NATIONAL RADIO ASTRONOMY OBSERVATORY

Charlottesville, Virginia

Engineering Division Internal Report No. 110

The Design of the Deformable Subreflector for the 140 ft. Radio Telescope

W-Y. Wong

January, 1979

Number of copies: 100

The Design of a Deformable Subreflector for the 140-ft. Radio Telescope

W-Y. Wong

TABLE OF CONTENTS

TABLE OF CONTENTS

(Continued)

I. SUMMARY

It was suggested that the strong gravitational astigmatism of large radio telescopes can be corrected for by a shaped subreflector [1], if the error profile of the main reflector is reproduced at the subreflector and the reflecting rays do not cross each other [2]. Experiments by Cowles and Parker, using a 0.3m shaped subreflector on a 2.8m reflector show that the aperture efficiency was improved by a factor of two at 34 GHz [3]. The gravitational astigmatism of the 140 ft telescope was first indicated by structural analyses, and subsequently verified by S. Von Hoerner, using an elongated feed horn [4]. A 3.2m subreflector was built, with the profile mechanically deformed and electronically controlled. The amount of deformation varies with the telescope pointing.

This second subreflector includes all the features of the first subreflector: it is asymmetrical and tiltable. This report describes the detailed mechanical design, and ground level evaluation, of the subreflector. The electronic aspect of this subreflector: design of controller and the interface to the computer, is described in a separate report by R. Lacasse [5]. The test on the telescope with seven unresolved radio sources at 22.3 GHz gives considerable amount of improvements, especially far south and east. The best result shows the aperture efficiency was increased by a factor between two and three [6].

II. THE SUBREFLECTOR

General Description

The deformable subreflector has to follow the criteria that 1) the subreflector is asymmetrical due to the off-set feed arrangement; 2) the subreflector is tiltable for beam switching technique in observation; and 3) the subreflector is deformable to correct the gravitational astigmatism.

The first consideration was entirely geometric, readily defined when the first subreflector was implemented to the telescope system. The second consideration means that the weight and the moment of inertia of the new system has to be kept equal to, if not less than, the previous system. For the load capacity of the sterling mount, nominally set to 2500 lbs., was already exceeded. The third consideration was new to the system. Four servo controlled motor-actuators are installed at four points behind the subreflector. The amount of actuation is determined by the elevation position of the telescope.

The fiberglass, aluminum honeycomb core sandwich construction method is proven acceptable, in terms of cost and surface accuracy, by the first subreflector. The force required for the deformation of the surface, solved analytically, was found reasonable. Hence the same material and fabrication approach for the new surface was adopted. The long term behavior of this composite material is not entirely known, however. Aging, temperature, environment and cyclic loading effects might degrade the surface accuracy and reduce its usable life time. The surface of the first subreflector has degraded from the original error of 0.25 mm rms to 0.43 mm rms after four years.

The stiffening rib arrangement of these two subreflectors are also different. The first subreflector has a stiffening ring, whereas the second

one has two stiff diagonals. For both cases, however, the main structural stiffness is derived from the back-up frame which serves as an interface between the beam-tilt system and the subreflector surface.

Due to the off-set receiver arrangement, the deforming feature is applicable only to the feed position of the K-band maser receiver (22.3 GHz, λ = 1.23 cm) plus three others, rotated by 90°. For other receiver positions, the subreflector's profile must be returned to its undeformed position.

The subreflector system can be dismounted from the telescope so that the optic of the system can be modified within a reasonably short period of time.

The front face of the subreflector is coated with a thin layer of aluminum film for reflection. The subreflector is painted white. The back-up frame is unpainted. All moving parts of the deforming system are protected by water-tight covers.

Geometry

A schematic arrangement of the Cassegrain system for the 140-ft telescope is shown in Figure 1. Some important parameters are given in the following list, describing the Cassegrain optics:

Average subreflector diameter = 3.213m (126.4")

Distance from the prime focus to secondary focus: a = 13.48m (530.89")

Distance from the prime focus to subreflector vertex: b = 1.30^{m} (51.262")

Magnification: m = 9.348

Eccentricity: e = 1.240

Angle between paraboloic axis and hyperbolic axis: 4.148°

The hyperboloid is given in eq. (1), describing the profile of the subreflector in an untilted system, with Z-axis as the hyperbolic axis.

$$z = (c - \tfrac{1}{2}a)\left\{\sqrt{1 + \frac{r^2}{bc}} - 1\right\} \qquad (1)$$

where $r^2 = x^2 + y^2$ and $c = a-b$.

The hyperbolic axis and the parabolic axis do not coincide. They subtend an angle of 4.148°. A cluster of feeds are arranged in a ring with a radius of 0.98m (38.58") and they are 4.57m (180") above the vertex [7]. The subreflectors' axis is aligned with the phase center of the feed in use by rotating the subreflector about the axis of the main reflector at the Sterling mount. The rim of the subreflector, which is the intersection of the hyperboloid and the cone defined by the line of sight of the prime system, is not a circle. The coordinates of the rim were computed by P. J. Napier and are listed in Table I for reference.

The rim is marked at every 15° with respect to the hyperbolic axis. A collimating mirror with cross-hair defines the hyperbolic axis. Another cross is marked 94.3 mm (1.714 in) from the hyperbolic axis along the plane of symmetry which defines the parabolic axis of the subreflector.

The arrangement of the actuators (A1 - A4) and the stationary connections (S1-S4) are also illustrated in Figure 1, showing their positions relative to the main reflector. The direction of the beam tilting is perpendicular to the parabolic and hyperbolic axes. The relative position of the subreflector to the main reflector is defined when a particular receiver is used. For example, the actuator A1 is aligned to the northern rim of the main reflector only when the K-band maser receiver is in use.

Material

The subreflector shell structure is composed of fiberglass cloths, and an aluminum honeycomb core. This sandwich construction is bonded together by epoxy rasin cured by hardener, or curing agent. There are three plies of fiberglass cloth laminates on the front and the back of the shell. The thickness of each ply is 0.28mm (0.011 in). The honeycomb core is 25.4mm (1.0 in) thick.

The shell structure is laid out in steps in a mold and bonded together with the epoxy. The curing takes place inside the mold. Suction was constantly applied between the mold and the front surface to insure the reproduction of the profile in the mold. The curing agent was selected so that room temperature curing is adequate to achieve the necessary material strength. Shrinkage of the curing agent is claimed to be less than 2%.

A thin layer (about 0.15mm) of aluminum film was flame-sprayed onto the mold before the epoxy-fiberglass was applied to make the surface reflective.

The quality of this composite construction depends on the material properties of the components as well as on the skill of the fabrication, the type of curing agent and the duration of the curing. Temperature, pressure and types of fiberglass cloth also affect the strength of the product. Moreover, there is a wide selection of size, foil thickness and alloy of the aluminum honeycomb core. Because of the many possible combinations, the material properties of the final product is best determined by direct measurements. On the other hand, tests on specimens are usually

made by the manufacturer only when a large quantity of the product is involved. There were no such measurements made for this subreflector. The material properties used during the analyses are based on the best available data provided by the manufacturer. The results of the calculations should be considered good within 20%. The components of the subreflector are listed in Table II. The Antenna System Incorporated of San Jose, California, was responsible for the construction of the shell structure.

Stress Analyses

The in-house feasibility studies started in 1975, determining the actuator arrangement and basic stiffening geometry. The studies also provided a prediction on the improvement of the telescope surface error due to the astigmatic correction. All detail computations are filed in reference [8].

The final analysis was done by an outside firm - Control Data Corporation. The CDC computations [9] used an accurate mathmatical model of the shell structure, with the geometry, material properties and loading conditions specified by N.R.A.O. The model has 180 finite elements and 108 nodes (Figure 2), made in one quadrant by assuming the subreflector is symmetrical. The model is constrained at node 57, and actuated at nodes 52 and 62. The four loading conditions considered in the analyses are:

1) Astigmatic deformation: node 52 and 62 were actuated in opposite directions by an amount of 1.57mm (0.062 in.);

2) Survival load: mode 52 and 62 were actuated in the same direction by 3.8mm (0.150 in.);

3) Acceleration due to tilting: the angular acceleration is 34.4 rad/sec^2, equivalent to the tilting of the subreflector at λ = 2cm observation, and

4) Thermal load of 10°F difference between the front and the back faces of the subreflector.

The analytical results are summarized in Table III. The large safety margin in the survival condition shows that the subreflector will survive an erroneous actuation, when all actuators actuate in the same direction. This mode of actuation is in fact a higher order astigmatic correction suggested in ref. [9], but presently not possible to produce.

The stiffness at the point of actuation are:

$$\left.\begin{array}{l}\text{1st mode astigmatic correction} = 201\ \text{N/mm} \\ \text{2nd mode astigmatic correction} = 417\ \text{N/mm}\end{array}\right\} \qquad (2)$$

The accuracy of the analyses was proven to be good. Figure 3 shows a comparison of the measured and the computed profiles. When the subreflector is actuated by 1.57mm, the difference of these two profiles is 0.14 mm rms.

Error Budget

The surface error distribution of the 140-ft for wavelength λ = 1.3 cm was discussed in detail by S. von Hoerner [11]. The error allocated specifically for the deformable subreflector is 0.42mm rms.

During the design of the subreflector, sources of error taken into account are:

1) the residual path length error after the astigmatic correction;
2) the deflection of the back-up frame and linkage causing an actuation differ and from the commanded magnitude;
3) the manufacturing inaccuracy of the subreflector surface and
4) the inaccuracy of the electronic control.

The first item was derived by analytical means. The difference of an astigmatic correction and the surface error of the main reflector is computed at various telescope positions. The average of these differences is found to be 0.26mm rms [1][11].

The second item was found by direct measurements after the subreflector was assembled. These measurements were obtained by placing dial indicators at various points of the frame, and recorded the deflection when the subreflector is deformed in a given amount. Due to the insufficient design of these parts, the average efficiency of the deforming system is only 60%. The averaged peak deflection of the linkage is 1.02mm, and the standard deviation is considered to be 0.34mm rms.

The surface error measured at the plant and in Green Bank produced an average value of 0.25mm rms. Refer to section "surface evaluation and measurement" for details.

The error derived from the electronics is small: 0.04mm rms. The error contributed mainly by the non-linearity of the position transducer, the temperature effect and the noise of the system. It was discussed in detail in a separate internal report by R. Lacasse [5].

All errors mentioned above are summarized in Table IV for quick reference. The combined error, a root sum square of the above mentioned contribution, is:

$$\text{Total error from the subreflector} = 0.50\text{mm} \tag{3}$$

Deforming the Subreflector Surface

The first order astigmatic deformation is defined by the rim deflection 'a' at four points 90° apart. The deflection of the circumference forms a two-cycle sine wave.

Figure 4 summarized the analytical and measured rim deflections in terms of the elevation positioning the telescope. The computations were based on the surface-panel adjustment at zero hour angle, +4.4° declination, with the dish illuminated with 9 and 15 db taper. There is good agreement between the analytical curves and the measurement with narrow feed horn, in which the edge taper of 8.7 db was used. The Cassegrain feed presently used has a 10 db taper. The rim deflection of the subreflector approximates the measured curve, with 'a' defined as the rim deflection, and ϕ the elevation angle of the telescope:

$$a(\text{mm}) = -1.72 + 8.70\ (1-\sin\phi) \tag{4}$$

The first term gives the absolute value of rim deflection at zenith position, and the constant of the second term denotes the total rim deflection required as the telescope moves from zenith position to horizontal position. The sign convention of the rim deflection is given in Table V.

Positive denotes a deflection towards the main reflector; whereas negative denotes a deflection away from the main reflector. Plus sign means a reduction of path length, and negative an addition to the path length.

The Motion of the Ideal Actuators

Actuators A1 thru A4 are arranged in a circle with a radius equal to 0.99m (39 in.) from the parabolic axis. The value 'a' from the analyses, also given in Figure 4, are based on a circular aperture, in which the radius of the rim equals 1.58m (62.28 in.). The amount of actuation required for each actuator are reduced by a geometric constant g where

$$g = 39/62.28 = 0.626 \tag{5}$$

so that the amount of actuation 'd' in mm at A1 thru A4 are

$$d = a \times g = -1.078 + 5.446\ (1-\sin \phi) \tag{6}$$

Since the most useful range of the telescope are positions above 20° elevation, the total amount of rim deflection A, taken from Figure 4, is

$$A_o = 6.8\text{mm} \tag{7}$$

The corresponding total amount of actuation D_o required at A1 thru A4 is

$$D_o = g \times A_o = 4.3\text{mm} \tag{8}$$

Evaluation and Measurements of the Subreflector Surface

The surface manufacturing tolerance specified was 0.17mm rms. The measurement at the factory during the acceptance tests was 0.24mm rms [12]. In spite of this discrepancy, the subreflector was accepted and we proceeded with the distorsion tests, which was, as agreed beforehand, no longer the responsibility of the manufacturer. These additional tests indicated no noticeable hysteresis. Surface contours repeated well after three cycles of deformation.

The surface was measured again in Green Bank with the in-home designed jig. The measuring accuracy is considered higher than that of the factory, and the data was processed with a best-fit program, included in Appendix E.

The results of the nine measurements was summarized in Table VI, with detail computations in ref. [13]. The first three measurements consistantly yielded a value of 0.21mm rms. The surface became slightly degraded after some cyclic distorsions. The nominal rms deviation from the best fit hyperbolid ΔZ is considered

$$\text{rms } (\Delta Z) = 0.25\text{mm} \tag{9}$$

It is advisable to re-adjust the subreflector at times so that the value 0.25mm rms or less can be maintained. Figure 5 shows the surface contours of three measurements.

There are six dials on the template which give six readings on each radius. The readings were taken on every 15°, providing 24 sets of radial readings. Each surface measurements hence produced 144 single dial readings. These readings were recorded manually, and data are punched on cards as input to the computer.

The zero-readings on the template profile were done prior to each measurement. These six additional data are required by the program.

The first subreflector of the 140-ft was also measured. The purpose of this measurement was to provide the proper position of the bearings on the beam-tilting shaft, so that the alignments of these two subreflectors are "identical" when installed on the telescope. It is interesting to note that the profile of this first subreflector had deteriorated from the original error of 0.25mm rms to 0.43mm rms after four years of service [14].

Evaluation and Measurement of the Experimental Actuators

Equation (6) represents the amount of actuation required for each actuator at a given telescope elevation ϕ. It is assumed that the actuators are supported by a perfectly rigid frame. In reality, the bending of the frame and the linkage cause a reduction of the efficiency. In order to match the input and the output, calibration constants are incorporated to each individual actuator.

The most direct way to measure the efficiency is to place a dial indicator at the surface underneath each actuator, and to compare the commanded (input through the controller) and the dial reading. The ratio of the output and the input of the four actuators are listed in the third column of Table VII.

The deflection of the frame was measured in the similar fashion. The ratio of output and input are listed in column no. 1 of Table VII.

The amount unaccounted for is considered the deflection of the linkage. These values are listed in column no. 2 of Table VII, so that the sum of the column 1 thru 3 is 100% for each actuator.

Figure 6 illustrates a comparison of commanded actuation and the measured deformation under the dial no. 4 and 6. The solid line is the measurements, whereas the dashed lines are the ideal readings, based on the locations of the dial, assuming the supporting frame and linkage are perfectly rigid. The ratio of the two different slopes at dial no. 4 indicates a reduction of efficiency. The results, called indirect measurements, are listed in the forth column of Table VII.

Figure 6 also illustrates the linearity of the subreflector surface when deformed, showing the structure is behaving in its elactic range.

The average values of the direct and indirect measurements are listed in column 5, with the invert listed in column 6.

It is clear that the linkage arms and hinges are not adequately designed. Improvement is possible and desirable..

The Motion of the Experimental Actuators

The actuators used for the experiment have a maximum travel D_o of 6.35mm. The range was reduced by half in order to increase the rated force from 445 N (100 lbs) to 890 N (200 lbs). Hence in theory the actuation at the output is D_2, where

$$D_2 = 6.35 \div 2 = 3.18\text{mm} \tag{10}$$

The amount given in (10) was not totally realized. The actual travel capabled through the controller is limited to a value D_3 where

$$D_3 = 2.06\text{mm} + 0.50\text{mm} = 2.56\text{mm} \tag{11}$$

The zero crossing of D_3 was set tp 0.50mm in the controller as well as at the actuators.

The efficiencies of these actuators were measured as tabulated in Table VII. The averaged efficiency is found to be 60%. Hence the maximum travel D_4, capable by the present actuators at the output is

$$D_4 = (2.06+0.50) \times 60\% = 1.24+0.30=1.54\text{mm} \tag{12}$$

The corresponding rim deflection A_3, due to the ranges of the actuators described in (12) is

$$\begin{aligned} A_3 &= (1.24 + 0.30)/0.626 \\ &= 1.98 + 0.48 \\ &= 2.45\text{mm} \end{aligned} \tag{13}$$

The amount of (13) is marked in Figure 4. Compared with demanded range of 6.8mm eq. (7), the present actuators are short of the design goal by a factor of 2.8.

III. DEFORMING SYSTEM

General Description

The deformation of the subreflector is controlled by four independent and identical systems. Each system controls the contour of one quadrant of the surface. Each system has its own position control. The influence of deformation in one quadrant to the others is reduced because of the two stiff diagonals on the back of the subreflector (Figure 1).

The deforming system has four major parts: an electronic control, a position transducer, a motor actuator and a lever arm linkage. The first three items are covered in the internal report 139 in detail by R. Lacasse [5]. The lever arm linkage is illustrated in a detail drawing, no. 30D00017 in Appendix C.

This deforming system is an experimental version. Once this deformable subreflector is proven workable, the deforming system should be modified to incorporate actuators with a larger amount of actuations. A reverse in the direction of actuation will make the subreflector more versitile: it will be usable for receivers placed $\pm$ 90° and 180° apart from the present K-band maser receiver location.

Electronic Control

For the completeness of this report, a block diagram of the control system is included for reference, illustrated in Figure 7. Each actuator's position is monitored by a LVDT (Linear Voltage Displacement Transducer). A position given by the computer is compared with the measured position.

The difference of these quantities is reduced to an amount of $\pm$ 0.01m. The commanded position is up-dated by the output of the computer while the telescope tracks a source. The up-date rate is set to be 1 new position per second.

Figure 8 shows the front panel of the controller. The controller is an interface between the H-316 on-line computer and the servo mechanism. It's functions are receiver and transmitter data. Presently this interface is connected to the power supply at the focus by cable and no. 45 and 34 in the control room.

The interface provides the option of manual and computer control. A switch for this selection is located in the upper center of the panel.

Upper travel limit of 2.56mm is displayed under this switch. A display of travel in mm is located below this 'limit' window, with the selecting switch located on the right.

It is possible to monitor the commanded and measured positions of actuators A1 thru A4. It is also possible to monitor the temperature at the LVDT in Celsius. Heating elements are provided to the position transducer to maintain continued level specified by the producer of the transducer.

The display of 'LOCK' is for debug purposes. It shows the differences between the sum of the commanded positions and the sum of the actual positions.

The manual control enables one to input values into the thumb wheel knobs, either individually or as a group (switch to 'ALL'). The step is executed by pressing the 'UPDATE ENABLE' button.

All actuators are travel from an undeformed position to a predetermined direction (see Table V). The deforming option is applicable only for the K-band maser receiver.

Force Requirement on Actuation

The size of the actuator depends on 1) the force required to produce the given amount of deformation on the subreflector; 2) the weight of the subreflector surface and 3) the dynamical force due to the periodic tilt of the subreflector in the beam-tilting mode.

As shown in eq. (8), the range of actuation from 20° elevation angle to 90°, based on the measured value of astigmatism, is 4.3mm. Refer to eq. (2), that the stiffness of the shell structure when actuated in a astigmatic deflection, is 201 N/mm. The force required for the full actuation is

$$F \text{ (astigmatic)} = 4.3\text{mm} \times 201 \text{ N/mm} = 864 \text{ N(194 lb)} \qquad (14)$$

The weight of the shell is 668 N (150 lbs). Assuming that the weight is equally distributed to the eight connections, the force on the actuator due to dead weight is

$$F \text{ (dead weight)} = 668 \text{ N}/8 = 83 \text{ N (19 lb)} \qquad (15)$$

The force due to the beam switching at $\lambda = 2.0$ cm is derived as follows:

B (HPBW) $= 1.2\ \lambda/D = 57.92\ \lambda$ (sec)

θ (beam tilt) $= 3B$

α (Rotation of the subreflector) = $\frac{\theta \times M}{1.15} = 8.13\ \theta$

τ (transition time) = 40 ms

T (available torque) = 7000 ft-lb

A (angular acceleration) = $\frac{4\alpha}{\tau^2} = 34.3\ \text{rad/sec}^2$

m (mass of the shell per connection) = 150lb/8 = 19 lb = $.049\ \frac{\text{lb} - \text{sec}^2}{\text{in}}$

d (distance to the axis of tilt) = 39 in

F (beam tilt) = m·A·d = 294N (66 lb) (16)

The total force required for each actuator is the sum of the force shown in eq. (14), (15) and (16), or

F (total) = 864+83+294 = 1241N(279 lb) (17)

The experimental version has actuators rated at 100 lbs. This force was amplified by a factor of 2 by lever arm linkage, having rated output of only 200 lbs.

Type of the Experimental Actuator

The size of the actuator is dependent on the required force as described in eq. (17) and the amount of actuation as in eq. (8). The available actuator for the experimental version of the deformable subreflector has the following properties:

Manufacturer:	AIRSEACH
Part No.:	34596
Weight (approx.):	2.5 lbs.
Max. Actuation:	0.25 in (6.35 mm)
Max. Load:	100 lb (445 N)
Motor Type:	Series wound
Electrical:	20 VDC

The Linkage

The necessity of this linkage is due to the motor actuator available, in conjunction with the requirement in force and displacement of the subreflector. The final version of the deforming system could by-pass this linkage to reduce the additional deflections.

The extension of the lever arm also provides an amplification in the travel of the LVDT.

The Switching Off of the Actuators

The subreflector deformation can be terminated either at the commencement of an observation; or if erratic behavior is detected, such as the monitored amount of actuation being different from the commanded quantities; or the temperature readings at the actuator motors show a level higher than a preset value.

The proper sequence is 1) to set all actuator command values to 0.50 mm at the four windows on the left hand side of the panel; 2) switch the "manual/computer" switch to the manual position; 3) put the update selection button to "all" position and 4) press the "update enable" button. This will activate all actuators to drive the surface back to the undeformed position. These positions will hold even if the power supply is turned off.

IV. BACKUP FRAME

General Description

The backup frame of the deformable subreflector provides the following functions: to add structural stiffness to the subreflector; to interface the subreflector to the existing beam tilting device; and to support the actuators-linkage combinations.

The subreflector is connected to the frame at eight points. Four of which are actuated by electro-mechanical devices, labeled as A1 thru A4. Another four connections are mechanically fixed with rods and brackets, and labeled as S1 through S4. Figure 1 shows the schematic arrangements of these connections, and appendix B shows the detail. The stationary and the actuated connections are alternately arranged for the astigmatic deformation of the subreflector.

The subreflector-frame combination is attached to the beam tilting device through two bearings and two connections to the push rods. These connections are designed for quick installation so that the telescope can be operated in either prime focus or Cassegrain system. All connections are positioned beforehand by pins so that calibration procedures are kept to a minimum after the subreflector is installed.

The Design

The material of the frame is aluminum. The limit of design is not governed by the allowable stress but by the total weight and moment of inertia. The detail of the design is included in Appendix B. The analyses of the structure is summarized from the detailed computation of reference [8] and listed as follows:

Properties of the Subreflector Frame

Weight	6 in. sch. 10 weldment	134.4 lb
	Brackets	63.0 lb
	Shaft & brgs	83.0 lb
	Total	286.4 lb
Moment of Inertia	About the Shaft	3×10^4 lb-in^2
Spring const. in the direction parallel to the telescope axis	A1, S2, A3, S4:	2.05×10^4 #/in
	S1, A2, S3, A4:	6.29×10^4 #/in
Max. bending stress		612 psi
Max. shear stress		84 psi
Resonance frequency	Frame alone	105 Hz
	With Subreflector	33 Hz

The frame was fabricated in the G.B. shop. It was shipped to ASI of California for the framing of the subreflector. The subreflector/frame combination was shipped back to G.B. as one integral part.

V. SOFTWARE PROGRAMMING

General Description

A subroutine was added to the 140-ft telescope control program in the Honeywell-316 computer by Bob Vance. This subroutine gives the user the option of using this deformable subreflector by switching on the power supply of the servo control electronics and setting the manual/computer switch (see fig. 8) to the computer control mode.

This subroutine was written based on the algorithm illustrated in figure 9.

The Input and The Output

The amount of actuations on the subreflector surface is dependent on the elevation angle ϕ of the telescope. The values ϕ are readily calculated by the main control program.

There are four outputs from this subroutine: a, b, c and d. Each is a eight-bit word, containing a max. value of 255. These outputs are channeled to the controller, and ultimately translated into displacements of the subreflector shell. The expressions in eq. (18) are modifications of eq. (6), with zero crossing set at 0.50 mm:

$$\begin{aligned} P_1 &= V_1 + V_2\ (1-\sin\phi) = -0.578+5.446\ (1-\sin\phi) \\ P_2 &= V_3 + V_4\ (1-\sin\phi) = 0. \\ P_3 &= V_5 + V_6\ (1-\sin\phi) = 0. \\ P_4 &= V_7 + V_8\ (1-\sin\phi) = 0. \end{aligned} \tag{18}$$

and subsequently, the four output words are expressed as:

$$
\begin{aligned}
a &= (P_1 + P_2 + P_3) \times C_1 \text{ at } A_1 \\
b &= (P_1 + P_2 - P_3) \times C_3 \text{ at } A_3 \\
c &= (-P_1 + P_2 + P_4) \times C_2 \text{ at } A_2 \\
d &= (-P_1 + P_2 - P_4) \times C_4 \text{ at } A_4
\end{aligned}
\tag{19}
$$

P_1 represents the first order astigmatic deformation. P_2 thru P_4 are values for higher order deformations, which are not being considered during the design phase of the subreflector. Presently P_2 thru P_4 are dummys and the values of P_2 thru P_4 (or V_3 thru V_8 in eq. (18)) are set to zero. These values can be redefined once it is proven that the subreflector is capable of these types of deformations.

C_1 thru C_4 are calibration constants, correspondent to actuators A_1 thru A_4. The numerical values of C_1 thru C_4 are listed in table VII, column 6.

VI. MEASURING JIG

Fig. 10 shows the general setup of the measuring jig, with the detailed design included in Appendix D. It consists of a supporting frame for the subreflector and a template. The measuring jig serves the following purposes.

1) to verify the subreflector's surface accuracy,

2) to measure the contour of the deformed subreflector,

3) to evaluate the efficiency of the actuators, and

4) to provide a reference system for future adjustments.

The subreflector was held in place and was so adjusted that its hyperbolic axis is aligned with gravity. The central axis of the jig is defined by an optical plumb, with the three adjustment screws at the supporting legs to correct the tilt, and mild forces required to shift the jig in lateral direction so that the axes of the subreflector and the jig are coinciding. Few repeating cycles are required until these two axes will actually meet. The optical plumb defines the direction of gravity, and with an eyepiece to collimate the central mirror of the subreflector.

The horizontal reference of the template was monitored by an optical level (Wild N3) to insure the line remains horizontal as the jig rotates. There is no fine adjustment for the counter weight, but good balance can be achieved by using heavy bolts or nuts clamped on the jig.

The 1/8 inch steel plate template has a cut-out hyperbolic profile. This profile was also measured by optical means so that the reference is well established. Fig. 11 shows the arrangements of the dial indicators, the reference profile and the proper corrections for the zero readings of each dial.

The dials are placed on the template vertically. The dials are product of MITUTOYO, part no. 3052. The graduation is 0.010 mm. The total travel is 30 mm.

There was no effort to evaluate the accuracy of the system. Rough estimates of the measure accuracy is about $\pm$ 0.050 mm.

ACKNOWLEDGEMENT

The author wishes to thank the Green Bank workshop, under the supervision of Albert Steinemann and Martin Barkley, for the assembly of the subreflector stiffening frame; the maintenance group with Ron Gordon, Don Gordon and Herb Hanes for the mechanical work; Sidney Smith for the help during the measurement of the surface. The Controller was designed and assembled by R. Lacasse of the Electronic Division. R. Vance was responsible for the programming. Also thanks for advice from R. Fisher. The moving force of the project shall be accredited entirely to Dr. S. von Hoerner.

REFERENCE

[1] S. von Hoerner and W-Y. Wong, "Gravitational Deformation and Astigmation of Tiltable Radio Telescopes", IEEE Trans. Antennas Propagat., Vol. AP-23, pp. 689-695, 1975.

[2] S. von Hoerner, "The Design of Correcting Secondary Reflectors," IEEE Trans. Antennas Propagat., Vol. AP-24, pp. 336-340, 1976.

[3] P. R. Cowles and E. A. Parker, "Reflector Surface Error Compensation in Cassegrain Antennas," IEEE Trans. Antennas Propagat., Vol. AR-23, p. 323, 1975.

[4] S. von Hoerner, "Measuring the Gravitational Astigmatism of a Radio Telescope," IEEE Trans. Antennas Propagat., Vol. AP-26, pp. 315-318, 1978.

[5] R. J. Lacasse, "Correctable Subreflector Controller," N.R.A.O. Electronic Division Internal Report No. 193, 1978.

[6] S. von Hoerner, R. Fisher, W-Y. Wong, "140-ft Observations with Deformable Subreflector," N.R.A.O. Engineering Division Internal Report No. 109, 1978.

[7] Peter J. Napier, "Cassegrain System for 140-ft Telescope," N.R.A.O. Memo, 1972.

[8] Computer output file no. DS-001, "N.R.A.O. Stren Analyses of the Subreflector".

[9] Computer output file no. DS-002, "C.D.C. Stress Analyses of the Subreflector, Dec. 28, 1976.

[10] C.D.C. Report, "Stress Analyses of A Deformable Subreflector", Control Data Corp., Dec. 28, 1976.

[11] S. von Hoerner, "140-ft Deformable Subreflector", N.R.A.O. Memo, October 28, 1975.

[12] Computer output file no. DS-004, "Antenna System Inc. Surface Measurement",

[13] Computer output file no. DS-005, "Green Bank Measurements of the Deformable Subreflector".

[14] Computer output file no. DS-003, "Green Bank Measurements of the First Subreflector".

LIST OF TABLES

TABLE I. SUBREFLECTOR EDGE COORDINATES. DEFINITION OF ANGLE ϕ IS SHOWN IN FIG. 1, LINEAR DIMENSIONS ARE IN INCH

ϕ	R	Z(R)	ϕ	R	Z(R)
-90.000	67.842	19.187	0.0	61.996	16.133
-88.000	67.839	19.185	2.000	61.803	16.036
-86.000	67.827	19.179	4.000	61.610	15.940
-84.000	67.809	19.169	6.000	61.419	15.844
-82.000	67.783	19.155	8.000	61.229	15.749
-80.000	67.749	19.137	10.000	61.040	15.656
-78.000	67.708	19.114	12.000	60.853	15.563
-76.000	67.660	19.088	14.000	60.669	15.472
-74.000	67.605	19.058	16.000	60.486	15.383
-72.000	67.543	19.025	18.000	60.306	15.294
-70.000	67.473	18.987	20.000	60.129	15.207
-68.000	67.397	19.946	22.000	59.954	15.122
-66.000	67.314	18.901	24.000	59.783	15.038
-64.000	67.224	18.853	26.000	59.614	14.956
-62.000	67.128	18.801	28.000	59.449	14.877
-60.000	67.025	18.746	30.000	59.288	14.799
-58.000	66.916	18.688	32.000	59.130	14.722
-56.000	66.801	18.626	34.000	58.976	14.648
-54.000	66.681	18.561	36.000	58.827	14.576
-52.000	66.554	18.494	38.000	58.681	14.507
-50.000	66.422	18.424	40.000	58.540	14.439
-48.000	66.285	18.350	42.000	58.403	14.374
-46.000	66.143	18.275	44.000	58.271	14.311
-44.000	65.995	18.197	46.000	58.143	14.250
-42.000	65.843	18.116	48.000	58.020	14.192
-40.000	65.687	18.034	50.000	57.903	14.136
-38.000	65.526	17.949	52.000	57.790	14.083
-36.000	65.362	17.862	54.000	57.683	14.032
-34.000	65.194	17.774	56.000	57.580	13.984
-32.000	65.002	17.684	58.000	57.484	13.939
-30.000	64.847	17.593	60.000	57.392	13.896
-28.000	64.668	17.500	62.000	57.306	13,855
-26.000	64.487	17.406	64.000	57.226	13.817
-24.000	64.30	17.310	66.000	57.151	13.783
-22.000	64.117	17.214	68.000	57.082	13.750
-20.000	63.930	17.117	70.000	57.019	13.721
-18.000	63.741	17.020	72.000	56.962	13.694
-16.000	63.550	16.921	74.000	56.911	13.670
-14.000	63.357	16.823	76.000	56.865	13.649
-12.000	63.164	16.724	78.000	56.826	13.631
-10.000	62.970	16.825	80.000	56.792	13.615
- 8.000	62.775	16.526	82.000	56.765	13.602
- 6.000	62.580	16.428	84.000	56.744	13.593
- 4.000	62.386	16.329	86.000	56.728	13.585
- 2.000	62.191	16.231	88.000	56.719	13.581
			90.000	56.716	13.580

TABLE II. MATERIAL LIST OF THE SUBREFLECTOR

Item	Product	Part No.
Epoxy rasin and curing agent	HEXCEL	Hexcelite HP-302
Fiberglass cloth	J. P. STEVENS	No. 141
Honeycomb core	HEXCEL	5052/F40-0013-2.1
Collimating mirror	BRUNSON	185-1

TABLE III. ANALYTICAL RESULTS BY CDC

Loading Condition	Reaction at actuator N(lb)
Astigmatic correction of 1.57 mm	16 (71)
Survival load of 3.8 mm	80 (356) *
Beam tilt at 34.4 rad/sec^2	4 (16)
$\Delta T = 10°F$	3 (12)

* Safety margin is 63% for the laminates, 54% for the core.

TABLE IV. ERROR CONTRIBUTION OF THE SUBREFLECTOR

Source	Error (mm rms)
Residue path length error after astigmatic correction	0.26
Deflection of the frame, linkage	0.34
Manufacturing error of the surface	0.25
Electronic	0.04
TOTAL	0.50

TABLE V. LOCATION & SIGN CONVENTION OF THE ACTUATORS

+: deflect towards the main reflector to shorten the path length

-: deflect away from the main reflector to lengthen the path length

Actuator	Location	Edge Movement
A1	North	-a
A2	West	+a
A3	South	-a
A4	East	+a

TABLE VI. G. B. MEASUREMENTS OF THE SUBREFLECTOR SURFACE

Date	Measurement	Error (mm rms)	Actuation (mm)
12- 5-77	GB2-2	0.21	--
12- 7-77	-3	0.21	--
12-13-77	-4	0.21	--
12-14-77	-5	--	2.54
12-14-77	-6	0.26	--
3-15-78	-12	0.22	--
3-15-78	-13	0.26	--
3-15-78	-14	--	1.94
3-15-78	-15	0.24	--

TABLE VII. EFFICIENCY OF THE ACTUATORS

Actuator	Frame Deflection 1	Linkage Deflection 2	Direct Measurement 3	Indirect Measurement 4	Avg 5	Calibration Constant 6
A-1	16%	26%	58%	63%	60%	$C_1 = 1.67$
A-2	17%	25%	58%	62%	60%	$C_2 = 1.67$
A-3	18%	15%	67%	69%	68%	$C_3 = 1.47$
A-4	15%	35%	50%	54%	52%	$C_4 = 1.92$
					Avg 60%	

FIGURE CAPTIONS

Figure 1 Schematic drawings of the arrangements of the 140-ft telescope Cassegrain system. The general dimension of the deformable subreflector is given, and its relative position to the main reflector when the K-band maser receiver is in use.

Figure 2 Finite element model used by C.D.C., contains 180 elements, 108 nodes. The subreflector is considered symmetrical, actuated at nodes no. 52 and 62, and held at no. 57.

Figure 3 Comparison of the deformed subreflector profile to the best-fit deviation of the main reflector at elevation angle of 28° with the radius of the main reflector normalized to 62.3 inches.

Figure 4 Comparison of rim deflection of the deformable subreflector with the telescope positions. Dashed line denote the analytical results of 0 db, 9 db and 15 db edge taper. Solid line shows the measured result with an elongated feed. A_0 denotes the desirable range of rim deflection; A_3 shows the range presently covered by the experimental actuators.

Figure 5 Profile contour of measurements GB2-4, GB2-14 and GB2-15, with contour level of 0.005 inch.

Figure 6 Comparison of the commanded actuations (vertical scale) and measured deflections (horizontal scale). Solid line is the measurement, dashed line is the ideal.

Figure 7 Deformable subreflector's electronic block diagram.

Figure 8 Panel arrangement of the controller.

Figure 9 Subroutine flow chart implemented to the telescope control program.

Figure 10 Green Bank measuring jig for the deformable subreflector. It includes a supporting frame (1), a template (2), 6 dial indicators (3), optical plumb (4) and a rotating arm with counter wt (5).

Figure 11 The error on the profile and the reference system of the template used in the measurement.

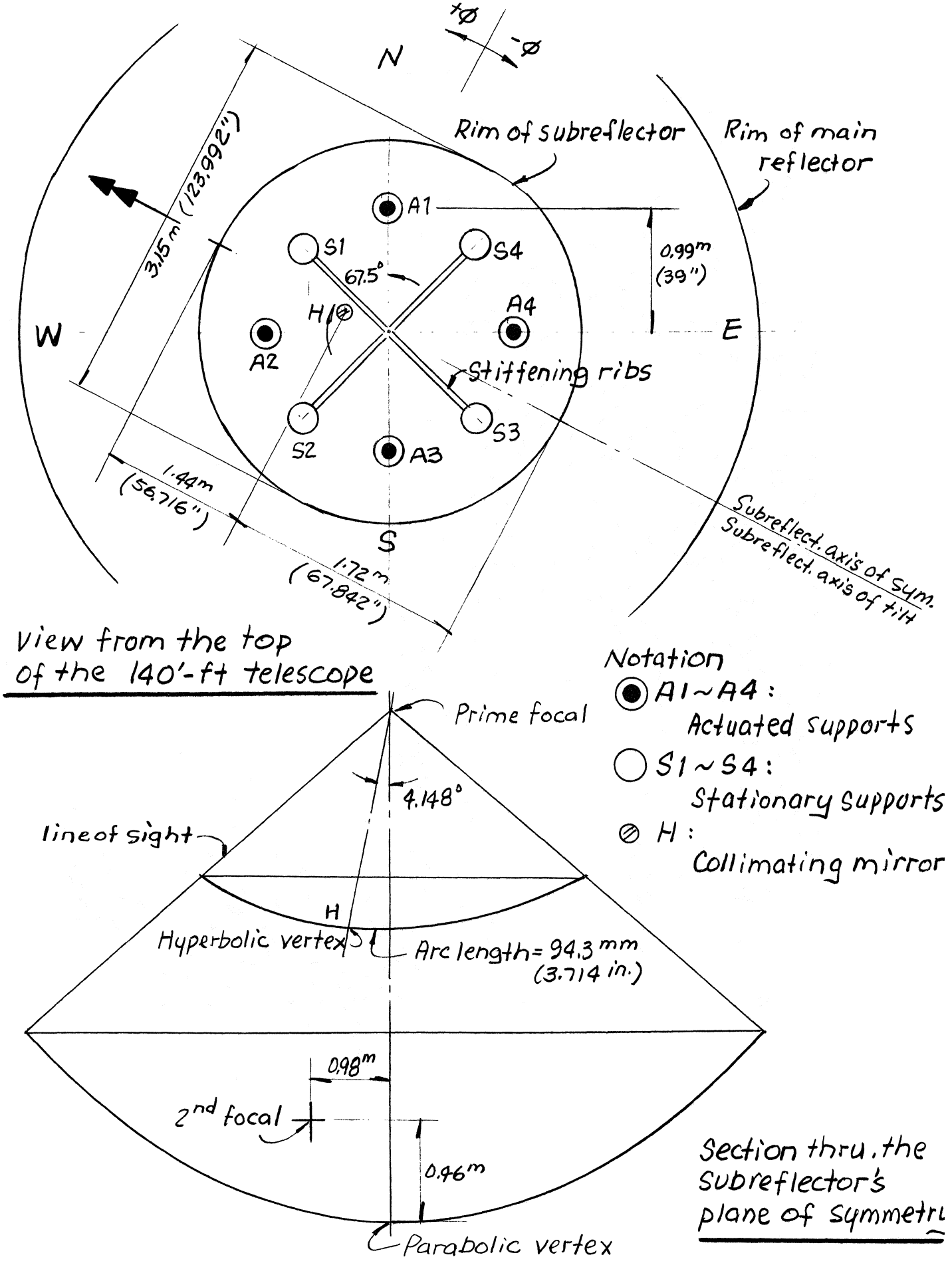

+Ø
-Ø
N
Rim of subreflector
Rim of main reflector
3.15m (123.992")
A1
S1
S4
67.5°
0.99m (39")
H
A4
W
E
A2
Stiffening ribs
S2
A3
S3
1.44m (56.716")
S
1.72m (67.842")
Subreflect. axis of sym.
Subreflect. axis of sym.
View from the top of the 140'-ft telescope
Notation
A1~A4: Actuated supports
S1~S4: Stationary supports
H: Collimating mirror
Prime focal
4.148°
line of sight
H
Hyperbolic vertex
Arc length = 94.3 mm (3.714 in.)
0.98m
2nd focal
0.46m
Parabolic vertex
Section thru. the subreflector's plane of symmetry

Fig. 1

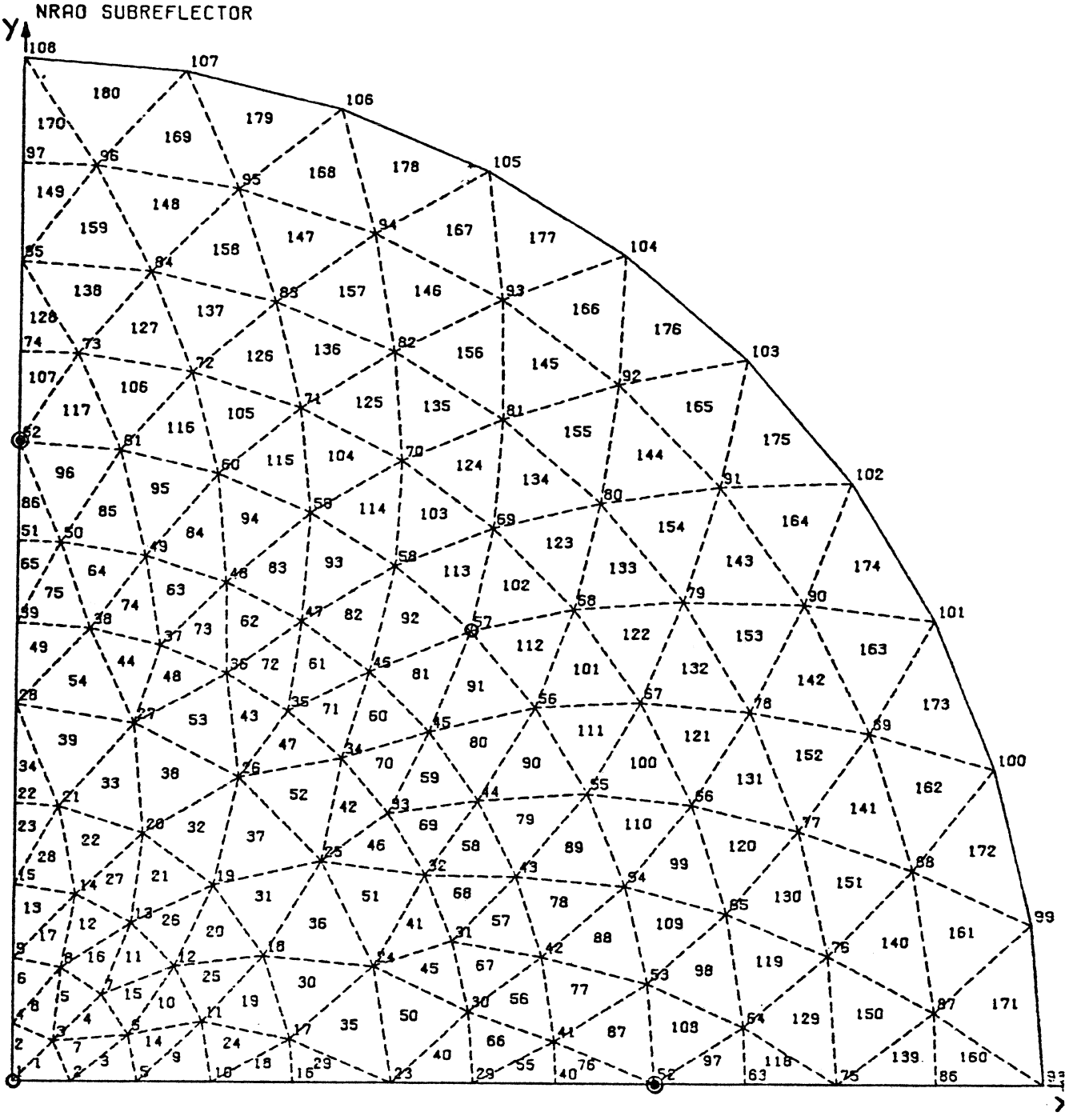


Figure 2 Finite element model used by C.D.C., contains 180 elements, 108 nodes. The subreflector is considered symmetrical, actuated at nodes no. 52 and 62, and held at no. 57.

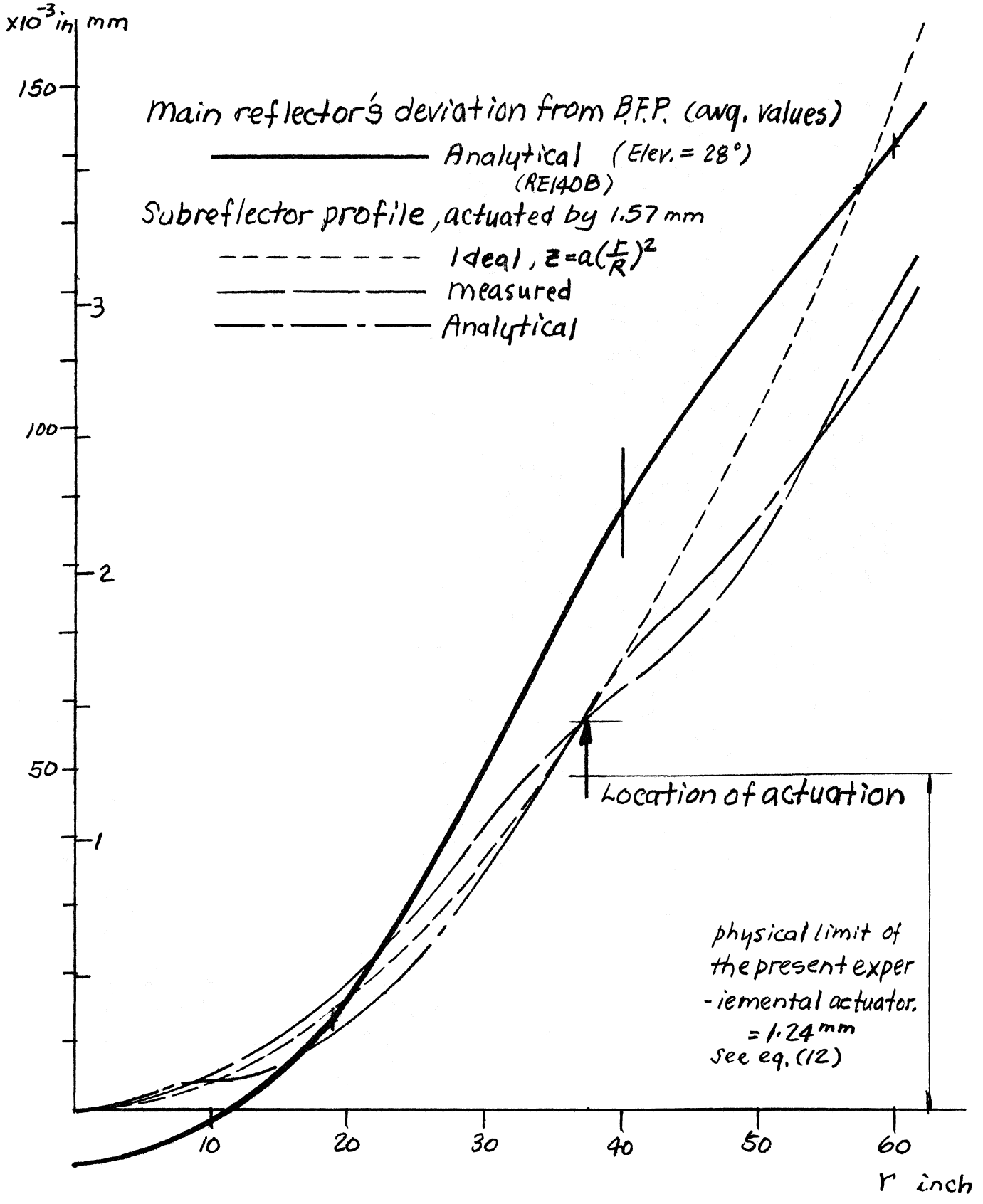


Figure 3 Comparison of the deformed subreflector profile to the best-fit deviation of the main reflector at elevation angle of 28° with the radius of the main reflector normalized to 62.3 inches.

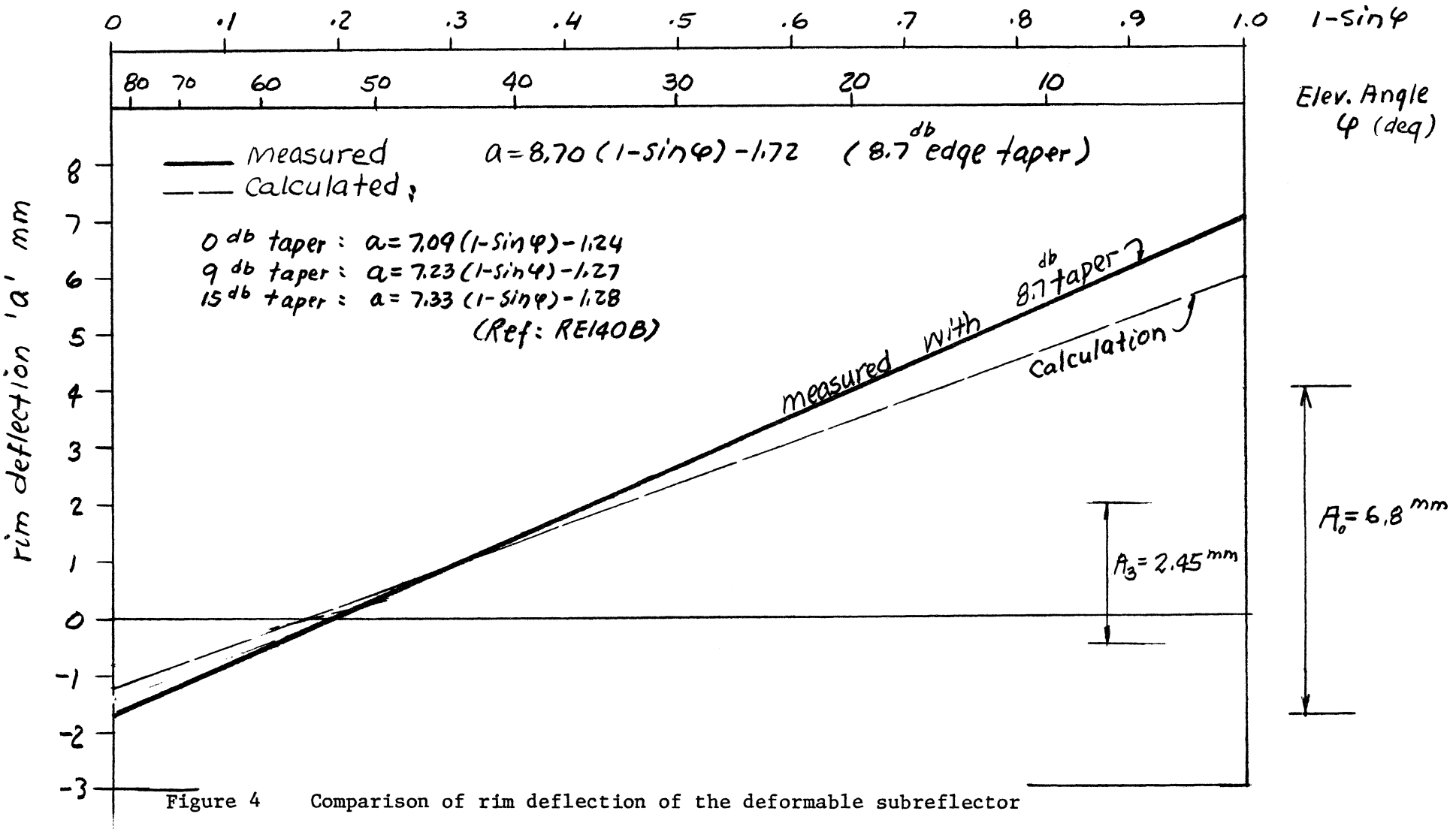
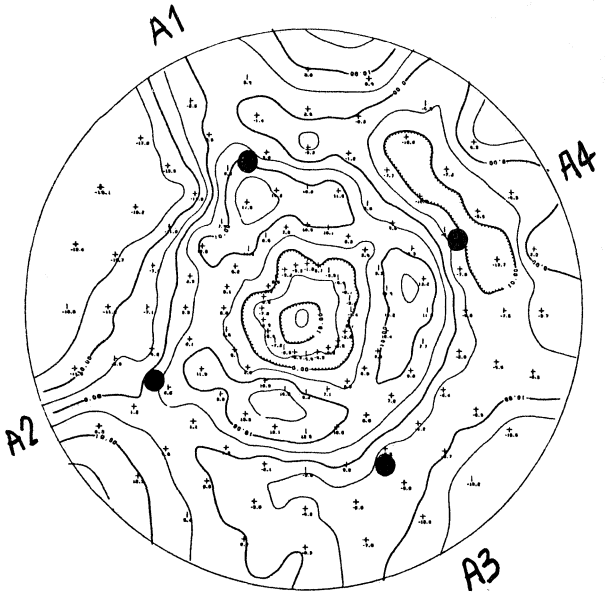
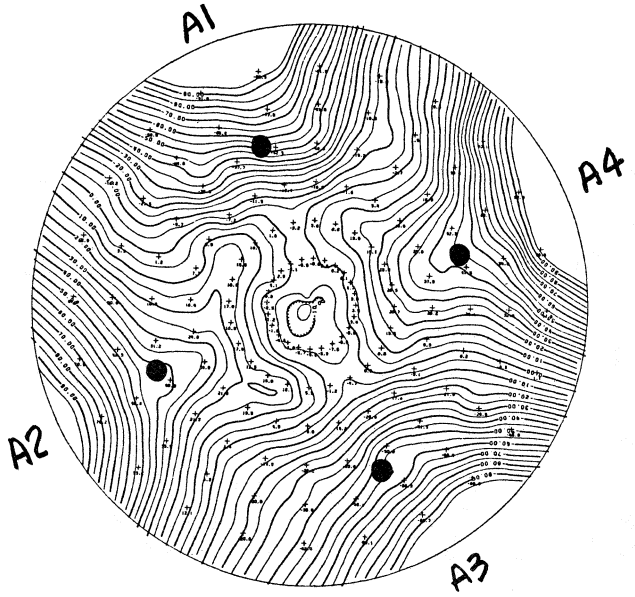


Figure 4 Comparison of rim deflection of the deformable subreflector with the telescope positions. Dashed line denote the analytical results of 0 db, 9 db and 15 db edge taper. Solid line shows the measured result with an elongated feed. A_0 denotes the desirable range of rim deflection; A_3 shows the range presently covered by the experimental actuators.



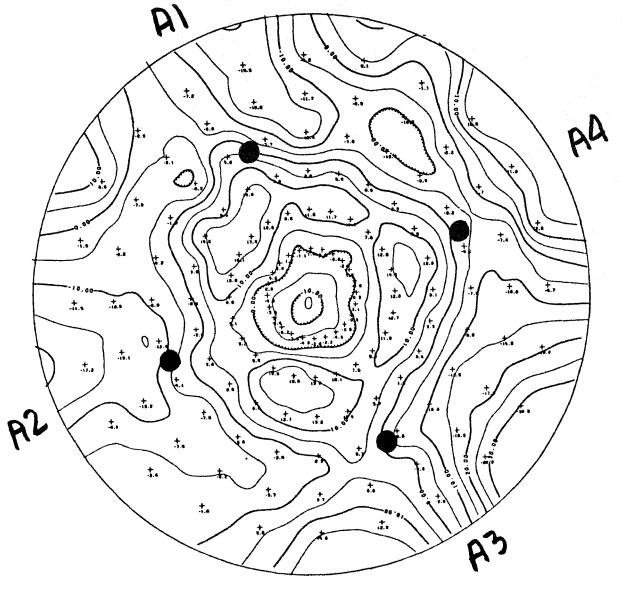
Measurement No.	Best-fit parameters
GB2-4	Relaxed surface

Surface error = 0.21 mm rms
ΔX = 0.04 mm
ΔY = 0.05 mm
ΔZ = 7.77 mm
θ_X = 27.1×10^{-5} rad.
θ_Y = 2.5×10^{-5} rad.



GB2-14 Deformed surface

A1, A3 = -1.94 mm
A2, A4 = +1.94 mm
Where '-' means lengthen of path length, and '+' shortening of Path length.



GB2-15 Relaxed surface

Surface error = 0.24 mm rms
ΔX = 0.02 mm
ΔY = 0.04 mm
ΔZ = 8.13 mm
θ_X = -40.2×10^{-5} rad.
θ_Y = 29.0×10^{-5} rad.

Fig. 5

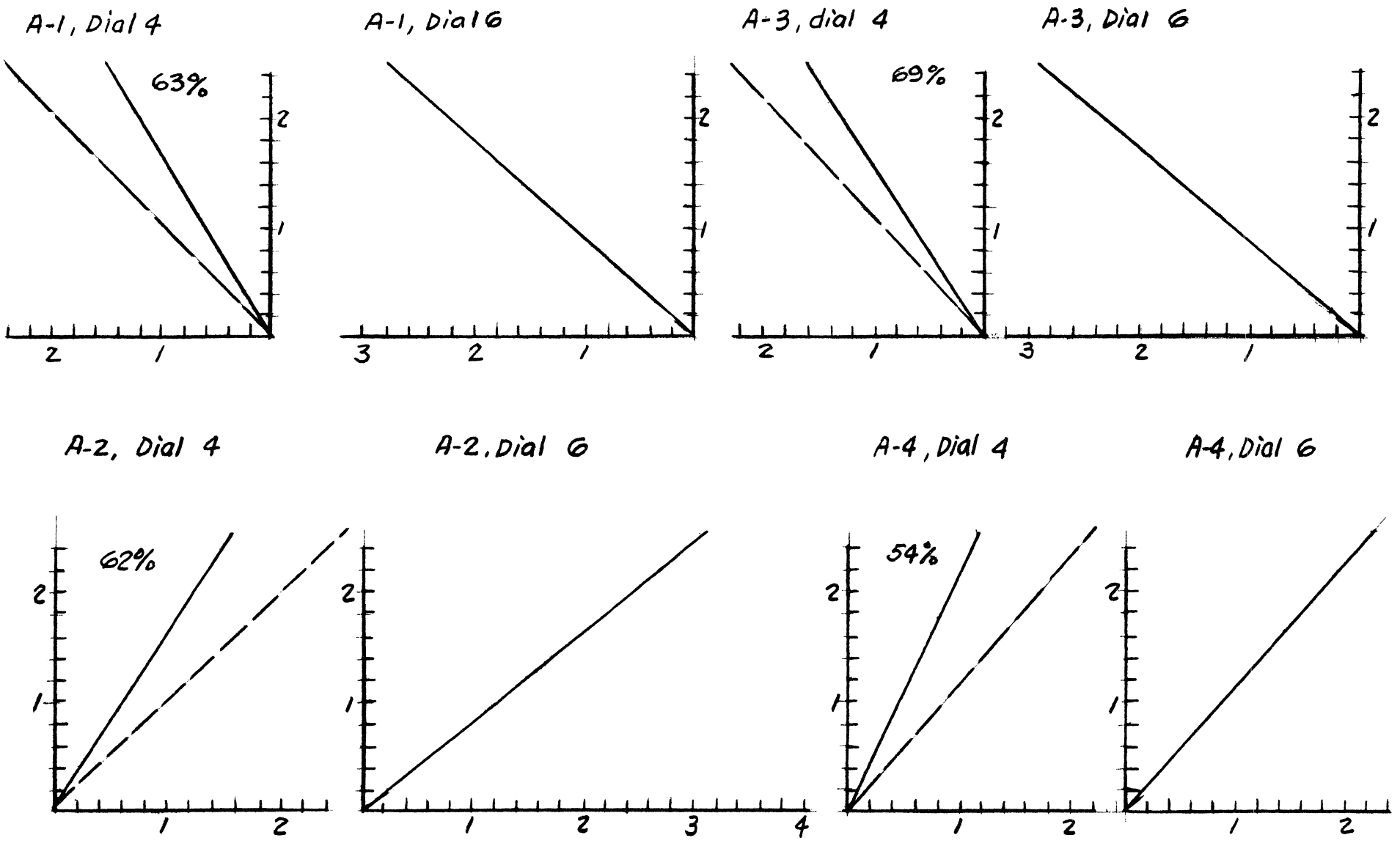


Fig. 6 Comparison of commanded actuations (vertical scale) & measured deflections (horizontal scale). Solid line is the measurement, dash line is the ideal.

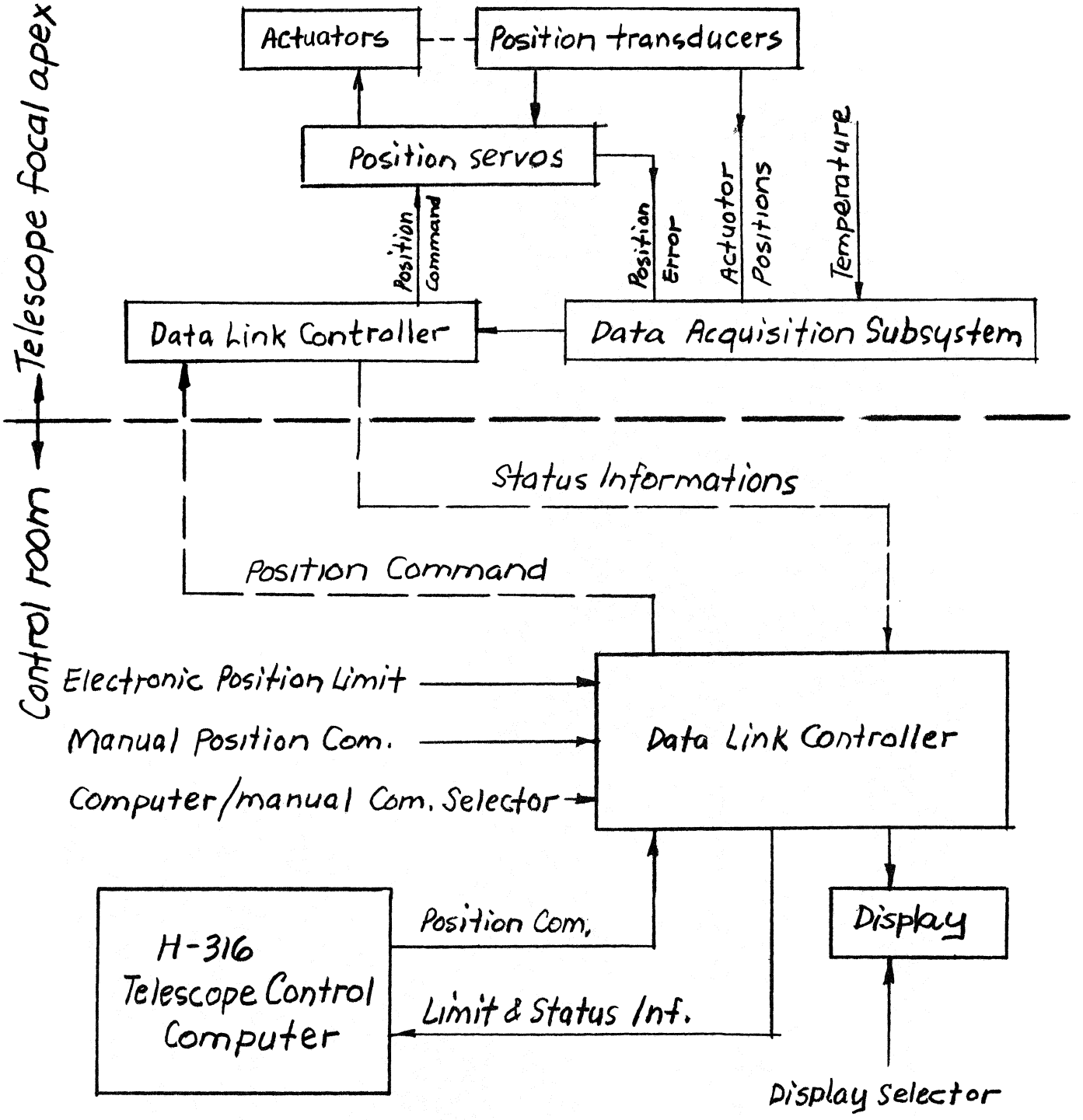


Fig. 7 Deformable Subreflector's electronic Block Diagram

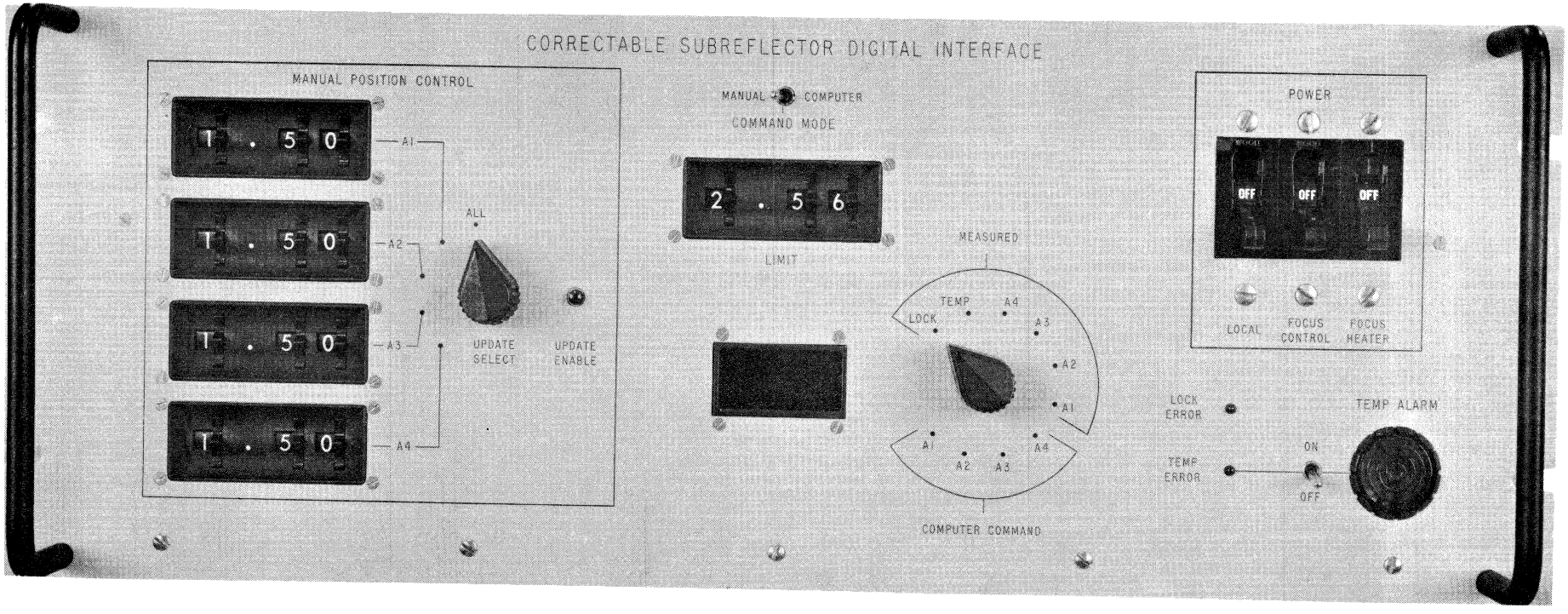


Figure 8 Panel arrangement of the controller.

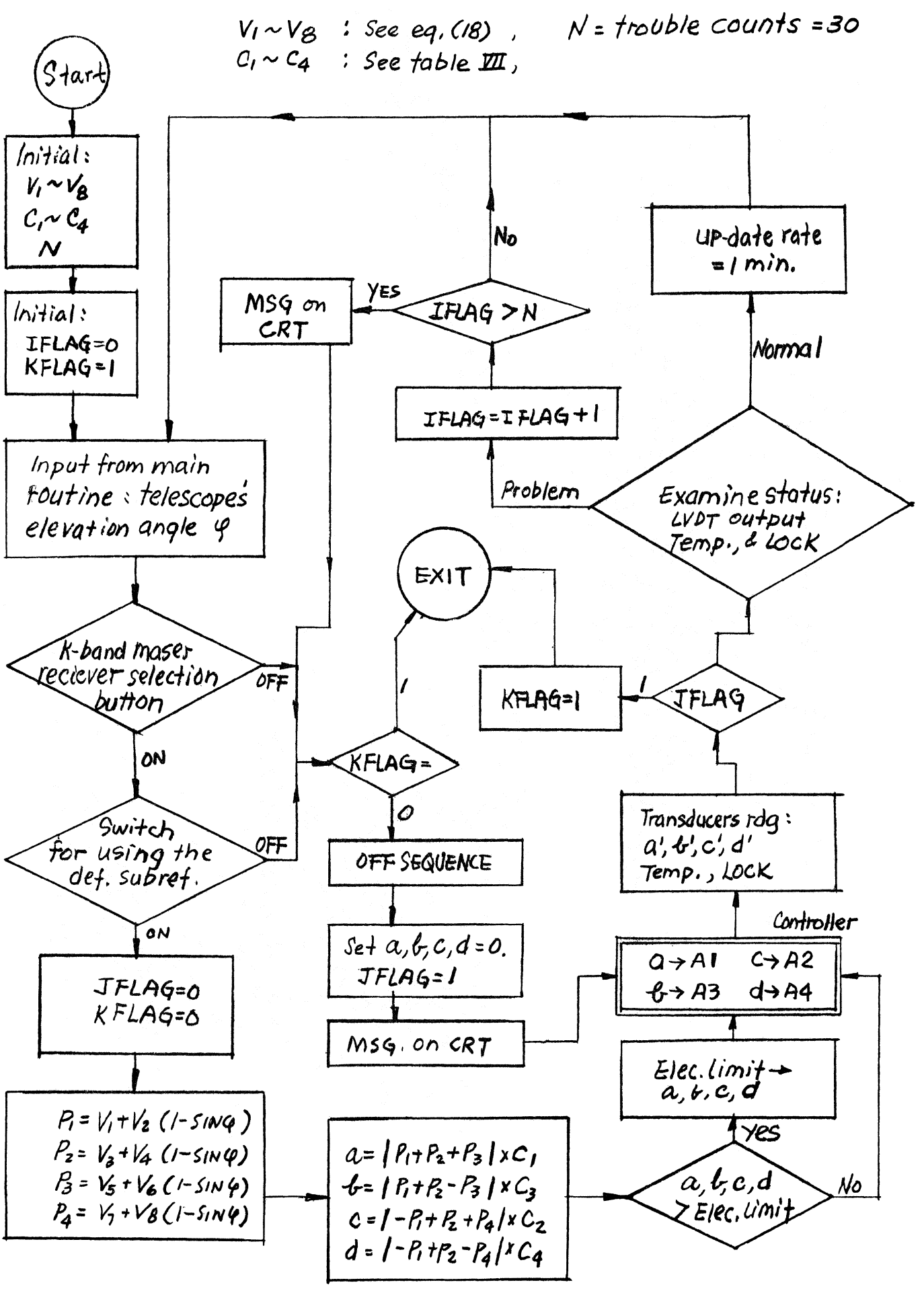

$V_1 \sim V_8$: See eq. (18), N = trouble counts = 30
$C_1 \sim C_4$: See table VII,
Start
Initial: $V_1 \sim V_8$ $C_1 \sim C_4$ N
Initial: IFLAG=0 KFLAG=1
Input from main routine : telescopes elevation angle φ
K-band maser reciever selection button
OFF
ON
Switch for using the def. subref.
OFF
ON
JFLAG=0 KFLAG=0
$P_1 = V_1 + V_2 (1-\sin\varphi)$
$P_2 = V_3 + V_4 (1-\sin\varphi)$
$P_3 = V_5 + V_6 (1-\sin\varphi)$
$P_4 = V_7 + V_8 (1-\sin\varphi)$
$a = |P_1+P_2+P_3| \times C_1$
$b = |P_1+P_2-P_3| \times C_3$
$c = |-P_1+P_2+P_4| \times C_2$
$d = |-P_1+P_2-P_4| \times C_4$
a, b, c, d ? Elec. limit
Yes
No
Elec. limit → a, b, c, d
Controller
a → A1 C → A2
b → A3 d → A4
Transducers rdg : a', b', c', d' Temp., LOCK
JFLAG
1
KFLAG=1
EXIT
1
KFLAG=
0
OFF SEQUENCE
Set a, b, c, d = 0. JFLAG=1
MSG. on CRT
MSG on CRT
YES
IFLAG > N
No
IFLAG=IFLAG+1
Problem
Examine status: LVDT output Temp., & Lock
Normal
UP-date rate = 1 min.

Fig. 9

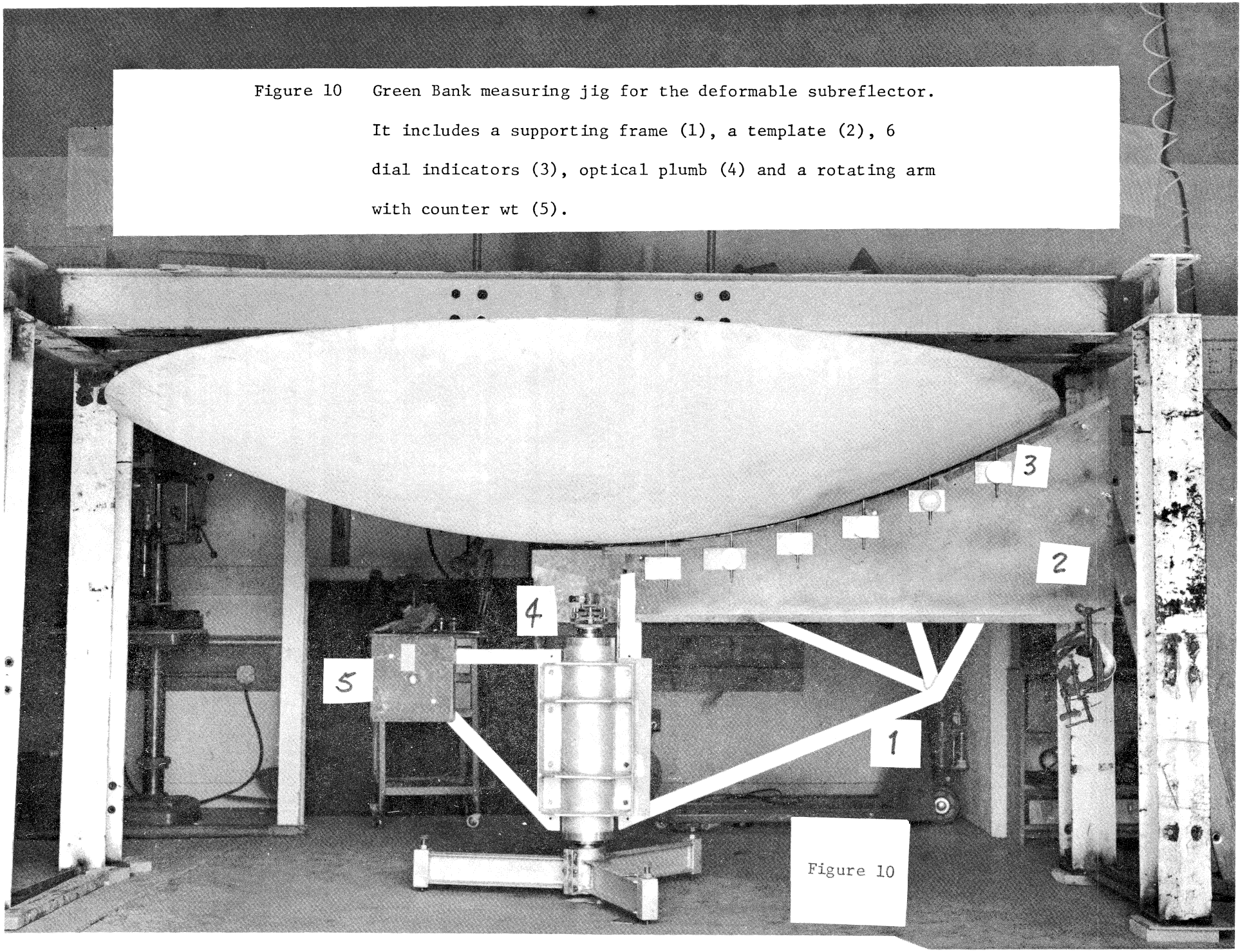


Figure 10 Green Bank measuring jig for the deformable subreflector. It includes a supporting frame (1), a template (2), 6 dial indicators (3), optical plumb (4) and a rotating arm with counter wt (5).

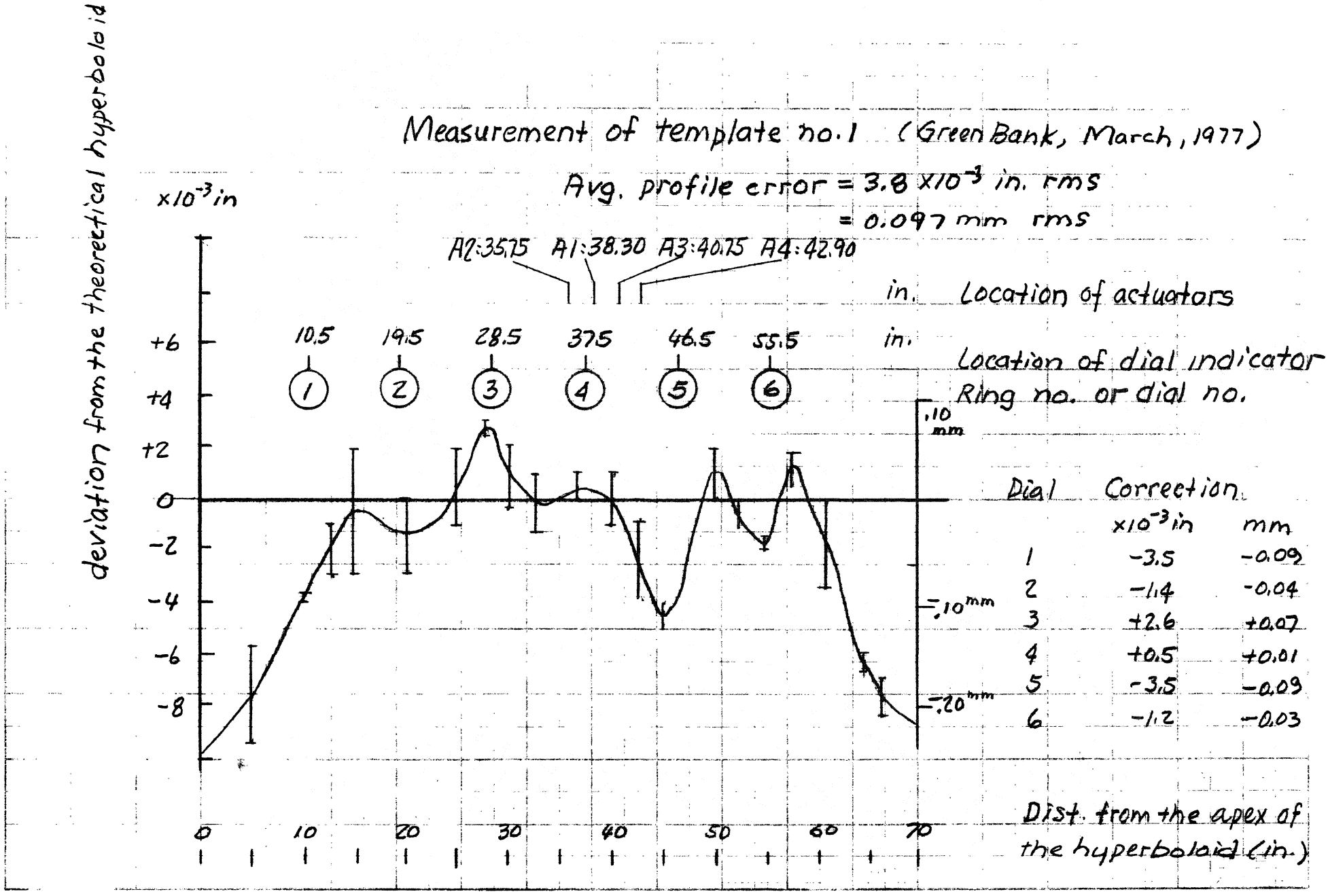
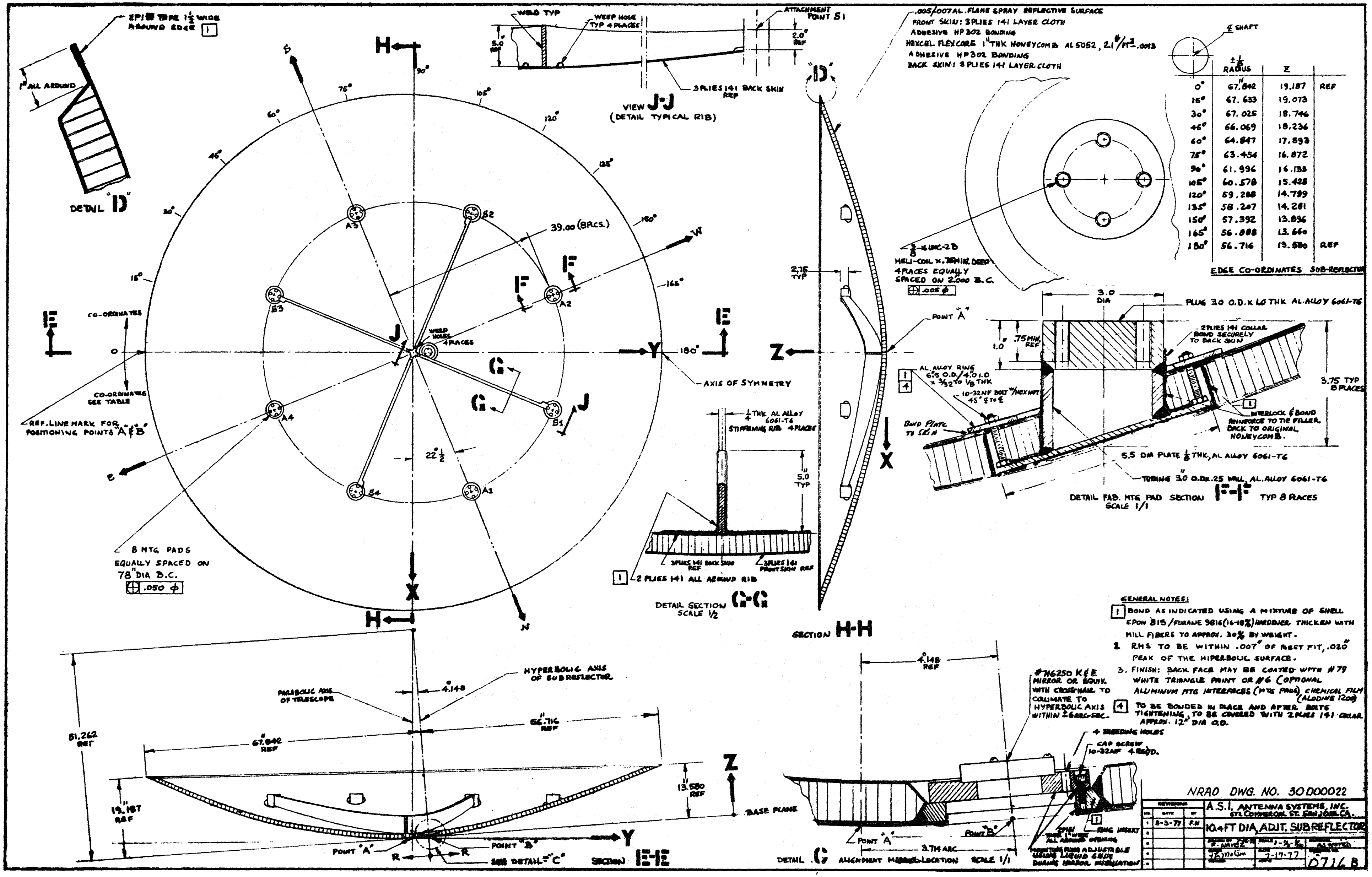


Figure 11 The error on the profile and the reference system of the template used in the measurement.

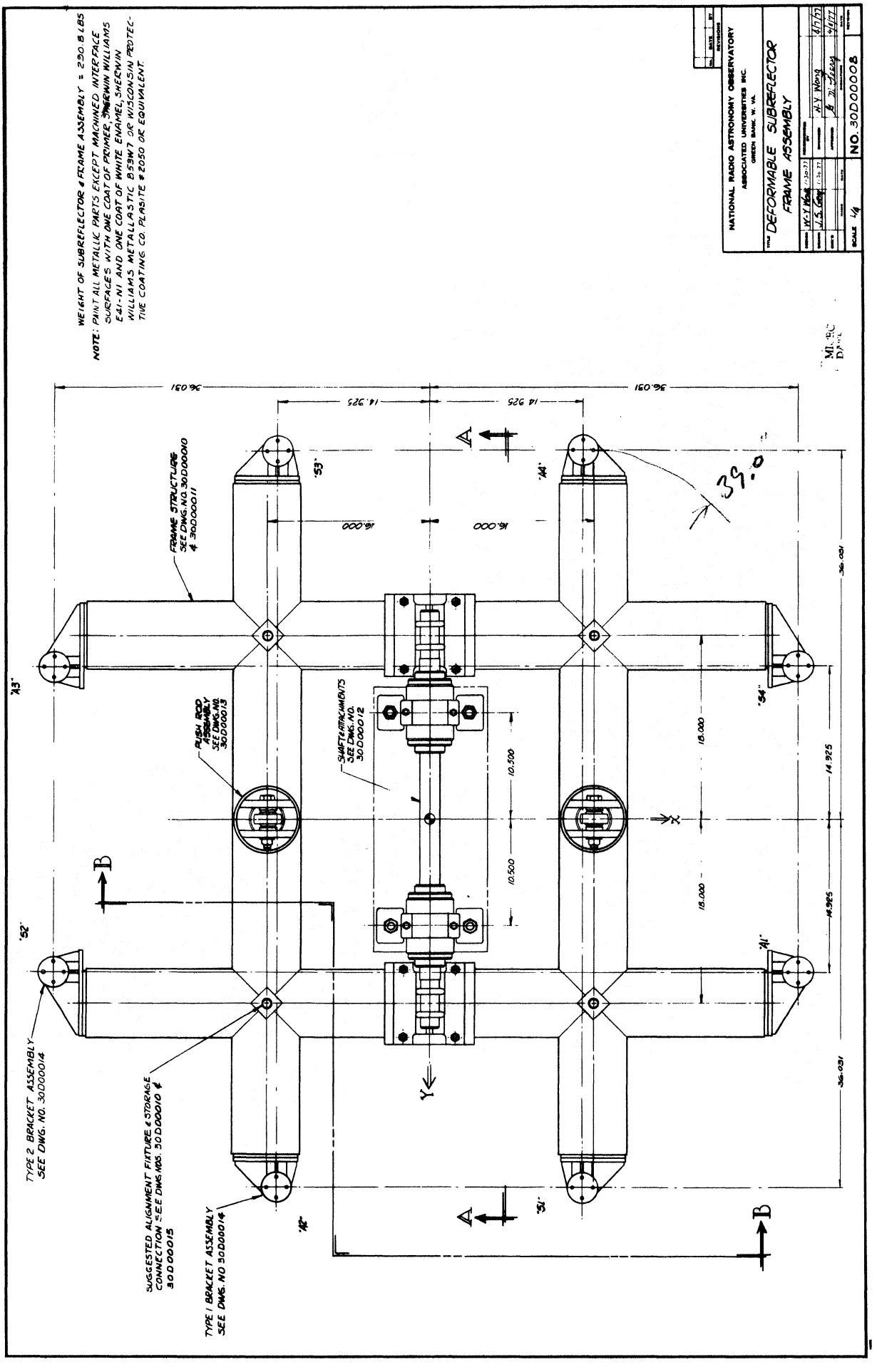
APPENDIX A

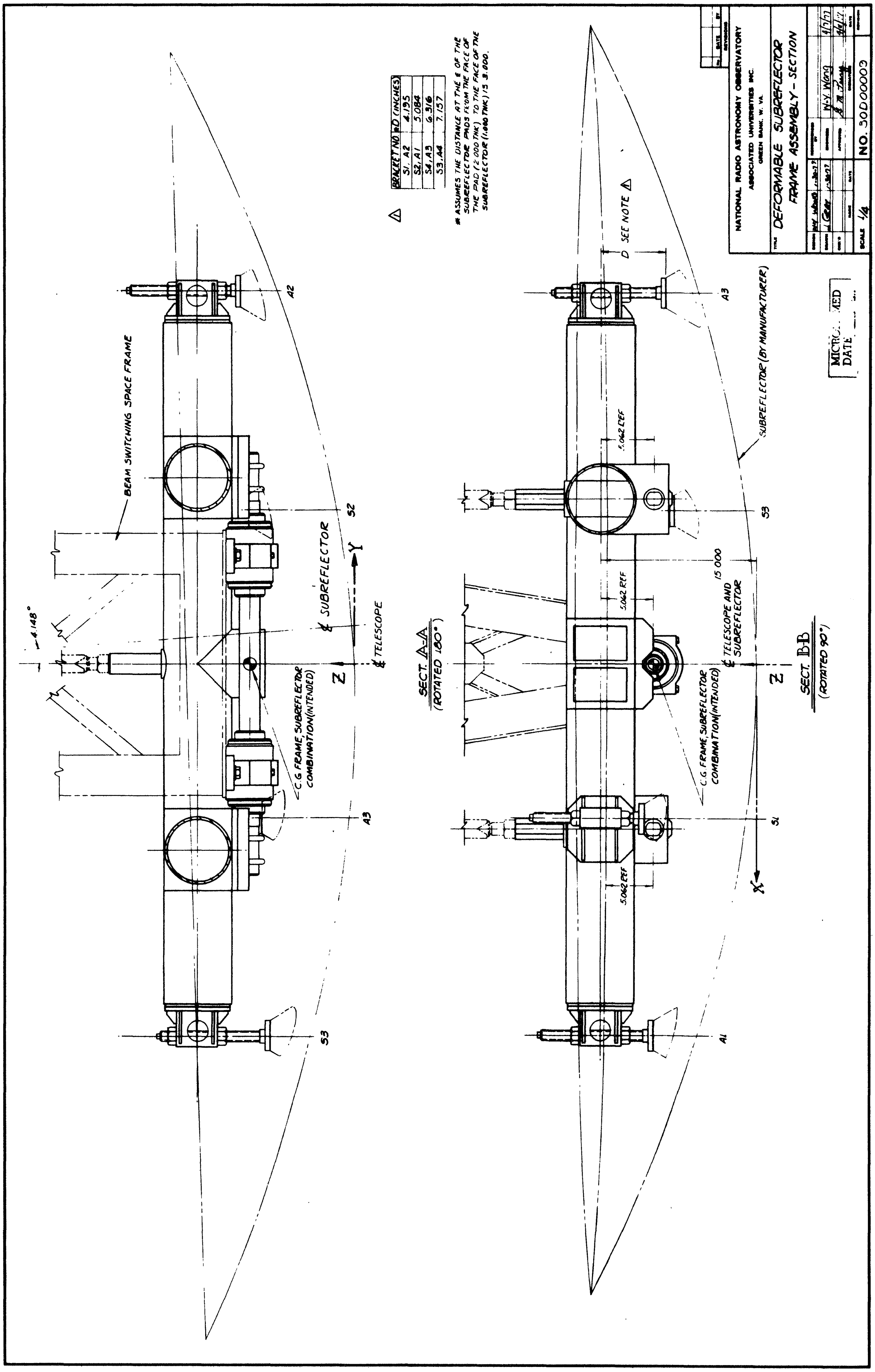
Engineering drawing of the Subreflector Surface

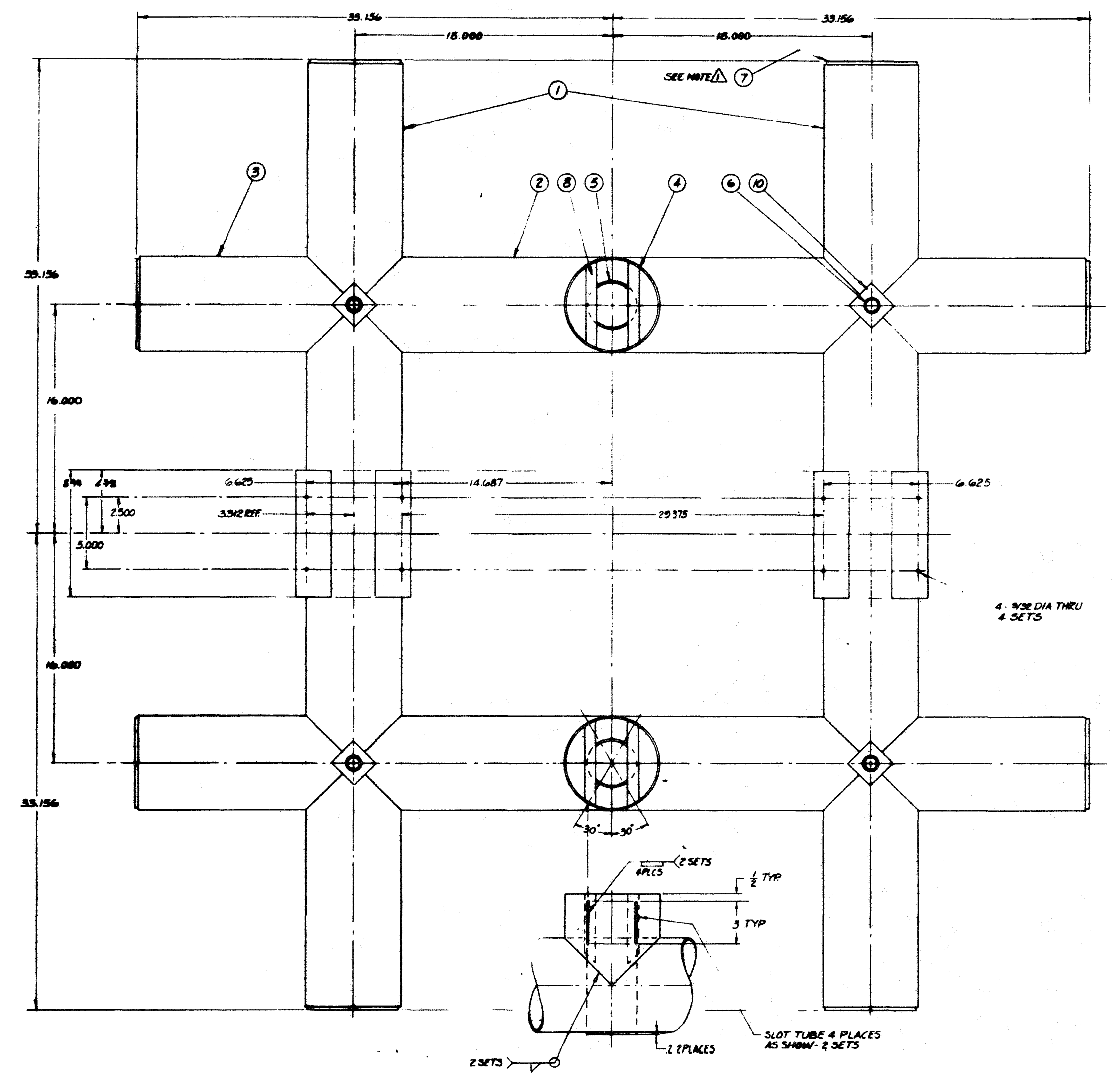

DETAIL "D"
1" ALL AROUND
VIEW J-J (DETAIL TYPICAL RIB)
3PLIES 141 BACK SKIN REF
ATTACHMENT POINT S1
WELD TYP
WEEP HOLE TYP 4 PLACES
.005/.007 AL. FLAME SPRAY REFLECTIVE SURFACE
FRONT SKIN: 3PLIES 141 LAYER CLOTH
ADHESIVE HP302 BONDING
HEXCEL FLEXCORE 1" THK HONEYCOMB AL5052, 2.1#/FT3 .0013
ADHESIVE HP302 BONDING
BACK SKIN: 3 PLIES 141 LAYER CLOTH
℄ SHAFT
EDGE CO-ORDINATES SUB-REFLECTOR
0° 67.842 19.187 REF
15° 67.633 19.073
30° 67.025 18.746
45° 66.069 18.236
60° 64.847 17.593
75° 63.454 16.872
90° 61.996 16.133
105° 60.578 15.428
120° 59.288 14.799
135° 58.207 14.281
150° 57.392 13.896
165° 56.888 13.660
180° 56.716 13.580 REF
39.00 (B.C.S.)
AXIS OF SYMMETRY
REF. LINE MARK FOR POSITIONING POINTS "A" & "B"
8 MTG PADS EQUALLY SPACED ON 78" DIA B.C.
DETAIL SECTION G-G SCALE 1/2
2 PLIES 141 ALL AROUND RIB
SECTION H-H
POINT "A"
PLUG 3.0 O.D. x 1.0 THK AL. ALLOY 6061-T6
2PLIES 141 COLLAR BOND SECURELY TO BACK SKIN
INTERLOCK & BOND REINFORCE TO TIE FILLER BACK TO ORIGINAL HONEYCOMB
5.5 DIA PLATE 1/8 THK, AL ALLOY 6061-T6
TUBING 3.0 O.D. x .25 WALL AL. ALLOY 6061-T6
DETAIL FAB. MTG PAD SECTION F-F TYP 8 PLACES SCALE 1/1
GENERAL NOTES:
1 BOND AS INDICATED USING A MIXTURE OF SHELL EPON 815 / FURANE 9816 (16-18%) HARDENER. THICKEN WITH MILL FIBERS TO APPROX. 30% BY WEIGHT.
2. RMS TO BE WITHIN .007" OF BEST FIT, .020" PEAK OF THE HYPERBOLIC SURFACE.
3. FINISH: BACK FACE MAY BE COATED WITH #79 WHITE TRIANGLE PAINT OR #6 (OPTIONAL ALUMINUM MTG INTERFACES (MTG PADS) CHEMICAL FILM (ALODINE 1200)
4 TO BE BONDED IN PLACE AND AFTER BOLTS TIGHTENING, TO BE COVERED WITH 2 PLIES 141 COLLAR APPROX. 12" DIA O.D.
HYPERBOLIC AXIS OF SUBREFLECTOR
PARABOLIC AXIS OF TELESCOPE
SECTION E-E
BASE PLANE
DETAIL C ALIGNMENT MIRROR LOCATION SCALE 1/1
NRAO DWG. NO. 30D00022
A.S.I. ANTENNA SYSTEMS, INC.
10.4 FT DIA ADJT. SUBREFLECTOR
D714 B

APPENDIX B

Engineering drawings of the Bakc-up Frame DWG No. 30 D 00008 to 30 D 00016.


WEIGHT OF SUBREFLECTOR & FRAME ASSEMBLY = 290.8 LBS
NOTE: PAINT ALL METALLIC PARTS EXCEPT MACHINED INTERFACE SURFACES WITH ONE COAT OF PRIMER, SHERWIN WILLIAMS E41-N1 AND ONE COAT OF WHITE ENAMEL, SHERWIN WILLIAMS METALLASTIC B53W7 OR WISCONSIN PROTECTIVE COATING CO. PLASITE #2050 OR EQUIVALENT.
NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.
DEFORMABLE SUBREFLECTOR FRAME ASSEMBLY
NO. 30D000008
SCALE 1/4
FRAME STRUCTURE SEE DWG. NO. 30D00010 & 30D00011
PUSH ROD ASSEMBLY SEE DWG. NO. 30D00013
SHAFT & ATTACHMENTS SEE DWG. NO. 30D00012
TYPE 2 BRACKET ASSEMBLY SEE DWG. NO. 30D00014
SUGGESTED ALIGNMENT FIXTURE & STORAGE CONNECTION SEE DWG. NOS. 30D00010 & 30D00015
TYPE 1 BRACKET ASSEMBLY SEE DWG. NO 30D00014
39.0
36.031
14.925
18.000
10.500
16.000


NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.
DEFORMABLE SUBREFLECTOR
FRAME ASSEMBLY - SECTION
NO. 30D00003
SCALE 1/4
BRACKET NO / D (INCHES)
S1, A2 4.195
S2, A1 5.084
S4, A3 6.316
S3, A4 7.157
* ASSUMES THE DISTANCE AT THE ℄ OF THE SUBREFLECTOR PADS FROM THE FACE OF THE PAD (2.000 THK) TO THE FACE OF THE SUBREFLECTOR (1.000 THK) IS 3.000.
D SEE NOTE
BEAM SWITCHING SPACE FRAME
SUBREFLECTOR (BY MANUFACTURER)
℄ SUBREFLECTOR
℄ TELESCOPE
℄ TELESCOPE AND ℄ SUBREFLECTOR
C.G FRAME, SUBREFLECTOR COMBINATION (INTENDED)
SECT. A-A (ROTATED 180°)
SECT. B-B (ROTATED 90°)
5.062 REF
15.000
4.148°



LIST OF MATERIAL			
ITEM	QTY	DESCRIPTION	REMARKS
1	2	TUBE 6" SCH. 10 PIPE 65.[illegible] LG 6061-T6 ALUM	SEE DWG. 30D00011
2	2	36.000 LG	
3	4	14.906 LG	
4	2	6.52 LG	
5	2	TUBE 3 SCH. 10 PIPE 6.82 LG	
6	4	TUBE 3/4 SCH. 40 PIPE 7.62 LG	
7	8	PLUG 3/4 THK × 6.36 DIA.	
8	4	BRACKET 3/4 THK × 5 × 6	
10	8	PAD 1/2 THK × 2 1/4 × 2 1/4	
11	4	PAD 3/4 THK × 8 × 8 3/4	BEFORE MACHINING
12	12	GUSSET 3/4 THK × 1 1/2 × 5 1/8	SEE DWG. 30D00011

NOTES:

CENTER LINES OF ALL 6" PIPE TO BE IN THE SAME HORIZONTAL PLANE & PERPENDICULAR TO EACH OTHER AT ALL INTERSECTIONS.

FRAME TO BE WELDED CONSTRUCTION THROUGHOUT UNLESS OTHER METHODS OF ASSEMBLY ARE DETAILED. ALL WELDS TO BE CONTINUOUS FOR ENTIRE LENGTH OF ATTACHMENT (MATCHING) SURFACES.

1. FACE OF PLUGS TO BE SQUARE TO THEORETICAL HORIZONTAL AND VERTICAL CENTERLINES WITHIN .005 INCH.
2. TAPPED HOLES IN PLUGS TO BE LOCATED OFF THE THEORETICAL HORIZONTAL AND VERTICAL CENTERLINES WITHIN .005 INCH.
3. SURFACES OF PADS TO BE IN THE SAME PLANE AND PARALLEL TO THE THEORETICAL HORIZONTAL CENTER LINE. FLUSH WITH TO .030 ABOVE THE O.D. OF THE TUBES. ONLY ON SIDE SHOWN.

MICROFILMED
DATE DEC 09 1977

UNLESS OTHERWISE SPECIFIED
TOLERANCES
$\frac{x}{x} = \pm \frac{1}{64}$
0.00 = ±.015
0.000 = ±.005
ALL DIMENSIONS ARE INCHES

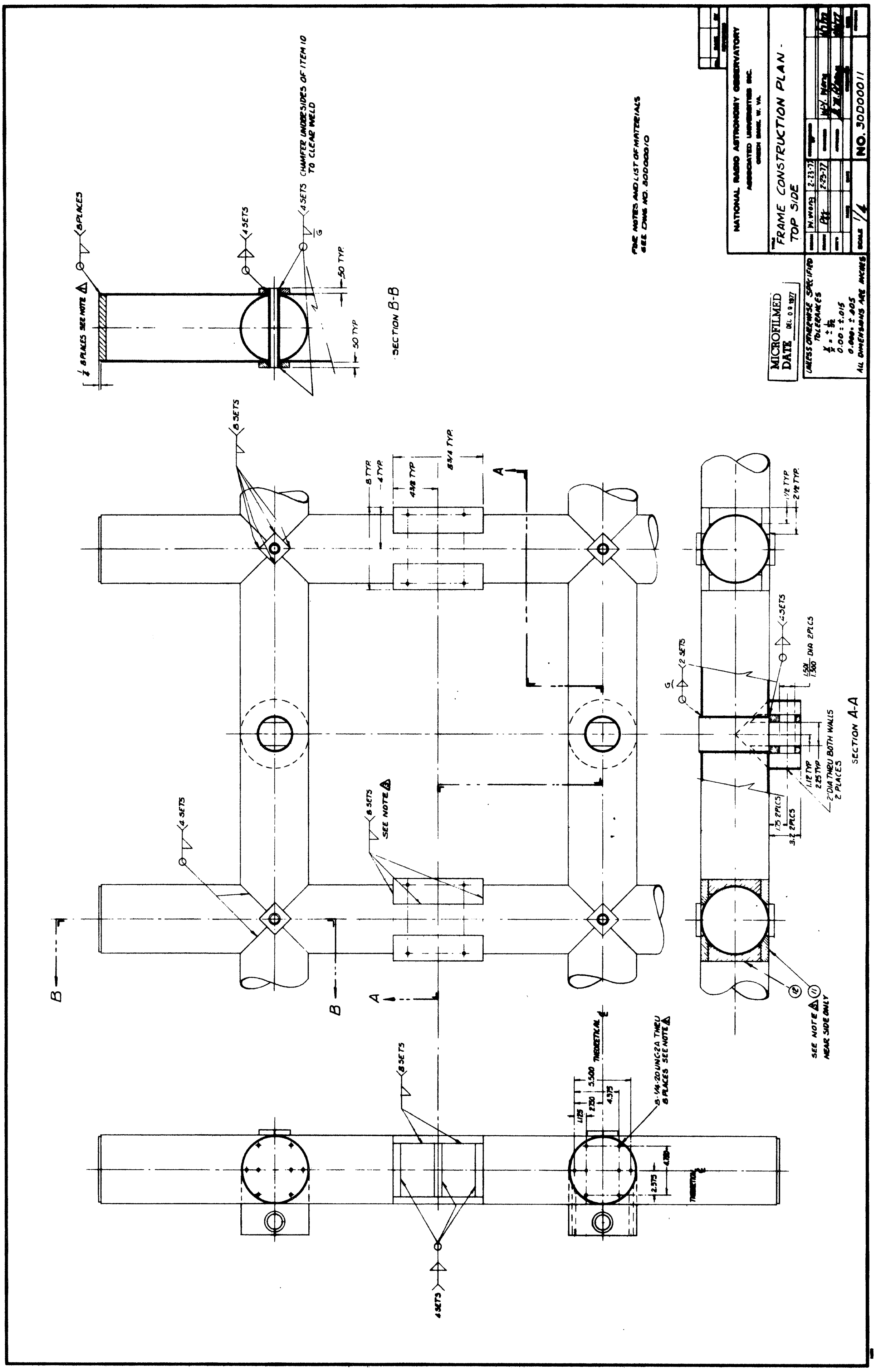
NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.

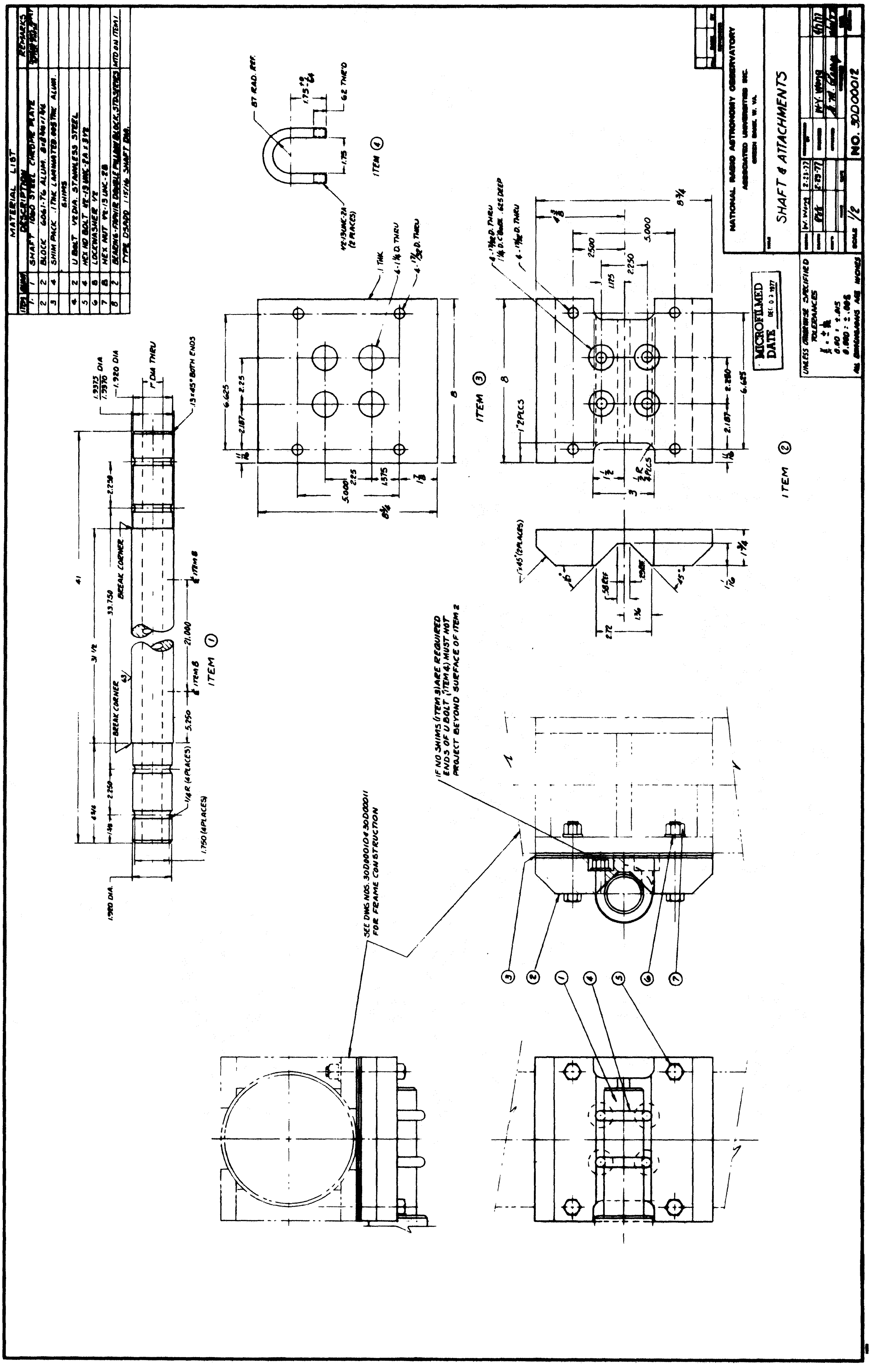
FRAME CONSTRUCTION PLAN - BOTTOM SIDE

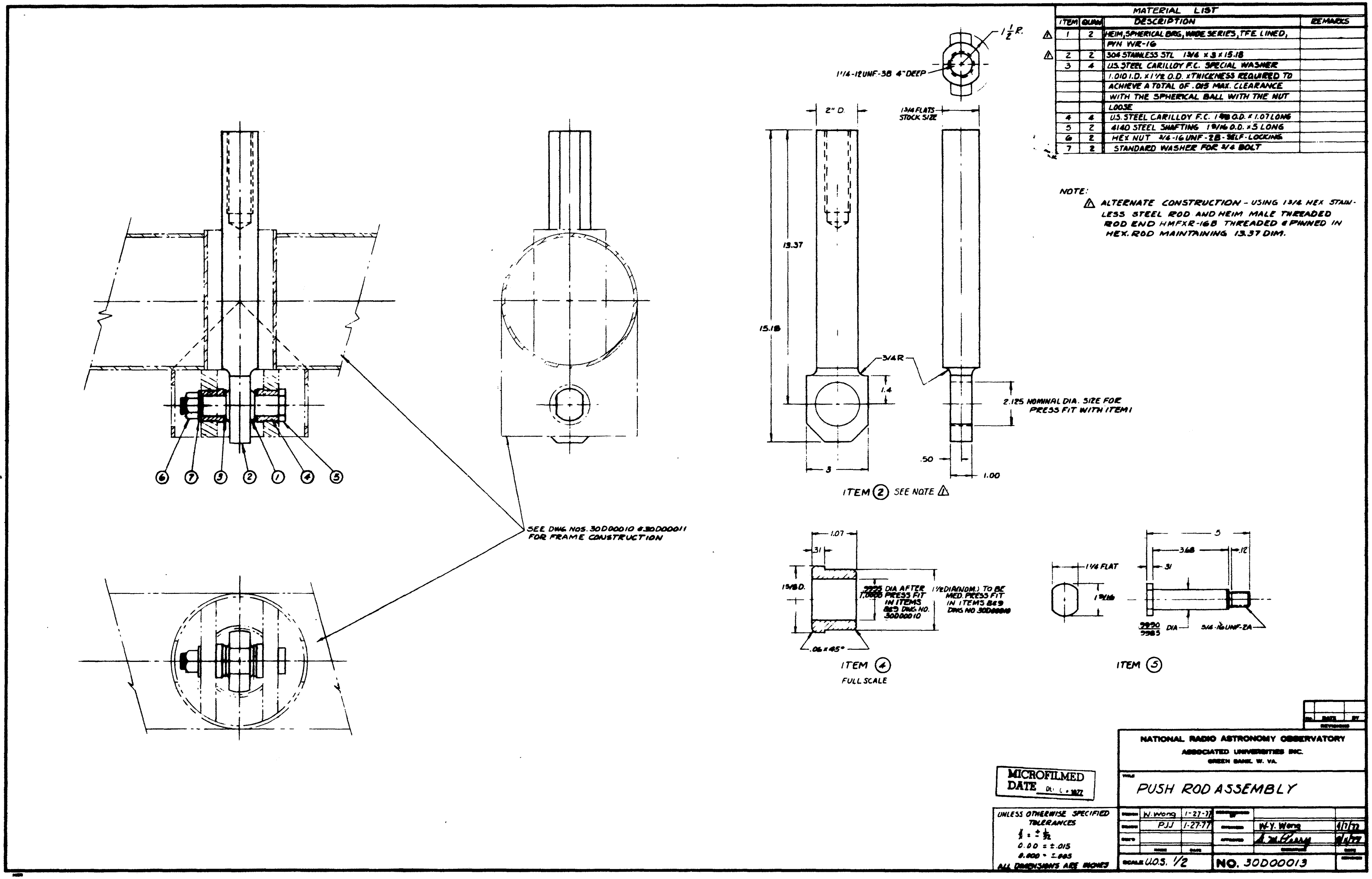
W. Wong 2-23-77
2-23-77
W.Y. Wong 4/7/77

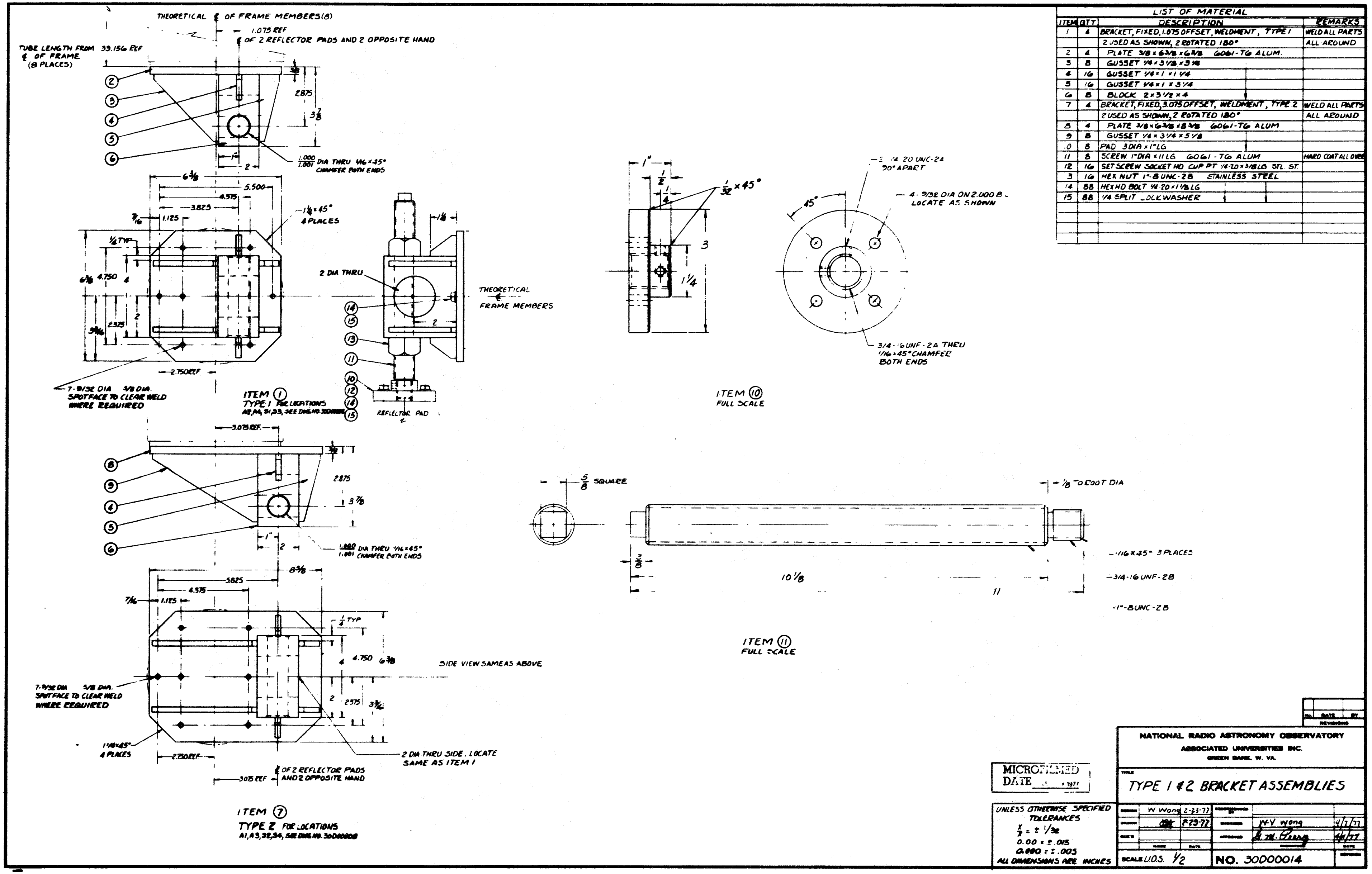
SCALE 1/4

NO. 30D00010


NATIONAL RADIO ASTRONOMY OBSERVATORY
FRAME CONSTRUCTION PLAN - TOP SIDE
NO. 30D00011
SECTION A-A
SECTION B-B
MICROFILMED DATE
FOR NOTES AND LIST OF MATERIALS SEE DWG NO. 30D00010
CHAMFER UNDERSIDES OF ITEM 10 TO CLEAR WELD
UNLESS OTHERWISE SPECIFIED TOLERANCES
ALL DIMENSIONS ARE INCHES

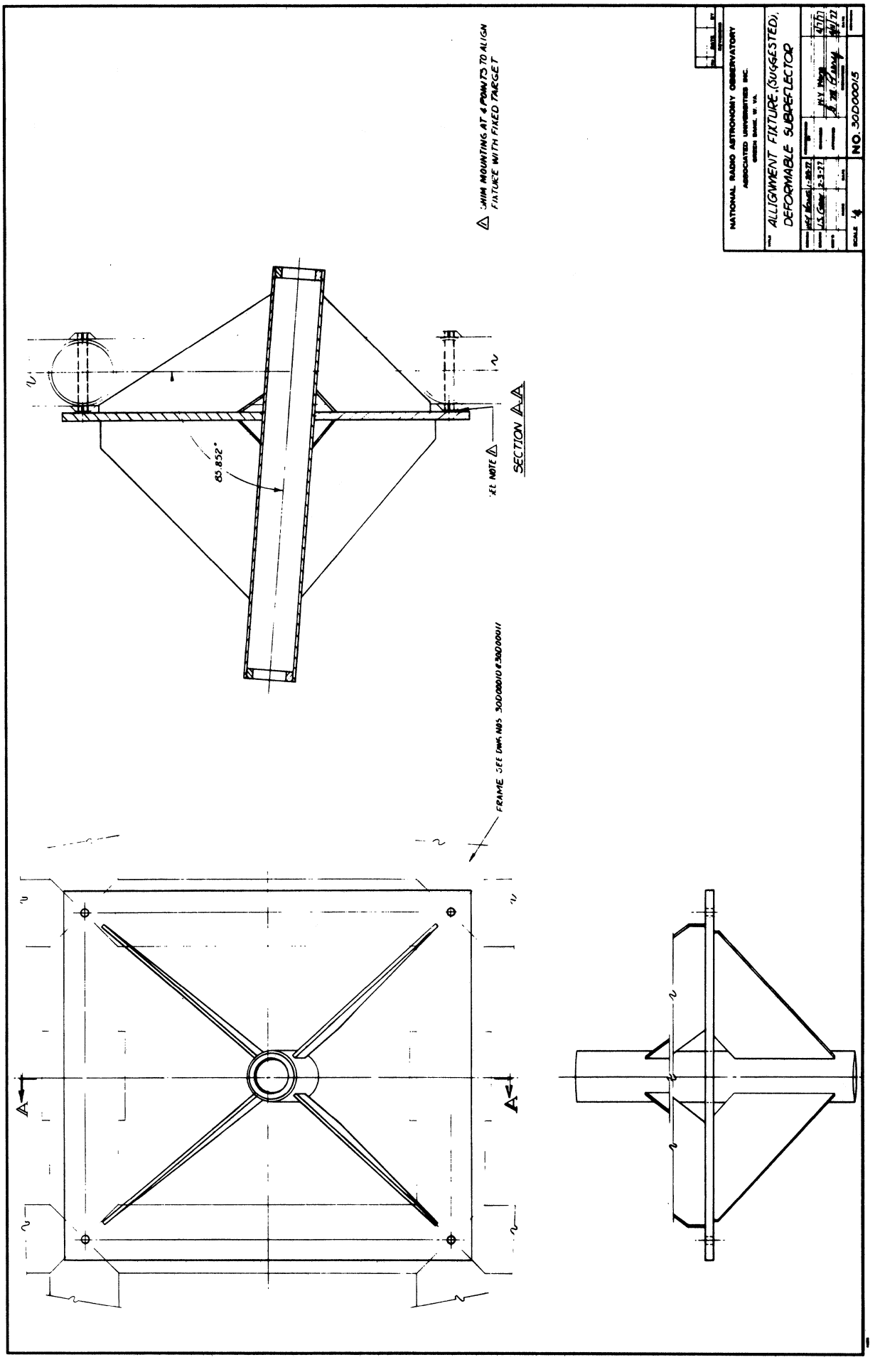

MATERIAL LIST
SHAFT & ATTACHMENTS
NATIONAL RADIO ASTRONOMY OBSERVATORY
NO. 30D00012
ITEM 1
ITEM 2
ITEM 3
ITEM 4
MICROFILMED

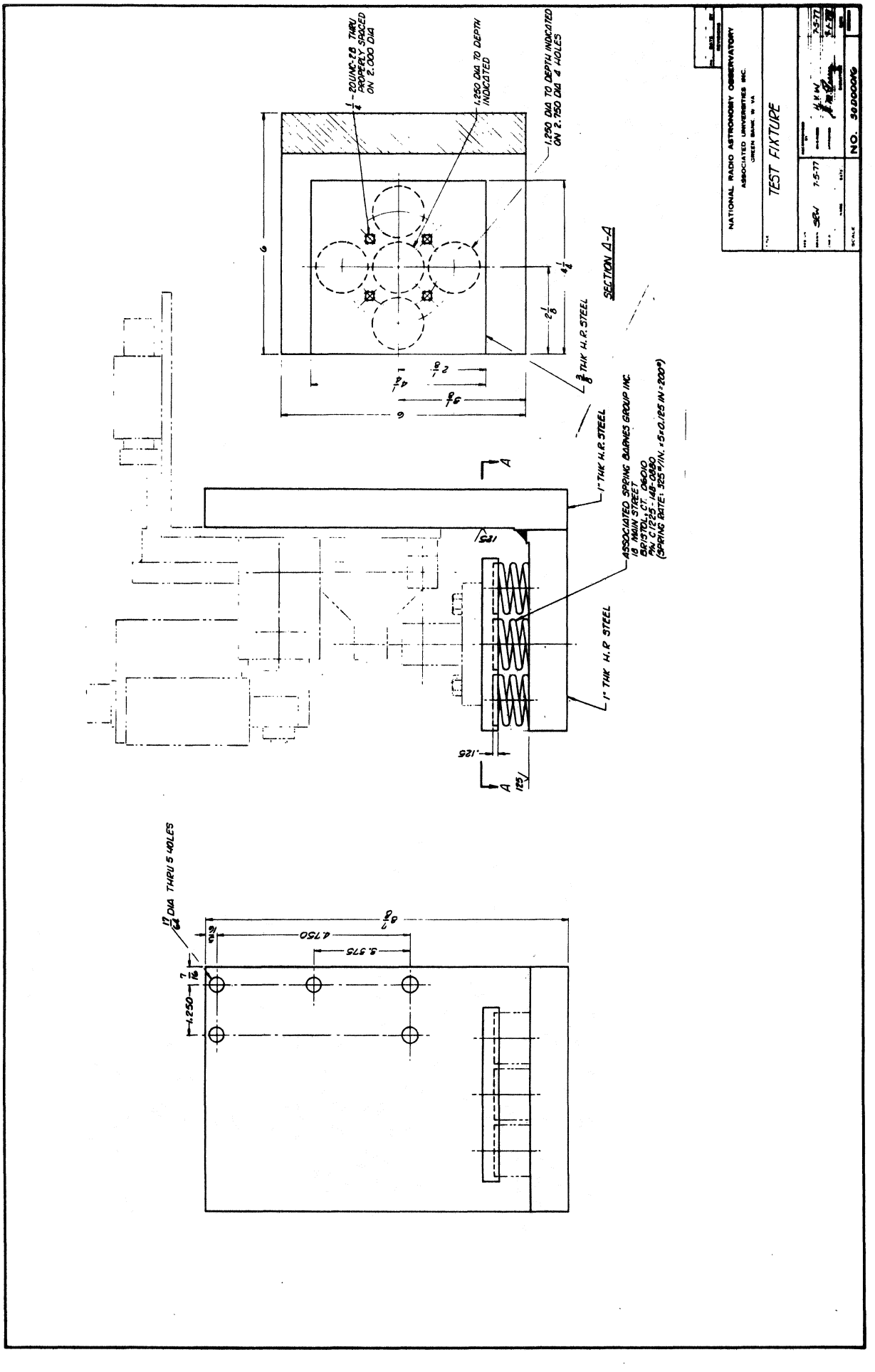

MATERIAL LIST
ITEM QUAN DESCRIPTION REMARKS
1 2 HEIM, SPHERICAL BRG, WIDE SERIES, TFE LINED, P/N WR-16
2 2 304 STAINLESS STL 13/4 x 3 x 15.18
3 4 U.S. STEEL CARILLOY F.C. SPECIAL WASHER 1.010 I.D. x 1 1/2 O.D. x THICKNESS REQUIRED TO ACHIEVE A TOTAL OF .015 MAX. CLEARANCE WITH THE SPHERICAL BALL WITH THE NUT LOOSE
4 4 U.S. STEEL CARILLOY F.C. 1.07 LONG
5 2 4140 STEEL SHAFTING 19/16 O.D. x 5 LONG
6 2 HEX NUT 3/4-16 UNF-2B SELF-LOCKING
7 2 STANDARD WASHER FOR 3/4 BOLT
NOTE:
ALTERNATE CONSTRUCTION - USING 13/4 HEX STAINLESS STEEL ROD AND HEIM MALE THREADED ROD END HMFXR-16B THREADED & PINNED IN HEX. ROD MAINTAINING 13.37 DIM.
SEE DWG NOS. 30D00010 & 30D00011 FOR FRAME CONSTRUCTION
ITEM 2 SEE NOTE
2.125 NOMINAL DIA. SIZE FOR PRESS FIT WITH ITEM 1
ITEM 4 FULL SCALE
ITEM 5
NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.
PUSH ROD ASSEMBLY
UNLESS OTHERWISE SPECIFIED TOLERANCES
0.00 = ±.015
ALL DIMENSIONS ARE INCHES
MICROFILMED DATE
SCALE U.O.S. 1/2
NO. 30D00013



LIST OF MATERIAL

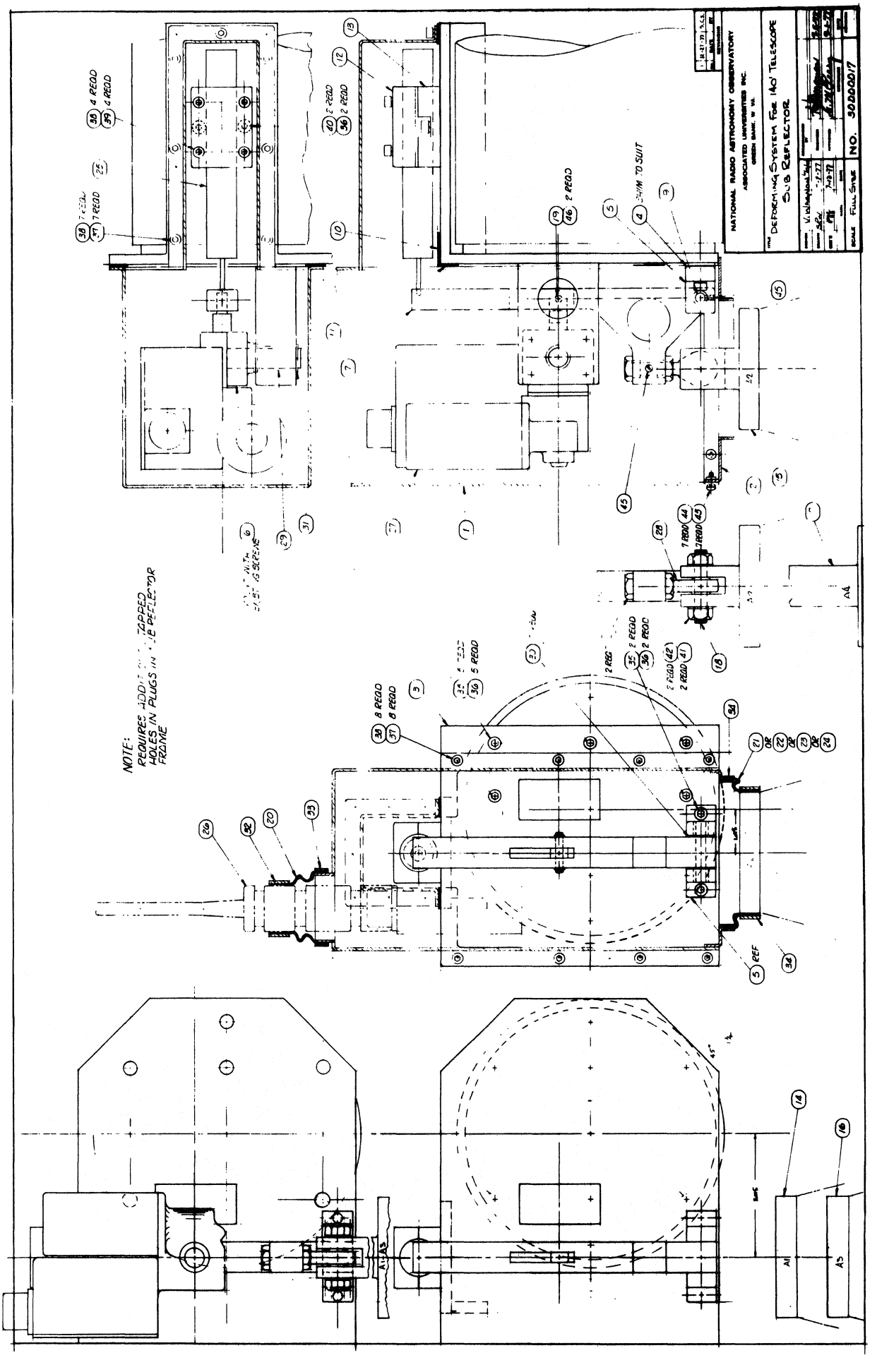
ITEM	QTY	DESCRIPTION	REMARKS
1	4	BRACKET, FIXED, 1.075 OFFSET, WELDMENT, TYPE 1	WELD ALL PARTS
		2 USED AS SHOWN, 2 ROTATED 180°	ALL AROUND
2	4	PLATE 3/8 × 6 3/8 × 6 3/8 6061-T6 ALUM.	
3	8	GUSSET 1/4 × 3 1/8 × 3 1/8	
4	16	GUSSET 1/4 × 1 × 1 1/4	
5	16	GUSSET 1/4 × 1 × 3 1/4	
6	8	BLOCK 2 × 3 1/2 × 4	
7	4	BRACKET, FIXED, 3.075 OFFSET, WELDMENT, TYPE 2	WELD ALL PARTS
		2 USED AS SHOWN, 2 ROTATED 180°	ALL AROUND
8	4	PLATE 3/8 × 6 3/8 × 8 3/8 6061-T6 ALUM	
9	8	GUSSET 1/4 × 3 1/4 × 5 1/8	
10	8	PAD 3 DIA × 1" LG	
11	8	SCREW 1" DIA × 11 LG 6061-T6 ALUM	HARD COAT ALL OVER
12	16	SET SCREW SOCKET HD CUP PT 1/4-20 × 3/16 LG STL ST	
13	16	HEX NUT 1"-8 UNC-2B STAINLESS STEEL	
14	88	HEX HD BOLT 1/4-20 × 1 1/2 LG	
15	88	1/4 SPLIT LOCKWASHER	

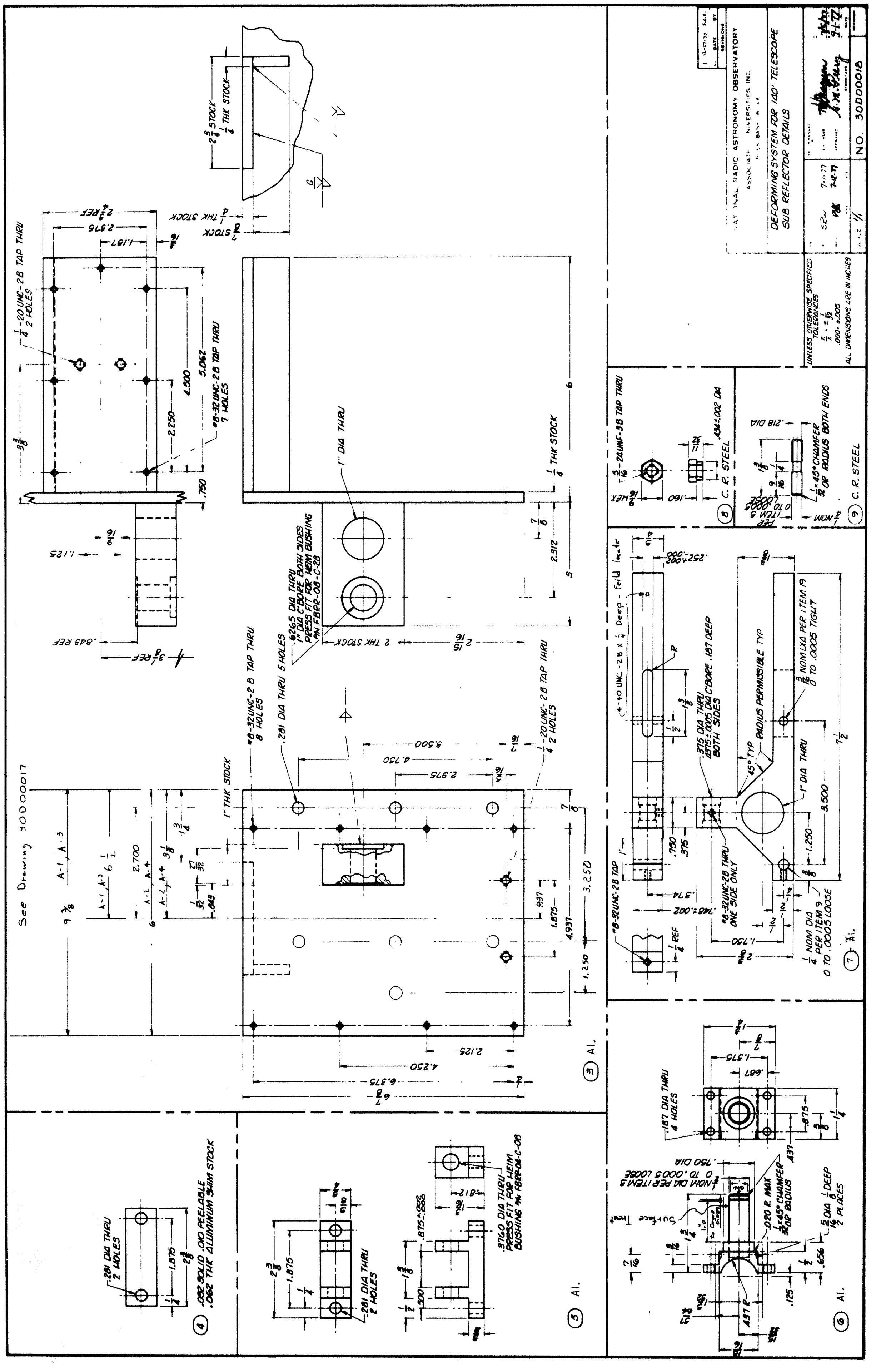

NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
ALLIGNMENT FIXTURE (SUGGESTED), DEFORMABLE SUBREFLECTOR
NO. 3000015
SECTION A-A
65.852
FRAME SEE DWG NOS 3000010 & 3000011
JAWS MOUNTING AT 4 POINTS TO ALIGN FIXTURE WITH FIXED TARGET

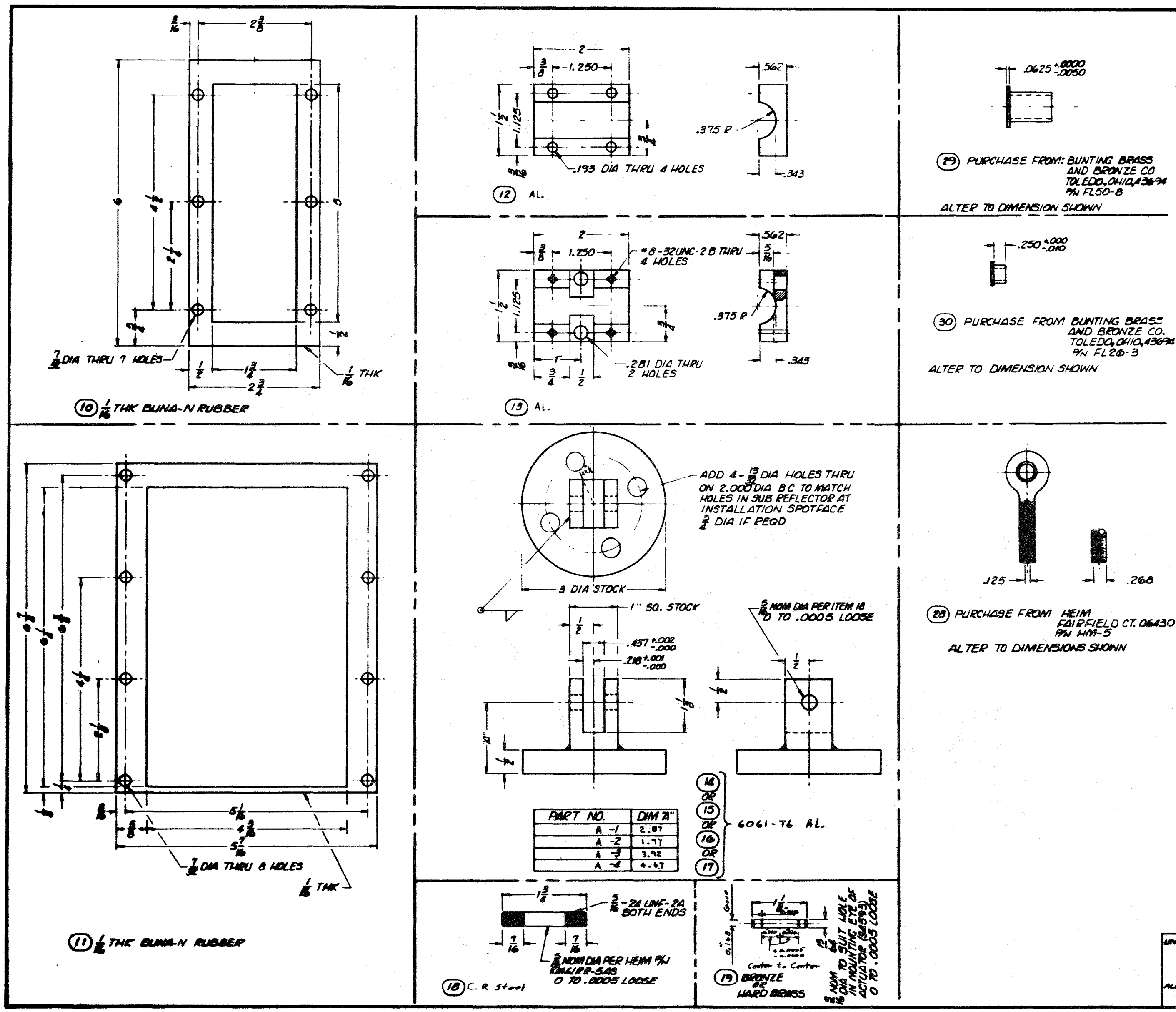

NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.
TEST FIXTURE
NO. 3000006
7-5-77
SECTION A-A
1 - ZDUNE-28 THRU PROPERLY SPACED ON 2.000 DIA
1.250 DIA TO DEPTH INDICATED
1.250 DIA TO DEPTH INDICATED ON 2.750 DIA 4 HOLES
3/8 THK H.R. STEEL
1" THK H.R. STEEL
ASSOCIATED SPRING BARNES GROUP INC.
18 MAIN STREET
BRISTOL, CT. 06010
PN. C1225-148-0880
(SPRING RATE: 325#/IN. ±5%@.125 IN-200#)
1" THK H.P STEEL
.125
17/64 DIA THRU 5 HOLES
4.750
8.575
1.250

APPENDIX C

Engineering Drawings of the Deforming System DWG No. 30 D 00017 to 30 D 00020.


NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
DEFORMING SYSTEM FOR 140' TELESCOPE
SUB REFLECTOR
NO. 30D00017
SCALE FULL SIZE
NOTE:
REQUIRES ADDITIONAL TAPPED
HOLES IN PLUGS IN SUB REFLECTOR
FRAME
4 REQD
2 REQD
8 REQD
5 REQD
SHIM TO SUIT
REF


NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC
DEFORMING SYSTEM FOR 140' TELESCOPE
SUB REFLECTOR DETAILS
NO. 30D00018
UNLESS OTHERWISE SPECIFIED
TOLERANCES
ALL DIMENSIONS ARE IN INCHES
See Drawing 30D00017
1" THK STOCK
#8-32UNC-2B TAP THRU 8 HOLES
.281 DIA THRU 5 HOLES
1/4-20UNC-2B TAP THRU 2 HOLES
1" DIA THRU
1/4 THK STOCK
2 THK STOCK
#8-32 UNC-2B TAP THRU 7 HOLES
C. R. STEEL
RADIUS PERMISSIBLE TYP
1" DIA THRU
.187 DIA THRU 4 HOLES
.281 DIA THRU 2 HOLES
.062 SOLID .010 PEELABLE .062 THK ALUMINUM SHIM STOCK
.3760 DIA THRU PRESS FIT FOR HEIM BUSHING



LIST OF MATERIAL

ITEM	QTY	DESCRIPTION	REMARKS
1	4	COVER	SEE DETAIL 1
2	4	COVER	SEE DETAIL 2
3	4	PLATE, MOUNTING	SEE DETAIL 3
4	4	SHIM	SEE DETAIL 4
5	4	PIVOT	SEE DETAIL 5
6	4	SUPPORT, SHAFT	SEE DETAIL 6
7	4	ARM	SEE DETAIL 7
8	8	NUT	SEE DETAIL 8
9	4	SHAFT	SEE DETAIL 9
10	4	GASKET	SEE DETAIL 10
11	4	GASKET	SEE DETAIL 11
12	4	MOUNT, TRANSDUCER } MAY BE BORED AS ONE	SEE DETAIL 12
13	4	MOUNT. TRANSDUCER } PIECE & SUIT WITH .062 SHIM	SEE DETAIL 13
14	1	SUPPORT (A1)	SEE DETAIL 14
15	1	SUPPORT (A2)	SEE DETAIL 15
16	1	SUPPORT (A3)	SEE DETAIL 16
17	1	SUPPORT (A4)	SEE DETAIL 17
18	4	STUD	SEE DETAIL 18
19	4	PIN	SEE DETAIL 19
20	4	GASKET 1 5/8 O.D. × 1/32 WALL RUBBER TUBE LENGTH TO SUIT	
21	1	GASKET 3 O.D. × 1/32 WALL RUBBER TUBE LENGTH TO SUIT (A1)	
22	1	GASKET 3 O.D × 1/32 WALL RUBBER TUBE LENGTH TO SUIT (A2)	
23	1	GASKET 3 O.D. × 1/32 WALL RUBBER TUBE LENGTH TO SUIT (A3)	
24	1	GASKET 3 O.D × 1/32 WALL RUBBE TUBE LENGTH TO SUIT (A4)	
25	4	TRANSDUCER Schaevitz 250 HCA	
26	4	CONNECTOR	
27	4	ACTUATOR AIRESEARCH MFG.CO. P/N 34596	
28	4	ROD END HEIM P/N KMXJPR-5AS	SEE DETAIL 28
29	4	BUSHING HEIM P/N FBRR-08-C-28	SEE DETAIL 29
30	8	BUSHING HEIM P/N FBRR-04-C-08	SEE DETAIL 30
31	4	GRIP RING WALDES P/N 5555-50	
32	4	CLAMP	
33	4	CLAMP	
34	8	CLAMP	
35	28	SCREW SOC. HEAD 1/4-20-3/4 LONG	
36	36	LOCKWASHER # 1/4	
37	60	SCREW SOC. HEAD #8-32-5/8 LONG	
38	76	LOCKWASHER #8	
39	16	SCREW SOC. HEAD #8-32- 3/8 LONG	
40	8	SCREW SOC. HEAD 1/4-20-1/2 LONG	
41	8	NUT 5/16-24	
42	8	LOCKWASHER 5/16	
43	28	SCREW PAN HEAD #4-40 × 3/8 LONG	
44	28	LOCKWASHER #4-40	
45	8	SCREW SET DOG POINT #8-32 × 1/4 LONG (3/16 KEYWAY IN ROD END)	
46	8	GRIP RING WALDES P/N 5555-18	

QTY. SHOWN FOR ONE (1) SUB REFLECTOR

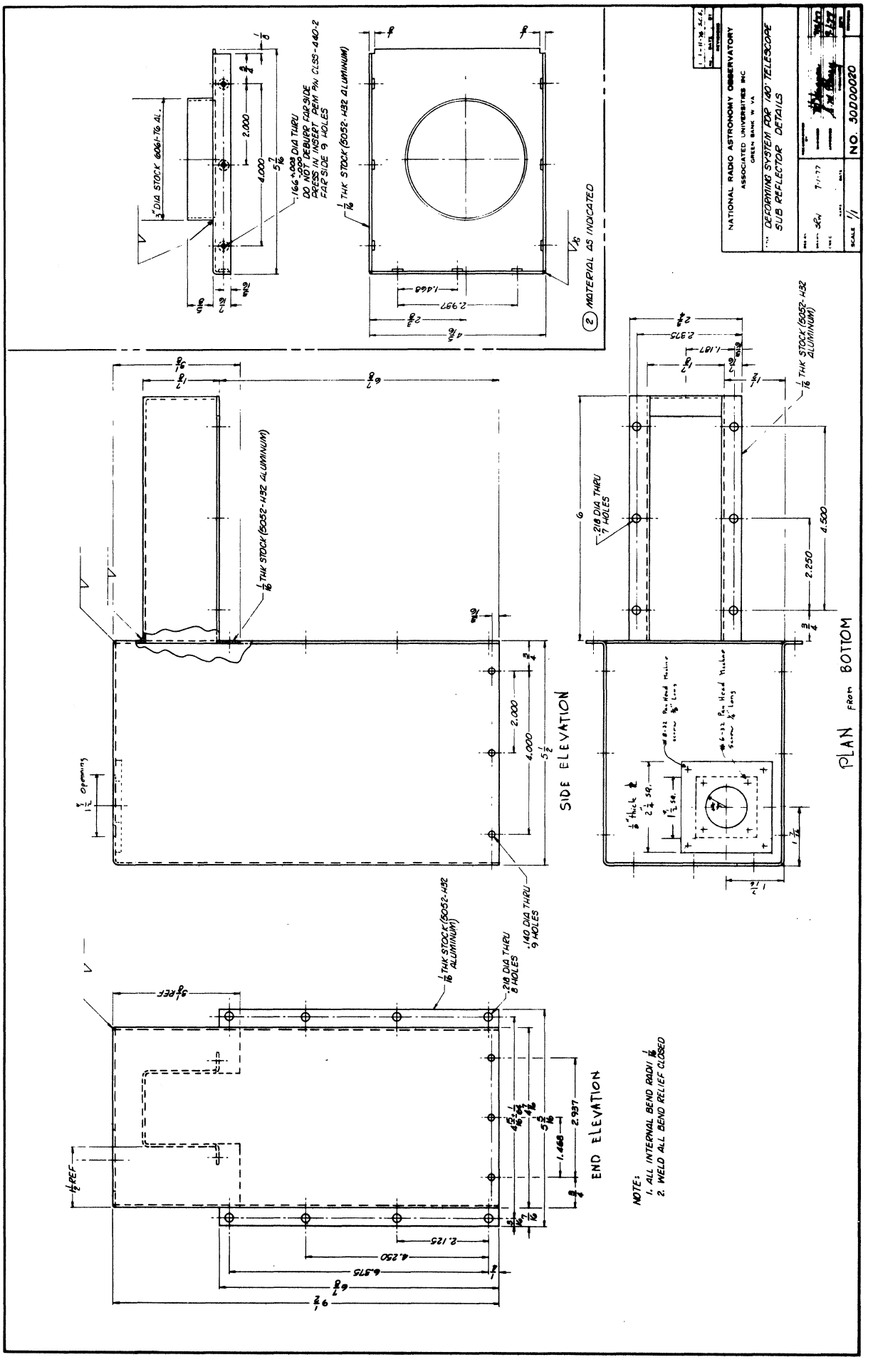
UNLESS OTHERWISE SPECIFIED TOLERANCES ± 1/32, .000 ± .005
ALL DIMENSIONS ARE IN INCHES

NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.

DEFORMING SYSTEM FOR 140' TELESCOPE SUB REFLECTOR DETAILS

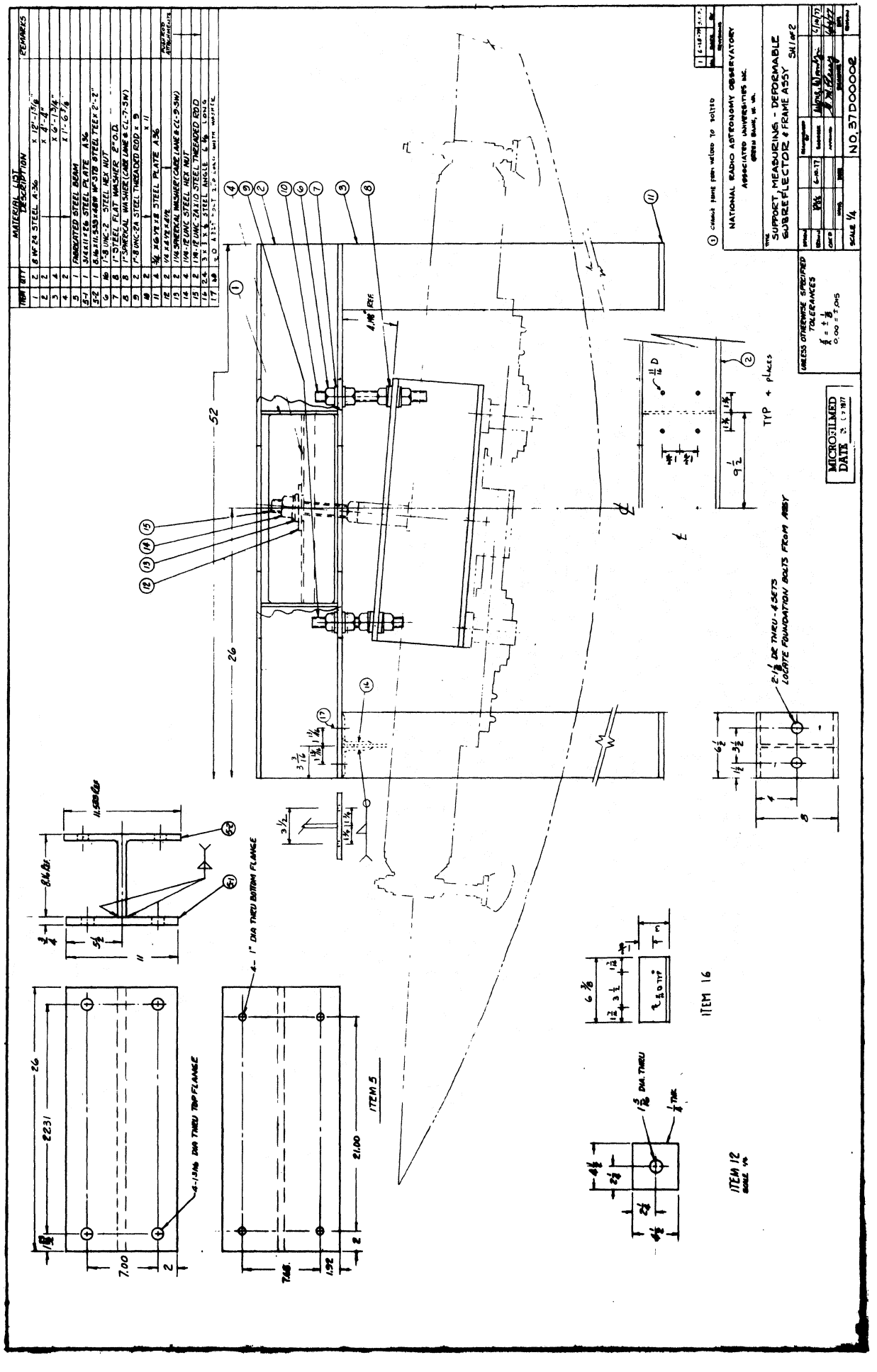
SCALE 1/1

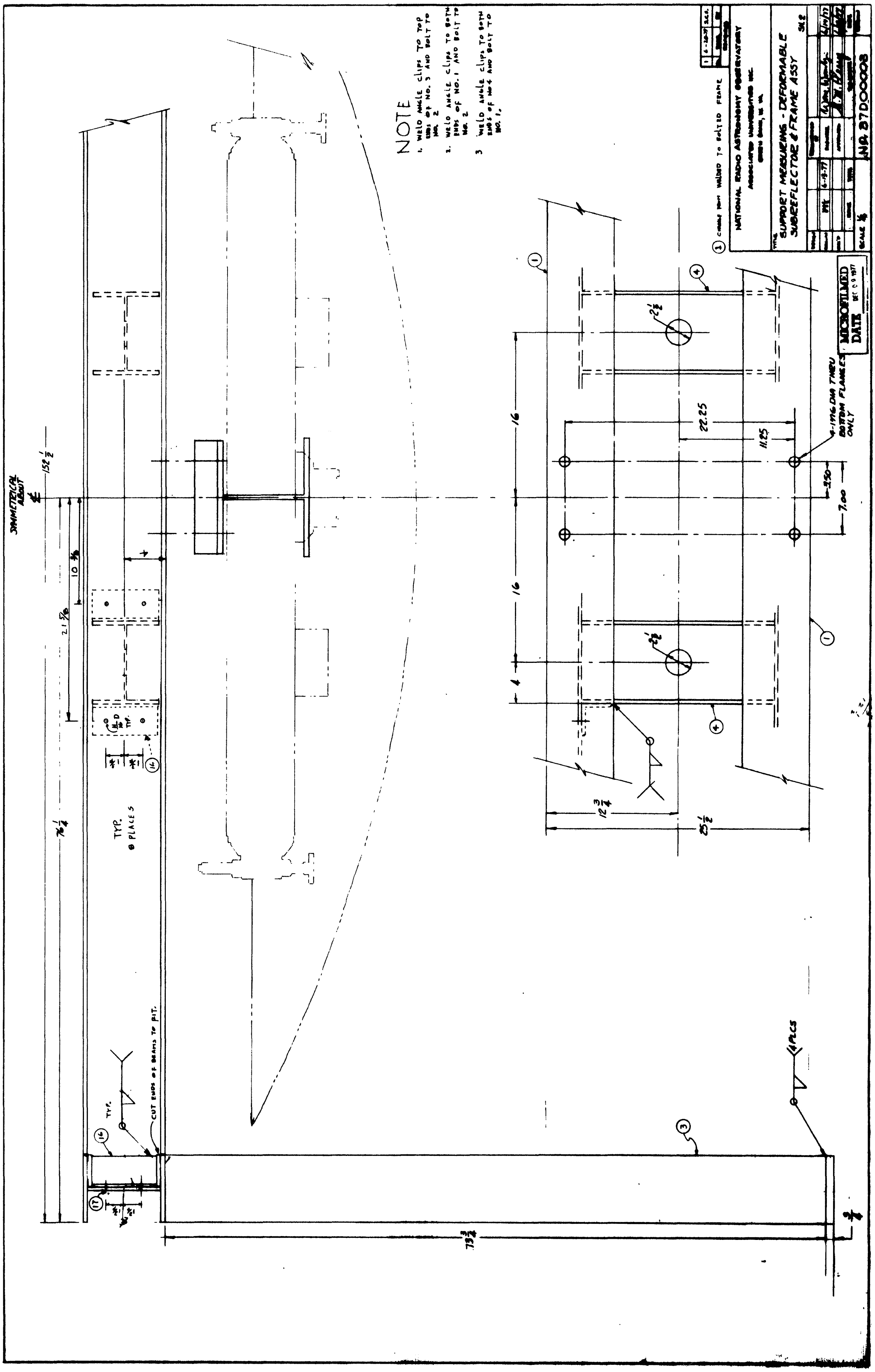
NO. 30D00019

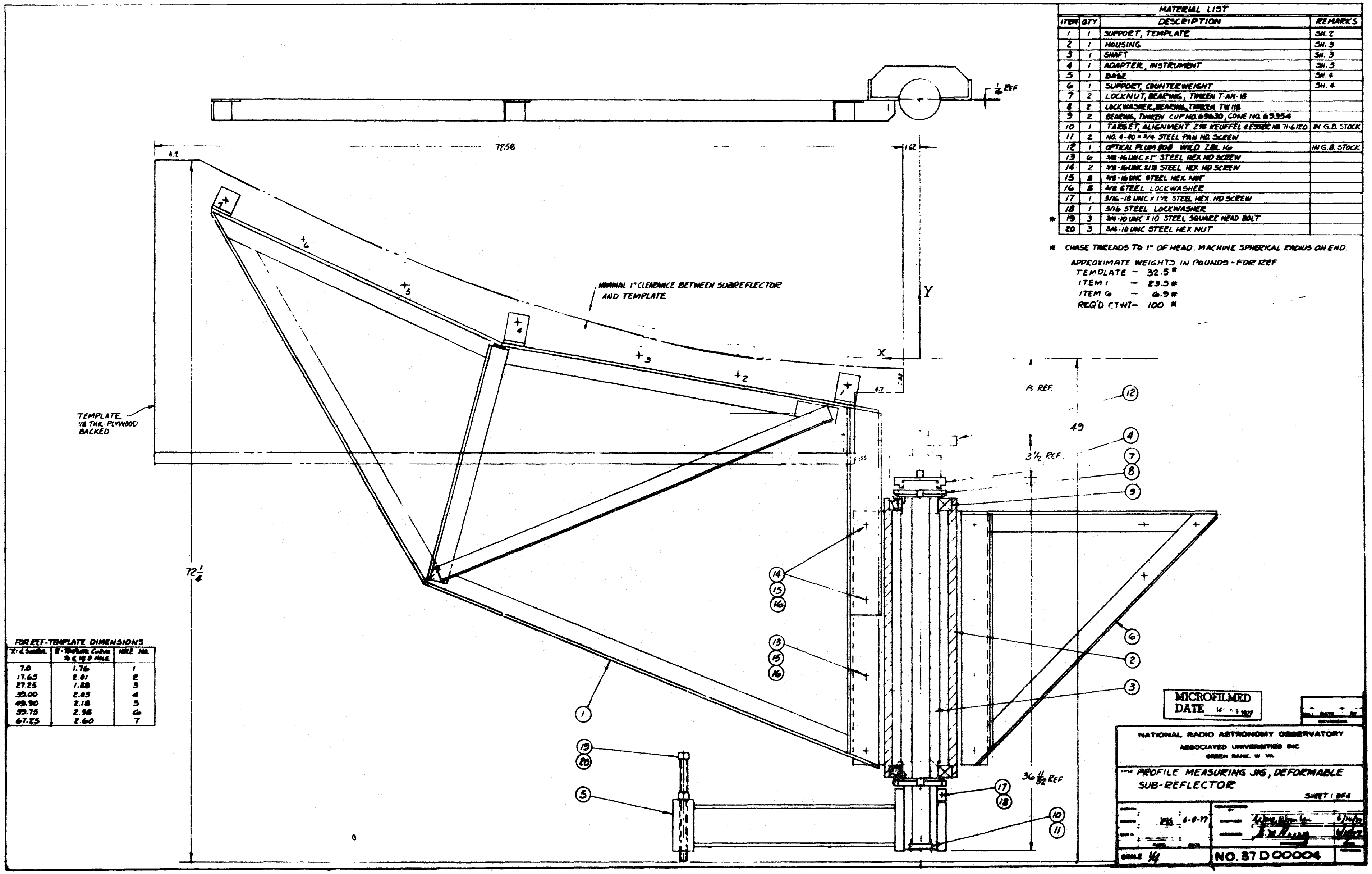

NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC
GREEN BANK W. VA
DEFORMING SYSTEM FOR 140' TELESCOPE
SUB REFLECTOR DETAILS
NO. 30D00020
SIDE ELEVATION
END ELEVATION
PLAN FROM BOTTOM
2 MATERIAL AS INDICATED
NOTE:
1. ALL INTERNAL BEND RADII 1/16
2. WELD ALL BEND RELIEF CLOSED

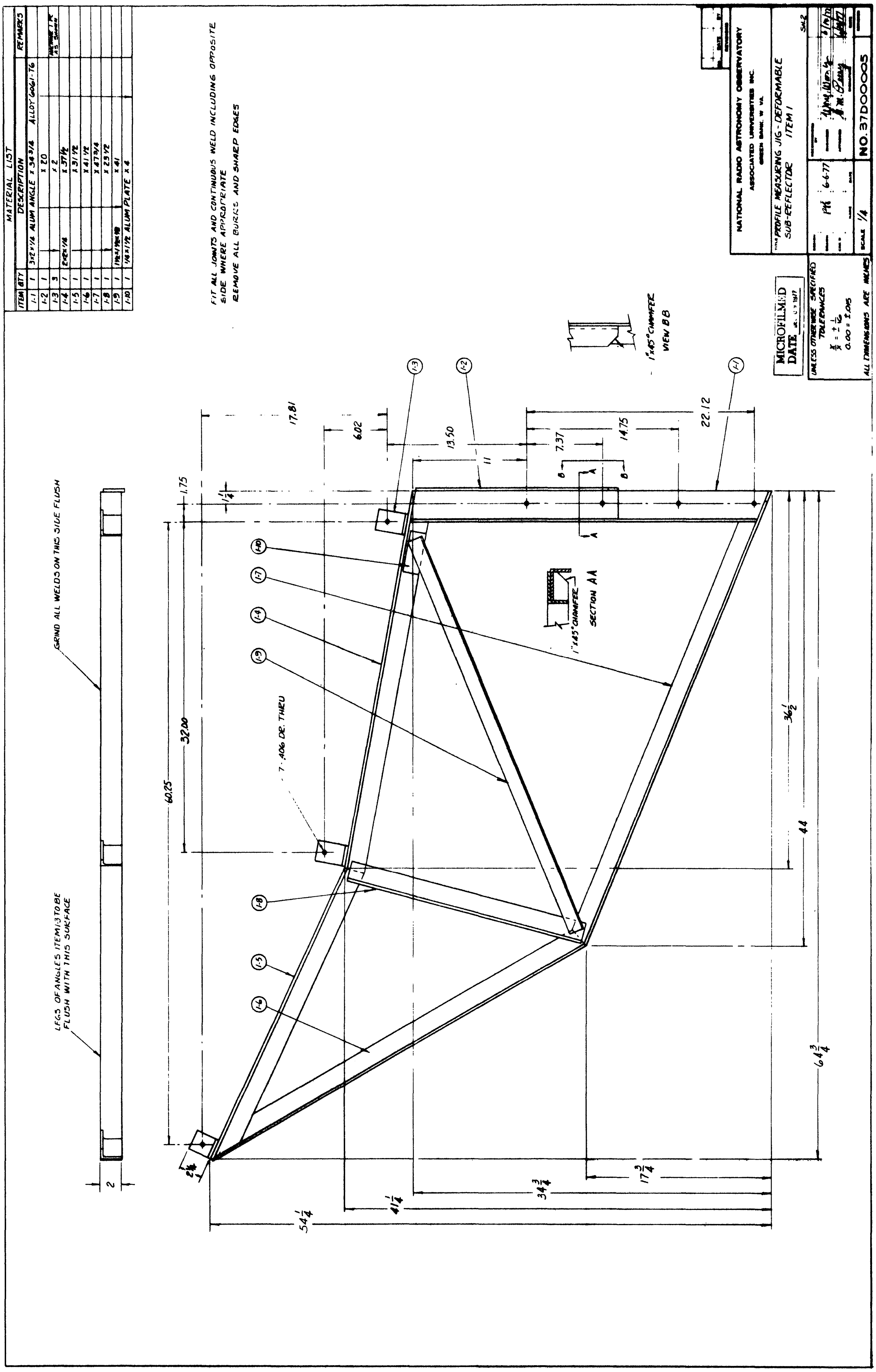
APPENDIX D

Engineering Drawings of the Measuring Jig and the Gentry DWG No. 37 D 00002 to 37 D 00007.

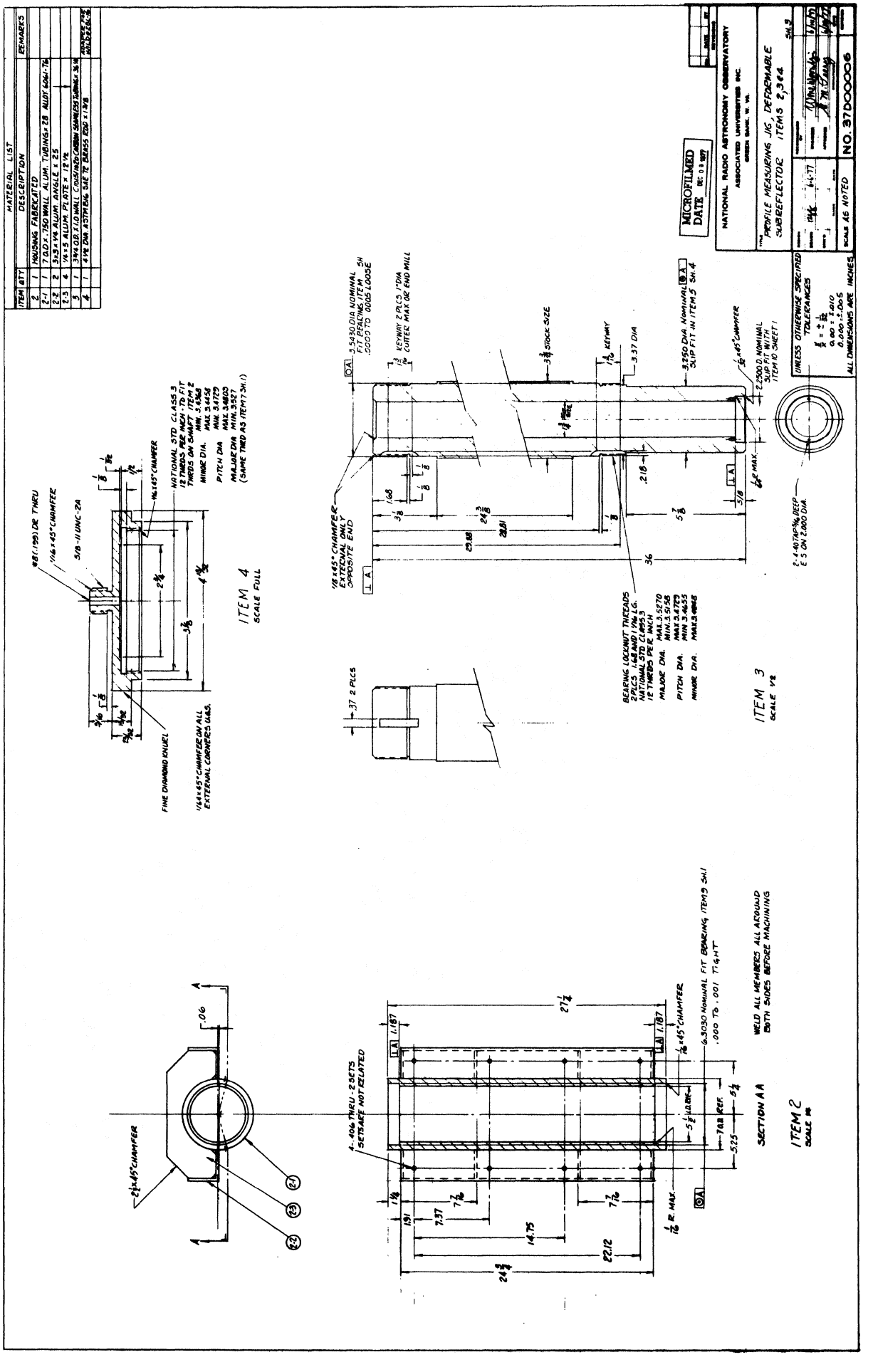

MATERIAL LIST
DESCRIPTION
REMARKS
NATIONAL RADIO ASTRONOMY OBSERVATORY
Associated Universities Inc.
SUPPORT, MEASURING - DEFORMABLE SUBREFLECTOR & FRAME ASSY SH 1 of 2
SCALE 1/4
NO. 37D0002
UNLESS OTHERWISE SPECIFIED TOLERANCES
MICROFILMED
DATE
52
26
4.118 REF
TYP 4 PLACES
2-1/2 DIA THRU - 4 SETS LOCATE FOUNDATION BOLTS FROM ASSY
ITEM 5
ITEM 16
ITEM 12


NOTE
1. WELD ANGLE CLIPS TO TOP ENDS OF NO. 3 AND BOLT TO NO. 2
2. WELD ANGLE CLIPS TO BOTH ENDS OF NO. 1 AND BOLT TO NO. 2
3 WELD ANGLE CLIPS TO BOTH ENDS OF NO 4 AND BOLT TO NO. 1.
NATIONAL RADIO ASTRONOMY OBSERVATORY
SUPPORT MEASURING - DEFORMABLE SUBREFLECTOR & FRAME ASSY
MICROFILMED
DATE DEC 09 1977
SYMMETRICAL ABOUT ℄
TYP. 8 PLACES
CUT ENDS OF BEAMS TO FIT.
4-.1996 DIA THRU BOTTOM FLANGES ONLY
4 PLCS

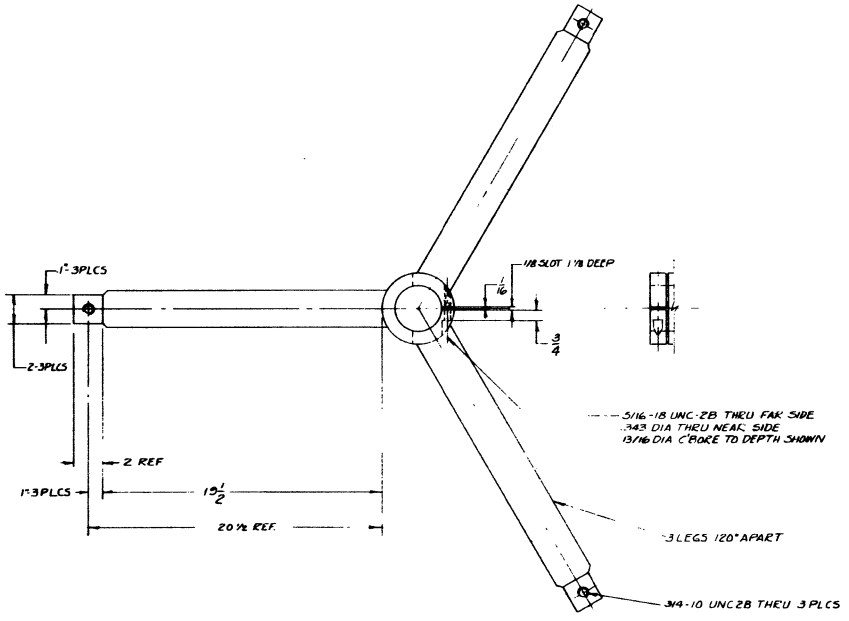

MATERIAL LIST
ITEM | QTY | DESCRIPTION | REMARKS
1 | 1 | SUPPORT, TEMPLATE | SH. 2
2 | 1 | HOUSING | SH. 3
3 | 1 | SHAFT | SH. 3
4 | 1 | ADAPTER, INSTRUMENT | SH. 3
5 | 1 | BASE | SH. 4
6 | 1 | SUPPORT, COUNTERWEIGHT | SH. 4
7 | 2 | LOCKNUT, BEARING, TIMKEN T-AN-18
8 | 2 | LOCKWASHER, BEARING, TIMKEN TW 118
9 | 2 | BEARING, TIMKEN CUP NO. 69630, CONE NO. 69354
10 | 1 | TARGET, ALIGNMENT 2⅛ KEUFFEL & ESSER NO. 71-6120 | IN G.B. STOCK
11 | 2 | NO. 4-40 × 3/4 STEEL PAN HD. SCREW
12 | 1 | OPTICAL PLUM BOB WILD ZBL 16 | IN G.B. STOCK
13 | 6 | 3/8-16 UNC × 1" STEEL HEX HD SCREW
14 | 2 | 3/8-16 UNC × 1⅛ STEEL HEX HD SCREW
15 | 8 | 3/8-16 UNC STEEL HEX. NUT
16 | 8 | 3/8 STEEL LOCKWASHER
17 | 1 | 5/16-18 UNC × 1½ STEEL HEX. HD SCREW
18 | 1 | 5/16 STEEL LOCKWASHER
* 19 | 3 | 3/4-10 UNC × 10 STEEL SQUARE HEAD BOLT
20 | 3 | 3/4-10 UNC STEEL HEX NUT
* CHASE THREADS TO 1" OF HEAD. MACHINE SPHERICAL RADIUS ON END.
APPROXIMATE WEIGHTS IN POUNDS - FOR REF
TEMPLATE - 32.5 #
ITEM 1 - 23.3 #
ITEM 6 - 6.9 #
REQ'D CTWT - 100 #
FOR REF-TEMPLATE DIMENSIONS
7.0 | 1.76 | 1
17.65 | 2.01 | 2
27.25 | 1.88 | 3
39.00 | 2.05 | 4
49.90 | 2.18 | 5
59.75 | 2.58 | 6
67.25 | 2.60 | 7
TEMPLATE 1/8 THK. PLYWOOD BACKED
NOMINAL 1" CLEARANCE BETWEEN SUBREFLECTOR AND TEMPLATE
72.58
1.12
4.2
72¼
49
3½ REF.
36 11/32 REF
X
Y
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
MICROFILMED DATE
NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC
GREEN BANK, W. VA.
PROFILE MEASURING JIG, DEFORMABLE SUB-REFLECTOR
SHEET 1 OF 4
6-8-77
SCALE 1/4
NO. 37 D 00004

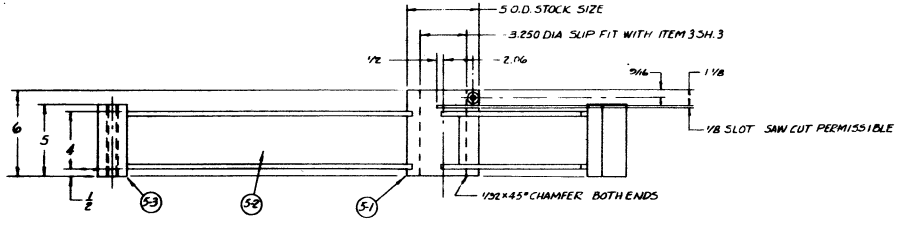


MATERIAL LIST
ITEM
QTY
DESCRIPTION
REMARKS
FIT ALL JOINTS AND CONTINUOUS WELD INCLUDING OPPOSITE SIDE WHERE APPROPRIATE
REMOVE ALL BURRS AND SHARP EDGES
GRIND ALL WELDS ON THIS SIDE FLUSH
LEGS OF ANGLES ITEM 1-3 TO BE FLUSH WITH THIS SURFACE
SECTION AA
VIEW BB
1"x45° CHAMFER
7: .406 DR. THRU
NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.
PROFILE MEASURING JIG - DEFORMABLE
SUB-REFLECTOR ITEM 1
MICROFILMED
DATE
UNLESS OTHERWISE SPECIFIED TOLERANCES
ALL DIMENSIONS ARE INCHES
SCALE 1/4
NO. 37D00005


MATERIAL LIST
ITEM QTY DESCRIPTION REMARKS
2 1 HOUSING FABRICATED
2-1 1 T.O.D.X .750 WALL ALUM. TUBING x 28 ALLOY 6061-T6
2-2 2 3.5 x V4 ALUM. ANGLE x 25
2-3 6 V4 x 5 ALUM. PLATE x 12 V4
3 1 3V4 O.D. X I.D WALL COLD DRAWN SEAMLESS TUBING x 36
4 1 4 V4 DIA ASTM B16, SAE 72 BRASS ROD x 1 V8
ITEM 4
SCALE FULL
ITEM 3
SCALE 1/2
ITEM 2
SCALE 1/8
SECTION AA
WELD ALL MEMBERS ALL AROUND BOTH SIDES BEFORE MACHINING
NATIONAL RADIO ASTRONOMY OBSERVATORY
ASSOCIATED UNIVERSITIES INC.
GREEN BANK, W. VA.
PROFILE MEASURING JIG, DEFORMABLE SUBREFLECTOR ITEMS 2,3&4
SCALE AS NOTED
NO. 3700006
MICROFILMED
DATE DEC 08 1977
UNLESS OTHERWISE SPECIFIED TOLERANCES
ALL DIMENSIONS ARE INCHES

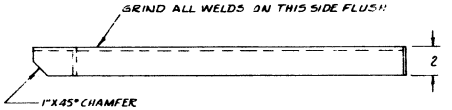
MATERIAL LIST			
ITEM	QTY	DESCRIPTION	REMARKS
5	1	BASE-FABRICATED	
5-1	1	30.D. x 1.0 WALL ALUM. TUBING x 6 1/2 ALLOY 6061-T6	
5-2	3	4 x 3.28 LB ALUM I BEAM x 20	
5-3	3	2 x 2 SQUARE ALUM BAR x 5 1/4	
6	1	SUPPORT, COUNTERWEIGHT-FABRICATED	
6-1	1	3 x 2 x 1/4 ALUM ANGLE x 25 ALLOY 6061-T6	
6-2	1	2 x 2 x 1/4 ALUM ANGLE x 26	
6-3	1	2 x 2 x 1/4 ALUM ANGLE x 35	

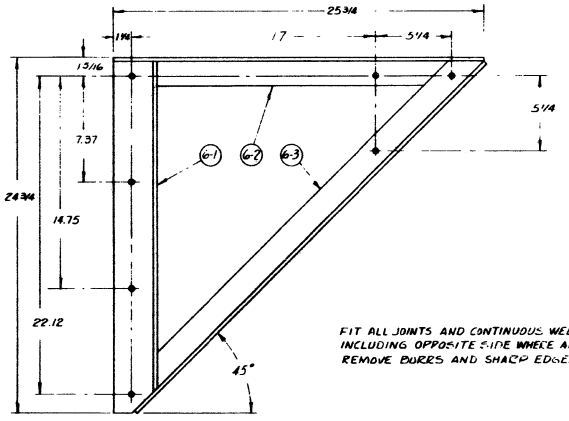




ITEM 5
SCALE 1/4

WELD ALL MEMBERS ALL AROUND BOTH SIDES





FIT ALL JOINTS AND CONTINUOUS WELD
INCLUDING OPPOSITE SIDE WHERE APPROPRIATE
REMOVE BURRS AND SHARP EDGES

ITEM 6
SCALE 1/4

MICROFILMED
DATE DEC 09 1977

PROFILE MEASURING JIG, DEFORMABLE SUBREFLECTOR ITEMS 5 & 6

SH. 4

UNLESS OTHERWISE SPECIFIED TOLERANCES
1/8 = ± 1/64
2.00 = ±0.015
ALL DIMENSIONS ARE INCHES

MS 6-6-77

AS NOTED

37D00007

APPENDIX E

The Best Fitting of a Hyperboloid

The equation of the hyperboloid described in eq. (1) is repeat here:

$$z_o = (c-\tfrac{1}{2}a)\left\{\sqrt{1+\frac{r^2}{bc}}-1\right\} \tag{1}$$

where $r^2=X^2+Y^2$, and the coordinate system illustrated in figure 1.

The subreflector is small in size and stiff in a sense that the defocusing type of deformation is considered neglegible. Hence, as a rigid body motion, the five degree of freedom are:

ΔX: translation in X direction

ΔY: translation in Y direction

ΔZ: translation in Z direction (2)

$\Delta\phi$: rotation about the X axis, where $Z=Y\Delta\phi$

$\Delta\psi$: rotation about the Y axis, where $Z=X\Delta\psi$

with small amount of motions, eq. (1) can be written as:

$$z_i = (c-\tfrac{1}{2}a)\left\{\sqrt{1+\frac{(x_i-\Delta x)^2+(y_i-\Delta y)^2}{bc}}-1\right\}+\Delta z+y_i\Delta\phi+x_i\Delta\psi \tag{3}$$

in which, without losing the accuracy, the mathematics is simplified by letting

$$\begin{aligned}(x_i-\Delta x)^2 &= x_i^2-2x_i\Delta x\\ (y_i-\Delta y)^2 &= y_i^2-2y_i\Delta y\end{aligned} \tag{4}$$

so the eq. (3) became

$$Z_i = (c-\tfrac{1}{2}a)\left\{\sqrt{1+\frac{x_i^2+y_i^2}{bc}-2\frac{x_i\Delta x+y_i\Delta y}{bc}}-1\right\}+\Delta z+y_i\Delta\phi+x_i\Delta\psi \tag{5}$$

Call

$$A = 1+\frac{x_i^2+y_i^2}{bc}\,, \qquad \varepsilon = -2\frac{x_i\Delta x+y_i\Delta y}{bc} \tag{6}$$

with a general expression that

$$\sqrt{A+\varepsilon} = \sqrt{A}\cdot\sqrt{1+\frac{\varepsilon}{A}} = \sqrt{A}\left(1+\frac{1}{2}\frac{\varepsilon}{A}\right) = \sqrt{A}+\frac{\varepsilon}{2\sqrt{A}} \tag{7}$$

combining (6) and (7)

$$\sqrt{A+\varepsilon} = \sqrt{1+\frac{r_i^2}{bc}}-\frac{1}{\sqrt{1+\frac{r_i^2}{bc}}}\cdot\frac{x_i\Delta x+y_i\Delta y}{bc} \tag{8}$$

substituting (8) to (5)

$$Z_i = (c-\tfrac{1}{2}a)\left\{\sqrt{1+\frac{r_i^2}{bc}}-\frac{1}{\sqrt{1+\frac{r_i^2}{bc}}}\cdot\frac{x_i\Delta x+y_i\Delta y}{bc}-1\right\}+\Delta z+y_i\Delta\phi+x_i\Delta\psi \tag{9}$$

Call

$$U = (c-\tfrac{1}{2}a)\,, \qquad V = (c-\tfrac{1}{2}a)\frac{1}{bc}\,, \qquad R_i = \sqrt{A}$$

Eq. (9) becomes

$$Z_i = UR_i-\frac{V}{R_i}(x_i\Delta x+y_i\Delta y)-U+\Delta z+y_i\Delta\phi+x_i\Delta\psi \tag{10}$$

Equation (10) can be rewritten as

$$\Delta z - \frac{\nabla x_i}{R_i}\Delta x - \frac{\nabla y_i}{R_i}\Delta y + y_i \Delta\phi + x_i \Delta\psi = z_i - \upsilon R_i + \nabla \tag{11}$$

Call

$$[F] = \begin{bmatrix} 1 & -\frac{\nabla x_1}{R_1} & -\frac{\nabla y_1}{R_1} & +y_1 & +x_1 \\ & & \vdots & & \\ 1 & -\frac{\nabla x_n}{R_n} & -\frac{\nabla y_n}{R_n} & +y_n & +x_n \end{bmatrix}_{n\times 5} \qquad [G] = \begin{bmatrix} \Delta z \\ \Delta x \\ \Delta y \\ \Delta\phi \\ \Delta\psi \end{bmatrix}_{5\times 1}$$

and the deviation from the design [H] is

$$[H] = \begin{bmatrix} z_1 - \upsilon R_1 + \nabla \\ \vdots \\ z_n - \upsilon R_n + \nabla \end{bmatrix}_{n\times 1}$$

So that eq. (11) is written as

$$[F][G] = [H] \tag{12}$$

Premultiplying the transpose of [F] to both sides of (12):

$$[F]^T[F][G] = [F]^T[H] \tag{13}$$

where

$$[\alpha] = [F[^T[F]$$

and the unknown [G] is solved,

$$[G] = [\alpha]^{-1}[F]^T[H] \tag{14}$$

The computer program listing, sample input and the result are also included.

```
      C
      C  BEST FIT FOR HYPERBOLOID WITH 5 DOF
      C
      C DEC. 12,1977 A4=-.100, A-2=-.100,A-1=+.100,A-3=+.100
      C
0001        DIMENSION A(144,5),B(144),AA(5,5),BB(5),S(5),C(144)
           X,         X(144),Y(144),Z(144),STM(72),R(144),RO(6)
           X,         RADI(6),LL(5),MM(5),ZBF(144),DEV(144)
           X,         ADJ(6)
      C
      C  INITIATION
      C
0002        DATA RADI / 266.7, 495.3, 723.9, 952.5, 1181.1, 1409.7 /
0003        DATA AA, BB, S /  25*0., 5*0., 5*0. /
0004        DATA ADJ / -.09, -.04, .07, .01, -.09, -.03/
0005        C1=214.185*25.4
0006        C2=24585.8338*25.4
0007        C3=C1/C2
      C
      C  X(I), Y(I)
      C
0008        K=0
0009        PH=0.
0010        ANG=15.*3.1416/180.
0011        DO 60 I=1,24
0012        COSP=COS(PH)
0013        SINP=SIN(PH)
0014        DO 50 J=1,6
0015        K=K+1
0016        X(K)=RADI(J)*COSP
0017        Y(K)=RADI(J)*SINP
0018     50 CONTINUE
0019        PH=PH+ANG
0020     60 CONTINUE
      C
      C  INPUT : STATEMENT & GAP READINGS
      C
0021        READ(5,900) (STM(I),I=1,72)
0022        PRINT 914,  (STM(I),I=1,72)
0023        READ(5,901) (RO(I),I=1,6)
0024        DO 70 I=1,6
0025        RO(I)=RO(I)+ADJ(I)
0026     70 CONTINUE
      C
      C  Z(I)
      C
0027        K=0
0028        AG=0.
0029        DO 200 I=1,24
0030        DO 100 J=1,6
0031        K=K+1
0032        READ(5,917) R(K)
0033        C(K)=SQRT(1.+(X(K)*X(K)+Y(K)*Y(K))/C2)
0034        Z(K)=C1*(C(K)-1.)+RO(J)-R(K)
0035        AG=AG+RO(J)-R(K)
0036    100 CONTINUE
0037    200 CONTINUE
0038        AG=AG/144.
```

```
0039                PRINT 916,(R(I),I=1,144)
                C
                C  PRINT X,Y,Z & AVG. GAP
                C
0040                PRINT 902,(I,X(I),Y(I),Z(I),I=1,144)
0041                PRINT 903,AG
                C
                C  BEST FIT BY PARALLEL SHIFT
                C
0042                SUM=0.
0043                DO 250 I=1,144
0044                ZBF(I)=C1*(C(I)-1.)+AG
0045                DEV(I)=Z(I)-ZBF(I)
0046                SUM=SUM+DEV(I)*DEV(I)
0047              250 CONTINUE
0048                RMS=SQRT(SUM/144.)
0049                PRINT 915,(DEV(I),I=1,144)
0050                PRINT 904,RMS
                C
                C  MAKING OF A(N,5) & B(N)
                C
0051                DO 300 I=1,144
0052                A(I,1)=1.
0053                A(I,2)=-(C3*X(I)/C(I))
0054                A(I,3)=-(C3*Y(I)/C(I))
0055                A(I,4)=Y(I)
0056                A(I,5)=X(I)
0057                B(I)=Z(I)-C(I)*C1+C1
0058              300 CONTINUE
0059                PRINT 905,((A(I,J),J=1,5),I=1,144)
0060                PRINT 906,(B(I),I=1,144)
                C
                C  PRE-MULTIPLY A BY A-TRANSPOSE
                C
0061                DO 400 L=1,5
0062                DO 400 J=1,5
0063                DO 400 I=1,144
0064                AA(L,J)=AA(L,J)+A(I,L)*A(I,J)
0065              400 CONTINUE
0066                PRINT 907,((AA(I,J),J=1,5),I=1,5)
                C
                C  PRE-MULTIPLY B BY A-TRANSPOSE
                C
0067                DO 500 L=1,5
0068                DO 500 I=1,144
0069                BB(L)=  BB(L)+A(I,L)*B(I)
0070              500 CONTINUE
0071                PRINT 908,(BB(I),I=1,5)
                C
                C  INVERT OF AA
                C
0072                CALL MINV(AA,5,D,LL,MM)
0073                PRINT 909,((AA(I,J),J=1,5),I=1,5)
                C
                C  SOLUTION
                C
0074                DO 600 L=1,5
0075                DO 600 I=1,5
```

```
0076                S(L)=  S(L)+AA(L,I)*BB(I)
0077              600 CONTINUE
0078                PRINT 910,(S(I),I=1,5)
                  C
                  C  NEW Z OR BEST FITTED Z
                  C
0079                DO 700 I=1,144
0080                XA=X(I)-S(2)
0081                YA=Y(I)-S(3)
0082                ZBF(I)=C1*(SQRT(1.+(XA*XA+YA*YA)/C2)-1.)+S(1)+Y(I)*S(4)
                   X       +X(I)*S(5)
0083              700 CONTINUE
0084                PRINT 911,(ZBF(I),I=1,144)
                  C
                  C  DEVIATION FROM THE BEST FIT
                  C
0085                SUM=0.
0086                DO 800 I=1,144
0087                DEV(I)=ZBF(I)-Z(I)
0088                SUM=SUM+DEV(I)*DEV(I)
0089              800 CONTINUE
0090                RMS=SQRT(SUM/144.)
0091                PRINT 912,(DEV(I),I=1,144)
0092                PRINT 913,RMS
0093                DO 850 I=1,144
0094                X(I)=X(I)/25.4
0095                Y(I)=Y(I)/25.4
0096                DEV(I)=DEV(I)*1000./25.4
0097                PUNCH 918,X(I),Y(I),DEV(I)
0098              850 CONTINUE
                  C
                  C  FORMATS
                  C
0099              900 FORMAT(72A1)
0100              901 FORMAT(6F10.4)
0101              902 FORMAT(' ',//,'  X,Y AND MEASURED Z ',//,(I5,3F10.4))
0102              903 FORMAT(' ',//,'  PARALLEL SHIFT = ',F10.4,' MM ')
0103              904 FORMAT(' ',//,'  RMS BEST-FIT BY ADJ. OF Z ONLY : ',F10.4,' MM')
0104              905 FORMAT('1',//,'  MATRIX A ',//,(5F10.4))
0105              906 FORMAT('1',//,'  MATRIX B ',//,(10F10.4))
0106              907 FORMAT('1',//,'  MATRIX AA ',//,(5E15.4))
0107              908 FORMAT('1',//,'  MATRIX BB ',//,(5E15.4))
0108              909 FORMAT('1',//,'  MATRIX AA-INV ',//,(5E15.4))
0109              910 FORMAT('1',//,'  DZ = ',F10.4,
                     X             /,'  DX = ',F10.4,
                     X             /,'  DY = ',F10.4,
                     X             /,'  RX = ',E10.4,
                     X             /,'  RY = ',E10.4)
0110              911 FORMAT('1',//,'  BEST-FITTED Z ',/,(6F10.4))
0111              912 FORMAT('1',//,'  DEVIATION FROM THE BEST FIT ',/,(6F10.4))
0112              913 FORMAT(' ',//,'  RMS DEVIATION FROM THE BEST FIT WITH 5 DOF = '
                     X,      F10.4,' MM ')
0113              914 FORMAT('1',///,72A1,/)
0114              915 FORMAT(' ',///,'  DEVIATION FROM THE AVG. ',/,(6F10.4))
0115              916 FORMAT(' ',///,'  ORIGIVAL READINGS ',/,(6F10.4))
0116              917 FORMAT(F10.3)
0117              918 FORMAT(2X,3F10.3)
                  C
```

75