NATIONAL RADIO ASTRONOMY OBSERVATORY
COMPUTER DIVISION
INTERNAL REPORT

300 FT. TELESCOPE SURFACE ANALYSIS

BY

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This article describes the analysis completed during September and October, 1966 on NRAO's 300 ft. telescope in Green Bank, West Virginia. It defines the problem, explains the approach used, the results obtained, and subsequent actions taken.

When using any scientific instrument it is necessary to know the precision of the instrument to derive a valid interpretation of its measurements. Parabolic antennas are no exception. Their condition and shape must be maintained within tolerances commensurate with those of their applications.

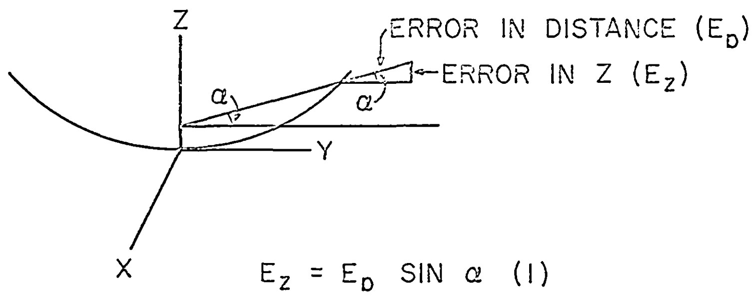
Here we are concerned with the shape of a parabolic antenna 300 feet in diameter. The problem falls into three categories: 1. How well does the shape of the dish conform to a paraboloid? 2. Which areas are out of shape and by how much? 3. Are the deviations severe enough to warrant correction, and if so, how are they going to be corrected?

The general approach was to survey points on the dish's surface, find the 'best fit' surface passing through these points, and use the deviations (of the surveyed points) from the 'best fit' to correct the dish itself. This will be referred to as the survey method.

There are approximately 15,000 locations on the surface of the dish which may be adjusted. Due to limited storage locations in the computer and available time to perform a survey, it was impractical to survey every adjustment location. Consequently, 504 strategic locations were selected to represent the surface of the dish. The 504 locations were symmetrically spaced to be compatible with the computed program which performs the 'best fit'. Three measurements define a point in space: azimuth, elevation, and straight line distance to the point. A survey consists of obtaining these measurements for each of the 504 points.

The first survey was conducted with the dish in Zenith position. Two other surveys were conducted with the dish tilted 30° then 60° to the south. These were conducted to determine the distortion occurring when the dish was positioned in other than Zenith position. In order to establish the degree of accuracy obtained in the surveys, another survey was conducted at 30°. Although four surveys were completed, distances were measured only once - that was in Zenith position. These distances were substituted in all subsequent surveys to complete the measurements for the respective survey.

There are two reasons why distances were measured only once. First, it would be difficult to measure distances accurately while resting at a 30° or 60° tilt. Second, the analysis is almost entirely concerned with deviation in the Z coordinate. It is evident from Figure 1 that an error in distance reflects little error in Z.



$$E_Z = E_D \sin \alpha \quad (1)$$

WHERE $0 \leq \sin \alpha < .25$

Figure 1

All surveys followed the same initial procedure, that is, the theodolite was positioned approximately the same each time before tilting the dish. This provided a reference system common to all surveys. To establish working tolerance it is necessary to know the precision of the measurements themselves. To derive this, the Z coordinates from one survey were differenced from those of another survey and the RMS of these differences were computed. The results of this analysis are shown in Table 1 where the RMS is in millimeters and 30°_1 and 30°_2 represent the respective surveys at 30° tilt.

Surveys

Surveys	0°	30°_1	30°_2	60°
0°		16.9		23.3
30°_1			9.0	11.8

Table 1

Of course, the most representative RMS from a precision standpoint is one derived from the two 30° surveys. But that is still not completely representative because it also reflects errors in the initialization procedure. Errors in the initialization procedure have no effect on a 'best fit' paraboloid though, because in that procedure the origin of initial measurements is arbitrary. Consequently, an RMS of Z coordinate differences between the two 'best fit' paraboloids at 30° would furnish a more valid criterion by which to establish tolerances. This turned out to be 6.9 millimeters; and considering the second 30° survey was done under adverse (windy) conditions, one might reduce that somewhat.

The program* which performs the 'best fit' also provides other information in addition to an RMS of Z coordinate differences between the 'best fit' surface and the measured surface. It also lists and maps the individual deviations, provides a focal length and focal point position for the 'best fit' paraboloid, and locates the vertex of the 'best fit' paraboloid with respect to the origin of the measurements. Also, built into this program was the facility to correct substituted distances in accordance with an indicated shift of the spool.

The theodolite is supported by the spool, and a shift in the spool's relative position seemed to occur when the dish was tilted. This would indicate that substituted distance measurements and angular measurements were no longer referenced to the same origin. Table 2 provides RMS's of individual surveys with and without these distance corrections. Again, RMS's are in millimeters.

*'300 Ft. Surface Analysis Program', August, 1966, by Paul Hitch available upon request.

Surveys

Distance Corrections	0°	$30°_1$	$30°_2$	30°avg.	60°
w		17.3			19.8
w/o	16.6	17.1	16.8	16.6	19.6

Table 2

In order to better understand the relative movement of the spool, the straight line distances to 42 selected points were measured while the dish was tilted 30° to the south. A comparison of these distances with those measured at Zenith appear in Figure 2. The results (with respect to equation 1) indicate that whatever the spool's movement it is neither systematically or significantly displacing the origin of the measurement system.

Before continuing with this analysis of the 300 foot telescope, let us consider another approach to the problem - the electrical measurement method. This method produces an RMS figure representing the entire surface of the dish. All theoretical material describing this method is referenced to pages 635 and 636 of the April, 1966 issue of <u>IEEE Proceedings</u>.

Given:

$$\frac{G}{G_o} = e^{-\left(\frac{4\pi\epsilon}{\lambda}\right)^2} \qquad (2)$$

And:

$$A = \frac{\lambda^2 G}{4\pi} \qquad (3)$$

From (2) and (3) one can obtain an equation relating aperture efficiency to the effective RMS (ϵ) of a radio telescope. That is:

$$\frac{A}{A_o} = e^{-\left(\frac{4\pi\epsilon}{\lambda}\right)^2} = \eta_A \qquad (4)$$

Where:

$$\epsilon = \frac{\lambda}{4\pi}\left(\ln\frac{1}{\eta_A}\right)^{\frac{1}{2}} \qquad (5)$$

Positive = Increase from Zenith
Negative = Decrease from Zenith

Distances at Zenith vs.
Distances at 30° Tilt in
in 32nds of an inch.

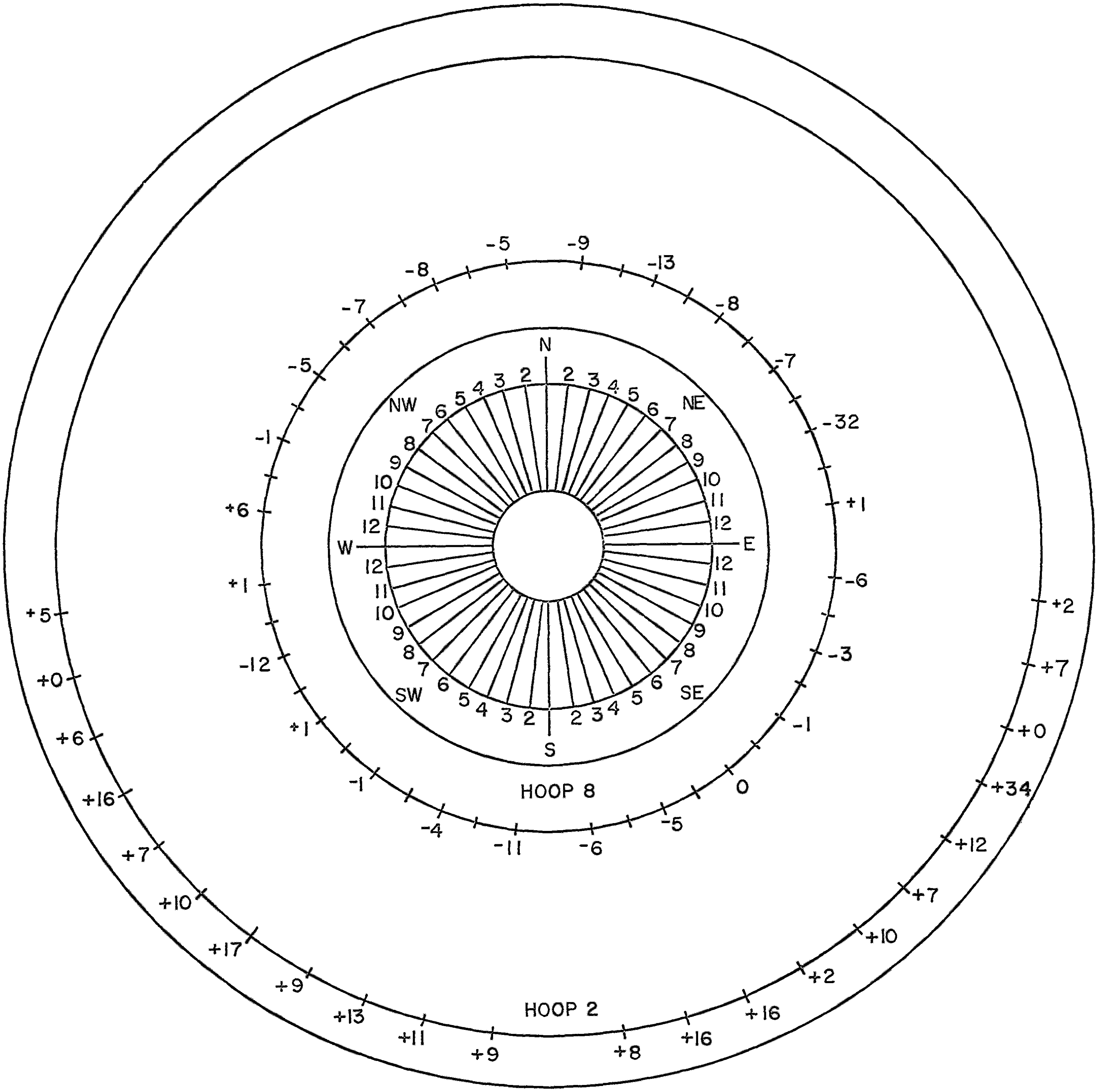


Figure 2

A_o was computed to be 0.67 for the 300 foot telescope. By observing a discrete radio source with small angular diameter and a known flux density, A was computed to be 0.29 at 21 cm. Substituting these values into equation (5), we find the effective RMS is 15.2 millimeters.

The effective RMS is smaller than either the Z RMS (as derived previously) or the normal RMS. However, there is a relationship between them as shown in the equation below:

$$Z = \epsilon(1+(r/2F)^2) \qquad (6)$$

Where:

F = Focal length
r = Telescope radius

When substituting ϵ from equation (5), the Z RMS of the entire surface is computed to be approximately 20 millimeters. The problems inherent in this method are accuracy and definition. It cannot provide the accuracy of the survey method and it does not define position or magnitude of the deviations.

Continuing with the survey method, we found that the RMS of the surface fits which utilized corrected distances were not significantly different from those utilizing substituted distances. Therefore, it was decided not to use corrected distances for the final surface fits.

Since there are a full set of adjustments for each survey, the question at this point is which one is the optimum set of adjustments. In order to gain some insight into the problem, the surface fit program was modified to do further analysis. Each survey was surface fit and corrected to a zero RMS. Then the dish was rotated (within the computer) to the two alternate positions and its RMS was computed in each. The results of this study are found in Table 3. This table served as a guide for all subsequent decisions.

It might be interesting to note how this was accomplished in the computer. Measurements from the survey to be adjusted were read into the computer. These measurements were surface fit and their Z deviations computed. Then measurements from alternate surveys were read in (one at a time) and their Z coordinates corrected according to the corresponding deviation computed from the first survey. The alternate surveys were then surface fit and their RMS computed. This, in effect, was correcting the dish to a zero RMS at one position and then rotating it to its alternate positions and computing its RMS at those positions.

Surveys Adjusted to Zero RMS

Alternate Positions	0°	$30°_1$	$30°_2$	60°	30° avg.
0°		9.0	8.3	13.5	7.9
30°	9.0			10.2	
60°	13.5	10.2	11.5		10.2

Table 3

The adjustments from the first 30° survey were chosen for two reasons: 1. Its results were more consistent than the other surveys. 2. The second 30° survey was conducted under adverse conditions, and this would not only indicate less reliability in its analysis but in the averaged analysis also.

Deviations in the Z coordinate cannot be applied directly to the adjustment points because the studs connecting the surface to the structure are normal to the surface and not vertical. Using the equation of the 'best fit' surface and the Z adjustments, the 'normal' adjustments can be derived as follows:

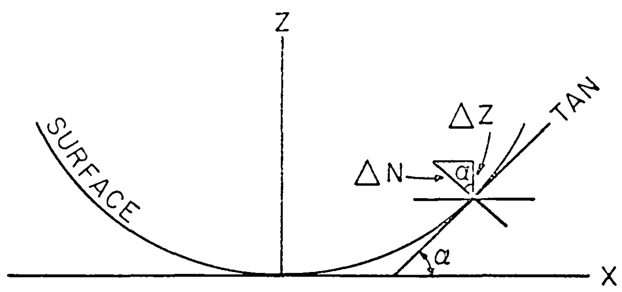


Figure 3

$$X^2 = 4FZ \tag{7}$$

$$Z = X^2/4F$$

$$\frac{dz}{dx} = \frac{x}{2F} = \tan\alpha \tag{8}$$

$$\Delta N = \Delta Z/\cos\alpha \tag{9}$$

The normal adjustments have been computed for 504 points out of 15,000, i.e., about three percent. Based upon the equation of the 'best fit' surface and the distance of a point from the Z axes, the location of the surface relative to a cord between adjusted points can be formalized. Graphs of these functions appear in Figures 4 and 5. The graphs will be used on site to adjust the points where adjustments were not provided.

In Figures 4 and 5:

X = horizontal distance from Z axes to point
ΔL = length of cord
$\Delta S'$ and $\Delta S''$ = vertical distance from center of cord to surface
F = focal length of telescope
α = tangent angle

Visual inspection revealed deformations of the surface between adjustment points. In order to correct these deformations, each panel was removed, straightened, and replaced before the support points were adjusted.

This project required the joint cooperation of the telescope operations, engineering division, and computer division. I feel the excellent cooperation of the telescope operations and engineering division greatly enhanced the computer division's ability to provide the designated information.

The entire operation was coordinated by Dr. Hein Hvatum who also very effectively participated in the development of analysis procedures.

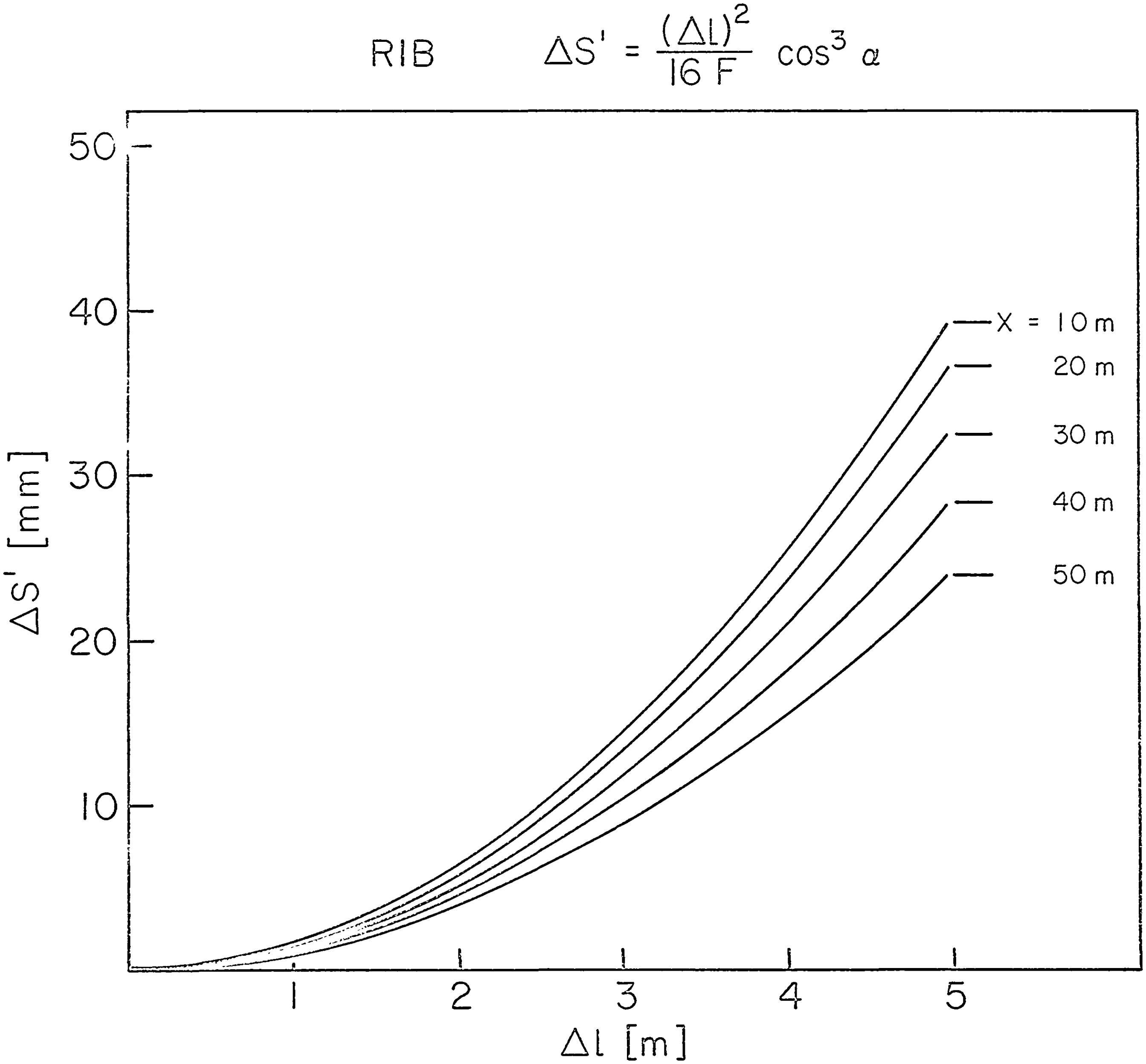

RIB
$\Delta S' = \frac{(\Delta l)^2}{16 F} \cos^3 \alpha$
50
40
30
20
10
ΔS' [mm]
1
2
3
4
5
Δl [m]
X = 10 m
20 m
30 m
40 m
50 m

Figure 4

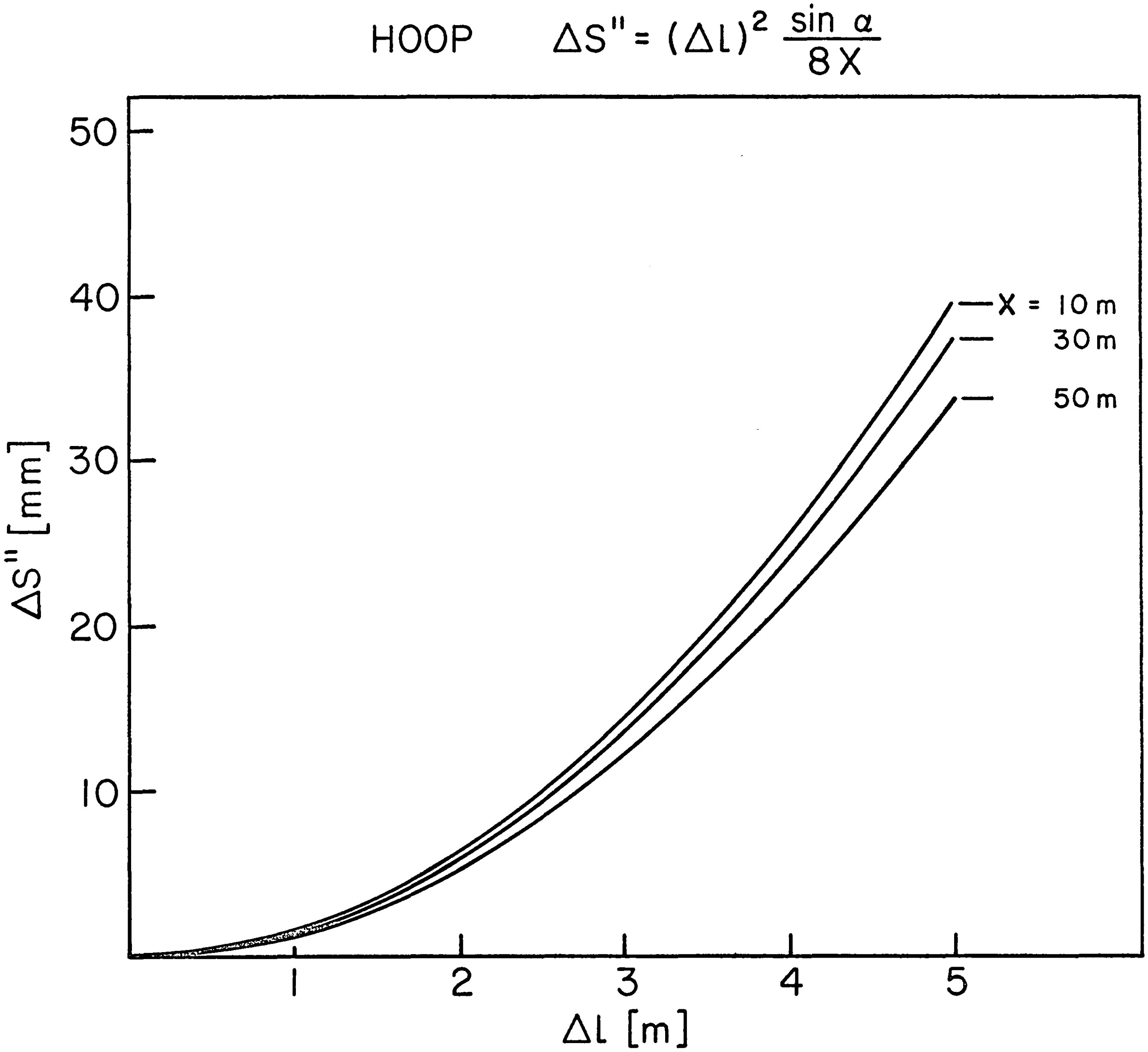

HOOP $\Delta S'' = (\Delta l)^2 \frac{\sin \alpha}{8X}$
50
40
30
20
10
$\Delta S''$ [mm]
X = 10 m
30 m
50 m
1
2
3
4
5
Δl [m]

Figure 5