Paired Antenna Phase Calibration: Residual Phase Errors and Configuration Study

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1 Introduction

In MMA Memo 68, it was shown how the MMA site survey data could be converted into atmospheric models which could be used in interferometer simulations, leading to estimates of the phase stability and the phase structure function. MMA Memo 84 developed a method of analyzing calibration schemes which can be summarized by the following steps:

- A calibration scheme implies a cycle time t and a distance between the lines of site to the calibrator and the target source in the atmosphere d.
- After calibration, the rms phase errors on baselines which are long compared to $\mathbf{v}t + \mathbf{d}$, will be approximately $\sqrt{2D_\phi(\mathbf{v}t + \mathbf{d})}$, where $D_\phi(\rho)$ is the phase structure function and $\mathbf{v}$ is the velocity of the atmosphere.
- The phase structure functions for a range of atmospheric conditions have been determined from the method of MMA Memo 68 and are presented in MMA Memo 84.
- Interferometric simulations using the method of Memo 68 and the residual phase structure function indicate the image quality which results from the calibration scheme.

Several novel approaches to phase calibration were presented in Memo 84, but because of hardware limitations, none of them are sure to work. This memo looks at another calibration scheme called *paired antenna calibration*, in which the MMA's 40 antennas are paired spatially, and in conditions of poor phase stability, one antenna from each pair will view a calibrator while the other 20 antennas view the target source. The phase fluctuations seen on the calibrator antenna are then applied to its paired antenna for the astronomical observations. The main advantage to this scheme is that there are no technical or hardware problems which will prevent it from working.

In the D, C, and B arrays, the antennas are already close enough to permit a paired antenna scheme to work. However, paired antenna calibration is incompatible with mosaicing in the D and C configurations, which requires complete Fourier plane coverage when imaging fields which are filled with emission, and hence, all 40 antennas are required for observing the astronomical source. This is not a problem since the phase stability in the D and C arrays will usually be quite good anyway, and calibration schemes like paired antennas calibration will not be required. In the A array, observations will be split between bright quasars, which permit selfcalibration, and very weak spectral line or dust continuum objects with fairly simple source structure, which may require hours of integration to get enough signal to meet the scientific objectives. A paired antenna array design must permit high quality imaging of the bright objects using all 40 antennas. Reducing the astronomical part of the array to only 20 antennas for observations of weak sources will half its sensitivity and degrade its (u, v) coverage. For these simple, weak sources, the sensitivity loss is important, but the loss of (u, v) coverage is probably not important. During the best atmospheric conditions, all 40 antennas can observe the astronomical source coherently and at full sensitivity.

In this memo, I analyze the improvement in phase stability due to paired calibration, present a prototype paired antenna A configuration, gauge the effects of the poorer (u, v) coverage, and point out various tradeoffs due to the paired calibration.

2 Analysis of "Pair Calibration"

Following the analysis scheme of MMA Memo 84, the quantity $\mathbf{v}t + \mathbf{d}$ becomes $\mathbf{v}t + \mathbf{d} + \mathbf{B}$, where $\mathbf{B}$ is the typical baseline between the paired antennas in meters. While $\mathbf{v}$, $\mathbf{d}$, and $\mathbf{B}$ are all vectors, we will not treat them as vectors in this document. Treating them as scalars is a worst case. With 20 antennas looking at the calibrator, there is no need for fast slewing, fast switching, or short setup times, so the "cycle time" is simply the time required to detect the calibrator. Using an expression appropriate to 20 antennas and the notation of Memo 84, we get that

$$vt + d + B = v\left(\frac{.002RT_{sys}}{\epsilon_a S}\right)^2 + \frac{h\sin(7.1S^{0.75})}{\sin\theta} + B. \tag{1}$$

R is the required signal to noise per calibrator visibility (R=1 leads to 12° phase errors in the gains for 20 antennas), T_{sys} is the system temperature, ϵ_a is the aperture efficiency, tabulated in Table 2 on MMA Memo 84, S is the brightness of the calibrator in Jy, h is the height of the turbulent layer of water vapor above the array, and θ is the elevation of the target source. T_{sys} depends upon frequency, water vapor column height, and elevation angle as indicated by Equation 15 of MMA Memo 84.

Optimal values for $vt{+}d$ and the minimum calibrator flux (assuming $h = 1000$ m, $v = 12m/s$ (Schwab, 1992), and $B = 0$) are presented in Table 1. The rms phase errors at 230 GHz can be determined for South Baldy from the phase structure functions given in Equations 1-3 of MMA Memo 84 for these values of $vt + d$ and various values of B and are listed in Table 2. With the paired antennas 100 m apart, phases are 30-40% worse than if the antennas are as close together as possible. Comparing the phase errors in Table 2 and those in Tables 4 and 5 in MMA Memo 84, we see that the paired antenna calibration scheme yields phase errors which are comparable to or better than position switching calibration (referred to as *fast calibration*

Elevation	$vt + d$	S
90.000	92	0.45
60.000	103	0.425
30.000	160	0.375
10.000	393	0.300

Table 1: Optimal values of $vt+d$ and S assuming $B = 0$. The calibrator list should be complete down to flux S to ensure calibrators are near enough. For nonzero B, the optimal value of S is unchanged and the optimal $vt + d$ is increased by B.

in MMA Memo 84), but not as good as beam switching phase calibration or simultaneous phase calibration.

Phase errors in Table 2 cannot be directly compared to the phase errors in Tables 4 and 5 in MMA Memo 84. From the imaging simulations reported in Memo 84, a 60° rms residual phase error will permit moderate dynamic range imaging (85:1) with the 40 element MMA. However, using 20 elements to image the target source would require 30° phase errors to obtain the same dynamic range. If we use a paired antenna approach and switch the antennas which view the source every couple of minutes, then phase errors go down by $\sqrt{2}$ and dynamic range goes up by $\sqrt{2}$ if the residual phase errors in each 20 element subarray are independent.

It is assumed that 60° phase errors are required for an image dynamic range of ~85:1 for position switching calibration, beam switching calibration, and simultaneous calibration; 43° for paired antenna calibration, and 38° for infrequent calibration (10 minutes). Using the interpolation scheme used in MMA Memo 68, we can estimate the fraction of time the array will be usable at high frequencies with each calibration method (see Table 3). We can also look at the fraction of time when high quality observations will be possible. 30° phase errors result in an image dynamic range of 250:1 for position switching calibration, beam

ASD(56 s)	Zenith			30 degrees		
	$B = 10$ m	$B = 100$ m	$B = 500$ m	$B = 10$ m	$B = 100$ m	$B = 500$ m
0.04K	2.9°	4.0°	7.4°	7.5°	9.5°	16°
0.07K	7.9°	11°	20°	21°	26°	43°
0.13K	19°	26°	39°	50°	59°	82°
0.20K	31°	43°	65°	83°	97°	135°
0.30K	48°	66°	98°	126°	147°	205°

Table 2: RMS phase errors after paired antenna calibration for atmospheres of various Allan standard deviation (ASD) at 56 s averaging time at zenith and at 30 degrees elevation, for the distance between paired antennas B of 10 m, 100 m, and 500 m.

switching calibration, and simultaneous calibration; 21° for paired antenna calibration, and 20° for infrequent calibration (~10 minutes). Brightness temperature calibration has not been included in these tables because the main contributors to the residual phase errors, namely fluctuations in the atmosphere temperature and the receiver temperature, have not yet been properly analyzed. We also list the increase in noise which goes along with each method and how certain we are that the calibration method will work. The increase in noise is due to three factors: less time, fewer antennas, and less bandwidth (one IF instead of two) on the target source. The assumptions which have entered into the increase in noise are:

- infrequent calibration, it is assumed that the loss in time due to calibration is insignificant.
- Paired antenna calibration utilizes both IFs all of the time for half of the antennas.
- Position switching calibration utilizes both IFs in all of the antennas, but only one third to one half of the time.
- Beam switching calibration uses only one IF since it requires observations at two different frequencies, and only one half to one third of the time will be spent on source.
- Simultaneous calibration allows us to observe the target source all of the time with all of the antennas, but only in one of the IFs.

These assumptions will need to be revised as the MMA design changes, but the basic truth of the matter is that we are trading sensitivity for phase stability. The "technical challenges" column indicates that some of these methods present technical problems in the antenna design which have not been solved at the present time:

- For position switching to be competitive, the antennas must switch position very quickly and remain stable.
- For beam switching calibration to work, the subreflector must be able to switch $1 - 2°$ quickly, point reasonably accurately to both positions, and not distort the antenna's shape too much.
- For simultaneous calibration to work, the optics must be designed to track two different beams separated by 1-2° on the sky.

The Springerville phase structure functions have also been determined. There has been some concern that the calibration techniques would not work as well at Springerville due to a flatter structure function on short baselines. The same analysis has been performed for the Springerville site's A array usability, and is presented in Tables 4 and 6. In this memo, I will deal with the South Baldy data since the residual phase errors after calibration are smaller at South Baldy. Note that this depends solely upon the phase structure function. A future comparison between the Springerville and South Baldy sites will take another look at the phase structure functions at the two sites.

Calibration Method	A array usability at zenith	A array usability at 30° elevation	noise increase	technical challenges?
infrequent calibration	22%	9%	1	no
paired antennas ($B = 10$m)	46%	24%	2	no
paired antennas ($B = 100$m)	38%	22%	2	no
paired antennas ($B = 500$m)	28%	14%	2	no
position switching calibration	46%	27%	$\sqrt{2} - \sqrt{3}$	yes
beam switching calibration	>52%	35%	$2 - \sqrt{6}$	yes
simultaneous calibration	>52%	37%	$\sqrt{2}$	yes

Table 3: **South Baldy.** Estimates of the fraction of time the A array will be usable at high frequencies at zenith and 30° elevation using various calibration schemes. "Usable" means that residual atmospheric phase fluctuations will not limit the dynamic range to below 85:1 at 230 GHz, or below 60:1 at 345 GHz. Much of the usable time will permit higher dynamic range.

Calibration Method	A array usability at zenith	A array usability at 30° elevation	noise increase	technical challenges?
infrequent calibration	15%	3%	1	no
paired antennas ($B = 10$m)	42%	17%	2	no
paired antennas ($B = 100$m)	38%	14%	2	no
paired antennas ($B = 500$m)	27%	6%	2	no
position switching calibration	38%	15%	$\sqrt{2} - \sqrt{3}$	yes
beam switching calibration	>50%	21%	$2 - \sqrt{6}$	yes
simultaneous calibration	>50%	22%	$\sqrt{2}$	yes

Table 4: **Springerville.** Estimates of the fraction of time the A array will be usable at high frequencies at zenith and 30° elevation using various calibration schemes.

Calibration Method	A array usability at zenith	A array usability at 30° elevation	noise increase	technical challenges?
infrequent calibration	10%	4%	1	no
paired antennas ($B = 10$m)	27%	14%	2	no
paired antennas ($B = 100$m)	21%	11%	2	no
paired antennas ($B = 500$m)	14%	7%	2	no
position switching calibration	29%	17%	$\sqrt{2} - \sqrt{3}$	yes
beam switching calibration	45%	21%	$2 - \sqrt{6}$	yes
simultaneous calibration	47%	22%	$\sqrt{2}$	yes

Table 5: **South Baldy.** Estimates of the fraction of time the A array will produce *high quality images* at high frequencies at zenith and 30° elevation using various calibration schemes. "High quality images" means that residual atmospheric phase fluctuations will not limit the dynamic range to below 250:1 at 230 GHz, or below 170:1 at 345 GHz.

Calibration Method	A array usability at zenith	A array usability at 30° elevation	noise increase	technical challenges?
infrequent calibration	~ 0%	~ 0%	1	no
paired antennas ($B = 10$m)	22%	4%	2	no
paired antennas ($B = 100$m)	16%	3%	2	no
paired antennas ($B = 500$m)	6%	2%	2	no
position switching calibration	17%	4%	$\sqrt{2} - \sqrt{3}$	yes
beam switching calibration	29%	6%	$2 - \sqrt{6}$	yes
simultaneous calibration	32%	7%	$\sqrt{2}$	yes

Table 6: **Springerville.** Estimates of the fraction of time the A array will produce *high quality images* at high frequencies at zenith and 30° elevation using various calibration schemes. "High quality images" means that residual atmospheric phase fluctuations will not limit the dynamic range to below 250:1 at 230 GHz, or below 170:1 at 345 GHz.

3 Configuration Feasibility

The current designs for the D and C configurations would not need to be changed as the shortest baseline from any antenna is much less than 100 m. If the B configuration is a randomized ring 800 m in diameter, the average distance between antennas will be 60 m. The configuration design might need to be tweaked up to ensure all antennas' nearest neighbor is less than 100 m away. Hence, such a scheme would result in no loss of flexibility of the D, C, and B arrays: during excellent atmospheric stability, all 40 antennas could be used to observe the source and calibration could be performed on timescales of tens of minutes. For the three smallest configurations, a different calibration method might be used: half of the array would observe the calibrator and solve for a 2-D image of the atmospheric phase as a function of time. The phase error and array usability analysis is the same as for paired antenna calibration, but the appropriate B must be used.

In the A configuration, the average spacing to the nearest neighbor is greater than 100 m, and some antennas will be several hundred meters away from their nearest neighbors. Some improvement will result from performing the paired antennas calibration on the nearest neighbor, but the A configuration can get much more benefit out of this scheme if it is redesigned to have closer nearest neighbors. This illustrates one of the fundamental tradeoffs inherent in the paired antenna calibration scheme: the array is usable for more time if the antennas are placed very close together, but the (u, v) coverage is optimized when the antennas are placed further apart.

To demonstrate the feasibility of a 3 km array of 40 paired antennas, we have performed a configuration study using Ge's configuration toolkit (1992b). The antennas in each pair are located about 100 m distant from each other. The resulting configuration is not intended to be an optimal configuration, but merely indicates that such an array is feasible. To find an optimal array of 40 paired antennas, a modification could be made to the simulated annealing array design scheme presented in Cornwell (1986) and Holdaway (1992b).

A possible paired array design is found in Figure 1. When all 40 antennas are used to look at a source, the (u, v) coverage has a nice banding due to the four correlations between any two pairs of antennas. The result is not unlike the (u, v) coverage which one gets from multi-frequency synthesis, except that the offset between the tracks in the band are not necessarily radial. Examples of the full tracking[1] (u, v) coverage obtained from all 40 antennas is shown in Figure 2. While the (u, v) coverage is good, it is clear that better (u, v) coverage can be obtained in a 40 element array of *unpaired* antennas (see MMA 80, Appendix B, Figure 2b). The snapshot coverage is not very good for the paired configuration.

If the paired antenna calibration scheme is employed, the (u, v) coverage will be much thinner. Some of the coverage can be made up by switching which antennas are in the target source subarray and which are in the calibrator source subarray. The easiest way to switch is to just switch subarrays. However, it is possible to obtain (u, v) coverage which approaches the 40 antenna coverage of Figure 1 by randomly switching antennas in each antenna pair.

[1] Full tracks are defined as observing while the maximum air mass is less than 1.4 times the transit air mass, or 9 hours at $\delta = 60°$, 7 hours at $\delta = 30°$, 6 hours at $\delta = 0°$, and 4.7 hours at $\delta = -30°$.

Array Used	Dynamic Range	Fidelity Index
40 unpaired elements	29000	121
40 paired elements	11800	112
20 element array	770	18
2 20 element subarrays switching every 5 min	1060	24

Table 7: Results of numerical simulations for a variety of arrays.

The antennas in a pair should switch about once every 20 seconds. The disadvantage to such a scheme is that it would complicate the online system and the data handling. So we show the worst case (u, v) coverage in which the two subarrays merely switch from target source to calibrator and vice versa in Figure 3.

4 Imaging Simulations

We now determine how much the image reconstruction is affected by the less complete (u, v) coverage obtained from the paired antenna calibration scheme. Consider four cases: a non-paired 40 element array, a paired antenna 40 element array, a 20 element array, and a paired 40 element array in which the 20 element subarrays take turns observing the target source, changing every 5 minutes. The arrays used in these four cases are closely related: the 20 element array is made up of one antenna from each pair of the paired 40 element array, and the unpaired 40 element array is made by moving one antenna in each pair to a more reasonable location. No errors, atmospheric or otherwise, have been introduced into the data: the questions which are asked are of the (u, v) coverage and how it limits the images. Full track simulated visibilities were calculated for a very complicated source (the M31 HII region used in all MMA imaging simulations) at $\delta = 30°$ every 5 minutes for 7 hours symmetric about transit. The resulting data were imaged and deconvolved using a maximum entropy deconvolution scheme (Cornwell, 1984), and the convolved, residuals added, maximum entropy images were gauged by dynamic range and fidelity index (Holdaway, 1990). The results are reported in Table 7. It is clear that an optimized 40 element array provides the best imaging. However, if the antennas in each pair are separated by $\sim$ 100 m, the 40 element paired array has imaging characteristics which are not that much worse than the unpaired 40 element array. If only 20 elements are used, the source is too complicated for the array, resulting in poor dynamic range and fidelity index. Switching between the target source array and the calibrator array increases both the dynamic range and the fidelity index by almost exactly $\sqrt{2}$. Higher dynamic range could be achieved for a simpler model brightness distribution.

A paired antenna array does not have very good imaging characteristics when performing paired antenna calibration. However, all bases seem to be covered. Assume the MMA is an a paired antenna A array. If the atmospheric conditions are excellent, then no paired antenna calibration is required. If the atmosphere is not good and a bright source is being observed, (ie, a source which is bright enough to permit dynamic ranges exceeding 1000:1 if the image were noise limited), selfcalibration will be possible and paired antenna calibration does not need to be used. In this case, the simulation results for *40 paired elements* apply and dynamic ranges of 10000:1 (or greater for simple source structure) are possible. If a weak source is being observed (noise limited dynamic range is less than 1000:1) the images will either be limited by noise or by residual phase errors. Hence, poor (u, v) coverage will only be a factor for the very brightest, most complicated sources, such as Cygnus A. For this reason, it may be desirable to have both a paired 40 element array and a more optimized 40 element array with 20 stations in common. In other words, the cost for this calibration scheme is the cost of 20 extra stations in the A array.

5 Evaluation of Paired Antenna Calibration

Paired antenna calibration provides a substantial improvement over infrequent calibration, especially at low elevations.

Using half of the array to observe a calibrator and the other half of the array to observe the target source reduces the array's sensitivity by a factor of 2. This must be compared to a sensitivity reduction of $\sqrt{2} - \sqrt{3}$ for the position switching and beam switching calibration methods and no reduction at all for simultaneous calibration and the traditional infrequent calibration.

Some decrease in array usability results when the paired antennas are separated by $\sim$100 m rather than 10 m. However, the great improvement in (u, v) coverage and the improved image quality which results when using all 40 antennas in a paired antenna A configuration with 100 m spacings between paired antennas probably outweighs the small loss in observing time. Larger pair separations should also be explored in future configuration studies.

The limited (u, v) coverage obtained from the paired antenna calibration is not a problem for the weak sources which would require such a scheme. Stronger sources which might be limited in dynamic range due to the limited (u, v) coverage can use selfcalibration to correct for the atmospheric phases, and can therefore use all 40 elements for astronomical observing.

6 References

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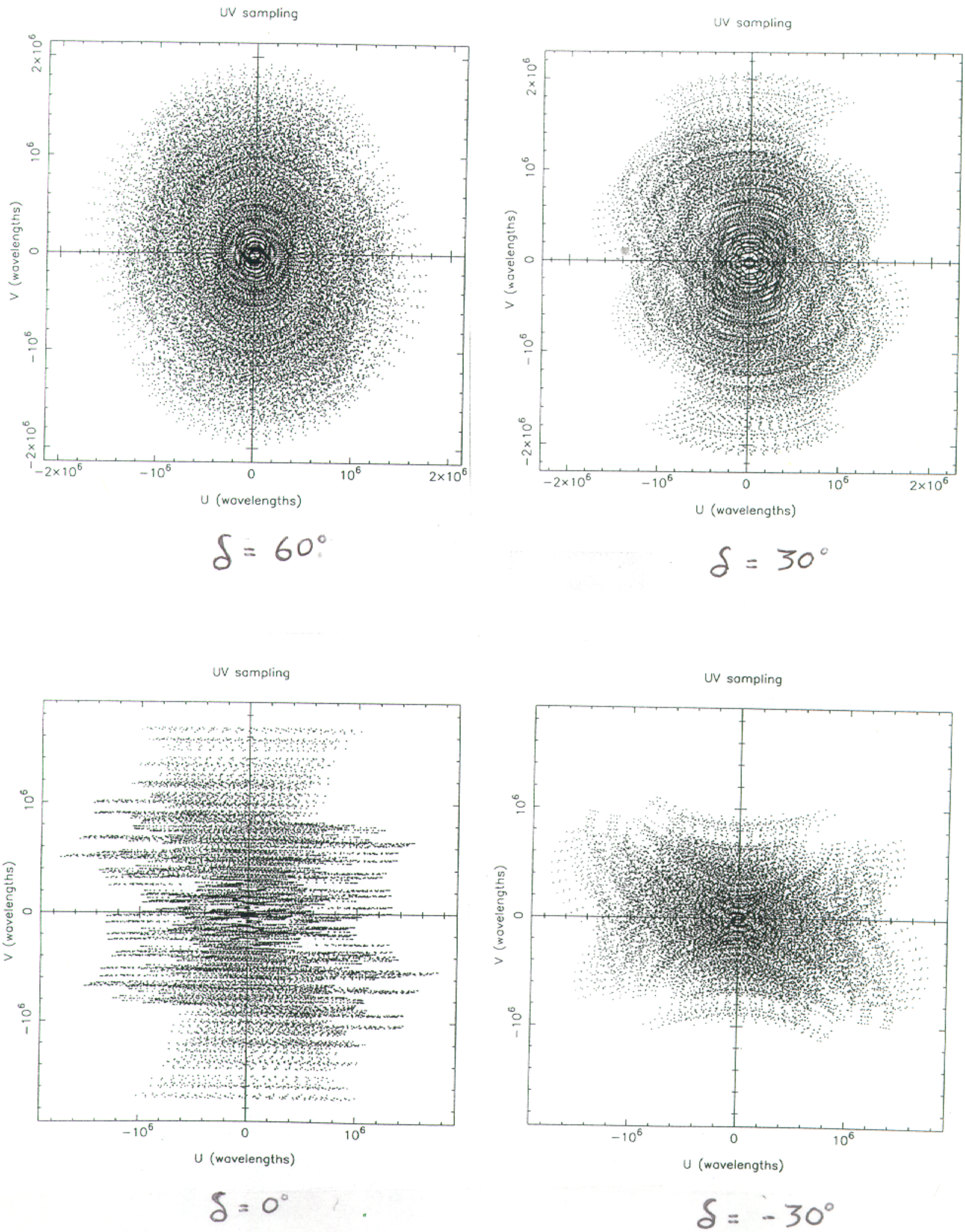

Figure 2: The full tracking (u, v) coverage for all antennas on source.

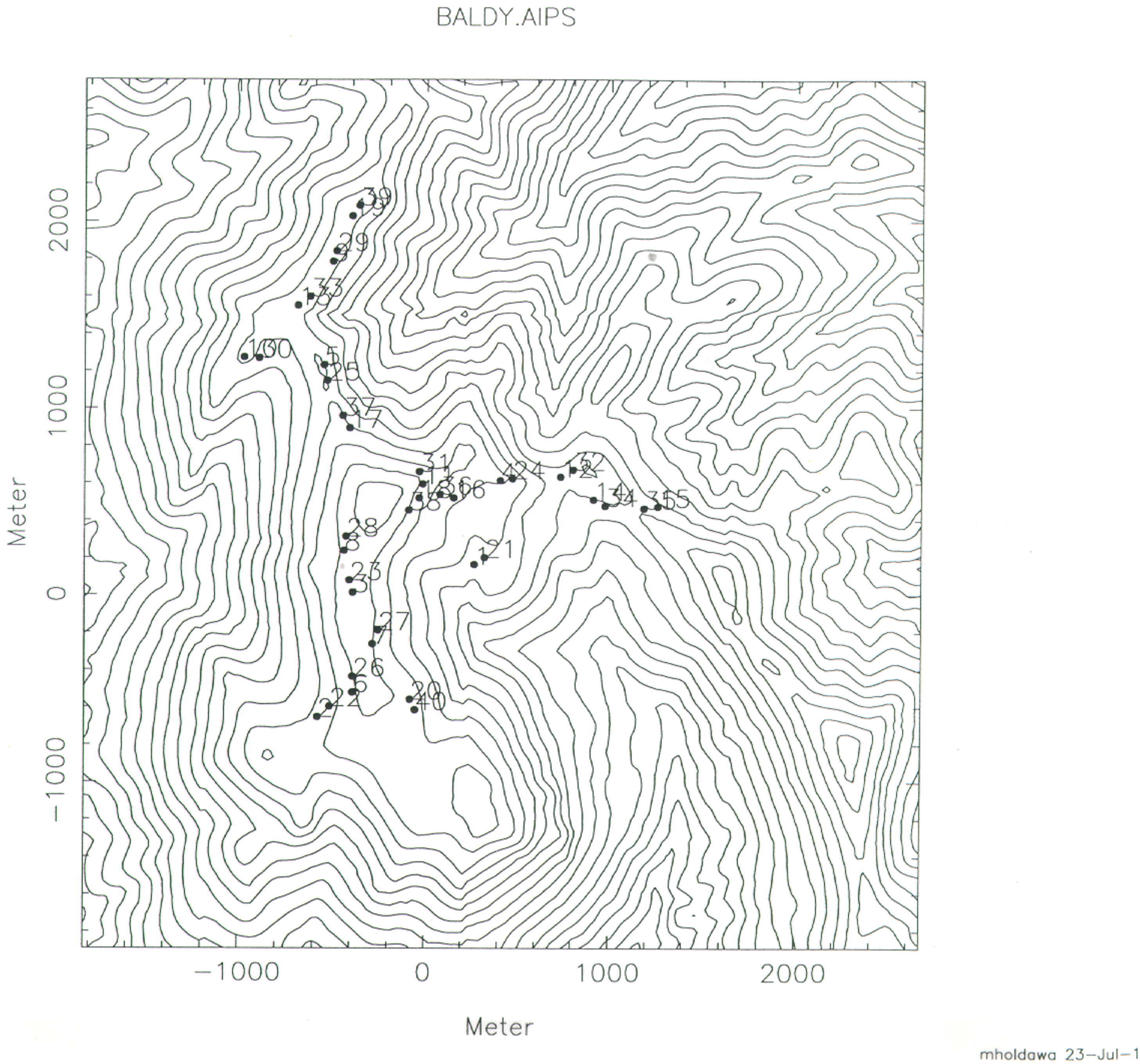

Figure 1: A possible design for an array of 40 paired antennas in the Magdalena Mountains.

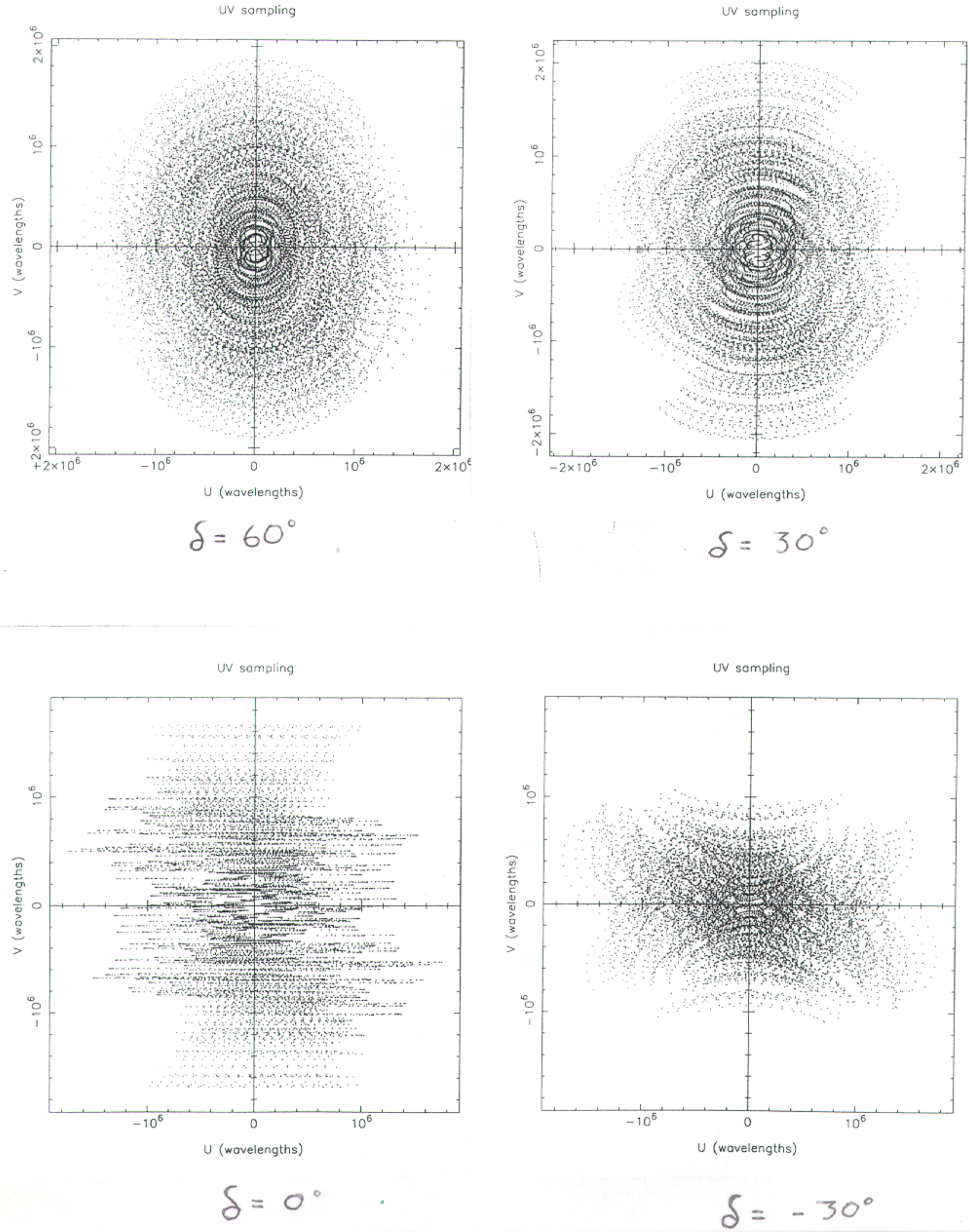

Figure 3: The full tracking (u, v) coverage for half of the antennas on the source and half tracking a calibrator, switching every 8 minutes.