ALMA memo 530
Coherence estimation on the measured phase noise in Allan standard deviation

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ABSTRACT

In this memo, a technique for signal coherence loss estimation for Interferometer is introduced. We discuss the coherence loss caused by the phase noise in order to find out the cause of dominant coherence-loss source, and introduce a concept of coherent integration time. The coherence loss can be calculated from the measured phase stability in Allan standard deviation. The key of an interferometer is to maintain the signal coherence. In a connected interferometer, the instability of the distributed common reference signal is compensated for as a common noise. On the other hand, the independent instability of each element decreases the signal coherence. For ALMA, the instability of LLC (Line length corrector), WMA (Warm multiplier assembly) and CMA (Cold multiplier assembly) are independent and/or mounted on independent antennas. The frequency standard (including signal transmission and multiplier chain) of ALMA must be stable over the long-time period (up to observation time) and the short-time period (coherent integration time for fringe detection) to maintain the coherence. We will discuss the required phase stability and the coherence loss in the Allan standard deviation. This coherence estimation is essential for the VLBI application. As a result, the ALMA specification is good enough to keep the coherence, the estimated instrumental coherence losses are 5 % in LO, and 10 % in total (exclude atmospheric scintillation) at 938 GHz with over 260 sec coherent integration time.

1. FREQUENCY INSTABILITY

We can analyze the behaviors of phase noise, by using the Allan standard deviation[1,2] (is commonly called Allan variance).

Frequency Instability refer to a spontaneous and/or environmentally induced frequency change within a given time interval. In other words, frequency instability represents the degree to what the output frequency of a frequency standard remains constant over a given period of time. Since the atomic frequency standard is usually operated as a clock over a long period of time in the time and frequency applications, a measure that can express instability in the short/long term is required to assess the frequency instability.

Noises can be classified into five types according to the noise generation mechanism.

White PM (phase modulation) noise (τ^{-1}) is caused by the additive noise that always overlaps signals generated with the oscillator. The additive noise in the low frequency region of electro-magnetic wave is thermal noise.

Flicker PM noise (τ^{-1}) is produced by phase modulation of flicker noise. The modulation is caused by the non-linearity of amplifiers.[4]

White FM (frequency modulation) noise ($\tau^{-1/2}$) is produced by disturbed oscillation caused by the noise in the oscillator loop within the oscillator. Its amplitude depends on the oscillator Q value, which represents the sharpness of oscillation.

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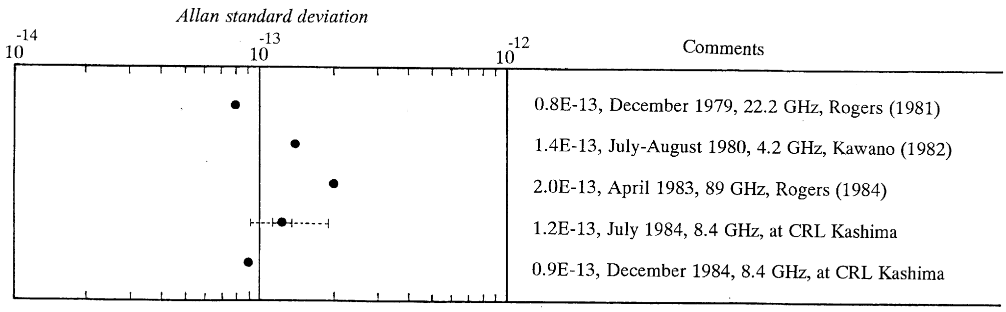

Figure 1. Atmospheric fluctuation in Allan standard deviation. σ_y (τ is less than 1000 sec) Atmospheric fluctuation is a flicker frequency noise less than 1000 seconds.

Flicker FM noise (τ^0) and Random walk FM noise ($\tau^{+1/2}$) are factors limiting the long-term stability. These types of noise are also produced in electronic circuits and frequency standards, according to the environmental conditions.

2. COHERENCE ESTIMATION BY ALLAN STANDARD DEVIATION

The fractional loss of coherence due to the instability in the frequency standard for *T-sec* integration times is estimated by Eq. (1)[8,5]

$$L_c = \omega_o^2 [\frac{\alpha_p}{6} + \frac{\alpha_f}{12} T + \frac{\sigma_y^2}{57} T^2] \tag{1}$$

where

L_c	the loss of coherence,
ω_o	the angular frequency of local oscillator,
α_p	the Allan variance [$(standard\ deviation)^2$] of white phase noise at 1 sec, $(1 \times 10^{-13})^2$: hydrogen maser*
α_f	the Allan variance [$(standard\ deviation)^2$] of white frequency noise at 1 sec, $(7 \times 10^{-14})^2$: hydrogen maser*
σ_y^2	the constant Allan variance [$(standard\ deviation)^2$] of flicker frequency noise, $(5.5 \times 10^{-15})^2$: hydrogen maser*
T	the integration time [sec].

* The hydrogen maser was produced by CRL (Communications Res. Lab. in former, now NICT: National Institute of Information and Communications Technology[13]), and the stability was measured[7] in 1984.

Another important point: the long-term stability of the frequency standard is necessary for regulating the results of each observation or incoherent integration during analysis. The typical stabilities of Atomic clocks are shown in Fig. 2.

3. PHASE NOISE AND TIME ERROR

To calculate the coherence loss and time error, it is convenient to use the Allan standard deviation. One of the stability measurement methods in time domain (Allan standard deviation) is the DMTD (Dual-Mixer Time Difference; Fig. 3[12]) method[2] which is used in NIST,[12] NICT[13] and other time/frequency standard institutes. The method is a double difference phase measurement method. Using this method the phase stability of a device-under-test isolated from the reference signal instability can be obtained. The DMTD enables time measurements as well as frequency and frequency stability measurements for

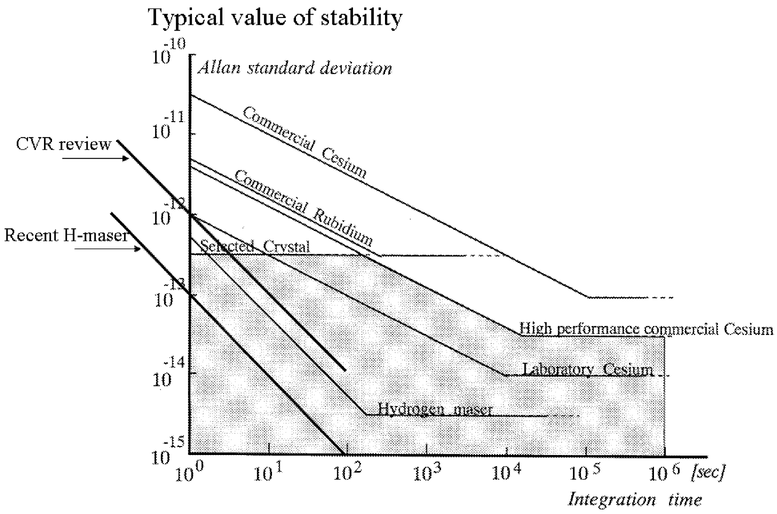

Figure 2. Typical Atomic clock stability.

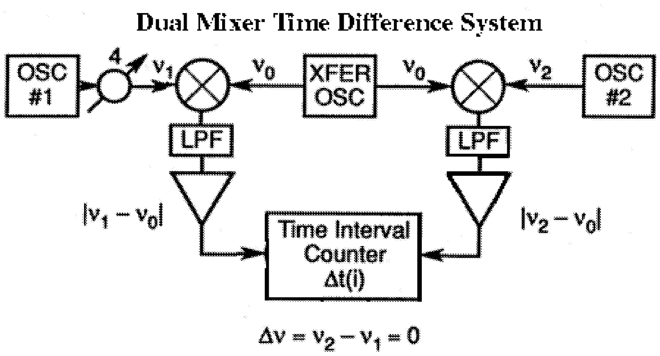

Figure 3. A block diagram of DMTD. Phase noise of the XFER OSC is cancelled as common-mode noise. We can use the DMTD in two ways. Case 1: OSC1 is a known stability oscillator and OSC2 is the DUT (device under test). Case 2: OSC1 and OSC2 are DUTs and they are the same specification devices.

sample times as short as a few milliseconds or longer, all without dead time. This method is contrived to cancel out the phase noise in the measurement system. When 10% of dead time is involved in integration time T, the standard deviation is displayed as $\sigma(2, T, 0.9T)$. If there is no dead time, $\sigma(2, T, T)$ equals to $\sigma_y(T)$, $\sigma_y(T)$ is usually called as the Allan standard deviation in the field of the time standard.

A total of the system instability is calculated by RSS (root sum square) of each component's Allan standard deviation. Time errors of phase noises are also calculated as follows;

$$\text{Time error} = \frac{T \times \sigma_y(\tau = T)}{\sqrt{3}} = \frac{\sigma_y(\tau = 1)}{\sqrt{3}} \quad \text{in white PM} \tag{2}$$

$$= \sigma_y(\tau = 1) \times \sqrt{T} \quad \text{in white FM} \tag{3}$$

$$= \frac{T \times \sigma_y(\tau = 1)}{\sqrt{\ln 2}} \quad \text{in flicker FM} \tag{4}$$

Table 1. Short baseline: Atmospheric scintillation is cancelled as a common-mode noise. Example of parameters and estimated coherence loss are calculated at 938 GHz. The stability of 125 MHz is not the measured value. DDS stability depends on output frequency, see Appendix B.

Function	Freq.	Effective Freq.	White PM	White FM	Flicker FM	Loss
Reference signal	5 MHz	938 GHz	–	–	–	Common noise
LLC,WMA,CMA	109 GHz	938 GHz	9.2×10^{-14}		1.56×10^{-16}	4.90×10^{-2}
1st LO DDS	30 MHz	270 MHz	3×10^{-13}			4.32×10^{-8}
2nd LO DDS	30 MHz	30 MHz	3×10^{-13}			5.33×10^{-10}
125 MHz	125 MHz	14 GHz	1×10^{-12}			1.29×10^{-3}
Atmosphere					–	Ignore

In the system-level technical requirements of the ALMA project, the instrumental delay/phase error about the 1st LO system should be 53 fs in the short-time period, and a drift of 17.7 fs in RMS integrated difference between 10 sec averages at intervals of 300 sec. We can convert the values to the Allan standard deviation, it is assumed that the noises are in the white PM and the flicker FM. As a result (according to Eq. (2)), $\sigma_y(\tau = 1) = 9.2 \times 10^{-14}$ (White PM)in the short-time stability is obtained. According to Eq. (4) and 10 seconds averaging, the required stability is equivalent to $\sigma_y(\tau = 1) = 1.56 \times 10^{-16}$(Flicker FM) in the long-time period.

4. ERROR BUDGET

In a connected interferometer, the instability of the common-reference frequency in each element is compensated for as a common noise. On the other hand, the phase noises of different parts are not common noise. In other words, the phase noises after reference distribution are not common noises for the ALMA system. Therefore it is indispensable to measure the phase stability of each component. According to Eq. (1), we can calculate the coherence losses of the following three situations.

4.1. Interferometry in a short baseline: atmosphere and atomic-clock instability are considered as common noise

In the case of interferometry in the short baseline, it is assumed that the stability of atmospheric scintillation can be ignored.
In the ALMA system, it is the independent system after signal distribution (from LLC to antenna) of each antenna that causes the coherence loss. According to Eq. (1), the estimated coherence loss is shown in Fig. 4. The coherence loss is estimated on various white PM noises. The right side values of the figure show the first LO stability ($\sigma_y(\tau = 1)$) of White PM noise. The other phase noise parameters are shown in Table 1. Because the effective frequencies are lower, effects of 2nd LO and DDS (direct digital synthesizer) are less severe than that of the first LO (which means LLC, WMA and CMA). The stability of 125 MHz is not the measured value. These assumptions are in common with following discussion.
Commonly used DDS and multiplier output signal behaviors are shown in Appendix.

4.2. Interferometry in a middle baseline: atomic-clock instability is considered as common noise

In the case of interferometry in the middle baseline, atmospheric scintillation can not be ignored. The middle baseline, if things come to the worst, mach for shorter than a few hundreds meters baseline. However, as altitude is as high as 5000 m or to use the WVR, it is assumed that the expected stability of the atmosphere is about 10^{-14} ($\tau < 1000$ sec) of flicker frequency noise. The parameters are shown in Table 2. According to Eq. (1), the estimated coherence loss is shown in Fig. 5 with an assumed integration time of 10 sec, and Fig. 6 with 100 sec coherent integration. The 100 sec coherent integration time might be over estimation for ALMA, but it is helpful for finding out the cause of dominant coherence

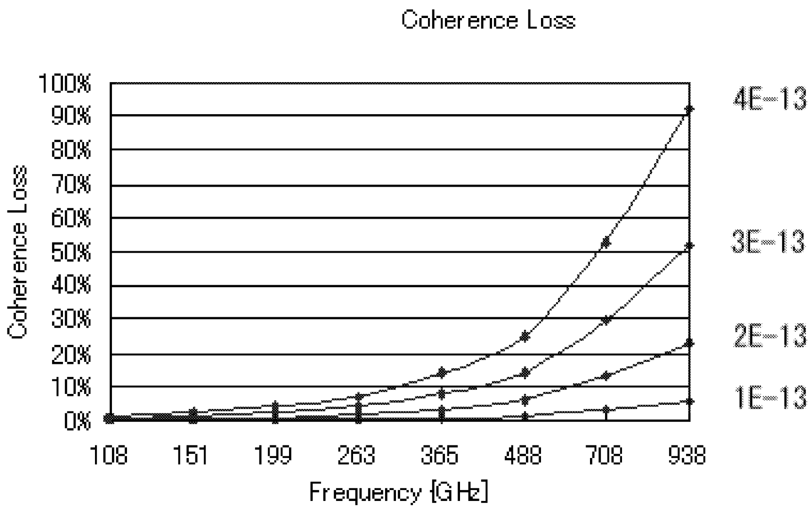

Figure 4. Short baseline: Estimated coherence loss in the short baseline interferometer. The atmospheric scintillation and signal (Atomic clock and distributors) noises deal with the common noise. In this Figure, the system phase stability of White PM is variable. The right-side parameter shows the stability of LLC, WMA, CMA which is the dominant value. The ALMA specification is the same as 1E-13 line.

Table 2. Middle baseline: Example of parameters and estimated coherence loss at 938 GHz, on the 10 and 100 sec integration. The stability of 125 MHz is not the measured value. DDS stability depends on output frequency, see Appendix B.

Function	Effective Freq.	White PM	White FM	Flicker FM	Loss(10s)	Loss(100s)
Reference signal	938 GHz	–	–	–	Common	Common
LLC,WMA,CMA	938 GHz	9.2×10^{-14}		1.56×10^{-16}	4.90×10^{-2}	4.91×10^{-2}
1st LO DDS	270 MHz	3×10^{-13}			4.32×10^{-8}	4.32×10^{-8}
2nd LO DDS	30 MHz	3×10^{-13}			5.33×10^{-10}	5.33×10^{-10}
125 MHz	14 GHz	1×10^{-12}			1.29×10^{-3}	1.29×10^{-3}
Atmosphere				1×10^{-14}	6.09×10^{-3}	6.09×10^{-1}

source. According to this estimation, main part of coherence loss in 100 sec integration is caused by the atmospheric scintillation. It's known that the stability of Atmospheric scintillation is flicker FM noise and cross over to white PM in long term. In this estimation, I assumed the cross-over time is longer than coherent integration time.

4.3. VLBI

It is desired that the ALMA system is capable of being expanded in the future to support the VLBI without serious difficulty. Here we check the current ALMA design from this view point. The reference signal phase noise can not be dealt with common noise for cross correlation between sub-arrays and VLBI stations. Therefore the reference signal and the Laser Synthesizer (LS) are required to maintain the signal coherence.

The frequency stability of the reference signal for the short-time periods is an important factor in maintaining the coherence of received signals in interferometer experiments. However, interferometer/VLBI observations from the ground always suffer from the effects of atmospheric scintillation, resulting in a loss of coherence. Therefore, the stability of the atmosphere determines the limitations of the frequency standard for the short-time period ranges.[7] The stability of the atmosphere has been determined to

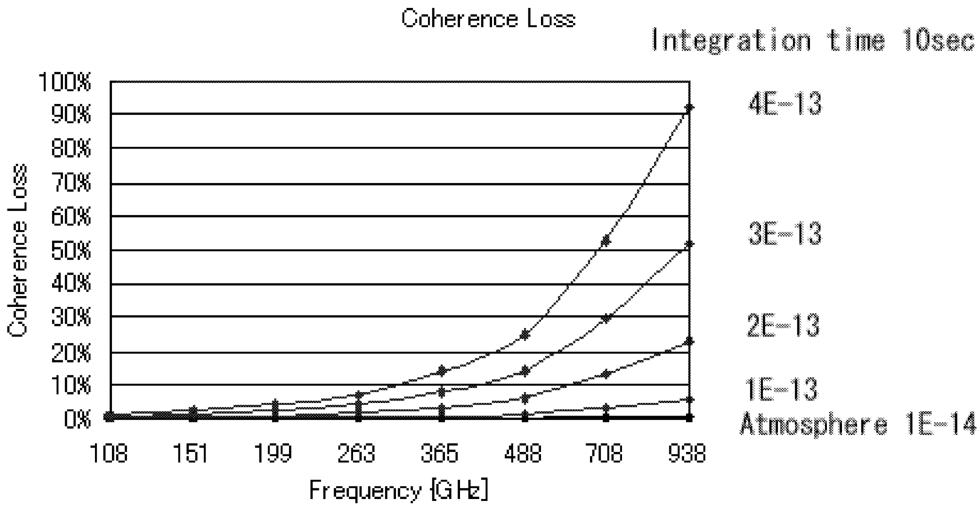

Figure 5. Middle baseline: Estimated coherence loss in the middle baseline interferometer. Integration time is 10 sec. The atmospheric scintillation is assumed as 10^{-14} ($\tau < 1000$ sec) of flicker frequency noise, and the signal (Atomic clock and distributors) noise deals with the common noise. The system phase stability (excluding the antenna structure) is variable. The value on the right shows the stability of LLC, WMA, CMA which is the dominant value. The ALMA specification is same as 1E-13 line.

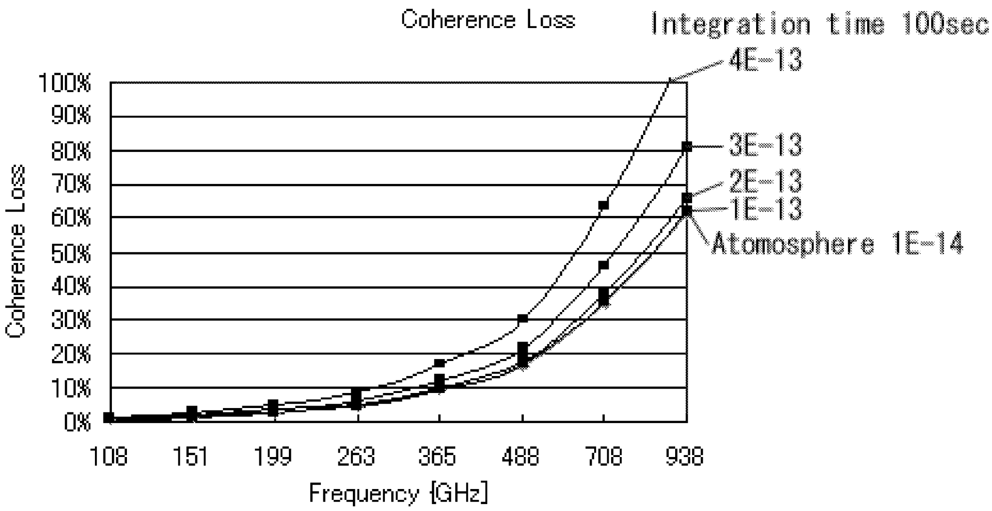

Figure 6. Estimated coherence loss in the middle baseline interferometer. Integration time is 100 sec. The atmospheric scintillation is assumed as 10^{-14} ($\tau < 1000$ sec) of flicker frequency noise, and the signal (Atomic clock and distributors) noise deals with the common noise. The system phase stability (excluding the antenna structure) is variable. The value on the right shows the stability of LLC, WMA, CMA. It is clear that the stability of atmosphere is the dominant value of coherence loss. Therefore the coherent integration time is restricted by atmospheric instability.

Table 3. VLBI: Example of parameters and estimated coherence loss at 263 GHz over 20-sec integration. The stability of 125 MHz is not the measured value. DDS stability depends on output frequency, see Appendix B.

Function	Freq.	Effective Freq.	White PM	White FM	Flicker FM	Loss
H-maser	5 MHz	263 GHz	1×10^{-13}	7×10^{-14}	5.5×10^{-15}	2.74×10^{-2}
LS,LLC,WMA,CMA	109 GHz	263 GHz	9.2×10^{-14}		1.56×10^{-16}	3.85×10^{-3}
1st LO DDS	30 MHz	90 MHz	3×10^{-13}			4.80×10^{-9}
2nd LO DDS	30 MHz	30 MHz	3×10^{-13}			5.33×10^{-10}
125 MHz	125 MHz	14 GHz	1×10^{-12}			1.29×10^{-3}
Atmosphere					1×10^{-13}	1.92×10^{-1}

be about 1×10^{-13} at 100 sec by VLBI (Fig. 1).[8,6,9] However, the atmospheric scintillation might be reduced dramatically at a high-altitude place such as ALMA, and/or by the fast beam switching and the WVR (Water-Vapor Radiometer) correction. The stability of the hydrogen maser is better than 1×10^{-14} at 100 sec, which was stable enough compared with that of the atmosphere. The atmospheric scintillation becomes a dominant factor in VLBI. It is assumed that the stability of the atmosphere is about 10^{-13} ($\tau < 1000$ sec) of flicker frequency noise. VLBI technology uses a stable independent reference oscillator at each station. To preserve the coherence, each station must use a superior reference oscillator, that is, a hydrogen maser. From estimated coherence (according to Eq. (1) and Fig. 2), the fringe can not be obtained with other atomic clocks (Fig. 2) even if 100 GHz band, the hydrogen maser is indispensable for VLBI.

In the ALMA project book, VLBI is/was planned at frequency in the lower equal 270 GHz (Band-6). The parameters are shown in Table 3. According to Eq. (1), the estimated coherence loss is shown in Fig. 7 with an assumed 20 sec integration. The stabilities of H-maser are not recent values; the latest model has better performances.

The coherence loss caused by the level of stability in Table 3 is about 20% in 20 sec coherent integration in Band 6, which is enough to ensure coherence in case of above assumption (this estimation exclude antenna instability). If the atmospheric scintillation is worse than assumed here, it causes a coherence loss explosion.

5. ESTIMATION OF THE OPTIMUM INTEGRATION TIME TO GET MAXIMUM SNR

In this section, we discuss the possible optimum integration time. The SNR of interferometer is calculated by Eq. (5)[8,5]

$$SNR = \frac{\pi}{8k} \frac{S_c\ D_1\ D_2\ \sqrt{\eta_1\ \eta_2}}{\sqrt{T_{s1}\ T_{s2}}} \sqrt{2BT}\rho \qquad (5)$$

where

S_c	correlated flux of source,
K	Boltzman's constant,
D_1	the diameter of the antenna (station 1),
D_2	the diameter of the antenna (station 2),
η_1	the antenna *efficiency* of station 1,
η_2	the antenna *efficiency* of station 2,
T_s	the system temperature,
B	the bandwidth,
T	the integration time.

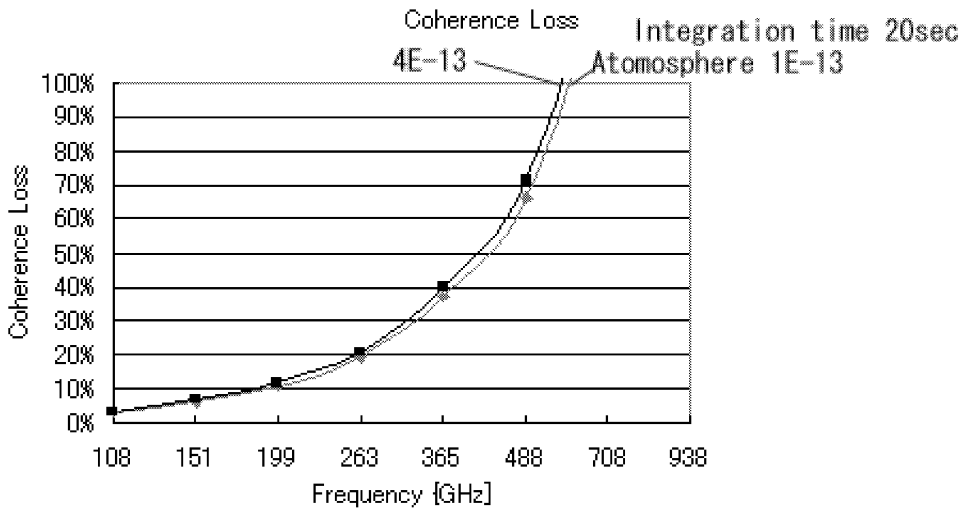

Figure 7. VLBI: Estimated coherence loss in the VLBI. Integration time is 20 sec. VLBI is/was planned at frequencies in lower or equal to 270 GHz. The atmospheric scintillation is assumed as 10^{-13} ($\tau < 1000$ sec) of flicker frequency noise. The system phase stability (excluding the antenna structure) is variable. The value on the right shows the stability of atmosphere which is the dominant value.

Table 4. The optimum integration time in seconds, when the LO stability is 9.2×10^{-14}(White PM) with 1.56×10^{-16}(Flicker FM).

Local frequency $[GHz]$	Middle baseline Atmosphere: 1×10^{-14}	VLBI Atmosphere: 1×10^{-13}
108	498	50
151	356	36
199	270	27
263	205	20
365	147	
488	110	
708	76	
938	58	

And signal coherence ρ can be calculated as follow.

$$\begin{aligned}\rho = &(Imperfect\ image\ rejection) \times (Phase\ noise) \times (Imperfect\ band - pass)\\ &\times (Aliasing\ noise) \times (Digitizing) \times (Fringe\ stopping)\\ &\times (Fractional\ bit\ correction) \times (Atmospheric\ scintillation).\end{aligned}$$

The coherence loss is expressed by Eq. (1). There is an optimum integration time which gives the maximum $(SNR) \times (coherence)$. The optimum integration time (maximum SNR) strongly depends on the stability of the atmosphere, that of frequency standard, Local frequency, and the type of phase noise.

Table 4 shows the optimal integration time in the middle baseline when the atmosphere stability is $\sigma_y(\tau = 1sec) = 1 \times 10^{-14}$ and the LO stability is 9.2×10^{-14}(White PM) with 1.56×10^{-16}(Flicker FM) as required in the ALMA specification. It also shows the optimal integration time for VLBI with hydrogen maser when the atmosphere stability is $\sigma_y(\tau = 1sec) = 1 \times 10^{-13}$.

Table 5. In the case of the White PM noise only, the coherence loss is independent on the integration time. However other cases, the coherence loss is strongly depended on the integration time. Estimated the time independent loss and the coherent integration times with 5, 10 % losses are shown in the Table. This estimation excludes the atmospheric scintillation loss.

Time independent (minimum) coherence loss			Time dependent coherence loss			
	Coherence loss LO only	Coherence loss Total		Integration time LO only		Integration time Total system
Flicker FM	none	none	Flicker FM	1.56×10^{-16}		2.20×10^{-16}
White PM	9.2×10^{-14}	1.3×10^{-13}	White PM	9.2×10^{-14}		1.3×10^{-13}
	Coherence loss	Coherence loss		5% loss	10% loss	10% loss
108 GHz	0.14%	0.18%	108 GHz	>10000 sec	>10000 sec	>10000 sec
151 GHz	0.18%	0.28%	151 GHz	>10000 sec	>10000 sec	>10000 sec
199 GHz	0.26%	0.46%	199 GHz	8470 sec	>10000 sec	8480 sec
263 GHz	0.41%	0.78%	263 GHz	6295 sec	9090 sec	6305 sec
365 GHz	0.75%	1.49%	365 GHz	4357 sec	6425 sec	4365 sec
488 GHz	1.33%	2.65%	488 GHz	3026 sec	4653 sec	3033 sec
708 GHz	2.79%	5.58%	708 GHz	1618 sec	2923 sec	1623 sec
938 GHz	4.90%	9.78%	938 GHz	260 sec	1856 sec	280 sec

6. ESTIMATED COHERENT INTEGRATION TIME CAUSE OF SYSTEM PHASE INSTABILITY.

In this section, we focus on the coherent integration time with 5, 10% coherence loss. The cause of the coherence loss is the system phase instability. During the coherent integration time, the observed phases (a signal phase vector and a noise phase vector) of cross-spectrum are vector integrated in order to detect a fringe. It is assumed that the phase noises are in white PM and flicker FM, as are 9.2×10^{-14}(White PM) with 1.56×10^{-16} (Flicker FM) in LO, and 1.30×10^{-13}(White PM) with 2.20×10^{-16} (Flicker FM) in Total. These values are gained by calculating the values of ALMA LO/Total and those of short/long term specifications with Equations (2)-(4). And the calculated coherent integration times in four cases are shown in Table 5. They show enough coherent integration time.

7. CONCLUSION

The coherence loss is possible to be estimated from the measured Allan standard deviation, and the coherence loss depends on the phase noise characteristics. The frequency standard (include signal transmission and multiplier chain) of the ALMA must be stable over the long-time period (incoherent integration) to regulate the duration of observations and over the short-time period (coherence integration time) to maintain the coherence. The required stability on the ALMA system is tight especially for the 1st Local system. The independent instability of each element decreases the signal coherence. We should consider that the phase noises after reference distribution (from LLC to antenna) are not common noises. The RSS Allan standard deviation stability of the LLC, WMA and CMA is required better than 9.2×10^{-14}(White PM) with 1.56×10^{-16} (Flicker FM) at $\tau = 1sec$ by the ALMA specification. In the case of 5% LO coherence loss in the Band 10, the coherent integration time of 260 sec is obtained. Also the total system phase stability is required better than 1.30×10^{-13}(White PM) with 2.20×10^{-16} (Flicker FM). In the case of 10% coherence loss in the Band 10, the coherent integration time of 280 sec is obtained. The ALMA specification is very satisfying for maintaining the coherence.

REFERENCES

1. D.W. Allan, "Statistics of Atomic Frequency Standards," Proc. IEEE 54, 221 (1966).
2. D.W.Allan,"Report on NBS dual mixer time difference system (DMTD) built for time domain measurements associated with phase 1 of GPS," NBS IR 75 827, Jan. 1976.

3. L.D'Addario and M.Holdaway,"Joint distribution of atomospheric transparency and phase fluctuations at Chatnantor," LAMA memo 801.
4. D.J. Healey III, "Flicker of frequency and phase and white frequency and phase; Fluctuations in frequency sources", Proc. 25th Annu. Sympo. On Frequency Control (ASFC), Jun., 1972.
5. N.Kawaguchi, "Coherence loss and delay observation error in Very-Long-Baseline Interferometry," J. Rad. Res. Labs., vol.30, no.129, pp.59-87, Mar. 1983.
6. N.Kawano, F.Takahashi, T.Yoshino, K.Koike, H.Kumagai and N.Kawajiri,"Development of real-time VLBI and measurements of scintillation," J. Rad. Res. Labs., vol.29, no.127, pp.53-102, Jul. 1982.
7. H.Kiuchi, "A highly stable crystal oscillator applied to the VLBI reference clock", IEEE Trans. Instr. and Meas., Vol.45, No.1, pp.177-183, Feb. 1996.
8. A.E.E.Rogers and J.M.Moran, "Coherence limits for very-long-baseline interferometry," IEEE Trans. Instrum. Meas., Vol. 30, No. 4, pp. 283-286, Dec. 1981.
9. A.E.E.Rogers, A.T.Moffet, D.C.Backer and J.M.Moran,"Coherence limits in VLBI observation at 3-millimeter wavelength," Radio Science, vol.19, no.6, pp.1552-1560, Nov.1984.
10. R.Vanblerkom and S.L.Aneman,"Considerations for the short term stability of frequency multiplication," IEEE AES-2, no.1, pp.36-47, Jan.1966.
11. B.Shillue, S.AlBanna and L.D'Addario,"The ALMA 1st local oscillator reference," ALMA memo 483.
12. http://ft.nist.gov/
13. http://www.nict.go.jp/
14. http://www.analog.com/

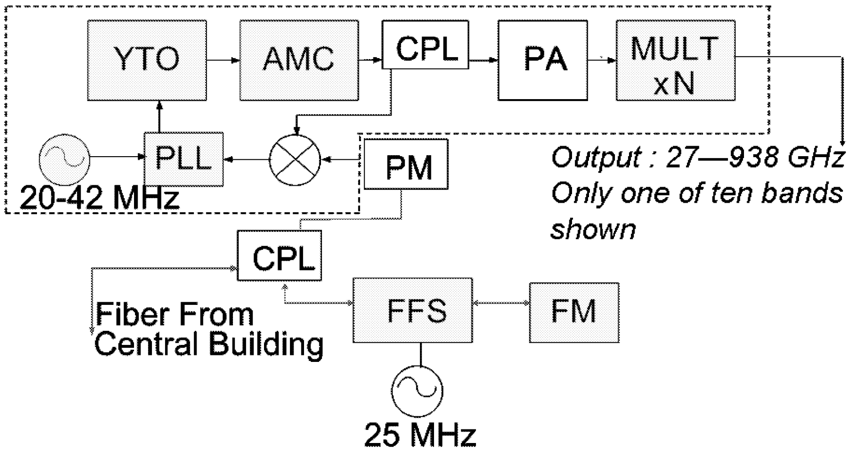

Figure 8. ALMA 1st LO Multiplier chain.[11] Where YTO: YIG tuned oscillator, AMC: active multiplier chain, CPL: coupler, PA: power amplifier, MULT: CMA, PLL: phase locked loop, FFS: fiber frequency shifter, FM: Faraday mirror, PM: photo-mixer.

APPENDIX A. FREQUENCY MULTIPLIER CHAIN

There is a noteworthy fact about multiplication, it is assumed that the multiple number is k, the input CNR (Carrier Noise Ratio) is $(C/N)_{in}$, and the output CNR of multiplier is $(C/N)_{out}$. Then, the frequency stability of the output of the ideal frequency multiplier is the same as the stability of the input[10] in the case of meeting the following conditions.

$$\sigma_y(\tau)_{out} = \sigma_y(\tau)_{in} \qquad in\ case\ of\ k < \sqrt{(C/N)_{in}} \tag{6}$$

Next, we estimate the required C/N_{0in} over the noise bandwidth of 10 MHz: as required in the ALMA specification. When the multiple number k (displayed as $\times N$ in Fig. 8) is 9 on Band-10, according to Eq. (6) the required C/N_{0in} is calculated as follow;

$$81 < \frac{1}{10^7}\frac{C}{N_{0in}} \tag{7}$$

Then C/N_{0in} is required better than 89 dB. In the ALMA local chain (Fig. 8), the PLL is used for reducing high-frequency noise as a clean-up filter. The output PLL signal spectrum is duplicated the reference signal spectrum within the PLL bandwidth. The required photo-mixer output signal purity became light in inverse proportion to the PLL bandwidth. On the other hand, the YTO signal spectrum is dominated over outside of the PLL bandwidth. Therefore, the high-signal purity of the YTO with AMC is required as above for keeping coherence in the short-time period.[7]

In other cases (where k is larger than $\sqrt{(C/N)_{in}}$), the stability declines.

APPENDIX B. COMMON DDS SIGNAL BEHAVIOR

Though may not be necessary, I show a behavior of common DDS signal. A DDS,[14] which consists of a phase accumulator and a phase-to-amplitude converter. The 1st LO (Fig. 8) of the ALMA, a 48-bit phase accumulator of DDS has a role of fringe stopping, 90-degree and 180-degree phase switching. The phase resolution is up to $360/2^{48}$ degrees. Sixteen bits of the phase accumulator (called as phase-words) is passed along to the phase-to-amplitude (12-bit amplitude accuracy) converter. And the remaining 32 bits are called as truncation bits. The DDS phase accumulator includes the delta-phase of every clock cycle. Therefore the accumulated phase is like a saw-tooth signal. Then the phase is converted to amplitude by the phase-to-amplitude converter. In the case of DDS (FLOOG), the clock reference is 125

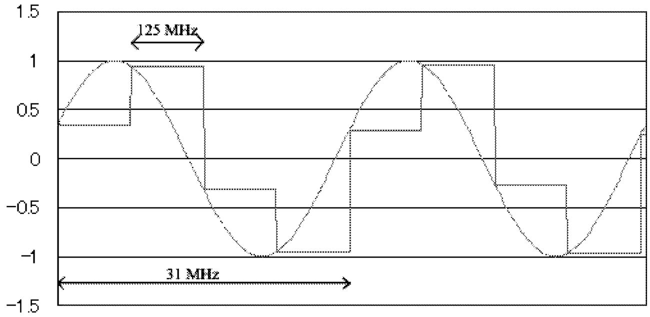

Figure 9. The SINE wave shows an ideal 31 MHz, and the rectangular shows the DDS output 31 MHz.

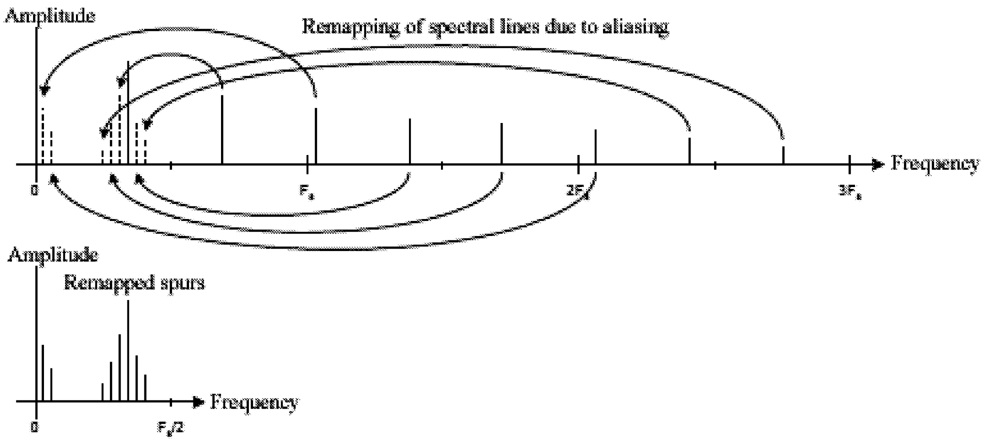

Figure 10. The inter-modulated signal.

MHz and the required DDS output frequency is from 20 to 42 MHz. In this case, the truncation bits are overflow. The nominal DDS frequency of the FLOOG is 31 MHz (shown in Fig. 9). The SINE wave in the figure shows an ideal 31-MHz signal, and the rectangular wave shows the DDS output 31 MHz signal. The rectangular approximation generates high spur. Then the maximum spur level turns out to be approximated by $-6.02 \times (\textit{phase words bits})$ [dB].[14] The estimated spur level is about -12 dB, and the spur signals are far over the clock frequency (125 MHz). The spur signals are re-mapped due to aliasing (Fig. 10).[14] The aliasing cause spurs in frequency bands that are Odd integer multiples of Nyquist-frequency of clock to map directly into region of the Nyquist-frequency. Then the DDS output signal is obtained as an inter-modulated signal. After that the DDS signal is filtered by 20-42 MHz band-pass filters. The filtered signal is passed along to the (digital) phase-frequency discriminator of the 1st/2nd LO PLL as a reference signal. This effect is especially taken account of the case of [a just-MHz] + [a few-Hz] signal, as Fringe stopping. Although the zero-crossing phase only makes sense in the digital phase/frequency discriminator, the zero-crossing points are also inter-modulated. In common case, the inter-modulated signal decreases the LO stability or brings the Flicker FM noise in LO signal. As I assumed that implications of this effect had been studied with adequate consideration in ALMA design, the coherence loss estimation in this memo excludes this effect.