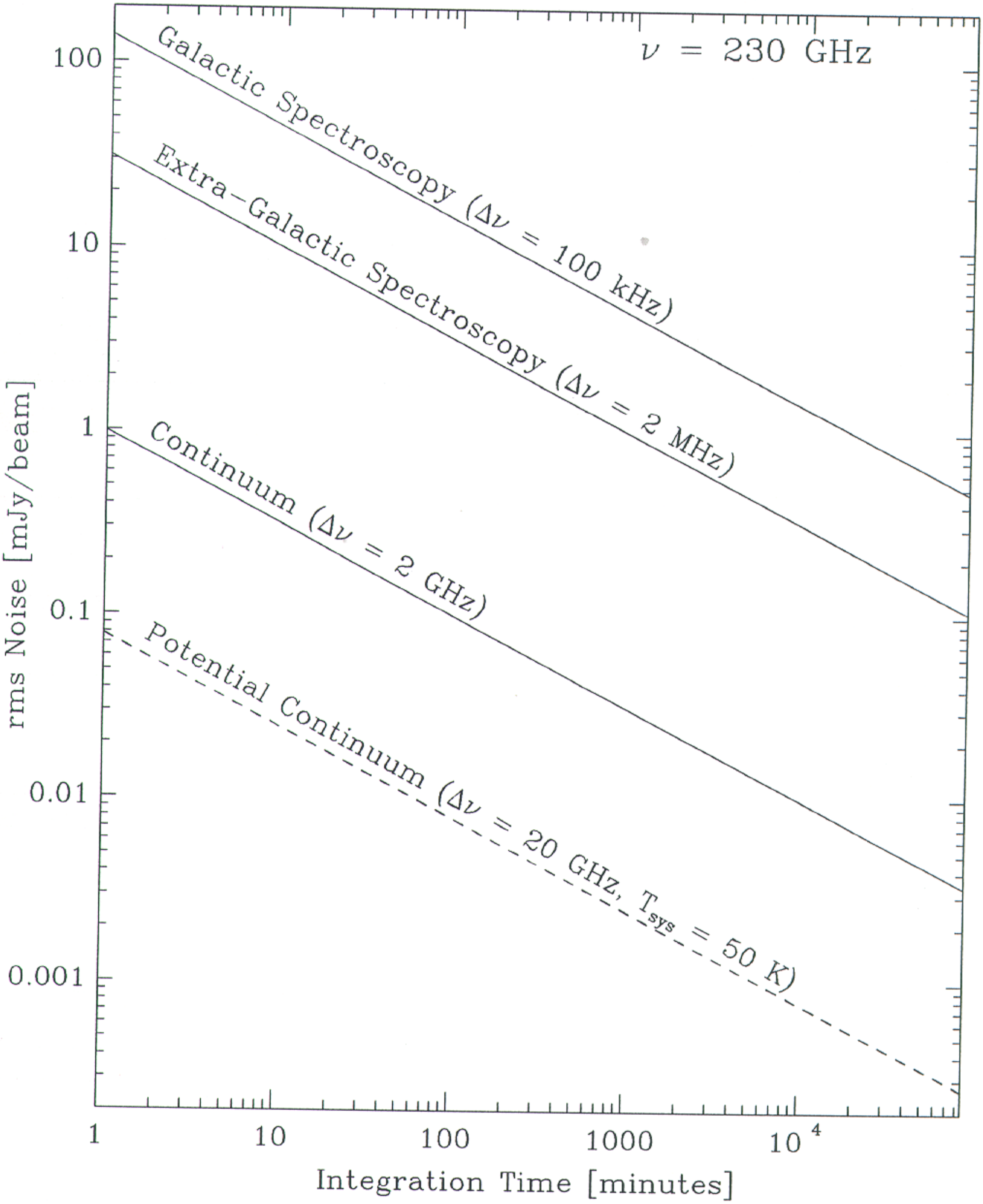


Figure 7



ν = 230 GHz
Galactic Spectroscopy (Δν = 100 kHz)
Extra-Galactic Spectroscopy (Δν = 2 MHz)
Continuum (Δν = 2 GHz)
Potential Continuum (Δν = 20 GHz, T_sys = 50 K)
rms Noise [mJy/beam]
100
10
1
0.1
0.01
0.001
1
10
100
1000
10 4
Integration Time [minutes]

Figure 1

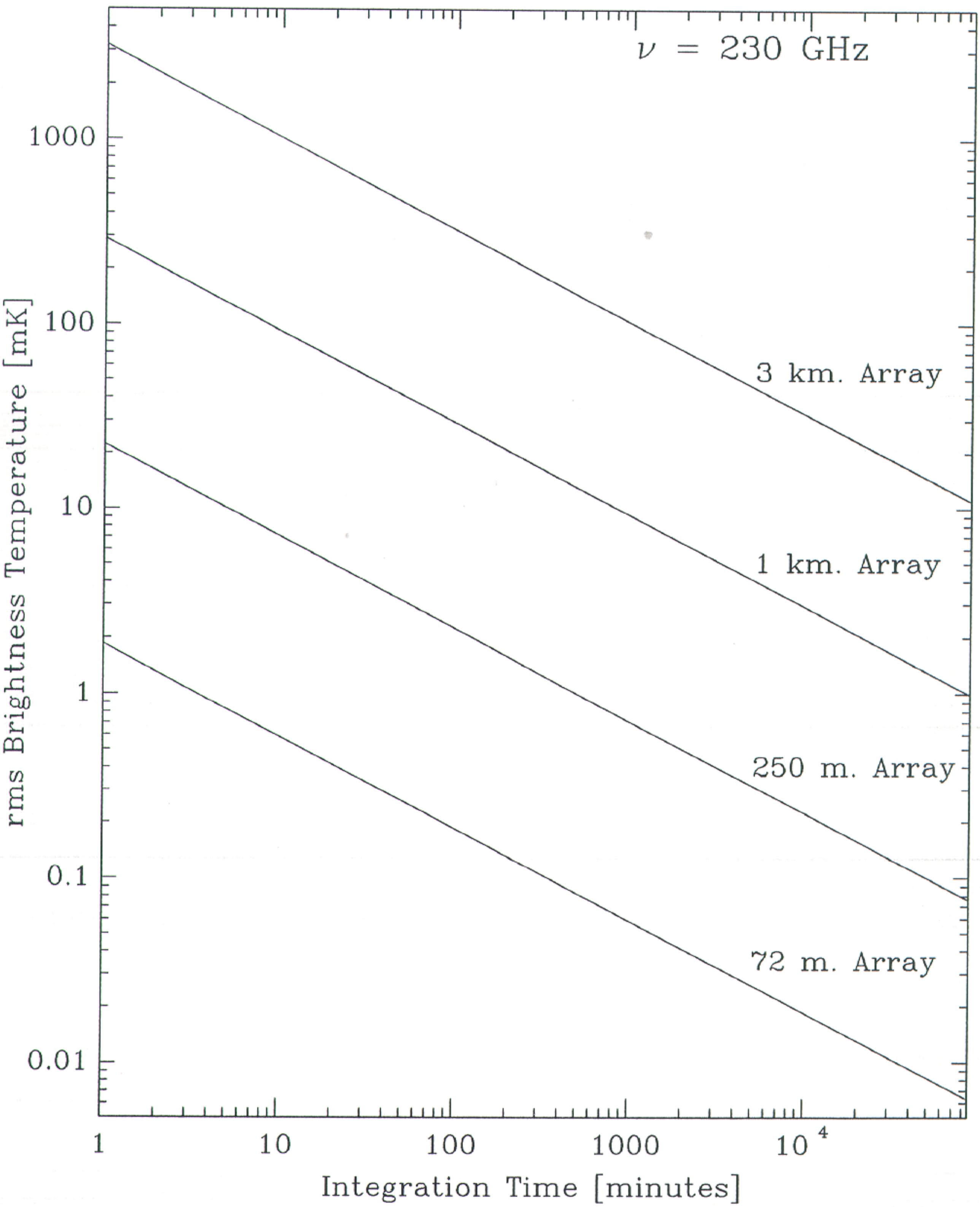

ν = 230 GHz
3 km. Array
1 km. Array
250 m. Array
72 m. Array
rms Brightness Temperature [mK]
1000
100
10
1
0.1
0.01
1
10
100
1000
10 4
Integration Time [minutes]

Figure 2

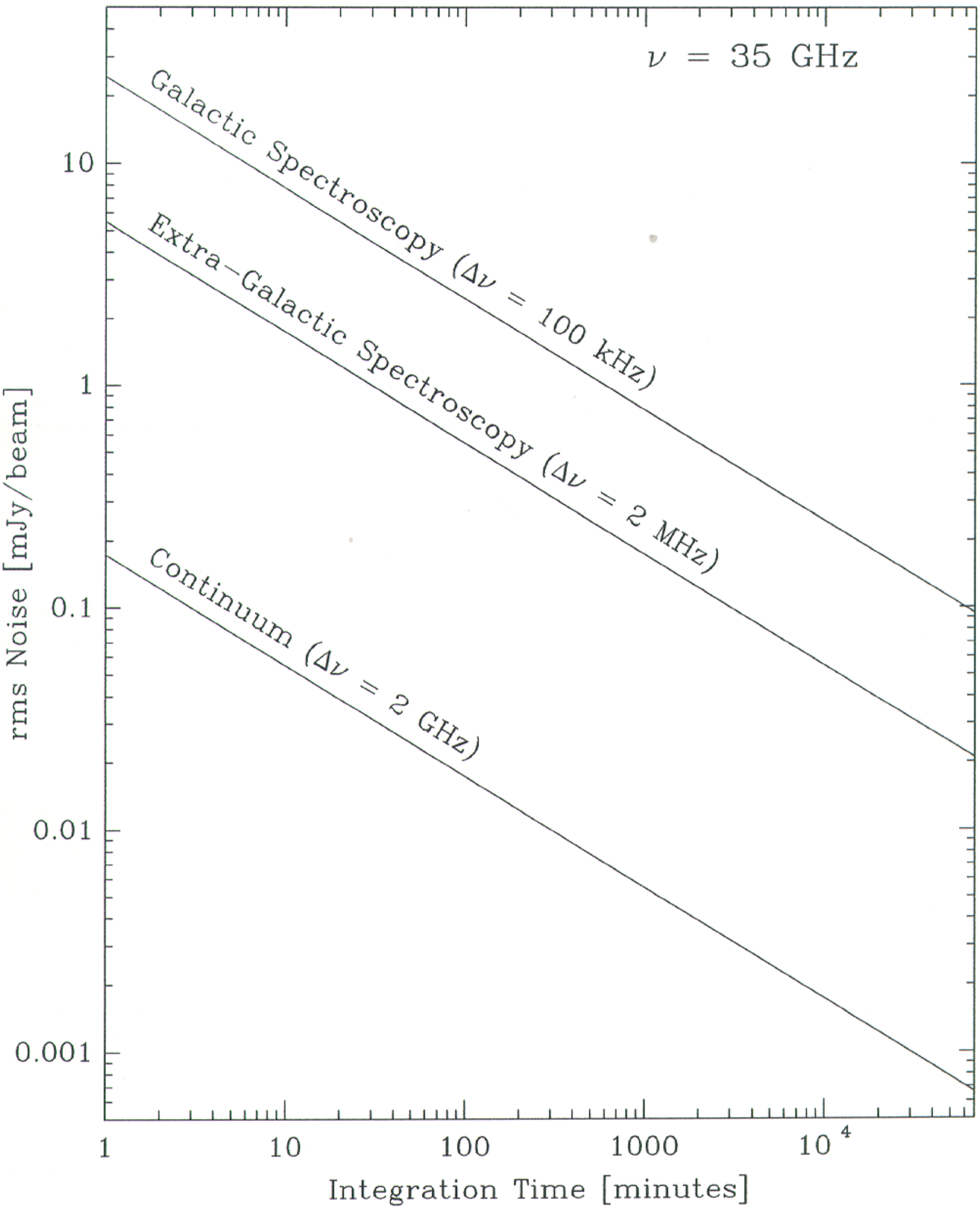
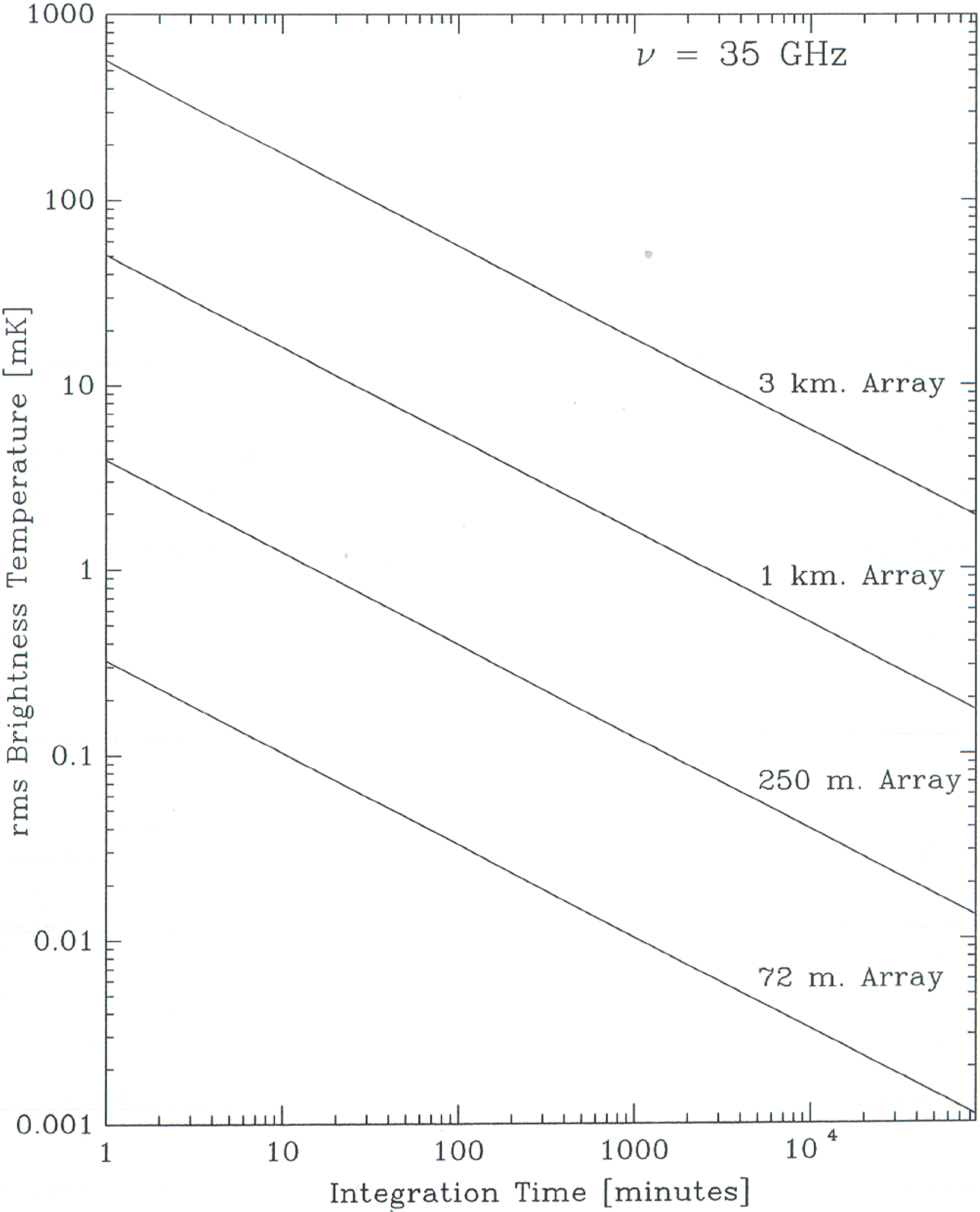

ν = 35 GHz
Galactic Spectroscopy (Δν = 100 kHz)
Extra-Galactic Spectroscopy (Δν = 2 MHz)
Continuum (Δν = 2 GHz)
rms Noise [mJy/beam]
10
1
0.1
0.01
0.001
1
10
100
1000
10 4
Integration Time [minutes]

Figure 3



ν = 35 GHz
3 km. Array
1 km. Array
250 m. Array
72 m. Array
rms Brightness Temperature [mK]
1000
100
10
1
0.1
0.01
0.001
1
10
100
1000
10 4
Integration Time [minutes]

Figure 4

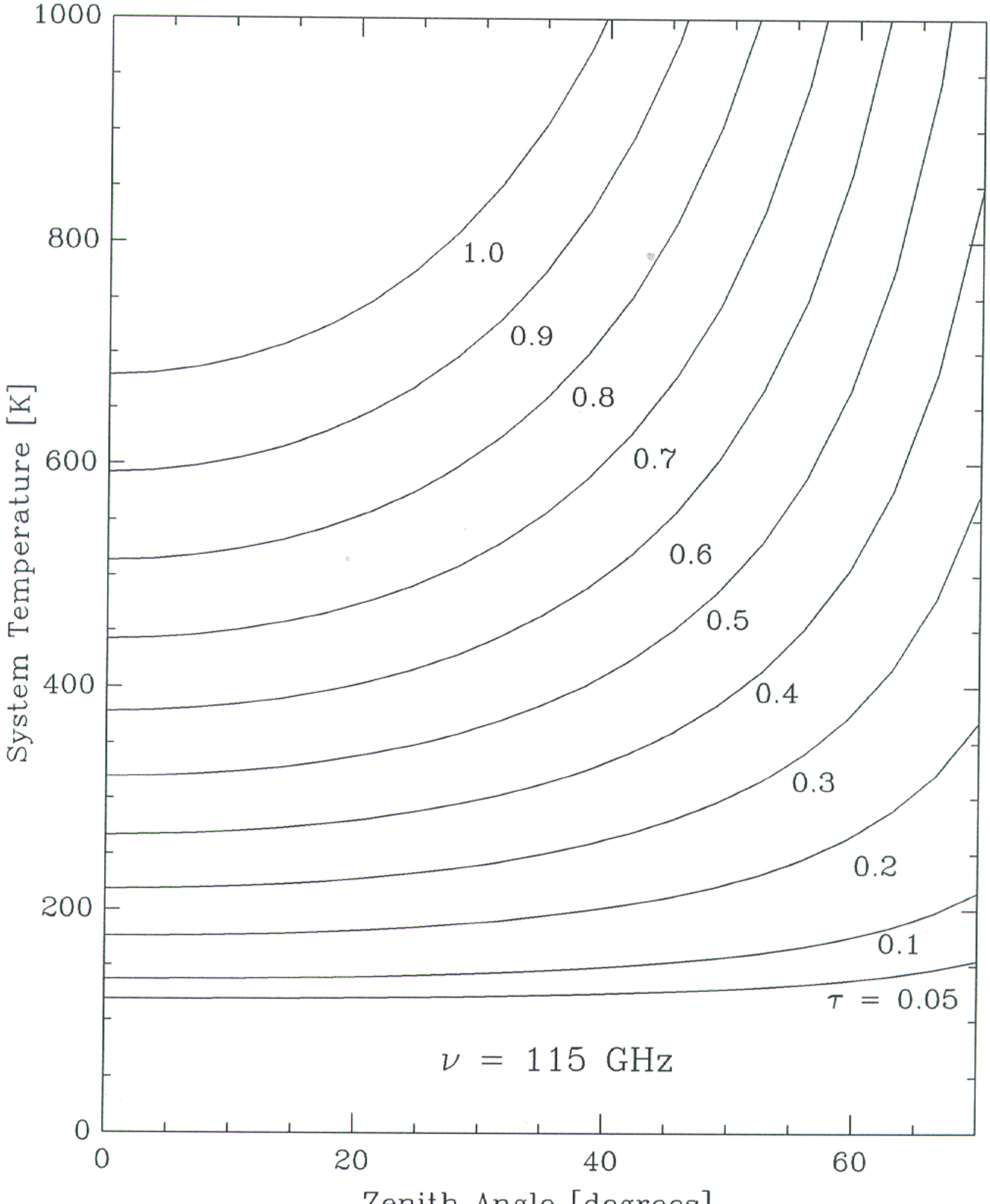

1000
800
600
400
200
0
System Temperature [K]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
τ = 0.05
ν = 115 GHz
0
20
40
60
Zenith Angle [degrees]

Figure 5

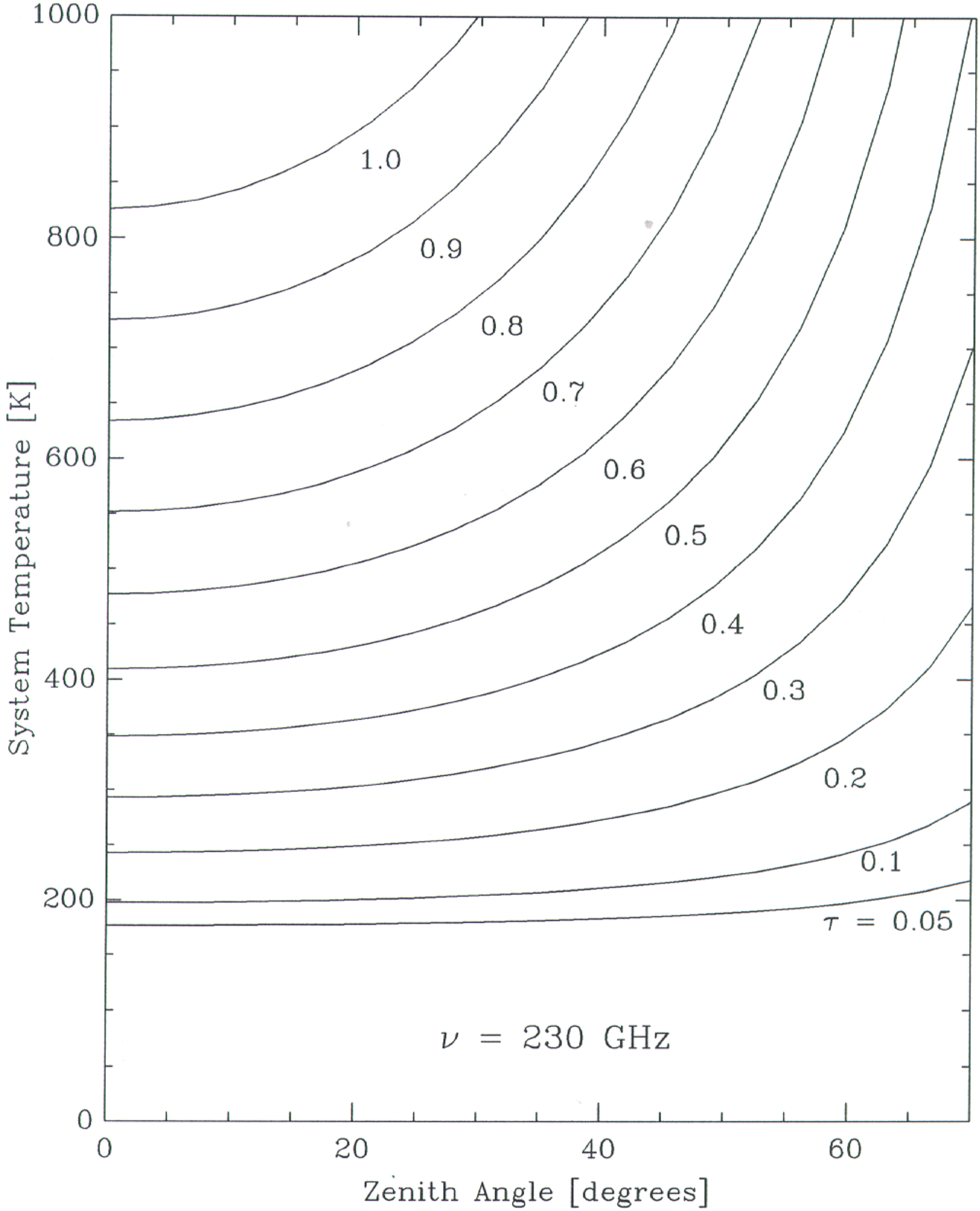

1000
800
600
400
200
0
System Temperature [K]
0
20
40
60
Zenith Angle [degrees]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
τ = 0.05
ν = 230 GHz

Figure 6

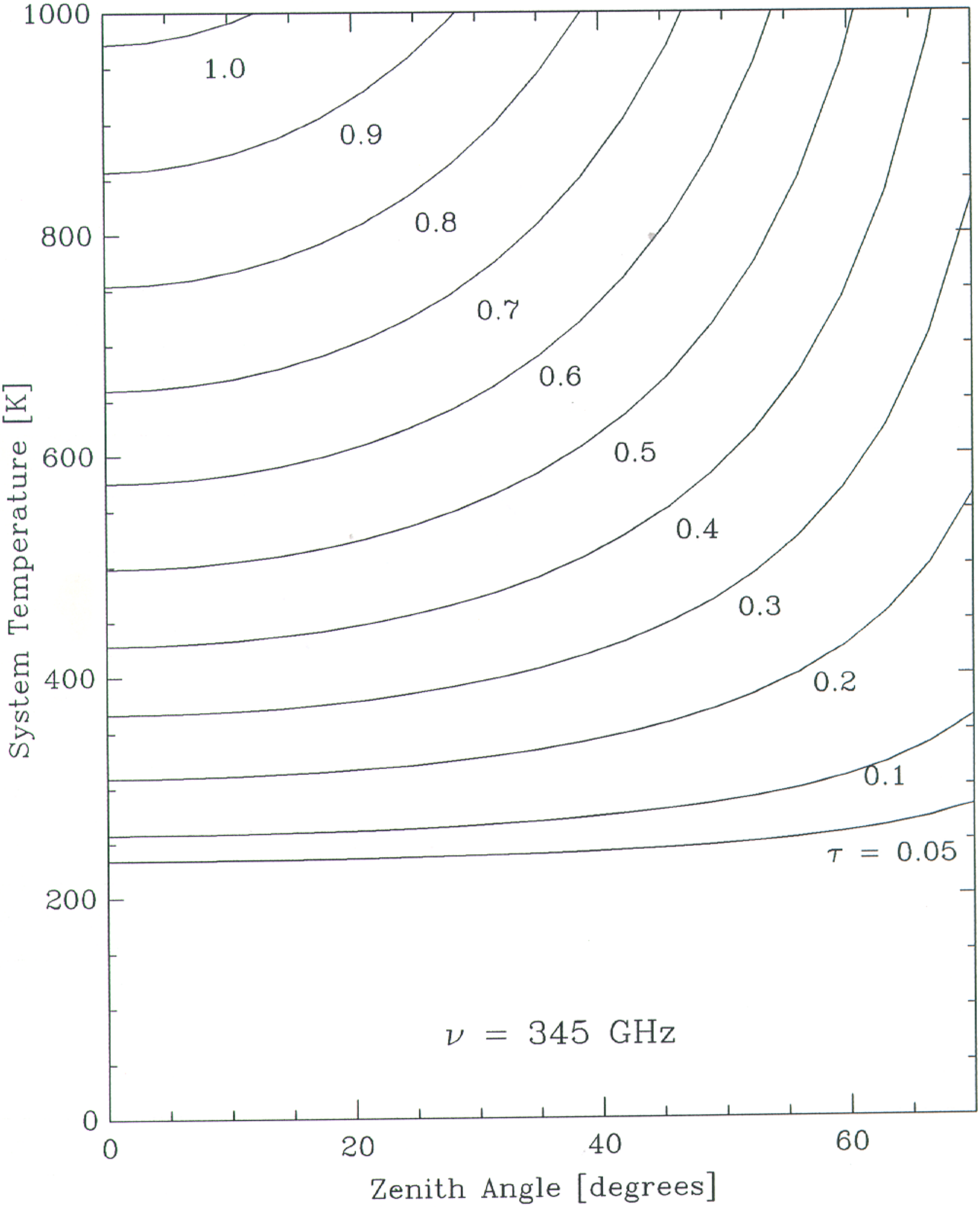

1000
800
600
400
200
0
System Temperature [K]
0
20
40
60
Zenith Angle [degrees]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
τ = 0.05
ν = 345 GHz

Figure 7