ALMA MEMO 339

Aspects of the Antennas for the ALMA Compact Array (ACA)

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Summary

The Atacama Compact Array (ACA) will supplement ALMA in order to provide the "missing spacings" in the uv-plane coverage, caused by the finite size of and the minimum distance between the elements of ALMA. ACA will consists of about a dozen small antennas of a diameter between 6 and 8 m. In this Memo I consider a number of aspects related to the antennas. After reviewing the basic requirements on the antennas, I look in some detail on the possibilities of using the ALMA antenna mounting for the ACA dishes. Minimizing the "collision radius" to enable close packing is important for the ACA. I suggest a straw man ACA antenna consisting of a 7 m diameter reflector with a short focal length (F/D = 0.3) on an ALMA mounting. A number of points for further study is listed.

1. Introduction

It has now been accepted that the participation of Japan as an equal partner in the ALMA Project enables us to supplement the ALMA array of 12 m antennas with a small array of smaller antennas, the Atacama Compact Array (ACA). The main purpose of this array is to provide the missing spacings in the uv-plane, caused by the size and minimum spacing of the basic ALMA element antennas. This is roughly the spacing interval from 6 to 12 m.

Additionally, the ACA might be used as a dedicated array for observations in the supra THz windows (1.1, 1.3 and 1.5 THz), where the site provides acceptable atmospheric transmission for interestingly long periods. This would require the ACA antennas to be equipped with reflectors of a higher surface accuracy, of the order of 10 µm.

There is not yet agreement on the optimum size and number of antennas of the ACA. These depend on the quality with which the central hole in the uv-plane can be filled, as well as on the need to provide sufficient sensitivity for the independent calibration of the array. ACA will mainly be operated as a separate, “stand alone”, array and alternately correlations with a number of 12 - m antennas of the main ALMA array will be possible. Current studies suggest the need for 10-15 antennas of a diameter between 6 and 8 m.

In this Memo I look at a number of practical and technical aspects of the realisation of the ACA antennas. In particular, I list some boundary conditions and evaluate the feasibility of using the mount of the ALMA 12 m antennas for the ACA elements. I consider reflectors of a size between 6 and 8 m.

2. Major requirements on the ACA antennas

2.1. Pointing. Jack Welch has illustrated in his Memo of 25 August 2000, how pointing errors seriously impair the ability to subtract missing spacing information from extrapolated mosaic and single dish data. An important requirement on the ACA antennas is an **absolute** pointing accuracy and stability which is better than that of the 12 m elements. This should be possible with the smaller size of the antenna. It will be interesting to evaluate the pointing behaviour of the small dishes, mounted on the mounting of the 12 m antenna. In particular, the wind induced pointing errors should be considerably reduced.

2.2. Reflector accuracy. Certainly, the reflector accuracy must be at least as good as that of the 12 m antennas. There are good reasons to improve the surface quality of the ACA antennas. Firstly, this will open the possibility of observations in the atmospheric windows between 1 and 1.5 THz. Secondly, the sensitivity of ACA in the standard ALMA bands will be increased, particularly at the higher frequencies, which will ease the calibration of the independent compact array. The number of elements may perhaps now be determined solely by the requirements of recovering missing uv-information and not by the need for calibration. Figure 1 illustrates the relative sensitivities quantitatively, where I have assumed a maximum aperture efficiency of 70 percent for a perfect reflector. It essentially shows the loss due to "Ruze" scattering.

For operation above 1 THz, the 12 m antennas with the specified 25 µm surface will be significantly surpassed by the 8 m antennas with 10 µm, while a 6 m/10 µm antenna will be more sensitive above 1.2 THz. A goal of 10 µm rms surface error for 6 to 8 m antennas appears achievable. Assigning 5 µm to each of the 4 major error components - panel fabrication; BUS deformation due to gravity, wind and thermals; similar for panels; surface measurement and setting - would add to 10 µm rms, providing two thirds of maximum aperture efficiency at 1.5 THz.

2.3. Packing density and shadowing. ACA will be configured as a two-dimensional array with a limited amount of reconfiguring to prevent excessive shadowing. Several of the antennas will be as close together as possible for optimum performance. Apart from evaluating the shadowing, it should be assured that the mountings of the antennas do not cause a smallest distance that is unacceptable for good performance. We shall find below that this will not be the case when the 12 m mountings are used for ACA.

In the remainder of this Memo, I will concentrate on the advantages and limitations connected to the use of the mounting of the ALMA 12 m antennas for the smaller dishes of ACA. In particular, I consider aspects of receiver mounting and servicing, close packing and shadowing issues and the expected pointing behaviour. I present some suggestions for the size and geometry of the ACA reflector also.

3. Use the ALMA mounting for the ACA antenna?

I evaluate the advantages and disadvantages of the use of the ALMA mount for the ACA antennas. Let me begin by listing the advantages.

3.1. Advantages

1. no need to design a new mount cost saving
2. fabricate to ALMA antenna drawings - extend production series economic
3. provides standard ALMA receiver cabin for frontend box operations
4. very good pointing behaviour because of “over-dimensioned mount” quality
5. movable, if needed, by standard ALMA transporter operations
6. maintenance and repair identical to ALMA mounts economic

Ad 1 and 2. These advantages are obvious. One should however consider the fact that a mount for a smaller antenna will normally be cheaper. We ought to make an estimate of the cost of a design and fabrication of the smaller mount and compare it to the pure fabrication cost of an additional number of ALMA mounts. The ESO group will make such an estimate.

Ad 3. In my opinion, there is a very strong argument in favour of providing a receiver cabin, which can accommodate the standard ALMA receiver frontend unit. The additional costs in design and construction of a completely different frontend will be considerable. In addition, the maintenance, repair and exchange of identical receiver units is always easiest.

In reactions on an earlier draft of this Memo by System Scientists and receiver designers alike, all conclude that the use of the "standard" ALMA receiver frontend with its optics package should be a ***specification*** *for the ACA.*

If a small antenna were to be designed from scratch, the mount would definitely be smaller, resulting in less space for the receiver cabin behind the Cassegrain focus. It is unlikely that a frontend with all bands could be accommodated in this space. Apart from the need to split the receiver bands over more than one frontend unit, this would require regular exchanges of sensitive equipment on the antenna, something to be avoided if at all possible on the 5000 m high site. The alternative of providing Nasmyth foci on the outside of the elevation bearings is probably undesirable from an optics viewpoint. It would also widen the antenna and increase the minimum distance between neighbouring antennas, perhaps making the antenna again as wide as the mount of the 12 m antennas.

On the other hand, the standard ALMA frontend unit will provide all bands on the ACA and in addition allow the installation of supra-THz inserts in the slots of one or more of the ALMA bands, for which no short spacings are needed during the period of THz observations.

This is the simplest, cleanest and cheapest solution for providing the ACA antennas with the necessary receivers.

Ad 4. Although the primary beam width of the ACA antenna will be proportionally larger than that of the ALMA antenna at any frequency, the requirement on the pointing accuracy (in arcseconds) cannot be relaxed because the quality of deriving missing spacing data is particularly sensitive to pointing errors. Thus the ACA antennas should exhibit a pointing precision at least as good or possibly better than that of the ALMA antennas. This will ensure also a good behaviour of the ACA antennas for observations above 1 THz.

It appears likely that the ALMA mount with a small reflector attached to it will meet these higher pointing specifications. In particular, the wind component, being inherently difficult to predict and hence to correct, will be significantly decreased by about a factor equal to the ratio of the reflector areas (between 2 and 4 for 8 to 6 m ACA antennas). Thus it might be possible to achieve the required performance without an additional "metrology" system.

The pointing requirements form a good reason to use the "over-designed" ALMA mount*.*

Ad 5 and 6. These advantages again are obvious, but they admittedly are of a lower importance and impact than the other points.

I now turn to the disadvantages of the selection of the ALMA mounts.

3.2. Disadvantages

1. Over-engineered and over-dimensioned mount costly
2. Mount dimensions might determine shortest spacing performance
3. Small reflector on big mount may look odd immaterial

Ad 1. This is the most obvious problem with using the ALMA mounts. The significance of this point can only be evaluated quantitatively by making a good estimate of the cost of designing and fabricating a new mount for the small ACA reflectors and comparing this to the fabrication costs of an additional set of ALMA mounts. (See advantages 1 and 2 above.)

Ad 2. If the ALMA mount is so large that it determines the closest spacing of the ACA, a loss in performance may ensue. This loss can be estimated by simulations of the imaging qualities of the ACA and ALMA arrays.

The largest dimensions of the ALMA mounts are about 7 m for both the EIE and Vertex designs. This occurs along a direction parallel to the elevation axis. However, the distance from the azimuth axis to the top of the quadripod will be larger and it determines the collision radius. Thus, strictly for collision avoidance, the minimum spacing will not be determined by the lateral dimension of the mount. We shall return to this aspects in greater detail below.

Ad 3. While a well designed antenna can be beautiful, I submit that the sheer number and layout of ALMA will overwhelm the viewer to the point that he will not pay much attention to the perhaps somewhat oddly looking ACA antennas. In other words, in my view this point should have no influence on the decision making process.

4. Additional aspects

4.1. For the small diameter of the ACA antennas, one should aim at supporting the quadripod at, or very near, the rim of the main reflector. With a smaller and lighter subreflector, the entire quadripod structure might be optimized with an aim to obtain a smaller height of the top section. Thus both the blocking area and the collision radius would become smaller.

4.2. It might well be possible, without significant financial consequences, to design and build the ACA antennas with such a stiffness and performance that an active position control of the subreflector will not be necessary. However, some of the “optics” solutions being studied for ALMA require a refocusing of the subreflector depending on the reception band. This would be needed for ACA too, if the ALMA frontends are used unchanged. In this case, a simplified positioner, providing only axial focus and perhaps some "vertical" (gravitational bending) shift, will probably be sufficient. Such a simplified focus stage could be low mass, would bring significant savings and allow the design of an optimized quadripod. The HHT focus translator could serve as example.

If a full 5-degrees-of-freedom focus stage will be necessary, economies of scale might lead us to using the ALMA positioner. This will be an "over-dimensioned" design, requiring the current ALMA top section of the quadripod. This complicates the quadripod design and leads to extra aperture blocking. I consider it uncertain whether this is the most economic solution.

4.3. Regarding the need for ACA antennas to be equipped with nutating subreflectors, the discussions have led to the following agreement:

i) no nutators are needed for the basic “short-spacing” task of the ACA,
ii) for independent supra THz observations, at least one of the ACA antennas should have a nutator for total power measurements.

It seems expedient to perform a complete design of the quadripod, subreflector, attachment and positioner system, including the option to add a chopping mechanism. I consider it unlikely that any ALMA hardware would be directly usable in this case, apart perhaps from control electronics and software.

5. Details about the antenna design

Here I present some details about the proposed route to define the ACA antennas. These could be used as a guideline for the conceptual design activities, proposed above. I summarize my understanding of, or proposal for some basic boundary conditions first. These have not been fixed and might well be changed on the basis of scientific or technical/financial arguments.

5.1. Specifications - primary requirements

- The receiver cabin must accommodate the ALMA frontend unit without changes in its basic layout and outside dimensions.
- The optics of the receiver will be identical of that of the ALMA system. This means that the angle subtended by the subreflector as seen from the Cassegrain focus is the same as in the ALMA antenna, 7.16 °.
- The reflector diameter will be between 6 and 8 m.
- The collision radius should be as small as possible.
- There will be at least two configurations for the array to prevent excessive shadowing at low elevation angles. Thus (some of) the antennas must be transportable over a short distance.
- It is desirable to realize good antenna performance up to 1.5 THz. Pointing and path-length performance must be at least as good as with the ALMA antenna.

From these requirements, I come to the following conclusions and suggestions:

5.1.1. The first two requirements essentially prescribe the use of the ALMA antenna mount for the ACA antennas. The ALMA "receiver cabin" is already pretty full with the necessary frontend equipment. Also, the area just in front of it is fully used by the quasi-optical components. It will be very hard to make that cabin smaller and still accommodate the ALMA frontend in it. Thus it is virtually impossible to design a new mount for ACA which would be noticeable smaller than the existing ALMA mount.

Using the ALMA mount removes the need to design a separate mount for the ACA.

5.1.2. With a reflector diameter between 6 and 8 m, the reflector will determine the collision radius. Therefore there is no need to minimize mount dimensions. It will however be advantageous to mount the reflector as close to the elevation axis as possible with due account for the space needed for the receiver optics.

Based on the performances of the KOSMA and HHT antennas (3 and 10 m diameter, respectively, both with a CFRP backup structure), it should be possible to design and build a 6-8 m reflector with an overall surface accuracy of 10 µm. Moreover, its stiffness could be made sufficient to enable a simplified subreflector focus stage (see Sec. 4.2 above).

5.1.3. It is advisable to allow for a nutating subreflector in the design of the quadripod and apex structure.

5.1.4. With an rms surface error of 10 µm, the ACA antennas will provide 75% of the maximum aperture efficiency at 1.5 GHz, an excellent performance. With the ALMA mount and the small reflector, wind influences on the pointing will be 2-4 times smaller than for the 12 - ALMA antennas. Thus the pointing should be excellent, better than 0.6", and be sufficient for both the ALMA short-spacing work and operation at 1.5 THz. Note that the reflector surface shape will be essentially perfect at all ALMA bands.

5.1.5. With the ALMA mount, the transportability of the ACA antennas is easy with the available ALMA transporter. However, because the ACA antennas will be moved only over a small distance, it may actually be easier to move them on rails. This also removes a possible spacing limitation, which could be caused by the size of the ALMA Transporter.

5.2. More considerations regarding the geometry

To clarify some of the following statements and parameters, I have summarized the basic geometry of the Cassegrain antenna, together with the major formulae describing the relations of geometrical parameters, in Figure 2.

The ALMA mount with receiver cabin is assumed, and the ALMA frontend will be used for the ACA. This sets the interface between the receiver frontend and the mount and determines a few dimensions of the ACA antenna geometry, based on the ALMA antenna specification:

i) the distance from the elevation axis to the Cassegrain focus is 0.803 m.

ii) above the focus there is a volume of 1.38 m round diameter, 0.60 m height, reserved for the receiver optics and for access to this optics by a technician (volume E in ICD 1).

iii) above this volume there is a clear aperture of a diameter at least equal to the central vertex hole of the primary reflector up to the vertex plane. In the ALMA designs the height of this clear volume is about 0.78 m (volume F in ICD 1).The total height of volumes E and F is mainly determined by the height of the BUS in the central area.

It will be particularly important to minimize this height in the ACA antennas, because it directly impacts on the collision radius.

iv) the half-angle subtended by the subreflector from the focus is phi = 3.58 °.

There are now, for a given diameter of the ACA primary reflector D, essentially three parameters to play with: the diameter of the subreflector d, the distance between primary and secondary focus f (f=2c, c the hyperboloid geometrical parameter) and the primary focal length F (or the primary F/D-ratio). A choice of two determines the third. The spreadsheet of Table 1 illustrates the situation for a number of cases.

The distance $f = F + x$,

where x = 1.38 m for the ALMA designs, unknown, but to be minimized for ACA; it is roughly the BUS height at the center of the reflector and it contains the minimum 0.6 m optics package height. In Table 1 the height **x** is **assumed** to be 0.8, 0.9 and 1.0 m for reflector diameters 6, 7 and 8 m, respectively, **including** the 0.6 m for the optics.

The collision radius will be $col = f + 0.803 + y$,

where 0.803 is the distance elevation axis to focus and y is the height of the quadripod central structure beyond the primary focus. This is 0.5 m for ALMA, it could probably be reduced to about 0.3 m for ACA. Assume y=0.3 m and thus we have

$$col = f + 1.1 \text{ m}.$$

If we scale the ALMA antenna in all aspects by a factor one half, we halve the subreflector diameter as well as the distance f, which now is f = 3.13 m (column 2 in Table 1). This is only 0.1 m more than the requirement for the optics volume. I do not believe that the BUS could be made that low for the stiffness required and I conclude that a direct scaling of ALMA is not feasible.

There are advantages in keeping the subreflector diameter small. This is particularly true, if it will be attached to a nutator. Scaling the subreflector diameter as the main reflector then suggests to choose a smaller primary F/D-ratio in order to gain some distance for the BUS height. In the remaining columns of Table 1, the secondary angle is kept fixed and **f** is set with an allowance for the BUS height in the center, as explained above.

The resulting subreflector diameter and collision radius are indicated in Italics. Lowering the primary F/D-ratio decreases f for a given D and also decreases the collision radius. The subreflector diameter decreases somewhat too. In Table 1, I have assembled the numerical data for dish diameters of 6, 7 and 8 m and primary focal ratios of 0.3, 0.35 and 0.4. Once a basic design of the ACA antenna structure has been made, the consequences of the actual BUS height can easily be seen from the Table.

For an f-ratio > 0.35 we do not reach the ALMA close packing of 1.25 times the dish diameter. This is however not primarily caused by the choice of the ALMA mount with the ALMA receiver package. In particular, note that the lateral size of the mount (along the elevation axis) is not the determining factor. Choosing a smaller primary focal ratio of 0.3 brings us to the goal, a value of 1.2 or slightly better being achieved. It is clear that the situation improves with increasing diameter of the ACA antenna.

With a subreflector of about 400 mm diameter, one would prefer to make the central hole in the primary equally large. Will this be compatible with the ALMA receiver box? In the current design of the receiver, the feeds are placed less than 300 mm from the axis. For the 7 m example of Table 2 below, this would require a central hole of about 530 mm for unvignetted illumination of the subreflector, slightly larger than required by the subreflector. This situation might be improved by further optimisation and does not seem to prevent the use of the ALMA receiver box.

The following should, however, be noted. Because the feeds are laterally displaced from the focus, the optics is slightly tilted with respect to the axis (maximally 2.8°) for a proper illumination of the secondary. This alignment will have to be adjusted to the different angle of the ACA antenna of about 5.5° maximally.

6. Tentative conclusion

Weighing all aspects, including a number of comments from the ALMA community, I suggest the following choice for the ACA antenna as basis for a preliminary design (also highlighted in the applicable column of Table 1):

Table 2. Proposed Parameters of the ACA Antenna

Diameter Primary Reflector D (m)	7.0	selected
Focal ratio of Primary (F/D)	0.3	selected
Primary half angle (deg)	79.61	
Depth of Primary (m)	1.458	
Magnification of Cassegrain M	26.66	result
Cassegrain focal ratio	8.00	result
Secondary half angle (feed illumination) (deg)	3.58	fixed
Distance primary - secondary focus f (m)	3.00	mixed*
Diameter secondary reflector (m)	0.371	result
Eccentricity secondary	1.07793	result
Collision radius (m)	4.1	result
Collision factor (col/reflector ratio)	1.17	result
Allowed BUS height in vertex (m)	0.9	fixed

* this number is determined by the choice of BUS height, the optics height and the calculated focal length of the primary.

The arguments in favour if this choice follow.

1. There is a continuing debate as to the optimum diameter of the ACA reflector, but the numbers are in the range 6 to 8 m. Most critical is the achievable calibration accuracy. The number of antennas, needed to reach a certain accuracy is inversely proportional to the fourth power of the diameter. For instance, if we require 7 antennas of 8 m, we'll need 12 of 7 m and 22 of 6 m diameter. My choice of 7 m could be considered a "Salomon's verdict/proposal"; it is not based on deep scientific reasoning.

2. It has however the distinction that the reflector diameter is now just as large as the maximum size of the (ALMA) mount. This will be more pleasing to the eye than a 6 m dish.

3. The pointing behaviour under wind will be barely different from that with a smaller (i.e. 6 m) dish, because a good deal of the wind forces act on the 7 m wide mount. This will be especially true for lower elevation angles.

4. The size of the subreflector is a good compromise; in fact it is somewhat smaller than the ALMA ratio, which would be advantageous in case a nutator will be applied.

5. The collision factor is smaller than that adopted for ALMA. This is of advantage for the specific goal of "filling the missing spacings". It should be noted that this applies to the current design of the EIE antenna only. The Vertex antenna platforms (5 m radius from azimuth axis) cause the collision factor to increase to 1.43, which is definitely too large. In the present design of the Vertex platform and stairs only an 8 m reflector with F/D=0.35 would give a collision factor smaller than 1.25.

If the choice of primary diameter is 6 m, the primary focal ratio must be 0.3 to achieve an acceptable collision radius for the EIE antenna, while for an 8 m primary a focal ratio of 0.35 could be used both for EIE and Vertex (Table 1). I see no problem with these shorter primaries and note that several antennas are operational with such F/D-values.

7. Action points

The ALMA Management requests a preliminary estimate of the cost of the ACA antenna in January 2001. To that end, I propose the following action points:

1. Estimate costs of a dedicated ACA mount design and fabrication and compare with cost of supplemental ALMA mount fabrication. The CDR documentation, due early December, will contain data to help make these estimates.

2. Perform a conceptual design of the reflector, 6-8 m diameter and 10 μm overall accuracy. Estimate the cost of such a reflector.

3. Evaluate performance of a small (6-8 m) dish on an ALMA mount (pointing, etc.).

These three tasks will be carried out by the ESO/ALMA Antenna Group. We welcome help from others within the ALMA Project.

4. Estimate manpower and additional cost for the design of alternative receiver box(es). If there is however general agreement that the ALMA receiver frontend must be accommodated without change, this point is moot. **Nevertheless, some thoughts from the receiver group on this issue will be needed for an overall proposal.**

5. Study consequences of different geometries (mount, reflector diameter) for the required performance of ACA (close packing, calibration).

The science-systems group is carrying out simulations in this area.

Acknowledgement.

A first draft of this Memo was distributed to a selected group of colleagues. Each of them provided useful comments and encouragement, for which I express my gratitude. More comments are certainly needed and are invited herewith.

Reference

The Interface Control Document Nr.1 (ICD 1), referred to in the text, clarifies the interface between antenna and receiver cabin and receiver box. It can be found at the URL:

http://nastol.astro.lu.se/~torben/alma/almaspecsframe.html

Table 1 - Geometrical parameters of ACA alternatives

	ALMA	ACA	ACA	ACA	ACA	
Diameter parab. (m)	12	6	6	6	6	
F/D	0.4	0.4	0.4	0.35	0.3	
Primary half angle (deg)	64.01	64.01	64.01	71.08	79.61	
Depth of Primary (m)	1.875	0.938	0.938	1.071	1.25	
Magnification-M	20	20	20	22.86	26.66	
Effective F/D	8	8	8	8	8	
Secondary half angle (deg)	3.58	**3.58**	**3.58**	**3.58**	**3.58**	
f=2c (m)	6.177	3.13	**3.2**	**2.9**	**2.6**	
ls=c-a (m)	0.294	0.149	0.152	0.122	0.094	
lf (focus to subrl.) (m)	5.883	2.981	3.048	2.778	2.506	
eccentricity	1.10526	1.10527	1.10527	1.0915	1.07793	
Diameter secondary (m)	0.75	**0.38**	***0.389***	***0.355***	***0.322***	
Collision radius (m)	7.477	***4.23***	***4.3***	***4***	***3.7***	
Coll. circle/diameter	1.25	1.41	1.43	1.33	1.23	
Maximum BUS height (m)	1.38	0.13	0.8	0.8	0.8	

	ACA	ACA	ACA	ACA	ACA	ACA
Diameter parab. (m)	7	7	**7**	8	8	8
F/D	0.4	0.35	**0.3**	0.4	0.35	0.3
Primary half angle (deg)	64.01	71.08	**79.61**	64.01	71.08	79.61
Depth of Primary (m)	1.094	1.25	**1.458**	1.25	1.429	1.667
Magnification-M	20	22.86	**26.66**	20	22.86	26.66
Effective F/D	8	8	**8**	8	8	8
Secondary half angle (deg)	**3.58**	**3.58**	**3.58**	**3.58**	**3.58**	**3.58**
f=2c (m)	**3.7**	**3.35**	**3**	**4.2**	**3.8**	**3.4**
ls=c-a (m)	0.176	0.14	**0.108**	0.2	0.159	0.123
lf (focus to subrl.) (m)	3.524	3.21	**2.892**	4	3.641	3.277
eccentricity	1.10527	1.0915	**1.07793**	1.10527	1.0915	1.07793
Diameter secondary (m)	***0.449***	***0.41***	***0.371***	***0.51***	***0.466***	***0.421***
Collision radius (m)	***4.8***	***4.45***	***4.1***	***5.3***	***4.9***	***4.5***
Coll. circle/diameter	1.37	1.27	**1.17**	1.32	1.23	1.12
Maximum BUS height (m)	0.9	0.9	**0.9**	1	1	1

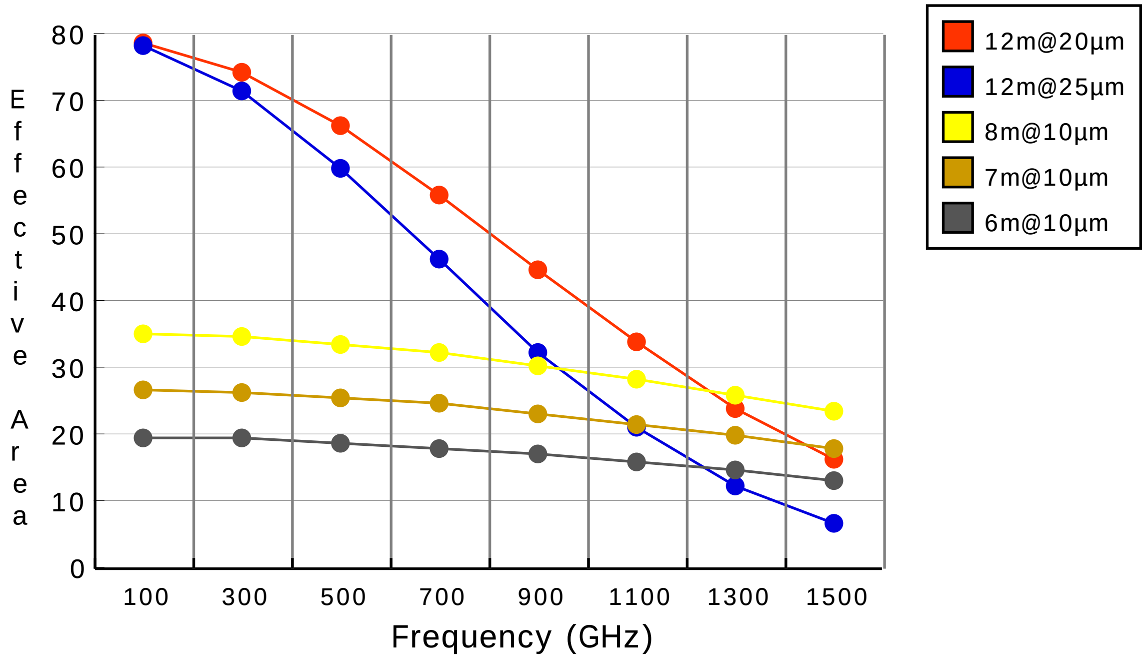
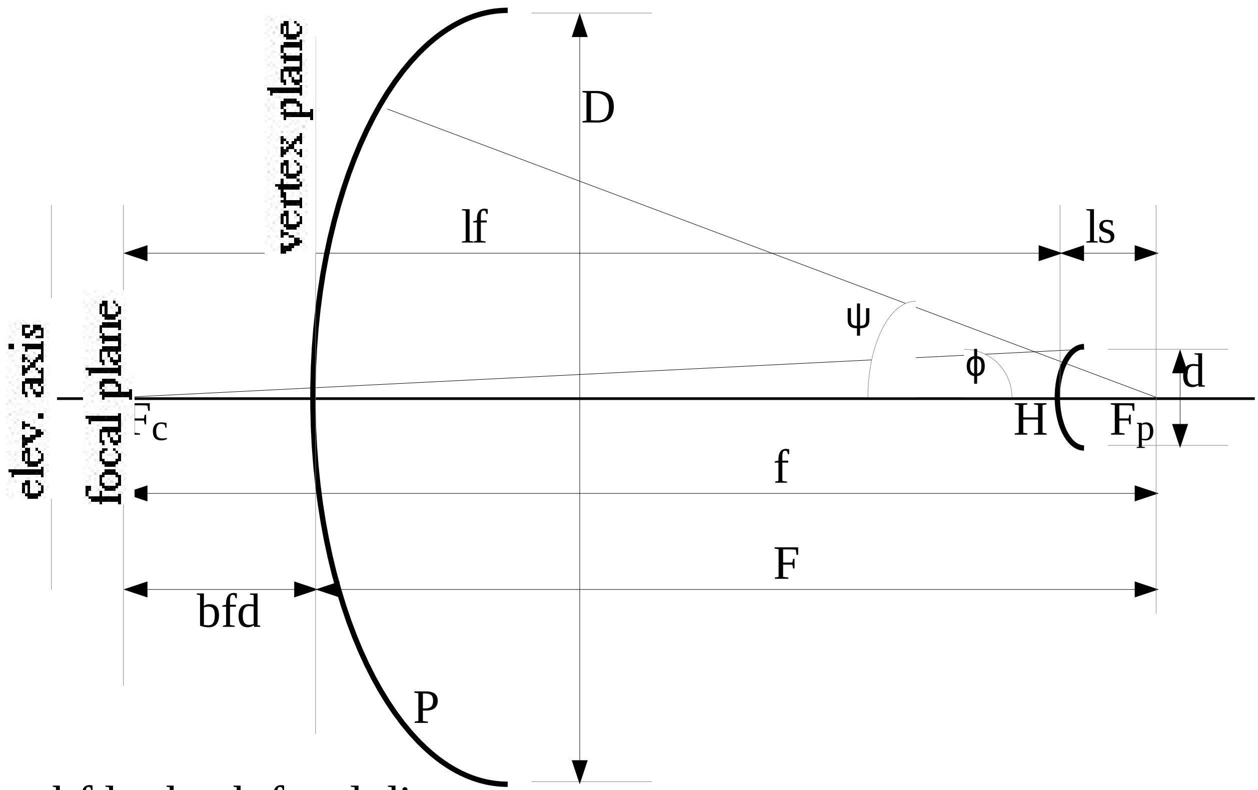


Fig. 1 ALMA-ACA comparison



bfd = back focal distance

main paraboloidal reflector P $\tan\frac{\psi}{2} = \frac{D}{4F}$ (1)

hyperboloidal subreflector H $\cot\varphi + \cot\psi = \frac{2f}{d}$ (2)

eccentricity subreflector $e = \frac{f/2}{f/2 - ls} = \frac{\sin((\psi+\varphi)/2)}{\sin((\psi-\varphi)/2)}$ (3)

cassegrain magnification $m = \frac{\tan(\psi/2)}{\tan(\varphi/2)} = \frac{e+1}{e-1}$ (4)

Prime focus to subreflector $ls = \frac{f}{2}.\frac{e-1}{e}$ (5)

Cass focus to subreflector $lf = \frac{f}{2}.\frac{e+1}{e} = f - ls$ (6)

(Eqs. 5 and 6 are easily derived from Eq. 3)

Fig. 2. Geometrical relations of the Cassegrain Telescope