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TESTS OF THE CART METHOD FOR MEASURING TELESCOPES

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TESTS OF THE CART METHOD FOR MEASURING TELESCOPES

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1. The Purpose and Plan of the Tests

The method of measuring a telescope by moving a cart over radial tracks on its surface and recording the position of the cart and the surface curvature was developed more than two years ago (Payne, Hollis and Findlay 1976*). It has been used several times to measure the NRAO 11-meter telescope. Although estimates of the accuracy which the method could achieve were made, these first measurements did not permit us to look carefully at the various possible sources of error. They showed the method could give good repeatability but they could not be used to determine whether, in fact, the method could be precise enough to be used to measure and set the surface of a 25-meter telescope intended to work at wavelengths as short as 1.2 mm.

The other uncertainty was whether it would be possible to determine the cart constants sufficiently well for the method to give an absolute measure of the surface profile. The cart constants which are most difficult to find are the reading which the depth sensor gives when the cart rests on a perfectly flat surface (the radius of curvature is infinite and the cart should give zero for the curvature), and, to a lesser extent, the relationship between the depth sensor reading and the radius of curvature.

For reasons which are fairly obvious, or which will soon become clearer, we chose to test the method by running the cart along an almost flat track. This could be built in the basement of the Green Bank laboratory, where the floor is a reinforced concrete slab resting on the ground and where the temperature,

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* Referred to through as PHF.

although not controlled independently of the general thermostat control of the whole building, in practice can remain stable to better than 1° K over several hours. The track was made 12.5 meters long, and it started and ended on good granite surface plates, each about 90 cm by 60 cm by 15 cm thick. These plates are flat to ± 10 microns and they could be levelled to be within a few arc minutes of the horizontal. The final form of this track is shown in Figure 1 and Plate 1. As we shall see later the first track that was built was unsatisfactory due to its bending under the cart weight.

The use of an almost horizontal track is convenient, and, provided it has enough variations in level of adequate size, it is quite satisfactory to test the reproducibility and accuracy of the method. Since the track was nearly level it was possible to measure it quite accurately with a precise optical level--thus direct comparisons of two independent profiling methods could be made.

The plan of the tests was to tow a cart along this track, record distance readings and curvature and study the reproducibility and absolute accuracy of the results. By using carts of different design with different sensors and by making other changes, the tests should allow better estimates to be made of the system error.

2. The Various Parts of the Test System

(a) The test track and towing system

The first track to be built was not satisfactory (in the section on errors we will return to this in more detail), because it bent under the cart weight. It was supported on wooden posts, and it had a wooden top surface carrying a thin (3 mm) aluminum sheet for the cart to run on.

The final track (Figure 1) has a running surface of 19 mm thick aluminum, 25.4 cm wide, the upper surface of which has been machined flat. The smoothness of the machined surface is 0.6 microns. This plate is supported on an aluminum H-beam which in turn is held above the floor by columns of adjustable height. The base of each column is anchor-bolted to the concrete floor. (Much of this track material was left over from the test stand frame used to measure the 300-foot telescope surface panels.)

The various carts were towed along the track by a thin steel model-aircraft control cable. This was wound around an electrically driven winch drum and its ends were fixed to the front and back of the cart. This cable was thus a continuous loop connecting to the cart and carried over the whole length of the track by pulleys and guides. The cable was kept under slight tension. The cart speed was about 10 cm per second; sometimes the towing wire was removed and the carts towed by hand. An operator always walked alongside the cart to give it gentle lateral steering forces and to carry the electrical connecting cable. Simple tests showed that the operator's weight on the floor did not alter the depth sensor readings.

(b) The carts and their sensors

Most of the carts which have been used have been the same length (50 cm between the front and back wheels) and most have been 3-wheeled. One cart of 25 cm length has been used (partly to see if there was any apparent reason to change the cart length and partly for its possible use on a short focal-length telescope). One cart with only two in-line wheels--kept upright by an outrigger wheel near the center--was also used. Most carts carried the depth sensor directly on the track surface, but many tests were made with the center-wheel cart (CWC) shown in Figure 2 and Plate 2. This cart was designed and built so that the depth sensor measured the up-and-down movement of the center wheel. By doing this, and by having all wheels of the same radius, effects of lateral gaps in the test track would be minimized.

All carts have used the same wheel sensors--a brief specification of which follows:

The Wheel Sensor

Made by:	Disc Instruments, Inc., Costa Mesa, California
Type:	Rotaswitch Incremental Shaft Encoder
Model:	821A-250
Output:	A +5 volt square wave with 250 complete cycles per wheel rotation.
Torque:	0.7 oz inches
Accuracy:	± 2.5 arc minutes

The wheel sensor square wave was divided by 2 in frequency for carts with a 76.2 mm diameter wheel. This gave a reading every 1.9151 mm of track. On one cart with a 50.8 mm diameter wheel, the output was divided by 3 to give the same result.

Most work has been done using the Schaevitz depth sensors; a brief specification of a typical one is:

A Schaevitz Depth Sensor

Made by:	Schaevitz Engineering, Pennsauken, New Jersey
Range of travel:	± 0.5 mm
Repeatability:	0.1 micron
Linearity:	± 0.2% of full range
Voltage Output:	± 5 volts for full range

The full-range DC voltage available from the depth sensor was not always ± 5 volts. The gain of a stable amplifier in the electronic interface was modified (according to the sensor in use) to set the voltage to about ± 5 volts before it was digitized and recorded.

Some tests have been made using a capacitative distance gauge instead of the Schaevitz depth sensor. The characteristics of this gauge are as follows:

<u>The Photocon Systems Capacity Gauge</u>

Made by:	Photocon Systems, Arcadia, California
Model:	PT-5 Proximity Transducers with Dynagage
Size of disk used as one plate of the capacity:	25.4 mm diameter down to 1.27 mm diameter
Linearity:	Depends on several factors, can be ± 2% of full-scale output.
Repeatability:	About 0.1 micron
Output:	± 5 volts for full range

(c) The data recording system

The data recording is based on the use of a 7-track Model DSR 1337 Digidata Stepping Recorder--purchased specially for this purpose. It writes a tape (at 556 BPI) which is compatible with the IBM 360 computer at Charlottesville.

The interface between the cart sensors and the DSR 1337 was designed by D. Schiebel and R. Weimer; we will not give the detailed circuit here, but only describe the method. The wheel sensor square wave, after being counted down by 2, provides the "write" command to the system. At this command the ± 5 V analog voltage from the depth sensor is read via a sample-hold circuit into the input of a 14-bit A/D converter. The output from the A/D is written (as a 16-bit binary number) onto the 7-track tape. This tape then has the same format as a standard DDP-116 7-track telescope tape and it can be read into the IBM 360 using the available program (RED116). The recording of 16 bits is done to stay compatible; it only means that one real bit writes the integer 4 on the computer print-out.

The cart must not run too fast, otherwise the DSR 1337 will be confused. The interface checks that the data flow is not too quick and provides a warning if false data is being recorded. A preset counter in the interface allows a choice of how many data points will be written as a single record for a single run of the cart. Usually, this has been 6400 points. When this number is reached the interface provides the inter-record gap signal to the DSR 1337, and the system waits for the next run of the cart to be made.

The interface also, by a D/A conversion of the wheel pulse count, writes an analog record on an X-Y plotter. Figure 3 is such a record, showing the analog voltage from the depth sensor as a function of distance along the track. Such analog records have been used only for visual checking. This is valuable, since the digital output cannot be seen until the tape has been carried to Charlottesville and read into the computer.

(d) Data reduction programs

A brief description will suffice. A single set of tests may be 10 or 12 runs of a particular cart under similar conditions over the test track. A program, RWTAPE, reads these records from the tape into the IBM 360, and stores them on disk. Each record should be (say) 6400 numbers long and should have no parity errors. (The DSR 1337 writes longitudinal and lateral parity onto the records.) RWTAPE rejects records with parity errors and labels any records whose length is wrong. (It also says what it has done!) So only good records get written into the IBM 360.

All records are then printed by LOOKAT. In one of its several forms this program checks that no numbers went out of the A/D range (± 32768). It prints the first 200 numbers and every 10th number, gives the mean and the RMS of the first 125 numbers (one wheel rotation) and the mean of all the numbers. It is often set to plot, on the printer, the first 200 numbers.

The integrations are done by MEASURE. Initial conditions and cart constants can be adjusted. Usually MEASURE prints the track elevation (Y) and the track angle (θ) as a function of distance (X), every 19.151 cm. It can give the mean of Y and the RMS of Y at each of the chosen X values for all the records in a single block. These mean and RMS values are, of course, essential to test the accuracy and reproducibility of the method.

Other programs can take and plot point-by-point differences between one run and another in the same block of runs and do other manipulations of the basic data. All the programs are in the PANDORA system and so, once the data for a given block of runs is in the IBM 360 it is simple to study the data by making changes either in the start conditions, the cart constants or in the program itself.

3. System Calibration and Checking

(a) The carts and their sensors

We have not made very precise checks on the performance of the wheel and its sensor as a distance measurer. During all runs we have used an independent counter to record the total wheel counts as the cart runs from a fixed start position, past the end-point where data is no longer taken to its final stop against a wooden end block. These total counts (usually about 13,000 since the cart travels 12.5 meters and we do not count down by 2 before taking these counts) on all runs rarely differ by more than ± 1 count (± 0.96 mm). Even if this represented a real distance error, which it probably does not, the track slope is never large and the derived Y values will not be wrong by more than 3 microns due to an error in X of this size.

The reproducibility of the Schaevitz depth sensors is excellent. We cannot confirm that it really is as good as the specification ($\sim$ 0.1 micron) because we have no surface on which the cart behaves this well. We calibrated the depth sensors while they were mounted on the cart by inserting feeler gauges of different sizes under the cart wheels or under the sensor. Figure 4 is one such typical calibration. Again, it is not precise enough to show the small non-linearity in the sensor response. The errors here are due to uncertainties in the exact size of the gauges used. We have not worried about this non-linearity, since it will not affect the repeatability of the results from run to run, and measures of this repeatability are our main goal. The depth sensor calibration is required to convert the recorded counts to measures of curvature, K, as follows:

$$V = C_1(d - d_0) \qquad (1)$$

where V is the sensor voltage output ($\pm$ 5 volts), C_1 is the calibration constant, d is the sensor position with reference to its V = 0 position, and d_0 is the sensor position with the cart on a perfect flat. For most of the sensors the distance range to give a $\pm$ 5 volt output change has been 1 mm, so that $C_1 \sim 10$ if d is in mms. The A/D converter gives a count (after going through the IBM 360) of $\pm$ 32768 for $\pm$ 5 volts so (1) becomes

$$\text{Count} = C_2(d - d_0) \qquad (2)$$

where C_2 is $\doteq$ 65536. If the wheel separation is L (and is nominally 500 mm) the curvature (K) is

$$K = \frac{8d}{L^2 + 4d^2} \doteq 3.2 \times 10^{-5} d \qquad (3)$$

where the error is neglecting $4d^2$ is 4 in 10^6 and is unimportant. Hence we convert counts to curvature by

$$K = C_3 \text{ (count - zero count)} \qquad (4)$$

where C_3 is about 5×10^{-10}. The zero count (the count with the cart on a flat) has to be determined, and since we have only calibrated the depth sensor to about 0.1%, we must remember C_3 is only known to about this accuracy. L of course is also known only to about 100 microns, but this uncertainty is also swallowed up in our calibration of C_3.

The Photocon Systems capacity sensor must be calibrated with the cart standing on the aluminum track (it obviously does not work on the granite), so it was bought mounted on a good micrometer head. This could be read to about 2.5 microns. Figure 5 is a typical calibration curve. Its shape depends on the size of the capacity plate used, its separation from the track and on the settings of the sensor electronics.

(b) Sources of error in the carts and sensors

We have already said that we do not consider the wheel sensor method of measuring distance to be a source of appreciable error. Wheel slipping is most unlikely; the torque required to turn the wheel sensor is very low.

(i) Errors due to lack of wheel roundness

All carts show the effects of lack of perfect roundness of the wheels, mainly as a lack of perfect concentricity of the wheel center and the bearing center. This can easily be studied with the cart on the granite slab. The center-wheel-cart (CWC) shows the effect most (see Figure 6), since errors in the center wheel show directly at the depth sensor. All wheels contribute, and the net result is an approximately sinusoidal variation of depth-sensor reading with distance. (It is of course really a cycloidal pattern.) It can easily be seen that the effect is not negligible (see Appendix I), but that it can be allowed for by studying the sensor readings with the cart on the granite slab and corrections then applied.

In practical use of the cart it might be wise for this, and other, reasons to use depth sensors at the end wheel positions as well as at the cart center. The curvature values derived from 3 sensors, so mounted, would not be affected by imperfect wheel roundness. We have not taken this more complex step at this stage in the study.

(ii) Errors in the electronics associated with the depth sensor

We will defer the subject of noise, and mention first a few possible sources of systematic errors. The voltage output of the depth sensor may, for a given depth, drift with time. We have observed such drifts, within the values expected by the manufacturers, for the first hour or so of switching on the sensors. We have confirmed that the drifts, after warm-up, are small throughout the duration of one set of runs--a time of perhaps an hour. We have made no attempt to check the day-to-day stability of the depth sensor outputs, since this is connected to properties of the cart itself, its ambient temperature, the mounting of the sensor and the behavior of the track. Equally, we believe that the gain of the amplifier in the data-interface is adequately stable, over periods of an hour or so. In the earlier tests, no circuit was present to sample and hold the analog voltage from the depth sensor while the A/D conversion took place. Later, as we were searching for causes for error, we added such a circuit. We did not confirm that the lack of this sample-hold feature introduced errors; it is clearly desirable that it should be present. We also, for some time, had a source of error in the digitizing process. This was tracked (by R. Weimer) to a faulty A/D converter; it was hard to find because it was only an occasional error at a level of a few bits.

We have studied the overall performance of the data recording system rather fully. Without going into too much detail we have looked to see whether the numbers written on tape for various constant voltage inputs are correct and free from noise. To see that the wheel pulses from the cart were not interfering with the data, we have run the cart and recorded the voltage of a 1.5 volt cell carried on the cart. We have also held the depth sensor fixed and run the cart, recording the constant voltage output from the depth sensor as read by the wheel pulses. From all these tests we have concluded that the final data taking system was accurate and noise-free, down to the 1-bit (in a 14-bit number) level. When the sensor is set to give ± 5 volts for a 1 mm movement one such bit corresponds to 0.061 microns movement of the depth sensor. We should note that, in principle, the depth sensor should be reproducible to 0.1 micron, so that it might appear that we have introduced a small digitizing noise. However, as will be seen in our discussion on noise, the system noise was equivalent to several recording bits, so that this digitizing noise was unimportant.

It may be thought that we have over-emphasized the testing of the data-recording. But the cart was connected to the interface by some 20 meters length of cabling, carrying power, the wheel-sensor waveform and the depth-sensor voltage.

Various possibilities of cross-talk existed and so the tests were made. In a more fully engineered system it would probably be preferable to complete the digitizing at the cart and send (by cable or even by a radio link) the digital information from the cart to the recording system.

(iii) Noise in the system

As the cart runs over the track the chief source of noise in the system is clearly due to the surface irregularities over which the depth sensor is moving. The test track surface was machined, so also was the surface of the 11-meter telescope. The end slabs of the test track were granite surface plates. It is of interest to see what the system noise is as the cart runs over various surfaces, and as different means are used to get the depth sensor readings.

The simple error theory given in PHF shows that the system noise may be the limiting factor in determining what the accumulated errors in Y are as the method is used to measure a telescope. That paper shows that, when N steps each of half a cart length have been made, the (1σ) error in Y will be related to the error in a depth sensor reading (σ_d) by:

$$\sigma_Y = \{4N^3/3\}^{1/2} \times \sigma_d. \tag{5}$$

In the present work, most carts were 50 cm long and thus N = 50 and

$$\sigma_Y = 408 \times \sigma_d. \tag{6}$$

We have attempted to estimate σ_d in two ways.

The first was to look at the depth sensor output as the cart moves over a flat surface. The best flat we have is the granite end slab. Figure 6 shows the results of such a test, for the center-wheel-cart. If we take the departures of the individual points from the sine curve to be a measure of σ_d, the results in Figure 6 suggest $\sigma_d = 0.67$ microns. This value is probably an overestimate, since it assumes the granite slab itself is perfect.

The second way to estimate σ_d is to take point-by-point differences between successive runs of the cart over the same length of track. If the RMS value of these differences is found, one could say that σ_d is approximately $1/\sqrt{2}$ of this RMS. We have applied this method to estimate σ_d for various carts and sensors, with the results shown in Table I below.

TABLE I

Estimates of σ_d, the Error in a Depth-Sensor Reading

Cart Used	Sensor Used	Conditions of Measurement	σ_d microns
Center wheel	Schaevitz - 1 mm	As shown in Figure 6.	0.67
Center wheel	Schaevitz - 1 mm	Difference of 2 runs on granite -- September 10, 1976	0.48
Payne 2-wheel cart	Schaevitz - 1 mm	Difference of 2 runs on granite -- December 21, 1976	0.38
Capacity sensor 3-wheel cart	Photocon, with 25.4 mm capacity plate	Difference of 2 runs on aluminum track -- December 9, 1976	0.38
Capacity sensor 3-wheel cart	Photocon, 12.7 mm capacity plate	Difference of 2 runs on aluminum track -- March 29, 1977	0.62

(c) Errors due to tracking

The simple theory given in PHF is true for the two-dimensional case, where the cart moves always in a straight line along the track. The errors introduced if the cart follows a wandering path along the track cannot be simply evaluated. In fact the lack of good tracking, combined with the fact that the track itself was not identical in profile for all straight lines drawn on its surface parallel to its length, introduced errors into the tests. The magnitude of these errors was not at first appreciated, and good results were only finally secured in the tests when the cart was constrained (by gentle steering) to run along paths which were identical in transverse position on the test track to $\pm$ 5 mm.

Some computer simulations of particular instances which might arise have been made to show the sort of errors which bad tracking might produce, but no general approach has been found to the problem of analyzing the effects of bad tracking.

(i) The finite cart length

It is clear that a cart of finite length does not accurately measure curvature, although Equation (3) implies that it does. Errors will arise, particularly if the curvature changes much over distances comparable with the cart length. These errors can be studied most easily by computer simulation. For example, the rapid curvature changes in the track at about 7.2 meters from the start (see Figure 3) have been simulated. As the cart of finite length (50 cm) passes through the rather deep hole at 7.2 m these simulations show that the depth of the hole (which is about 1.2 mm) is underestimated by 69 microns. However, as the cart leaves the hole (which was symmetrical in the computer model) the elevation error reduces to zero. In this symmetrical case the sampling errors cancel. They may not, however, cancel when the paths followed by the cart take somwhat different tracks on different runs through the hollow.

(ii) Differences in the track

The shape of the hollow at 7.2 m has been measured with the optical level, and attempts made to simulate errors due to the cart taking different tracks through it. The main source of error will arise if the values of θ which exist at the end of different tracks through the hollow are themselves different. Since the integral for Y has still 5 meters to run after this hollow, a difference of 10^{-5} radians in θ will give a Y error at the end of the track of 50 microns. Various computer simulations have been made to estimate whether errors of this magnitude can arise when the cart follows different tracks.

One such simulation assumed that the curvature as measured on two tracks through the hole had the same shape (an error function was fitted to the observations) but the greatest curvature measured differed by 0.5%. The difference in θ after traversing these two tracks was 1.9×10^{-5} radians, leading to a difference of 94 microns in the values of Y at the end of the track. Such curvature errors could occur if paths through the hollow differed by about 2 cm (measured transverse to the track), and until the magnitude of these effects was appreciated, the tracking was sometimes as poor as this.

(iii) A real telescope

It will be appreciated that these difficulties arose because the test track was imperfect--its profile was not the same for all straight parallel lines along its length. In a real measurement of a telescope the cart must be constrained to follow a straight radial track. This was finally done on the test track and the tracking errors were much reduced.

(d) The effect of cart weight

The weight of the cart bends the track and thus changes the curvature measurements. Clearly this effect must be kept small or the method will fail. At first sight, for a given cart always following the same track, the track bending should not affect the reproducibility of the results of several runs. Also, it might be argued that, since the cart weight always adds the same curvature to the track, the effect can be removed by a mere alteration of the cart zero count. These statements are, however, too simplistic. The calculation of how a particular cart might bend a surface will depend, for its accuracy, on a detailed knowledge of the cart wheel loads and of the track stiffness; we have not attempted such a computation. R. E. Hills (private communication) has worked out the simplest case; and the results show how errors can accumulate. So we have used empirical methods to discover whether track bending was important and then made the track so stiff as to produce no measurable effects.

The test of whether track bending is serious was straightforward. We assumed that the cart weight does not bend the end granite slabs. This assumption was tested by adding weights to the slab on either side of the cart center and testing that the measured curvature changes were negligible. Then we derived the mean depth-sensor reading over one wheel rotation as the cart moves on the granite, and then used this value as our zero count (Equation 4 in paragraph 3a). The resulting track profile should then come out about right--we permit our choice of the zero count to change within our estimated errors of its measurement--and if all is well we can conclude that track bending was not important. We also can examine track-bending by loading the track alongside the cart.

As a result of a long period of such tests, we found it necessary to build the strong track shown in Figure 1. The first track gave notably wrong results, which differed depending on whether a 2-wheel (no center-wheel load) or a 3-wheel cart were used. Loading tests showed the first track also had deflexion hysteresis.

We have not carried this part of the study through to the point where we could say exactly how strong an antenna surface should be. We have confirmed that the 11-meter Tucson telescope results (in PHF) were not in error due to surface bending. We have also confirmed that the surface of the Green Bank 42.7 meter telescope bends too much for the cart method to be used on it.

In principle the effects of cart weight could be estimated and corrections made if runs were made with and without loads added to the cart. However, the added complication particularly with the CWC where individual wheel loads must be considered would be a serious disadvantage.

4. The Results of Tests

(a) Optical measurement of the test track

Since one of the objectives of the tests was to determine the absolute accuracy of the system, we intended to measure the profile of the track using a good optical level. When the level differences are small and the measurement conditions good (a firm floor and a stable atmosphere) we estimated that the optical level measurements were probably good to ± 20 microns. This estimate is somewhat subjective, and is based on the following considerations:

(i) The level itself (the Wild N3 Precision Level) should, according to its makers, level accurately to ± 0.25 arc seconds. However, its elevation scale is marked only at 100 micron intervals and can be interpolated between graduations to about 10 or 20 microns.

(ii) Repeatability tests of the level made near the test track showed (1σ) standard deviations of about 18 microns at a range of 6 meters from the target. At 20 meters the s.d. was about 60 microns.

However, measurements of the track suggested that, over periods of weeks, the track itself might be moving up and down by perhaps 100 microns. Comparisons of optical measures with cart measures would thus only be good if made close together in time.

(b) Initial conditions

The initial conditions which need to be known are:

(i) The (X,Y) coordinates of the start point on the granite slab. These were always set at zero; the same start point was used for all sets of runs.

(ii) The angle (θ_0) that the start slab makes with the gravity horizontal. This was measured with various levels. It is needed to relate the cart results to the optical level results.

(iii) The cart zero count. This was estimated from the cart readings over a full wheel-rotation on the granite slab. However, when a capacity sensor was used, the zero count could not be found in this way, and it had to be treated as a fitting parameter.

(iv) The phase of the wheel-roundness (Appendix I). This was usually kept constant by marking the wheel edges.

(v) The depth sensor calibration. As in Equation 4, we include in this our knowledge of L, the wheel separation.

(vi) The wheel diameter. This was known from the wheel-maker, but was checked by a tape measurement of the distance travelled by the cart after (say) 6400 wheel counts.

(c) Repeatability of cart runs

Over the 18 months of tests of the system, many runs have been made with various carts under different conditions. We will report only examples of the results when, as far as we know, errors due to tracking, track-bending, etc., were small. The testing technique has remained constant. A number of cart runs (at least 5) have been made and recorded. The integrals which derive X and Y for each run have been evaluated, and then, for a given X, the RMS of the n Y-values has been found. We have not taken great care, in testing reproducibility, to get the values of θ_0 and the zero count correct, but this does not, of course, affect the reproducibility test. By RMS, for n values of Y at a given X, we mean

$$\mathrm{RMS} = \left\{ \frac{\sum_1^n \left(Y_i - \bar{Y}\right)^2}{n} \right\}^{1/2} \tag{7}$$

(i) Tests with the original cart

The "original" cart uses the depth sensor in contact with the track surface. It was first used to measure the Tucson 11-meter telescope, and has been subsequently modified to run on two in-line wheels only with a center outrigger wheel. Table II below gives the results of one such test.

TABLE II

Results from Five "Original" Cart Runs of March 28, 1977

X mm	1883	5650	7553	9416	11299	12241
RMS of Y microns	7	81	113	167	117	145

(ii) Tests with the center-wheel cart (CWC)

Much of the testing was done with the CWC (shown in Figure 2 and Plate 2) since this design minimizes errors due to gaps between the telescope panels. In the test track, one such gap existed at the start of each run between the granite and the aluminum. It was filled, as well as possible, with epoxy, but was an adequate simulation of a panel gap. Table III below gives the results of two tests with the CWC:

TABLE III

Two CWC Tests

Date	Number of Runs	RMS of Y in microns at X = (mm)					
		1915	5554	7469	9576	11299	12257
1976 Sept. 10	5	17	47	57	70	86	123
1977 Jan. 17	8	9	51	75	110	143	167

(iii) Tests with the capacity-sensor cart (CSC)

Here the runs had to start and finish on the aluminum track and so they were shorter in length than the CWC tests.

TABLE IV

Nine Runs with the CSC on December 9, 1976

X mm	1915	5554	7469	9576	10534
RMS of Y, microns	4	21	63	71	92

(d) Summary of repeatability tests

We may first note from Tables II, III and IV that there is no great difference between the measurement errors for the different carts. This conclusion is not unexpected. The main difference would show if one sensor were much more precise or well-behaved than another--and they are not. The capacity sensor obviously integrates surface roughness, but so does the computer for all sensors. (The first integral is merely the sum of all sensor readings, taken every 1.9 mm along the track.)

It is interesting to see whether the simple error theory (PHF Equation 8 and paragraph 3, Equation 5) gives a good description of the errors. In our experiment, the half-cart length was 25 cm, so if X is the distance traveled in meters, Equation 5 becomes:

$$\sigma_Y = 9.24\ (X)^{3/2} \times \sigma_d \tag{8}$$

Taking mean values for σ_Y at the different values of X from Tables II-IV, we can plot σ_Y against $X^{3/2}$, as in Figure 7. The fit to a straight line is good (the RMS departure is 7 microns) and from the slope we can derive a value of 0.35 microns for σ_d. If we look back at Table I we see that this estimate looks quite reasonable as compared to the values suggested in the table.

(e) Comparison with optical measures of the track

We have compared the results of the five September 10, 1976 CWC runs with measures of the track made, by the Wild optical level, on September 23, 1976. In reducing the CWC results, we adopted the following constants:

(i) The slope of the starting granite slab (θ_0) was taken as -4.39 arc minutes with respect to the horizontal. Level measures of the slab had given -4.32 arc minutes as the slope; the difference was within the errors of slope measurement.

(ii) The cart zero count was taken as -5890 counts. The mean count for the first 125 wheel counts over the five runs (one wheel rotation on the granite slab) was -5889.6 $\pm$ 38 counts, so that the chosen zero count was well within the error of measurement. (One count is equivalent to a depth-sensor movement of 0.015 microns, so $\pm$ 38 counts is $\pm$ 0.58 microns.)

(iii) The depth sensor calibration constant was taken to be 5.246 x 10^{-10}. (This is the number C_3 of Equation 4.) Our best estimate of C_3 from calibrating the sensor and measuring L was (5.246 $\pm$ 0.005) x 10^{-10}, and so our assumed constant is within the error limits of our measured value.

In Figure 8 we show the comparison in two ways. The lower curve shows as a continuous line the 65 optical level measures of the track. The points are the mean values of 5 runs of the CWC cart on September 10, and the error bars show $\pm\ \sigma_Y$, where σ_Y is the estimated error of a single measurement of elevation by the cart.

The upper curve shows Δ, the difference between the two sets of measurements. The close agreement is seen from the values of Δ:

$$\left.\begin{array}{l}\underline{\text{Mean value of } \Delta = -14 \text{ microns}}\\ \underline{\text{RMS value of } \Delta = 26 \text{ microns}}\end{array}\right\} \qquad (9)$$

(f) Discussion

(i) Reproducibility

We suggest that the tests summarized in (d) above allow of the conclusions that our elementary error theory is adequate and that, in that theory, σ_d is about 0.35 microns. On this basis, we can easily compute the sort of measuring repeatability we should expect if single radii of a 25-meter reflector are measured. In the case where 50 readings are taken, equally spaced along a radii, this gives an average error of 57 microns.

(ii) Agreement with optical measures

The agreement shown in (9) above is between the mean of 5 cart runs and one set of Wild level measures. The optical measures, as we have said in 4(a), may have an average error of $\pm$ 20 microns. Since this is included in our difference Δ, (9) leads us to a very low estimate of the cart error. Let us suppose it is, in fact, about 20 microns. Then the average value of σ_Y would be $20 \times \sqrt{4}$ or 40 microns, not too different from our 57 micron estimate.

(iii) Conclusion

The above numbers cannot clearly be taken as firmly fixed. However, we feel able to conclude that the method, both in reproducibility and in absolute accuracy, appears to be able to meet our need for an average measurement accuracy of 40 microns over a 25-meter diameter telescope.

5. Application to a Real Telescope

The following elements of the system would need further study and perhaps development before the system could be used to its best advantage on a telescope.

(a) The telescope itself

First, it is clear that the method is suited only to measure precise, strong and stable telescopes. We assume that, before the cart method is used, the telescope will have been measured and set to a precision of around 200 microns. Individual telescope panels will be known to a much higher accuracy. The telescope design should allow of a precise cart-moving system to be used. It should also allow of well-known and stable initial conditions for the cart to be provided.

(b) The cart constants

We have made it clear throughout that it has been difficult to determine the cart zero-count, as we have called it. On a real telescope, this can be even harder. The range of the Schaevitz transducers may be too small to measure this zero-count on a flat and still use the cart on a curved telescope. The ideal would be to have, at the center of the real telescope, a circular disk of known surface curvature. This should be about 75 cm in diameter (a good optical mirror blank would be fine). It should be figured to about 0.1 micron, so that each time the cart starts it would record its "zero-count" over a track of known curvature. The origin of the (x,y,z) coordinate system would be the center of this disk; the z-direction would be the dish axis.

It may be that the cart, as a whole, would not keep its long-term stability as a measuring device. We have only tested this over periods of an hour or so. In practice, two ways of checking this seem possible.

(i) A reference radius on the telescope

One or more tracks on the telescope could be a reference radius. Such a radius would be measured by the cart from time to time. It would also be monitored by an independent system. For example, a check by a modulated-laser range measurement (J. M. Payne (1973)) of distances over two paths to the outer edge of the radius could be used. Thus a check on the cart calibration could be kept.

(ii) The cart as a transfer instrument

A second way of using the cart would be as a transfer device. A single test radius of the real telescope would be built, on the ground in good atmospheric conditions, near the telescope to be measured. This test radius could be measured with high precision by, for example, the HP 5526A Laser Measurement System. The cart would be run on this track to establish its constants (exactly as we did). Then it would be used on the telescope--returning regularly to the track as a check. Used this way, the cart becomes a method of carrying a template to the telescope and measuring the shape differences.

(c) Tracking and weight

We believe the tracking was more difficult to achieve on our test track than it was on the 11-meter antenna, and we do not believe getting good tracking to be difficult. Nevertheless, it needs to be done.

Similarly, we do not see the cart-weight as a problem. We at present are asking that our 25-meter telescope should have a strong surface, capable of being walked on by a 100 kgrm man on one foot. It may be that a final cart should be designed to spread its load, but this can be done.

(d) Other problems

On a real telescope we imagine the panel gaps will be small. It may be desirable to fill them with epoxy, as we did for the one gap near the start. But we saw no evidence of errors from this gap, whether we used the center-wheel cart or the "original" cart.

Trailing cables are a nuisance. However, the cart power needs are small, and there is no reason why the data should not be digitized at the cart and sent back by a short radio link. Similarly, the cart could be self-propelled or towed/pushed by a small controlled tractor.

6. Acknowledgements

The work has been done over a period of two years at Green Bank. We have been helped by many, from electronics, central shop, site maintenance, engineering and administration. We acknowledge particularly the encouragement from J. M. Payne from Tucson.

7. References

Payne, J. M., Hollis, J. M. and Findlay, J. W. (1976). Rev. Sci. Inst. 47, 50-55.

Payne, J. M. (1973). Rev. Sci. Inst. 44, 304-6.

APPENDIX I

The Effect of Lack of Perfect Wheel Concentricity

For simplicity, consider the case of a perfectly flat track. Then an imperfect cart will generate curvature (K) readings of:

$$K = a \sin (s/r + \phi) \tag{1}$$

where s is distance along the track, r is the wheel radius, and ϕ is an angle between 0 and 2π which describes the starting condition (s = 0). The function is, of course, periodic in $2\pi r$, and also is not exact, but the sine is an adequate approximation to the cycloid.

The first integral for θ is:

$$\theta = a\int_0^s \sin (s/r + \phi)\, ds = -ra \cos (s/r + \phi)\Big|_0^s$$

i.e., $\theta = ra \cos \phi - ra \cos (s/r + \phi)$. (2)

Note that θ is always of the order of size ra. In our experiments $r \sim 40$ mm and a is about 10^{-7}. Thus, in evaluating the second integral we will set $\sin \theta = \theta$. The Y values are then derived from

$$\begin{aligned} Y &= \int_0^s ra \cos \phi \cdot ds - \int_0^s ra \cos (s/r + \phi) \cdot ds \\ &= ras \cos \phi - r^2 a \sin (s/r + \phi)\Big|_0^s \\ &= ras \cos \phi - r^2 a \sin (s/r + \phi) + r^2 a \sin \phi \end{aligned} \tag{3}$$

APPENDIX I (continued):

Equation (3) shows that the lack of wheel concentricity produces two errors. The second two terms of (3) are of the order of r^2a, and one is periodic as s/r. However, r^2a is, for our experiments, about $(40)^2 \times 10^{-7}$ mm or less than one micron, and these terms are of no interest. The first term, however, grows linearly with distance, and depends also on ϕ, the conditions at the start. Unless ϕ is kept fixed (which in our experiments we usually attempted to do), the first term can give errors as great as $\pm$ ras. These are not negligible; for example, the CWC (as Figure 6 shows) gives a value for $a = 1.66 \times 10^{-7}$; $r = 38.1$ mm and so at 12.25 meters ras is 77 microns.

Two steps have been taken in practice to avoid these errors. All runs of a given cart in a particular set of runs have been started with all cart wheel positions the same. The program LOOKAT prints and plots the counts from the first wheel rotation for each run so a and ϕ can both be measured and the value of the correction applied to the results.

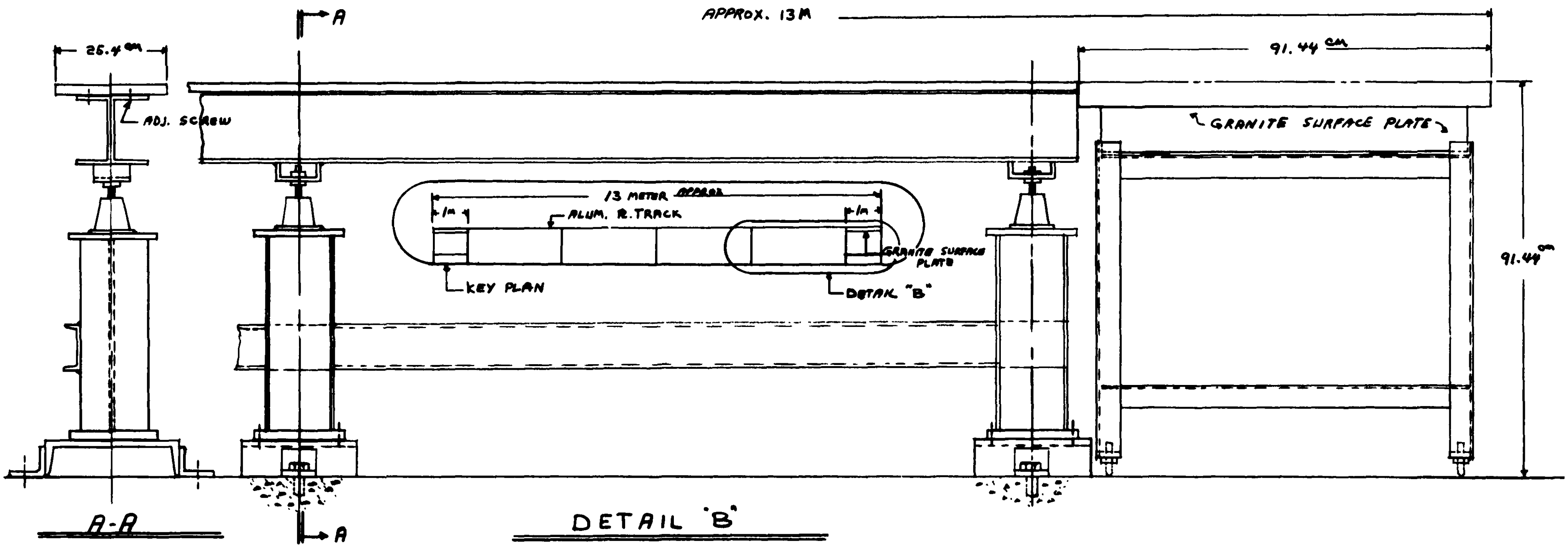

FIGURE 1: TEST TRACK FOR SURFACE MEASURING, MAY 1976.

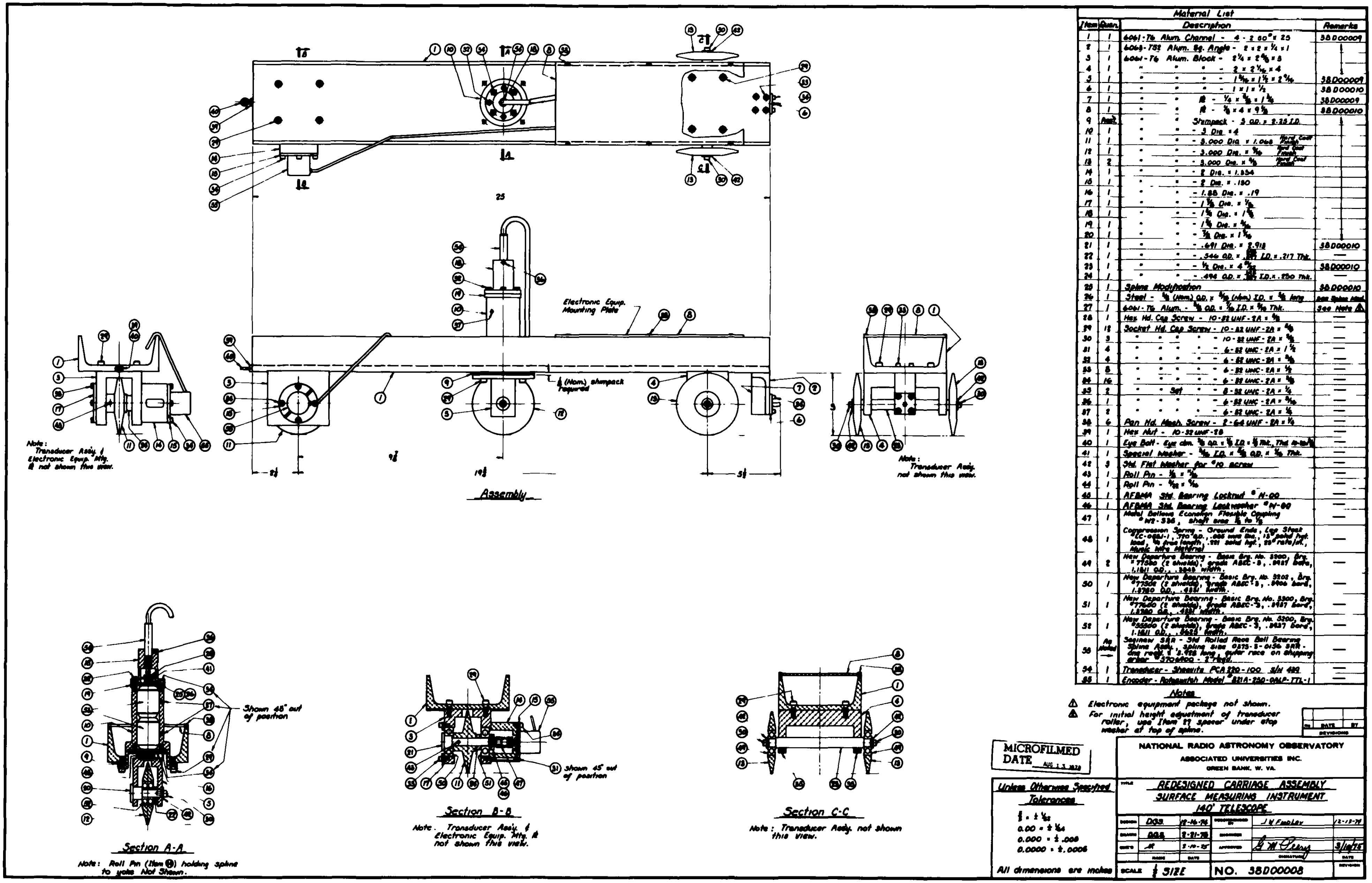

FIGURE 2: THE DESIGN OF THE CENTER-WHEEL CART.

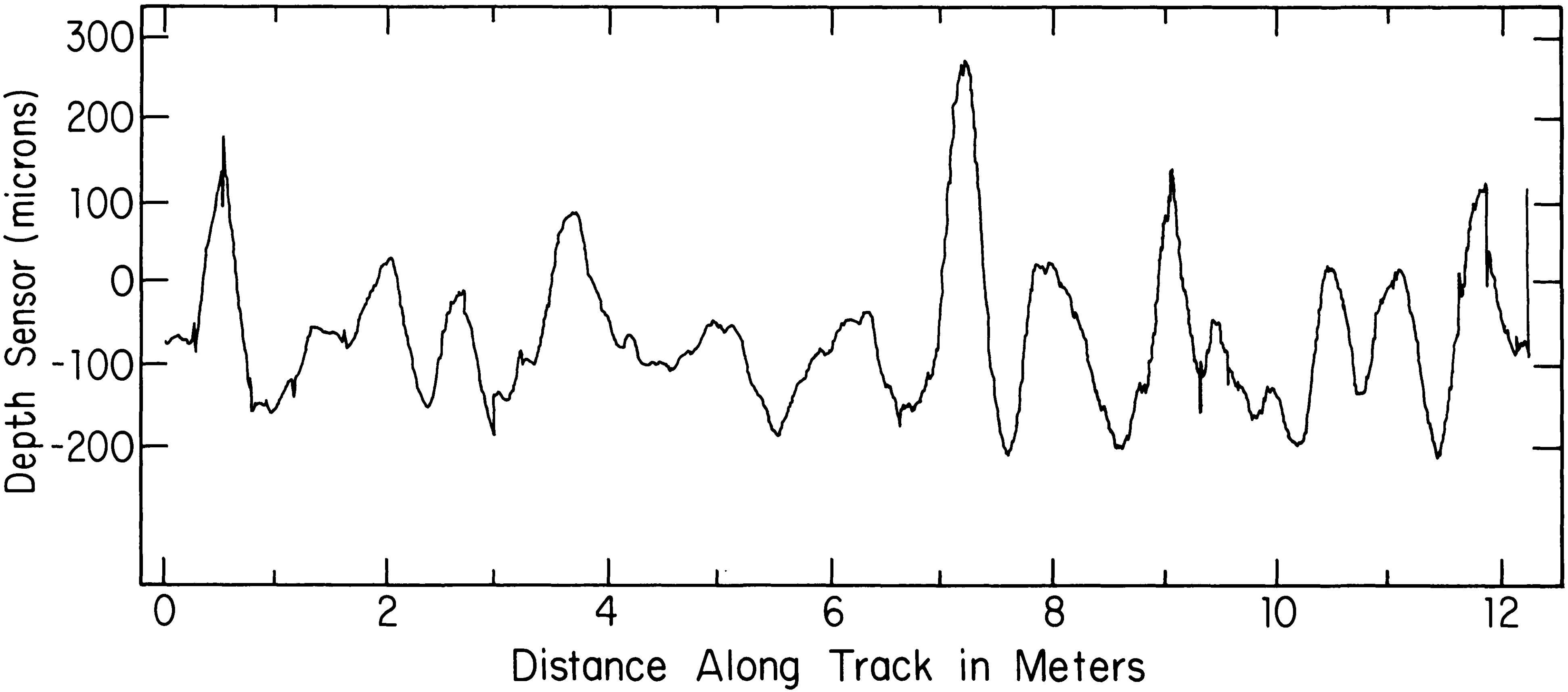

FIGURE 3: A TYPICAL X-Y PLOTTER RECORD.

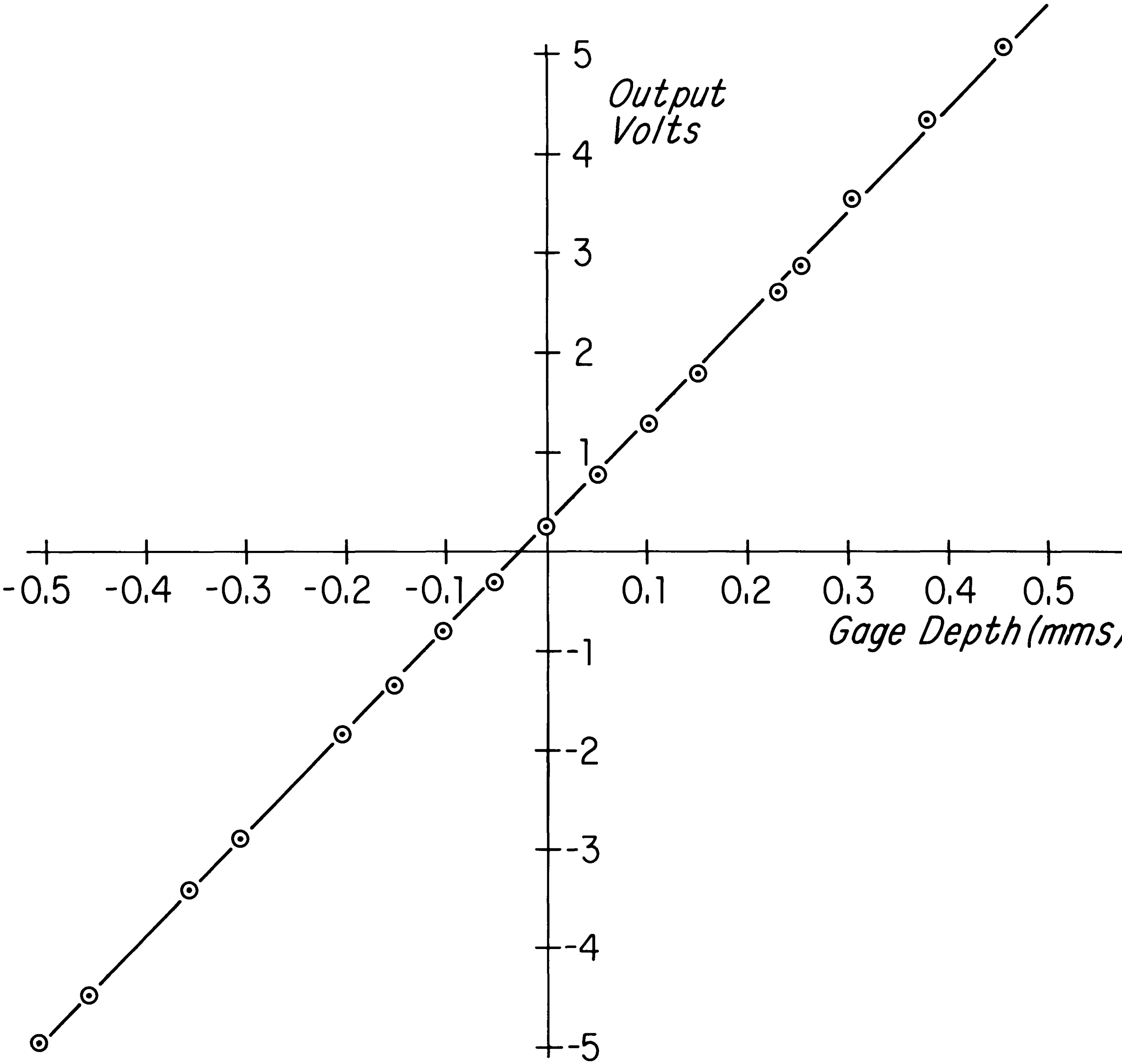

FIGURE 4: CALIBRATION OF A SCHAEVITZ DEPTH SENSOR. THE LINE IS $V = 10.396\,x + .2686$.

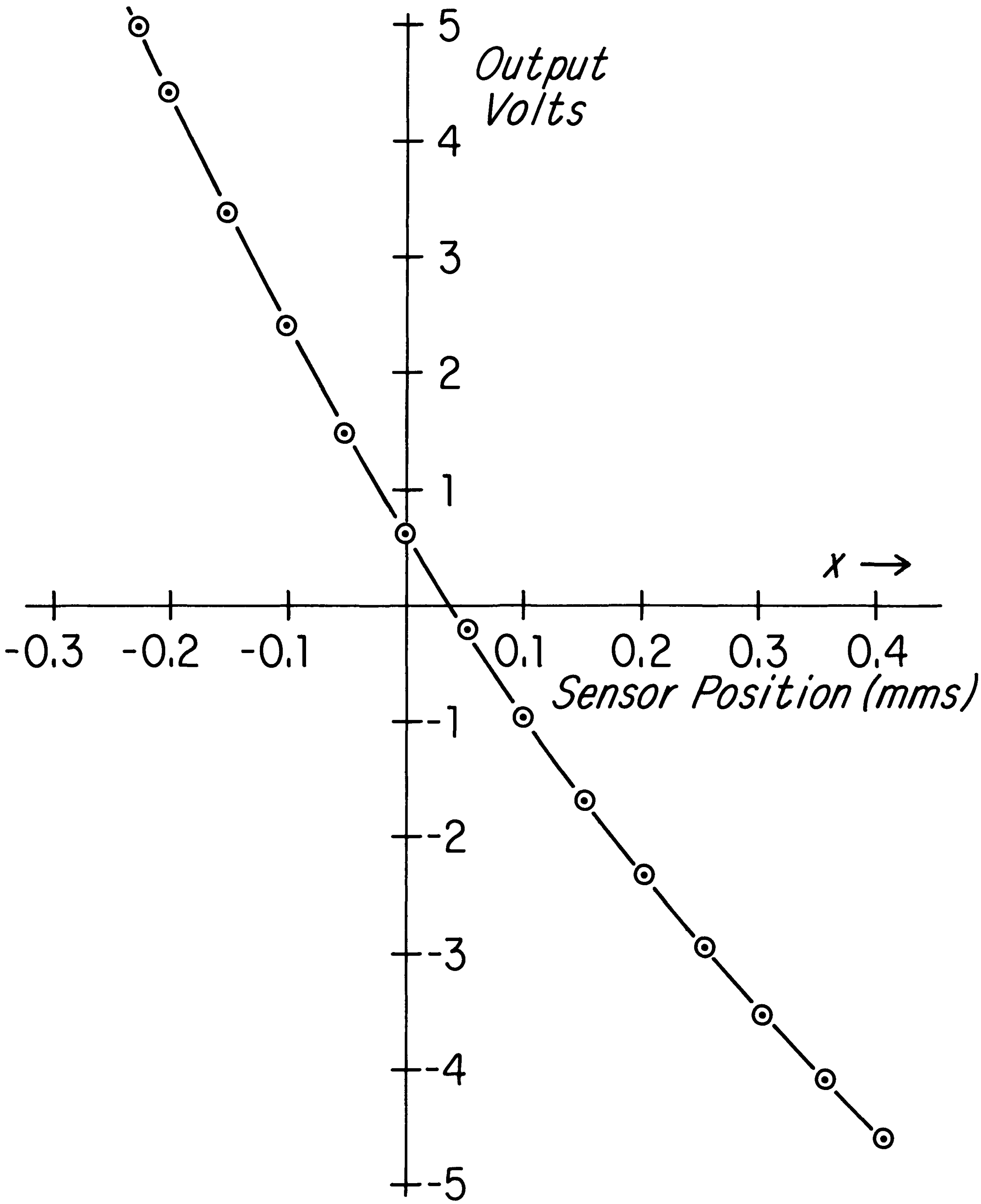

FIGURE 5: CALIBRATION OF THE PHOTOCON SYSTEMS CAPACITY DEPTH SENSOR. THE CURVE IS $V = 0.6342 - 16.625\,X + 9.570\,X^2$.

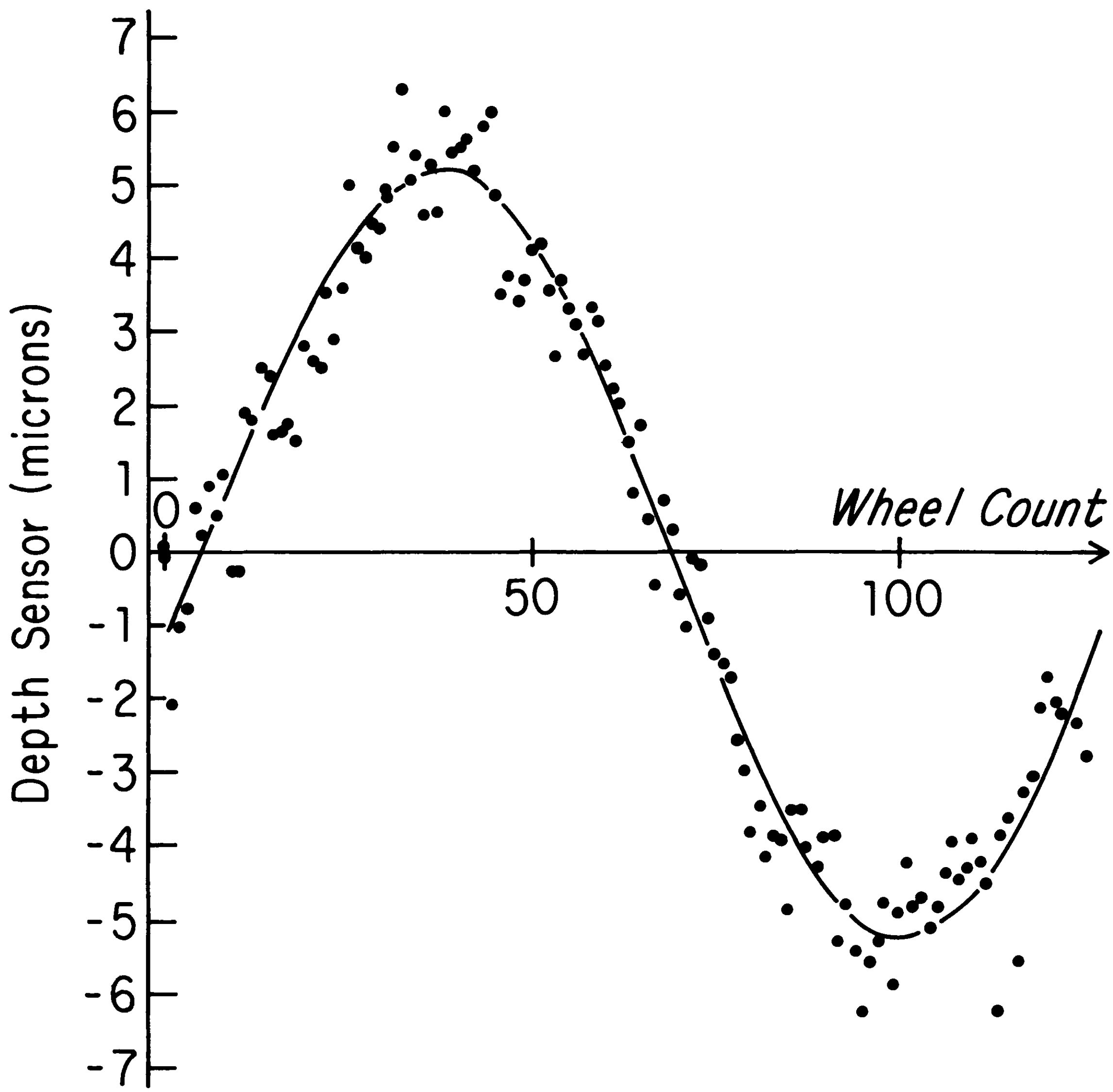

FIGURE 6: THE EFFECTS OF IMPERFECT WHEELS.

THE CURVE IS $Y = 5.2 \sin \left\{ \frac{\text{COUNT} - 6}{125} \times 2\pi \right\}$.

THE RMS DEPARTURE OF Y = 0.67 MICRONS.

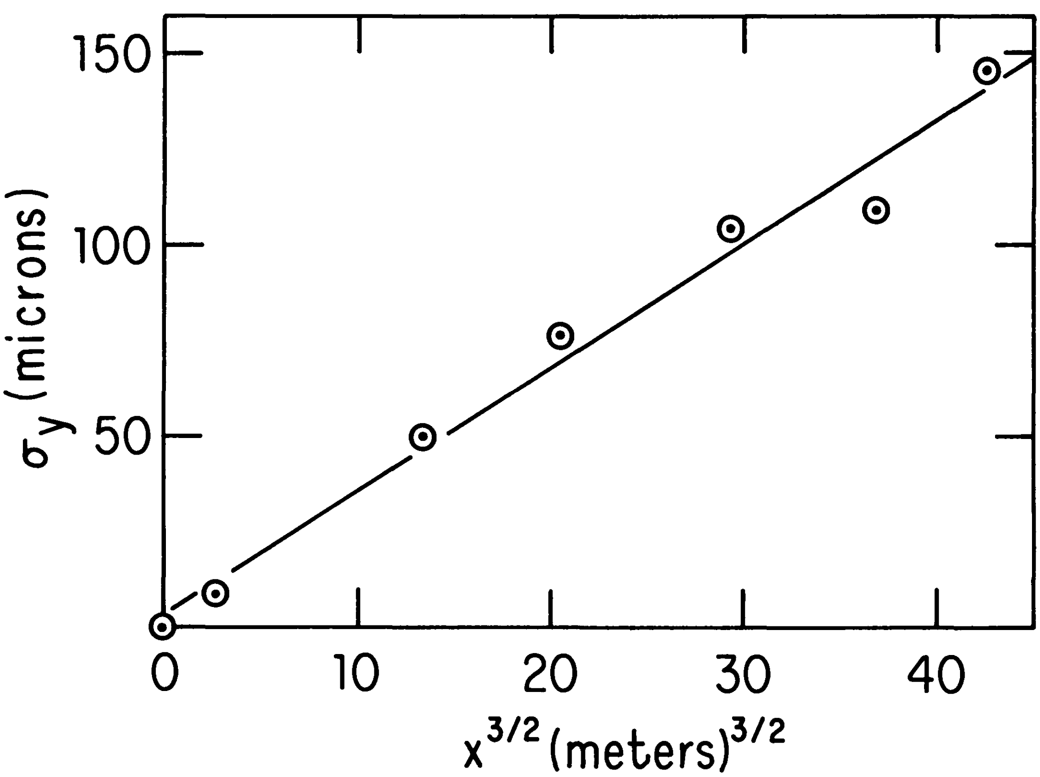

FIGURE 7: Y-ERROR PLOTTED AGAINST $X^{3/2}$ FOR RESULTS FOR ALL THREE CARTS. THE LINE IS $\sigma_Y = 3.71 + 3.220\ (X)^{3/2}$.

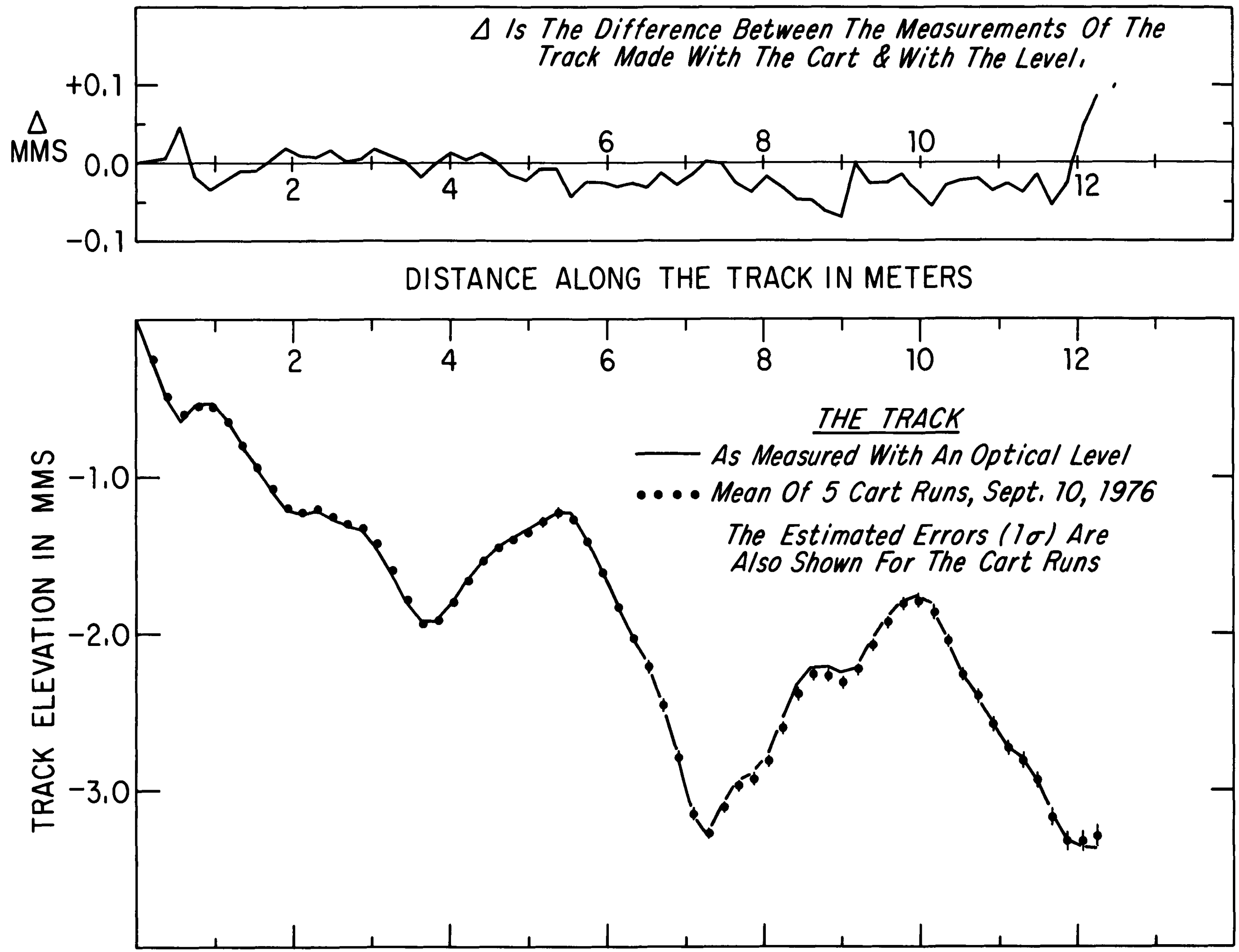

FIGURE 8: COMPARISONS BETWEEN OPTICAL AND CART MEASUREMENTS OF THE TRACK, SEPTEMBER 10, 1976.

THE UPPER PLOT SHOWS THE DIFFERENCES BETWEEN THE MEASURES (CART-OPTICAL), IN MICRONS. THE LOWER PLOT SHOWS THE OPTICAL RESULTS AS A SOLID LINE AND THE CART RESULTS AS FILLED CIRCLES. THE ERROR BARS ARE AN ESTIMATE OF THE ERROR OF A SINGLE CART OBSERVATION.

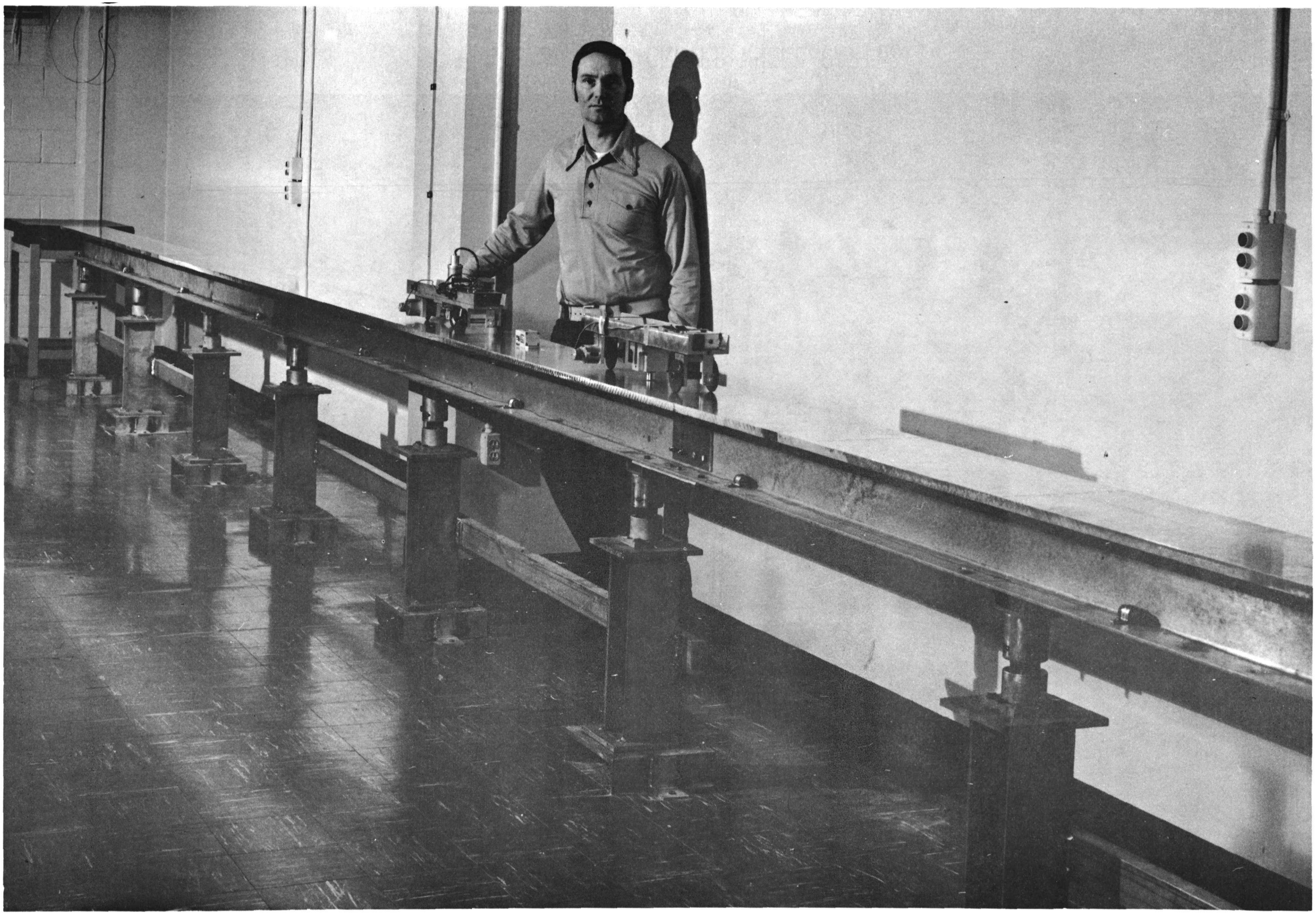

PLATE 1: THE TEST TRACK AND TWO CARTS.

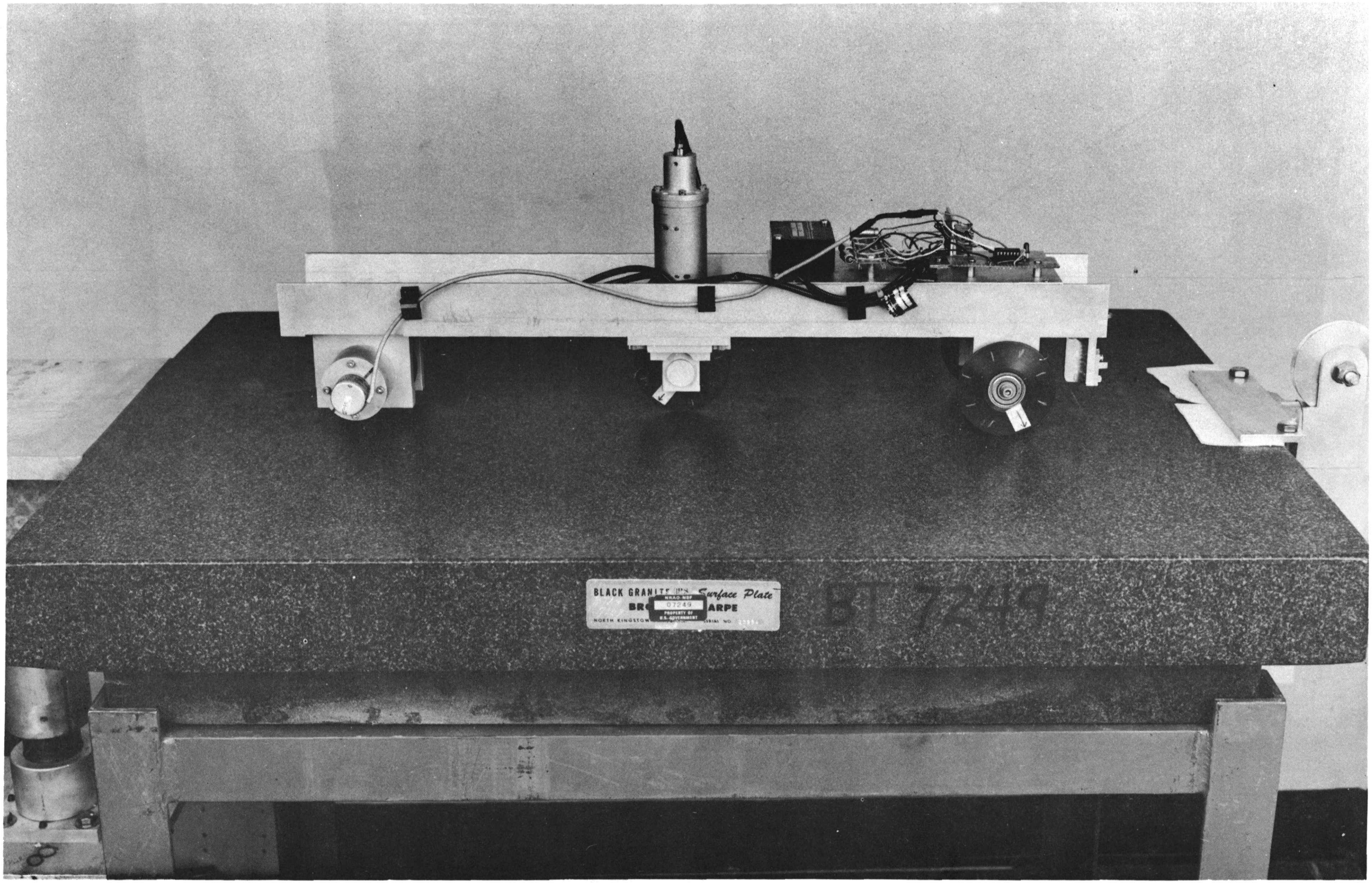

PLATE 2: THE CENTER WHEEL CART.