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REFRACTION CORRECTION FOR THE 140 FT-POINTING

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In Electronics Division Internal Report No. 164 (Dec. 1975) a weather-dependent refraction correction was suggested, using the weather data taken at the interferometer; but it was left open how to obtain the needed water vapor pressure from these data. Furthermore, the term correcting for the curvature of the Earth was found unsatisfactory but was not changed. Both will be done in the following.

1. Refractive Index

In Report 164, I took the formula for the refractive index of air from a 1972 NRAO Memo of Victor Herrero, who quoted Froome and Essen ("The velocity of light and radio waves", Academic Press, New York 1969). Similar formulas are given by Ed Fomalont ("A new Intcorr Program", NRAO Memo of Jan. 9, 1974) and Allen ("Astrophysical Quantities", Athlone Press, London 1964, page 120). All these formulas can be written as

$$(n - 1)\ 10^6 = B_1\ P/T - B_2\ P_w/T + 10^5\ B_3\ P_w/T^2 \qquad \textbf{(1)}$$

where n = refractive index (according to Froome and Essen, for wavelengths $\lambda \geq 7.5$ mm, and maybe even to $\lambda \geq 2.0$ mm), P = total barometric pressure in mmHg, P_w = partial water vapor pressure in mmHg, and T = air temperature in ^oK. A comparison gives

	B_1	B_2	B_3	
Fomalont 1974	103.5	0	4.97	(2)
Allen 1964	103.6	13.3	5.001	(3)
Froome & Essen 1969	103.49	17.23	4.958	(4)

A check with prevailing ranges of P_w and T showed that B_2 is rather unimportant, contributing at most 1% to the total. Thus, we do not worry about the difference, and we shall use the values (4) from Froome and Essen.

2. Water Vapor Pressure

At the interferometer, the dew-point temperature is measured with a Dewcel element of Foxboro Co. This is an electric thermometer, covered with a woven glass tape (wick) which is impregnated with lithium chloride (a hygroscopic salt). A pair of wires along this wick is connected to a stabilized power supply. If the air gets more humid, the wick absorbs more moisture from the air; then its conductivity increases and so does the current, which heats up the wick (and the thermometer), evaporating some moisture and so reducing the current until equilibrium is reached. The thermometer output then is electrically transformed (by a properly calibrated Controller) into the dew-point temperature.

Since several different temperatures are used in the literature which are somehow connected to humidity and water pressure, and since reading the literature gets sometimes quite confusing, I would like to explain some of these temperatures, following the Handbook of Meteorology (Berry, Bollay and Beers; McGraw-Hill, New York 1945).

Dew-Point Temperature D is reached at saturation (starting of fog or condensations), if a parcel of air is cooled down isobarically, which means by heat removal at constant pressure. Since with P = constant also P_w = constant during this cooling, the saturated water vapor pressure $P_w(t)$ as a function of saturation temperature t, as provided by tables in handbooks and also by Foxboro, is equal to the partial water vapor pressure P_w as needed for equation (1), just letting t = D when using the tables.

Condensation-Level Temperature T_c is reached at saturation, if a parcel of air is cooled down adiabatically; which means by expanding the air in a cylinder by pulling out a piston, where both cylinder and piston are thermally insulated against the air. This is different from D.

Wet-Bulb Temperature T_w is shown by a thermometer whose bulb is covered by a wet wick and is blown at by the air. This is different from both D and T_c. If humidity is measured this way, then the vapor pressure P_w does not only depend on T_w, but on T and P as well.

3. Approximation for $P_w(D)$

The saturated water vapor pressure as a function of temperature is given in handbooks either in tables or as a complicated formula with logarithms and exponentials. Ed Fomalont (1974) provided a very simple exponential formula; as we checked, it gives a maximum error of 2.2 mmHg for the range $-30 \le D \le +30$ °C, and an rms error of 0.9 mmHg.

Regarding the limited computer space at the 140-ft, Tom Cram asked me to deliver a formula for $P_w(D)$ which does not need logarithms, exponentials, or long tables. Thus I tried power series. The following one of fourth order, adjusted at D = 0.0, ±17.5, and ±27.5 °C, gives P_w in mmHg from D in °C:

$$P_w(D) = 4.58 + 3.369\ (D/10) + 1.029\ (D/10)^2 + 0.2080\ (D/10)^3 + 0.02778\ (D/10)^4. \qquad (5)$$

Compared to a handbook table, the errors are only

range	error (mmHg) max	rms
$-30.0 \le D \le +30.0$ °C	0.12	0.06
$-32.5 \le D \le +37.5$ °C	0.24	0.08

(6)

The accuracy of the Dewcel element is not given in its description. If we assume D ± 1.0 °C, then from the steepness of $P_w(D)$ we obtain, including (6),

$$P_w \pm 0.82 \text{ mmHg}. \qquad (7)$$

4. Curvature of Earth

Our present pointing program at the 140-ft uses for the refraction angle Δz, as a function of the zenith distance z,

$$\Delta z = A_3 \; (1 - 0.0011 \tan^2 z) \tan z \qquad (8)$$

where

$$A_3 = n - 1 = (1.04 \pm 0.05) \text{ arcmin} \qquad (9)$$

is one of the 15 parameters, obtained by best-fits to observations of sources with known positions. The term in parentheses, correcting for the curvature, gets bad close to horizon and even diverges to $\pm \infty$ at 1.90° elevation.

How close to horizon do we need to go? Although observations at low elevations get unreliable because of the strong atmospheric noise and absorption, and the large pick-up of ground noise, they cannot be avoided for some southern galactic sources seen only at low elevation and close to south azimuth. The actual limit is given by the hilly horizon as seen from the 140-ft, and by the hardware limit of the telescope drives, as shown in Table 1.

Table 1. Elevation limits of the 140-ft

azimuth	elevation (degrees)		
	horizon H	drive limit L	actual limit max(H,L)
135 SE	3.1	-5.3	3.1
150	2.6	-5.8	2.6
165	2.4	+1.6	2.4
180 South	2.1	+3.7	3.7
195	1.7	+1.6	1.7
210	1.1	-5.8	1.1
225 SW	1.8	-5.3	1.8

It would be a good policy to have in our pointing program a curvature correction which may be used down to any possible elevation angle (1.1°, Table 1) without blowing up. After several trials, the following "new correction" of equation (13) was found satisfactory (see Table 2). If we ask for accuracy even down to the horizon, we must be more specific regarding what we mean by z in equation (8). We define:

$$\left.\begin{aligned} z_a &= \text{apparent zenith angle} \\ z_t &= \text{true zenith angle of source} \end{aligned}\right\} \quad \Delta z = z_t - z_a \; . \tag{10}$$

For z_a and z_t we use a table given by Allen, "Astrophysical Quantities", page 120. The following Table 2 compares three cases (where $A_3 = 0.973$ arcmin is the best-fit to Allen's table for small z):

$$\text{uncorrected} \qquad \Delta z = A_3 \tan z_t \; , \tag{11}$$

$$\text{old correction} \qquad \Delta z = A_3 \,(1 - 0.0011 \tan^2 z_t) \tan z_t \; , \tag{12}$$

$$\text{new correction} \qquad \Delta z = A_3 \, \frac{\sin z_t}{\cos z_t + 0.00175 \tan(z_t - 2.5^{\circ})} \; . \tag{13}$$

Table 2. The atmospheric refraction, $\Delta z = z_t - z_a$, as a function of zenith distance z.

appar. elevation	zenith distance: appar. z_a	zenith distance: true z_t			error of Δz: uncorrected			error of Δz: old correction			error of Δz: new correction		
90°	0°	0°	0'	0"	0°	0'	0"	0°	0'	0"	0°	0'	0"
70	20	20	0	21			0			0			0
50	40	40	0	49			0			0			0
30	60	60	1	41			0			0			0
20	70	70	2	39	+		1			0			0
15	75	75	3	35	+		4	+		1			0
10	80	80	5	19	+		15	+		3			0
8	82	82	6	34	+		27	+		4			0
6	84	84	8	29	+	1'	0	+		1			0
4	86	86	11	47	+	2	51	-		47	+		2
3	87	87	14	27	+	5	44	-	3'	49	+		1
2	88	88	18	27	+	14	56	-	27	1	-		14
1	89	89	24	44	+ 1°	10	6	- 15°	21	2	-	1'	57
0	90	90	35	22	- 2°	9	57	+ 14	48	31	-	12	18
			= Δz										

If we allow a maximum error of 15 arcsec, say, then the uncorrected formula should not be used for apparent elevations below 10°, the old correction not below 5°, while the new correction may be used down to 2° apparent elevation. Even at the lowest point of Table 1, at 1.1°, equation (13) does not "blow up" but has an error of 2 arcmin.

5. Suggested Procedure

In Report 164 we suggested to leave the constant A_3 of equation (9) still open, as one of the 15 parameters to be solved for by observational data. First, for checking purpose; and second, because A_3 may actually be slightly dependent on the observational wavelength and the geographic location, and it must depend on the altitude. Maintaining this philosophy, we regard (1) as a weather-dependent correcting term, called K. Since it happens that K = 1.00 arcmin for a Green Bank average of $t = 10\ ^{\circ}C$, $P = 700$ mmHg, and $P_w = 6$ mmHg, we do not need a normalization. Rewriting (1) with values (4) as a correction to (13), we have, in minutes of arc:

$$\Delta z = A_3 \frac{K \sin z_t}{\cos z_t + 0.00175 \tan(z_t - 2.5^{\circ})} \tag{14}$$

with

$$K = 0.354\ P/T - 0.0585\ P_w/T + 1701\ P_w/T^2 \tag{15}$$

where z_t is the true zenith distance (from precessed catalogue position of the source, location of 140-ft, aberration, and sidereal time), and where $z_a = z_t - \Delta z$ is the apparent zenith distance to which the telescope must be pointed.

From the interferometer we obtain (per wires) the barometric pressure P in mmHg, the air temperature t in $^{\circ}C$, and the dew-point temperature D in $^{\circ}C$. The 140-ft computer then calculates the absolute temperature in $^{\circ}K$ as

$$T = t + 273.15 \tag{16}$$

and the water vapor pressure $P_w(D)$ in mmHg from equation (5). With these values of P, P_w and T, it calculates K from equation (15). For safety reasons, just in case, we suggest to ask whether

$$0.75 \leq K \leq 1.50 \tag{17}$$

which should cover the extreme possible range of Green Bank weather conditions. If yes, use K as calculated; if not, use K = 1.00 and give alarm to the operator. In any case, calculate the refraction Δz from equation (14).

6. Accuracy

The accuracy of Δz as a function of z is given in Table 2; down to 3° elevation we have only 2 arcsec maximum error. The accuracy of A_3 will depend on the method and the number of observations, preferably to be done at night only, for avoiding large thermal deformations or their corrections. Actually, a good new pointing run can only be done after the thermal shieldings of yoke arms, polar shaft, platform and tower have been completed and tested (Engineering Report No. 100), because the present thermal pointing errors are much too large even at night, 11 arcsec rms for a clear sky as it would be needed for short wavelengths (small beam). The expected error of A_3 is difficult to estimate, but it seems that about 2-3 arcsec rms could be achieved with several good nights' observing.

The accuracy of the weather-dependent term K depends on the reading accuracy at the interferometer. Assuming uncorrelated rms errors of $\Delta t = \Delta D = 1.0\ ^{\circ}C$ and $\Delta P = 2$ mmHg, we obtain from (6), (7), and (1) a combined rms error of 0.9 arcsec for K.

The largest error so far discussed comes from the empirical determination of A_3. But in addition, we have the fact that we measure t, D, and P only at ground level, while the observed radio waves travel through the whole atmosphere which may temporarily deviate from the standard atmosphere as assumed in equations (14) and (15). Furthermore, very close to the horizon, we look not only through our own local atmosphere but anybody elses as well. It seems very difficult to estimate these errors. In total, we still hope for a few seconds of arc, say 3 arcsec rms of refraction error at 45° elevation where tan z = 1; but to be multiplied by tan z for other elevations. For example, tan z = 2.75 for elevation 20°, and 5.67 for 10°.