MILLIMETER ARRAY
MEMO NO. 116

The Effect of Beam Offsets on Polarization Measurements

A. R. Thompson
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The pages that follow are a reissue of an early VLA Scientific Memorandum that was written at the time that the offset of the oppositely circularly polarized beams of the VLA antennas was discovered. This same phenomenon is a serious potential problem with the Open Cassegrain design proposed for the MMA antennas, as shown by Peter Napier in MMA Memo No. 115. For this reason it has been suggested that it would be useful to reissue this early memo in the MMA series. It takes a rather simple look at the basic problem of measurements with the beam offsets that occur with circularly polarized feeds, and the corresponding sidelobes that occur with linearly polarized feeds. Since the percentage polarization of radio brightness is often larger at millimeter wavelengths than at the centimeter wavelengths considered for the VLA, some of the approximations in the memo may not be directly applicable to the MMA case.

For measurements of the total intensity with circularly polarized feeds, it is clear that a small pointing offset is not a major problem. One can analyze the RR and LL responses independently, correct the images for the antenna beams using the appropriate pointing in the two cases, and as a final step sum the brightness distributions to obtain Stokes parameter I. For measurements of linear polarization, for which one requires the LR and RL correlations, the effective beams are the geometric means of the power responses for left and right circular polarization. One would like these effective beams to have circular contours to avoid problems as the antenna rotates relative to the sky. In section II of the memo, it is shown that if the contours of the individual right and left polarization beams are circular, and if the beams are represented by a Gaussian model, the contours of the effective RL and LR beams are also circular. The Gaussian model is a good representation near the beam centers, but becomes inaccurate at the beam edges. Thus in practice the contours of the effective LR and RL beams will deviate from circularity towards the outer edges, and one effect of the offsets will be limit the fraction of the main beam in which linear polarization can usefully be mapped. The variation of the circularity of the antenna polarization over the beams will also limit the useful beam area for polarization mapping, and a detailed model calculation would be required to determine which effect produces the more serious limitation. For measurements of the circular polarization parameter V the difference between the RR and LL maps is required, so the effect of the beam offsets is serious unless the percentage polarization is large.

NATIONAL RADIO ASTRONOMY OBSERVATORY

SOCORRO, NEW MEXICO

VLA Scientific Memorandum No. 125

THE EFFECT OF THE BEAM OFFSETS ON POLARIZATION MEASUREMENTS

A. R. Thompson
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I. Introduction

In this memorandum an attempt is made to understand how the offsets of the beams with opposite circular polarization could affect the performance of the VLA. A number of the points mentioned result from discussions with B. G. Clark, P. J. Napier, E. Raimond and L. R. D'Addario. At the time of writing the cause of the beam offsets is not definitely known, but the most probable one is surmised to be the offset feed geometry, since a similar offset for prime-focus fed paraboloids has been described by Chu and Turrin (1973).

As a beginning it may be helpful to recall the response of an interferometer to the four Stokes' parameters with various arrangements of linearly and circularly polarized antennas. These are given in Table I and a derivation of the expression for the general case will be found in Morris, Radhakrishnan and Seielstad (1964). The linear polarization parameters, Q and U, are usually 1-10% of I, and to measure them it is therefore advantageous to use crossed polarization arrangements that do not respond to I. The use of opposite circular feeds dates from Conway and Kronberg (1969) and is more convenient than the use of crossed linears. With the latter it is necessary to maintain accurate orthogonality between the feeds of different antennas and also to be able to rotate the feeds through 45° angles. The circular polarization parameter V is usually very small for observations beyond the solar system, and it is often assumed to be zero to simplify measurement of linear polarization. To separate V from I there is some advantage in using crossed linear feeds.

II. Measurements with Circularly Polarized Feeds

Consider the case where opposite circularly polarized beams are used and one is interested in mapping out to the half-power contour. Within this area it will be assumed that the polarization characteristics remain constant and close to circular, and only the effect of the offset is considered. A gaussian function will be used to represent the beams and this should be fairly realistic within the same beam area. Each beam has an offset Δ as shown in Figure 1. The beam functions in voltage are given by

$$\exp - h^2\{(x+\Delta)^2 + y^2\} \qquad \text{(left)}$$

$$\exp - h^2\{(x-\Delta)^2 + y^2\} \qquad \text{(right)}$$

where the assignment of left and right senses is arbitrary. The power responses, in (r, θ) coordinates, are

$$\exp - 2h^2\{r^2 + 2r\Delta\cos\theta + \Delta^2\} \qquad \text{(left-left)} \qquad (1)$$

$$\exp - 2h^2\{r^2 - 2r\Delta\cos\theta + \Delta^2\} \qquad \text{(right-right)} \qquad (2)$$

$$\exp - 2h^2\{r^2 + \Delta^2\} \qquad \text{(left-right and right-left)} \qquad (3)$$

Let r_o be the radius of the half-power contour of a beam, and this is equal to $0{\cdot}5887/h$. The circle $r=r_o$ represents the half-power contour for zero beam offset. The measured value of Δ for VLA antennas 1 and 2 is very close to 1/30 of the half-power beamwidth, so $\Delta=0{\cdot}0392/h$.

For measurements of I (including cases where the observer is not interested in polarization) it is appropriate to use the mean of the LL and RR responses, so the effective beam is the arithmetic mean of (1) and (2). On the circle $r=r_o$ this mean response for $\theta=0$, 45° and 90° has values 0·5006, 0·4995 and 0·4985 respectively. Clearly the half-power response is so nearly circular that one need not worry about the beam rotation on the sky resulting from the altazimuth mounts of the antennas. In fact, the ellipticity in the mean response introduced by the offsets is small compared with the residual ellipticity one would expect to find in the individual beams.

For measurements of Q and U the LR and RL beams are used and expression (3) shows these to be circular with a response of 0·4985 at $r=r_o$. When using oppositely polarized feeds the residual cross polarization in the antennas always results in some response to all four Stokes' parameters. Of the unwanted responses, only that to I is of significant magnitude. Thus, for example,

$$R_{RL} = V_Q + jV_U + CV_I \qquad (4)$$

where R is the interferometer response and V is the fringe visibility corresponding to the subscripts. C is a complex constant which is generally small, but since V_I is generally large all 3 terms in (4) are likely to be of the same order of magnitude. The constant C is different for each antenna pair in an array, and correction for it is therefore usually applied in the visibility data. This is possible if V_Q V_U and V_I

are measured with the same beam and if the cross polarization does not vary greatly across the beam. The first of these conditions presents no problem as has been determined above. The value of C for each antenna pair is usually obtained by observations near the beam centers, using a strong source with known polarization.

It appears that the arithmetic and geometric means of the offset beams are sufficiently circular and sufficiently similar that the offsets alone should introduce no difficulties in measurements of I Q and U. Of course, the above conclusion depends to some extent on the choice of the gaussian function to represent the beams, but there is some margin for less cooperative behaviour by the actual antennas. Measurement of V, which involves differences between RR and LL responses, is clearly very badly affected by the offsets. Since these differences depend on I to an extent that varies greatly across the beam, there is no straightforward way of removing the I contribution in the visibility data. Measurements of circular polarization can therefore be made only near the central part of the beams.

III Observations with Linearly Polarized Feeds

With linearly polarized feeds the beam offsets do not occur. Would an effect that causes the beam offsets in the circular case have some unwanted consequences when linear feeds are used? This question can be investigated by representing the circular polarization by orthogonal linear vectors (in phase quadrature) and seeing what happens to them. A pure beam offset indicates that a phase gradient exists across the antenna aperture. With circular polarization a phase change is equivalent to a rotation of the orthogonal vectors which represent it. A progressive rotation of the linear vectors across the aperture is therefore to be expected. The sense of rotation reverses in the two halves of the aperture as shown in Figure 2a. The description of the beam offsets by Chu and Turrin shows that such rotations can be caused by reflection at the curved surface of a paraboloid. In the case of the VLA we have a different reflector geometry, and prediction of the rotation caused by reflection would require a complicated computation. In this discussion therefore it will be assumed that the observation of beam offsets indicates that rotation does occur. It is instructive to see how this affects the linear feed case, even though more complex perturbations may also be present in the VLA antennas.

A rotation of the vectors shown in Figure 2a is equivalent to the generation of cross polarized vectors as shown in Figure 2b. For horizontal polarization from the feed a vertically polarized component is generated, and vice versa. The magnitude of the cross polarized components projected onto the s axis in Figure 2c is proportional to the magnitude of the main component multiplied by s to give the linear dependence. The radiation patterns in voltage are given by the Fourier transforms of the above functions, and from the derivative theorm for Fourier transforms* (Bracewell, 1965) the pattern function for the cross polarized component is purely imaginary and proportional to the first derivative of the pattern for the main component. Thus the cross polarization pattern consists of sidelobes which are in phase quadrature with the main beam radiation and lie in the plane in which the offsets occur for circular polarization. They have opposite phases on either side of the main beam, and peak at the points of maximum slope of the main beam which are close to the half-power points.

The level of the cross polarized radiation can be roughly estimated as follows. For a beam tilt Δ with circular polarization, the phase at the edge of the aperture relative to that in the plane of symmetry is $2\pi\Delta a/\lambda$, where a is the radius of the aperture. For the present VLA antennas this is equal to 0·11 radians. Thus at the edge of the aperture the cross polarized component is 0·11 of the amplitude of the main component. The average amplitude over the antennas must be about 75% of that at the edge, which corresponds to an average power in the cross polarized component of -22dB relative to the main component.

What effect would such sidelobes have on measurements with linearly polarized feeds? For measurements of I with parallel feeds the sidelobes have very little effect. With crossed feeds the sidelobes introduce a response to I as follows

$$R_{VH} = V_U + jV_V + D_1V_1 \tag{5}$$

$$R_{HV} = V_U - jV_V + D_2V_1 \tag{6}$$

The constants D_1 and D_2 should have magnitudes of about 0·08 at the peak of the -22dB sidelobes. Since the sidelobes are in quadrature with the main beam D_1 and D_2 are mainly imaginary, and consideration of the directions of the vectors

*If F(s) has a Fourier transform f(x), sF(s) has a Fourier transform $-jf'(x)/2\pi$.

in Figures 2a and 2b leads to the conclusion that D_1 and D_2 have opposite signs. Thus if (5) and (6) are added to obtain V_U the D terms largely cancel. So it appears that the effect of the sidelobes should be small for measurements of Q and U but large for measurements of V. The situation is very much the same as was found in section II for the use of circularly polarized feeds. This may be taken as an illustration of a statement made by B. G. Clark to the effect that the problem is fundamental to the antenna and feed, and is not affected by the polarizer or later stages of the system.

IV Conclusions and Comparison with Observed Polarization Behaviour

Only the effects of a pure beam offset have been considered above. In section II the state of polarization was assumed to remain constant over the beam area of interest, and this means that the rotation of the polarization vectors at any point in the aperture must be independent of the polarization angle of the incident radiation. In section III only the effect of rotation varying linearly across the antenna was included. As already mentioned, these simplifying assumptions may not necessarily apply to the VLA antennas.

The best present evidence of how the VLA antennas respond is found in VLA Test Memorandum No. 111 by B. G. Clark. This shows the results of observations of an unpolarized source (3C147) with circularly polarized feeds at 6 cm wavelength. Data were taken at the beam center and at half-power points in both azimuth and elevation. When all the data are corrected for the instrumental cross polarization at the beam center, the half-power values correspond to circular polarization between +15.6 and -13.5% and linear polarization between 0.7 and 5.8%. The apparent circular polarization has opposite signs on opposite sides of the beam, and is of about the same magnitude as the difference between expressions (1) and (2). It is therefore readily understood. (Note that the direction of the 6 cm feed offset on the antennas makes an angle of about 25^{o} with the vertical so some offset occurs in both the horizontal and vertical directions). The apparent linear polarization would not be predicted from the above discussion or the characteristics of the offset beams given by Chu and Turrin. The case for the VLA antennas is evidently more complex, and possible reasons for this include the use of a Cassegrain system, the non-parabolic shape of the reflector, and the fact that Chu and Turrin consider only a small section of a paraboloid which does not extend around the axis. The pattern of the variation of cross polarization across the beams is unlikely to be circularly symmetrical, so, as Clark points out, with altazimuth mounts its effects cannot be simply subtracted from the final maps.

One can conclude that with the present feed system, circular polarization can be measured only for sources close on the beam center, and the usefulness of this possibility at the shorter wavelengths is restricted by pointing errors. Linear polarization performance out to the half-power contour may be marginally acceptable with the use of rather clumsy correction techniques. These characteristics, if not improved, could well be a hindrance to development of more refined observing techniques in future years.

Addendum (7/8/76)

B. G. Clark has repeated his measurements with the new 6 cm feed designed by J. J. Gustincic, observing 3C273. The apparent circular polarization values are slightly less. The linear values are much reduced and lie between 0.7 and 2.2%. The performance with the new feed is much closer to the pure beam-offset case discussed in sections II and III. Evidently the old feed introduced cross polarization which varied across the beam, so that a combination of two undesirable effects was present.

References

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TABLE I

INTERFEROMETER RESPONSES WITH VARIOUS FEED ARRANGEMENTS

	DESCRIPTION	RESPONSE
||	Parallel Linear, Vertical, VV	I + Q
=	Parallel Linear, Horizontal, HH	I - Q
+	Crossed Linear, VH and HV	U ± jV
×	Crossed Linear, 45°	Q ± jV
	Identical Circular, LL	I + V
	Identical Circular, RR	I - V
	Opposite Circular, LR and RL	Q ± jU

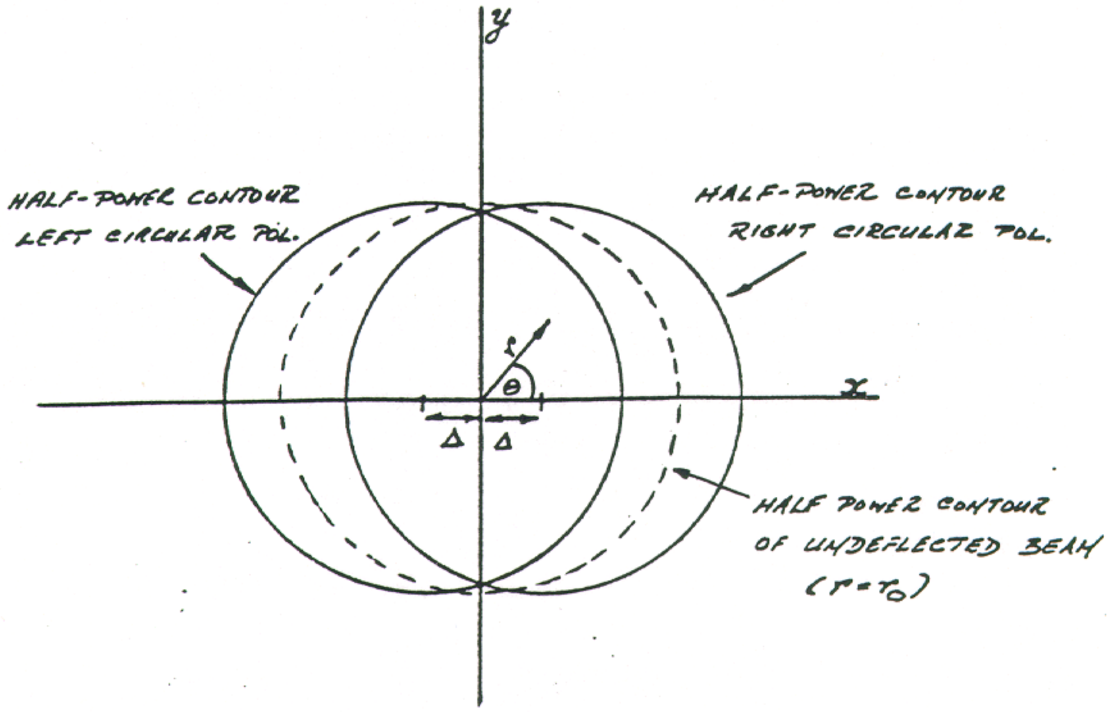

Figure 1. Contours of offset beams. The y-axis lies in the plane of symmetry of the antenna.

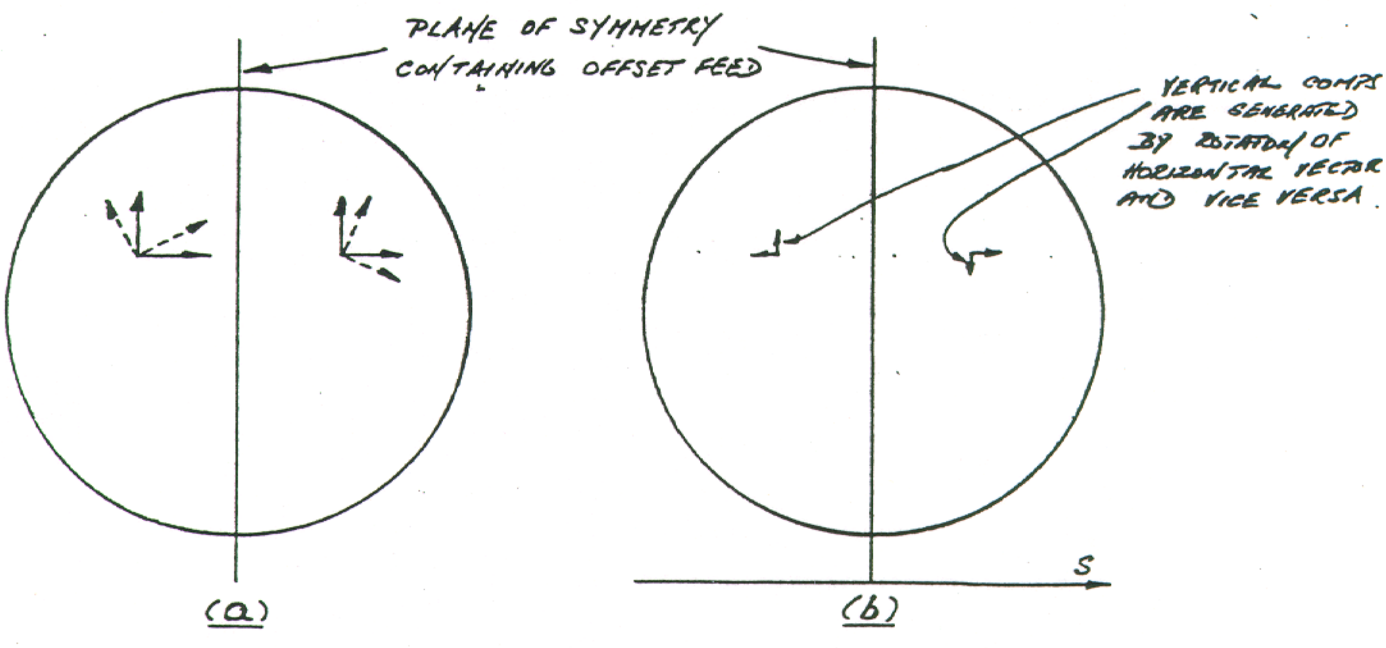

Figure 2. Aperture of antenna. (a) Full-line arrows show E-vector as transmitted from feed, and broken arrows as rotated after reflection. (b) Rotation is equivalent to the generation of the cross polarization vectors shown.

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