NATIONAL RADIO ASTRONOMY OBSERVATORY
Green Bank, West Virginia

Electronics Division Internal Report No. 18

POSSIBLE DESIGNS FOR A VERY LARGE ARRAY OF ANTENNAS

Nigel J. Keen

SEPTEMBER 1963

NUMBER OF COPIES: 100

POSSIBLE DESIGNS FOR A VERY LARGE ARRAY OF ANTENNAS

Nigel J. Keen

I. Introduction

The immediate prospect of the NRAO interferometer [1] poses many technical problems which must be solved before a large unfilled array of dishes can be contemplated. The two outstanding problems which will arise on the interferometer are (a) the relative phase stability of the local oscillator signal (< 10°) and (b) the determination of fringe amplitude and phase, with a view to restoring source brightness distributions.

The problems which will not be solved on the interferometer are more numerous, although all these problems are summed up in the question "which array pattern?". An empirical approach to this problem is given, with alternative arrays.

II. Preliminary Discussion

In order to map an area of sky without ambiguity we consider the minimum detectable flux for the array of single paraboloids, assuming parametric amplifiers (giving system noise temperatures T_S = 150 °K), and hence determine the number of detectable sources which will occur in the area of ambiguity (i.e., the single-paraboloid beam). Adapting the estimates of von Hoerner [2] to total power reception with parametric amplifiers, we find that the number of detectable sources for 60 fifty-foot paraboloids is $\sim 4 \times 10^5$ per steradian; assuming 20 arc seconds of resolution at λ = 11 cm, we are brightness- rather than confusion-limited. We see that there are $\sim$ 200 beam solid angles per source. The confusion limit would not have concerned us anyhow, since the counting of "sources" in a given area is irrelevant to the problem of unambiguous resolution of detail within the area. For this and other reasons, not attempt has been made to take into account the numerical distribution of observable source fluxes.

Ideally, we would like a pencil beam with low sidelobes, using a reasonable number of dishes (60, say). The only arrangements that could give such a beam would be random. These patterns have been studied [3] [4], but for individual antennas separated by distances greater than one wavelength, grating sidelobes begin to appear. The reduction of grating sidelobes <u>and</u> the intermediate sidelobes appears both complex and extravagant on antennas and receivers. Hence we consider multibeam arrays. There are two types of high resolution array — the grating array and the synthesis array. In fact these two arrays are similar in function; we merely analyze their performance in different ways. Since we have no intention of proposing a continuous array (dishes touching, or overlapping!) there will always be ambiguities, i.e., several array beams within the beam of an individual antenna.

A further consideration is the individual antenna sidelobe level, since a grating lobe can always occur here. The half-power width of the main beam of a paraboloid is $1.22\,\frac{\lambda}{d}$, which has been determined empirically, for several dishes, by Wade [5]. Using this result, and the calculations of Silver [6], we obtain a tapering index $p \simeq 0.75$ and a 30-minute (half-power width) sidelobe at -22.5 db for a 50-foot dish at $\lambda = 11$ cm. The equations of Silver were derived assuming the reflecting surface to be exactly a paraboloid. However, according to Ruze [7] a $\frac{\lambda}{16}$ rms error over the reflector surface will raise the first sidelobe to -21 db for a paraboloid of diameter $\simeq 150\,\lambda$. The individual antenna pattern is shown in Figure I. Since we are forced to accept some sidelobe level, we take -20 db as the highest permissible for the individual dish and for the array response. This is approximately equivalent to a one percent fringe amplitude or phase error in the correlated signal obtained from two unit antennas in a synthesis array. The positions of the first grating sidelobes are equivalent to the boundary of the "area of no ambiguity" in the synthetic array.

Let us consider an array response $G(\theta, \phi)$ whose Fourier transformation is $g(u, v)$, the "aperture function" of the array. If $R(\theta, \phi)$ is the record of a source whose brightness distribution is $T(\theta, \phi)$, then [8] we have the convolution

$$R = G * T$$

From the properties of a convolution integral

$$r(u, v) = g(u, v) \cdot t(u, v)$$

where t is the Fourier transformation of T, and r is the Fourier transformation of R. Hence we may consider R, G and T (as in the grating) or r, g and t (as in the synthetic array).

III. The Grating Array

Ultimately we wish to determine T. For an unfilled one-dimensional array with regular spacing, d, the grating lobes will appear every $\frac{\lambda}{d}$ radians. Furthermore, there will be intermediate sidelobes due to the "missing orders" being incompletely suppressed. These intermediate sidelobes may be considerably reduced by weighting the contributions from each element in the array. In order to use the techniques of a correlation interferometer [1], the output of one portion of the array may be correlated with the remainder of the array. This procedure gives the same grating response, however the elements may be summed and multiplied (see Appendix I), although intermediate sidelobes and maximum gain of grating lobes may vary. It is possible to suppress some grating lobes (e.g., alternate grating lobes), although this suppression raises intermediate sidelobe levels and widens the remaining grating lobes.

A two-dimensional grating array, of the type constructed by Chrisiansen [9], has the signal from one arm cross-correlated with the signal from the other arm. This array was only required to have intermediate sidelobes down to 2.5 percent, so a cosine-squared current taper was adequate. Figure II is the one-dimensional array response pattern for a crossed grating with 32 dishes in each arm, and a gaussian taper. The separation of adjacent dishes is 6860 λ, which is 75.5 m at 2.7 kMc. Hence this could be a useful array since the grating half-power beamwidth is 8.6 arc sec., and all intermediate sidelobes are < 22 db. The first grating sidelobe is 5 arc min. from the central lobe. The relative gain (compared with zero taper) is 0.52.

IV. The Synthesis Array

The technique of aperture synthesis [10] is to employ the equation

$$t(u, v) = \frac{r(u, v)}{g(u, v)}$$

and to determine t for each value of g corresponding to a pair of unit antennas. Where no pair of antennas gives a component of t, there will be a space on the u-v plane. Also, a uniform spacing of the antennas in the synthesis array gives the dimension of the area of no ambiguity. A synthesis "tee" is shown in Figure III; while the values of g derived from this tee are shown in Figure IV. It should be noted that we neglect the individual dish responses, in the same way that they were neglected in calculations on the NRAO interferometer [1]. The weighting of the components of t is equivalent to tapering a grating array. Mapping the two-dimensional fringe pattern requires some interpolation between the points in Figure IV.

V. Comparison of Gratings and Synthesis Arrays

Both of the approaches to high-resolution, unfilled apertures are for transit instruments. The choice between the two is based on several criteria, which appear to be

(a) The system delay stabilities, if many signals are to be added in a grating array;
(b) The accuracy with which amplitude and phase are retained when pairs of antennas have their outputs cross correlated;
(c) The phase stability of LO links in either system;
(d) The information obtainable;
(e) The complexity of the information processing.

Criteria (a), (b) and (c) will be studied on the NRAO interferometer. Neither system has a significant advantage from the point of view of (d), while (e) militates against the synthesis array in the amount of information to be processed. This is briefly considered in Section VIII.

VI. A First Generation VLA

Assuming that a suitable site has been found, from the point of view of available space, low interference levels and reasonably low tropospheric path fluctuations, an instrument is described which could give useful radio-astronomical information, and could be augmented at a later date to form a more complete array.

A parabolic cylinder oriented in a N-S direction would sweep out areas (on the u-v plane) shown in Figure V for various declinations, as the earth rotates from -5 HA to +5 HA. Nearly half of the Fourier transformation of the brightness distribution is obtained.

Hence we can say that, if a N-S array of dishes has the delay tracking facility from -5 HA to +5 HA, a one-dimensional grating pattern rotates through nearly 90 degrees over the source. Furthermore, for n regularly spaced dishes in the N-S array the phase ambiguity is reduced by a factor (n - 1). This would be the simplest high-resolution system to operate, and would not preclude extending the array for higher resolution, or interposing more dishes to increase grating lobe spacing. Furthermore, this first generation VLA could be switched to a synthesis instrument if required, permitting a comparison between the synthesis and grating approaches.

VII. Conclusions

It has been shown that a one-dimensional grating array could be a useful instrument, without requiring a commitment to the form of a more complete VLA. No detailed design studies have been considered. Rather, we have been concerned with the fundamental capabilities of first and second generation arrays.

VIII. Additional Considerations

1. Information Processing

A complete synthesis of all spatial components from 50 dishes will yield $\frac{50 \times 49}{2} = 1225$ pairs, the outputs of which can be treated in two different ways:

(a) The undetected signals may be recorded, and all 1225 pairs cross-correlated in a computer. For a 10 Mc bandwidth and a one minute scan time this will give > 1,000,000,000 points per scan per antenna. Even neglecting high resolution components in the direction of major arm of the synthesis tee, this still gives $\sim$ 1000 cross-correlations. Furthermore, existing magnetic tapes would be required to travel at 1000 in/sec, which is well beyond present capabilities.

(b) The signals may be instantaneously cross-correlated in $\sim$ 1000 multipliers and integrators, and the outputs be combined in a computer for reconstitution of the source brightness distribution. Such analog multipliers and integrators (voltage to frequency converters + counters) could feed a central scanner and magnetic tape recorder; this equipment (excluding booster amplifiers and cables) could cost $1.0 million, which is not excessive for such a system. Also, the outputs of all pairs of antennas would be on one tape.

2. Reduction of Ambiguities in Grating Arrays

Where instantaneous correlation of array signals is employed, we are interested in the ambiguities due to grating sidelobes. For a two-dimensional grating we may reduce these sidelobes by cross correlating the output of a very large paraboloid with the sum of all outputs from the elements of the grating. This is the system employed (in one dimension) at Fleurs [10]. A 400-foot transit instrument at $\lambda = 11$ cm would reduce the first of 8 minute sidelobes by $\sim$ 10 db, and the others by more.

References

[1] "Design Considerations for the NRAO Interferometer," N. J. Keen, NRAO Electronics Division Internal Report No. 14 (1963).

[2] "Very Large Antennas for the Cosmological Problem: I. Basic Considerations", S. von Hoerner, NRAO Pub. 1, 2 (1961).

[3] "Statistical Design of Space Tapered Arrays", S. J. Rabinowitz and R. F. Kolar. Proc. A. F. Symp. on Antennas, Univ. of Illinois, October 1962 (unclassified).

[4] "Space Tapering of Linear and Planar Arrays", R. E. Willey, Trans. IRE, AP-10, p. 369 (July 1962).

[5] C. M. Wade - private communication.

[6] "Microwave Antenna Theory and Design", S. Silver. Rad. Lab. Series, Vol. 12, Ch. 6 (1949).

[7] "Physical Limitations on Antennas", R. Ruze. Tech. Report 248, Res. Lab. of Electronics, MIT (1952).

[8] "Sensibilité des Radiotélescopes et Récepteurs à Corrélation", E. J. Blum. Ann. d'Ap., 22, 2 (1959). Translation at NRAO, p. 13.

[9] "The Crossed-Grating Interferometer: A New High-Resolution Radio Telescope", W. N. Christiansen, et al. Proc. IEE (Brit.), 108B, 37 (January 1961).

[10] "A Compound Interferometer With a 1.5 Minute of Arc Fan Beam", Labrum, Harting, Krishnan and Payten. P.I.R.E. Anst., 24, 2, p. 148 (1963).

APPENDIX I

We consider six uniformly spaced dishes in a straight line. We wish to determine the antenna response when the outputs, V, from the individual dishes are combined in various ways. All dishes are assumed identical. Hence, we have

$$\left.\begin{array}{ll} 1. & (V_1 + V_2 + V_3) \; + \; (V_4 + V_5 + V_6) \\ 2. & (V_1 + V_2) \; + \; (V_3 + V_4 + V_5 + V_6) \\ 3. & V_1 + (V_2 + V_3 + V_4 + V_5 + V_6) \end{array}\right\} \quad \text{Cross-correlation}$$

$$\begin{array}{ll} 4. & (V_1 + V_2 + V_3 + V_4 + V_5 + V_6)^2 \end{array} \quad \text{Square law detection}$$

1. Assuming amplitude of V is unity, we have

$$W = (1 + e^{i\delta} + e^{2i\delta})\;(e^{i\delta} + e^{2i\delta} + e^{3i\delta})$$

where

$$\begin{aligned} \delta &= 2\pi d \sin \Theta/\lambda \\ &= 2x \end{aligned}$$

$$W = \frac{(1 - e^{3i\delta})}{1 - e^{i\delta}} \times e^{i\delta} \frac{(1 - e^{3i\delta})}{1 - e^{i\delta}}$$

$$\mathrm{Re}[W] = \frac{\cos 3\delta\,(\cos 3\delta - 1)}{-2 \sin^2 \delta/2}$$

$$= \cos 6x \frac{\sin^2 3x}{\sin^2 x} \qquad \text{———————} \qquad (1)$$

2.

$$W = \frac{(1 - e^{4i\delta})}{1 - e^{i\delta}} \times e^{i\delta} \frac{(1 - e^{2i\delta})}{1 - e^{i\delta}}$$

$$\mathrm{Re}[W] = \frac{\cos 3\delta\,(\cos 3\delta - \cos \delta)}{-2 \sin^2 \delta/2}$$

$$= \cos 6x \frac{\sin 2x}{\sin x} \cdot \frac{\sin 4x}{\sin x} \qquad \text{———————} \qquad (2)$$

3. $$W = \frac{(1 - e^{5i\delta})}{1 - e^{i\delta}} \times e^{i\delta} \frac{(1 - e^{i\delta})}{1 - e^{i\delta}}$$

$$\mathrm{Re}[W] = \frac{\cos 3\delta\,(\cos 3\delta - \cos 2\delta)}{-2 \sin^2 \delta/2}$$

$$= \cos 6x \frac{\sin 5x}{\sin x} \qquad \text{———————} \qquad (3)$$

4. $$W = \left(\frac{1 - e^{6i\delta}}{1 - e^{i\delta}}\right)^2$$

$$\mathrm{Re}[W] = \cos 10x \frac{\sin^2 6x}{\sin^2 x} \qquad \text{———————} \qquad (4)$$

The relative gain in equation (1) is 9, that in (2) is 8, and that in (3) is 5. The relative gain in (4) appears to be 36, but it should be remembered that no correlated receiver noise occurs in (1), (2), and (3), so the apparent gain advantage of (4) will be lost.

We see that all grating lobes always occur when

$$x = n\pi$$

$$\text{i.e.,}\ n\lambda = d \sin \Theta$$

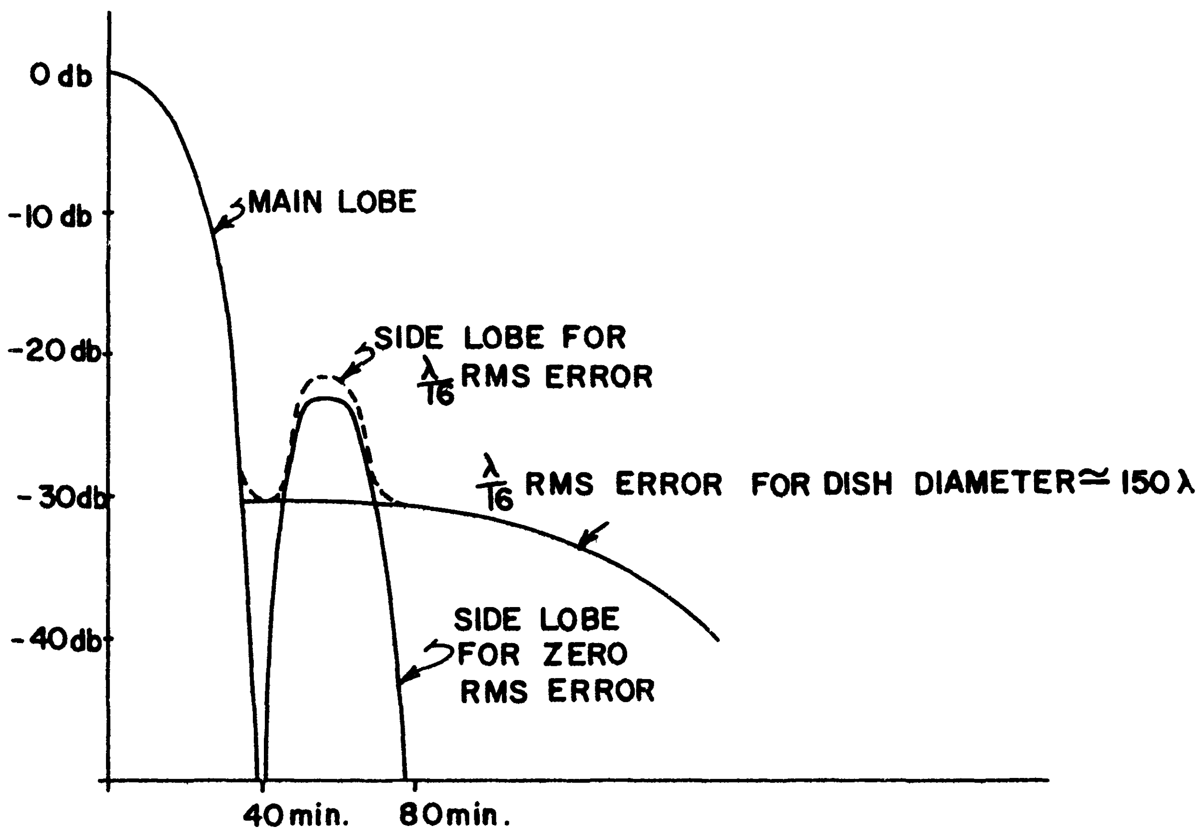
0 db
-10 db
-20 db
-30 db
-40 db
MAIN LOBE
SIDE LOBE FOR
$\frac{\lambda}{16}$ RMS ERROR
$\frac{\lambda}{16}$ RMS ERROR FOR DISH DIAMETER $\simeq 150\lambda$
SIDE LOBE
FOR ZERO
RMS ERROR
40 min.
80 min.

FIGURE I

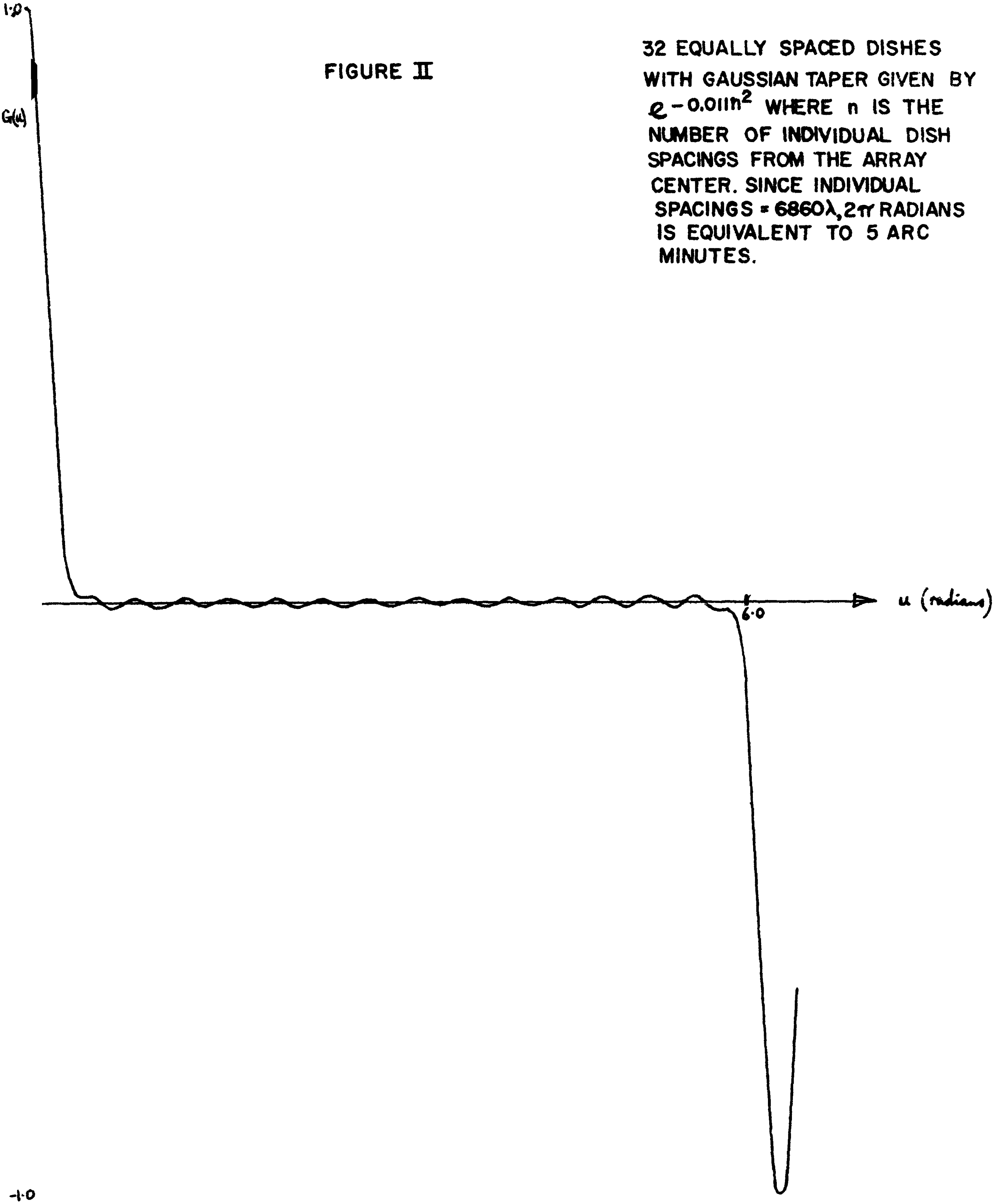
1.0
G(u)
32 EQUALLY SPACED DISHES WITH GAUSSIAN TAPER GIVEN BY $e^{-0.011n^2}$ WHERE n IS THE NUMBER OF INDIVIDUAL DISH SPACINGS FROM THE ARRAY CENTER. SINCE INDIVIDUAL SPACINGS = 6860λ, 2π RADIANS IS EQUIVALENT TO 5 ARC MINUTES.
6.0
u (radians)
-1.0

FIGURE II

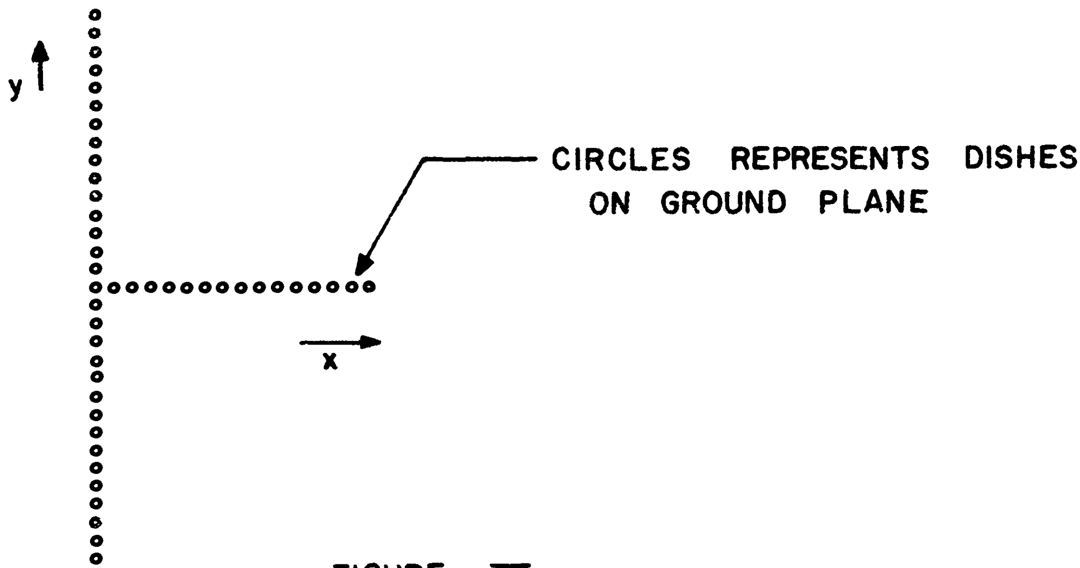

FIGURE III

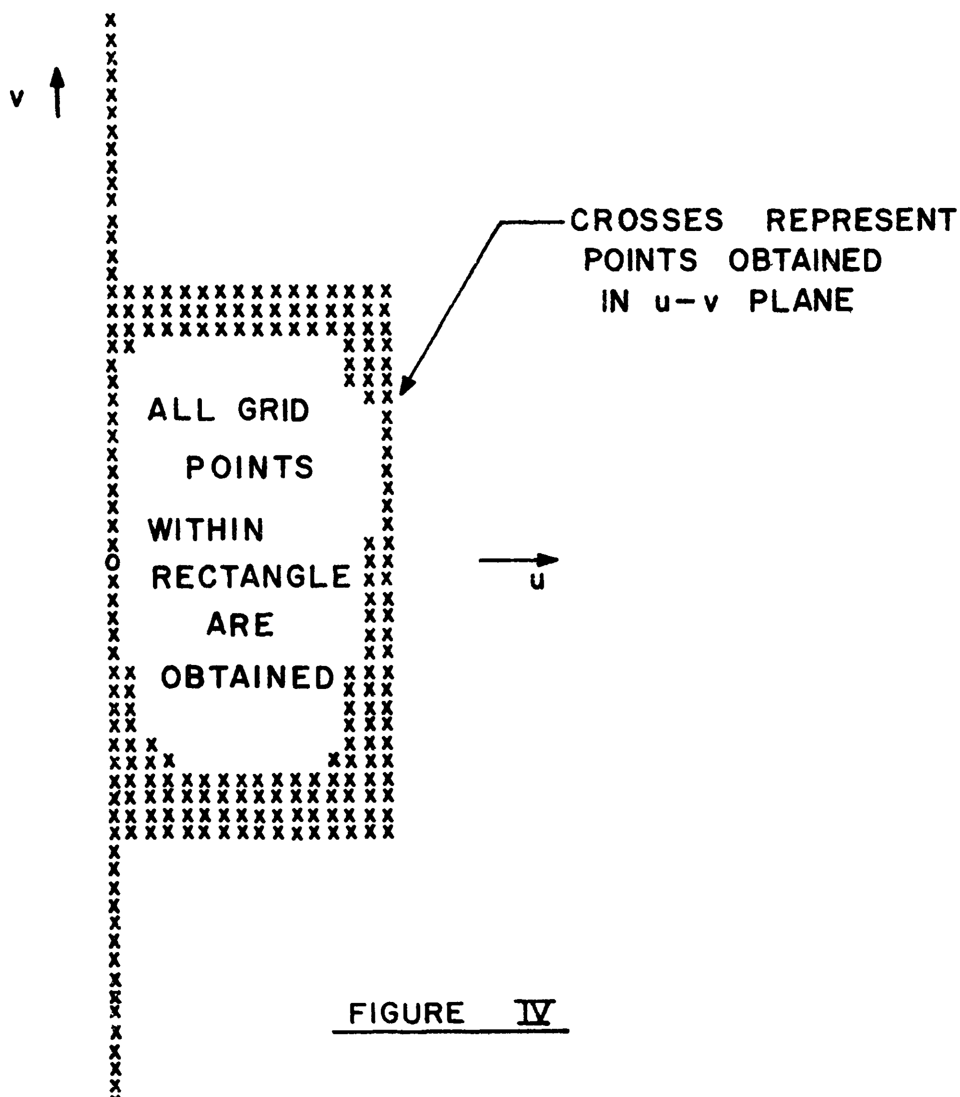

FIGURE IV

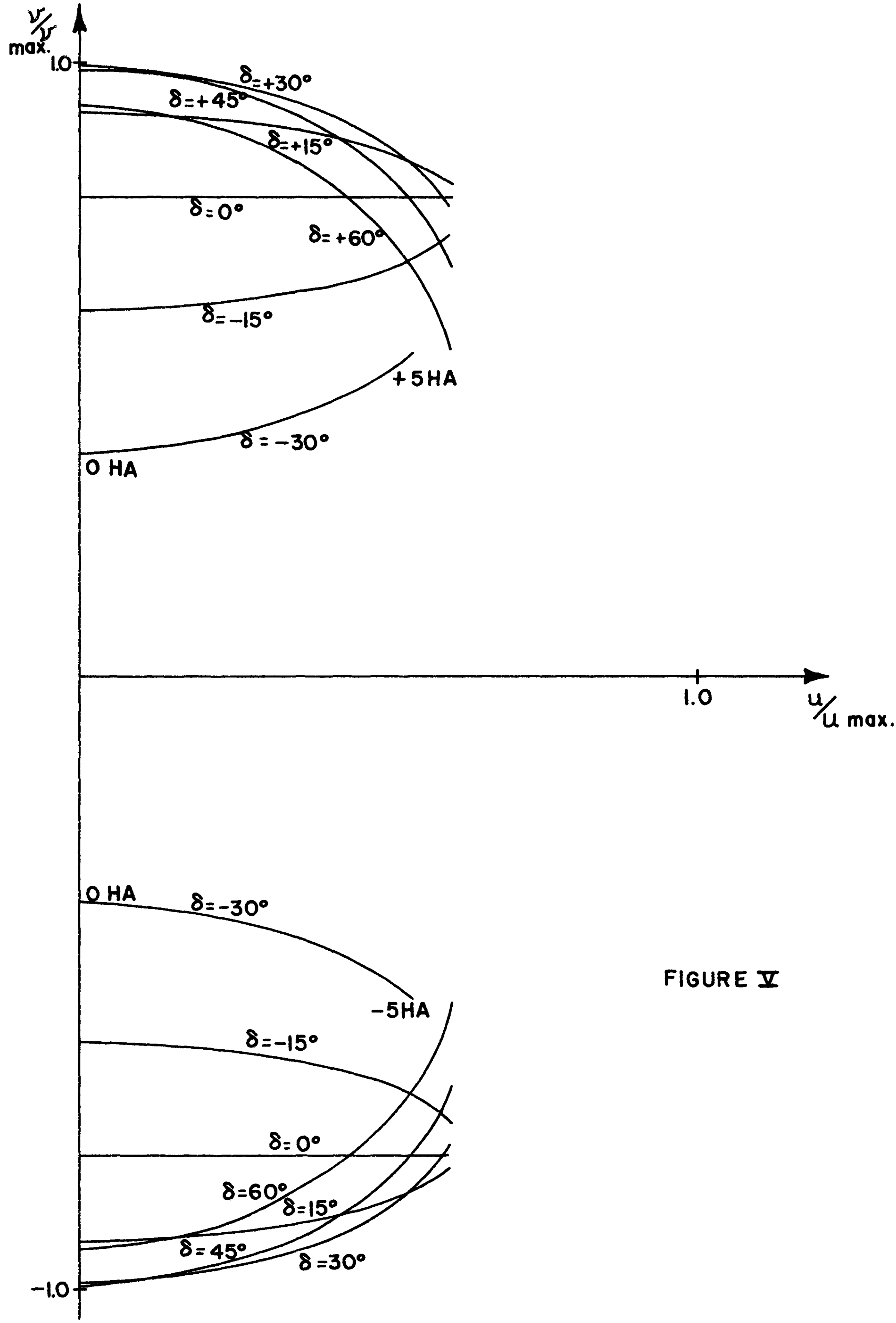
v/v max.
1.0
δ=+30°
δ=+45°
δ=+15°
δ=0°
δ=+60°
δ=-15°
+5HA
δ=-30°
0 HA
1.0
u/u max.
0 HA
δ=-30°
-5HA
δ=-15°
δ=0°
δ=60°
δ=15°
δ=45°
δ=30°
-1.0

FIGURE V