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ARECIBO THREE-MIRROR SYSTEMS, II:
WEIGHT, WIND FORCE, AND FEASIBILITY

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Summary

This a continuation of Report 112 which presented several optimized mirror systems, with 700 ft aperture diameter, and center offsets of 0, 50, 125 ft. The present report adds four systems: first, the largest possible aperture without vignetting, which has 726 ft diameter and 137 ft offset. Second, three systems, one each for 50, 125, 137 ft offset, where all mirror surfaces have axial symmetry (but asymmetrically cut rims for the aperture offset) in order to facilitate the manufacture of surface plates.

Weight estimates are based on three available designs of 25-m telescopes, scaled to the size of the Arecibo secondary mirrors, replacing steel by aluminum for weight-saving; plus simple estimates for the tertiary and its support legs. Wind force estimates are based on calculated shapes of mirror rims, suggested shapes of backup structures, plus conventional shape factors, for both side and front view. The smaller side view is suggested as stow position for survival wind, the larger front view as worst case for observation. In addition, we calculate illumination ratio, feed diameter, possible feed multiplicity, aperture shadow from mirrors and feed, and clearance between secondary backup structure and platform tiedown cables.

The results indicate that the three-mirror systems are well feasible, regarding all criteria. For the 50 ft offset (some vignetting) this holds already for the specified limit of 35,000 lb of the present combined structure of azimuth arm, platform, and cables. It holds also for the 125 ft offset (no vignetting for 700 ft aperture) if the carrying capacity of the present structure can be improved by a factor 1.26. With an improvement of a factor 1.75 which seems well possible, even the 137 ft offset is feasible (no vignetting for 726 ft aperture). Symmetric surfaces, however, would each demand an additional factor of 2.

The next step for this project needs a detailed structural analysis of the present structure: finding the weakest link, suggesting inexpensive improvements, calculating wind-induced pointing errors, and calculating (and improving) the dynamics.

I. GENERAL REMARKS

1. Largest Aperture

So far, we have always treated a circular aperture of 700 ft diameter as suggested by Frank Drake, with offsets of 0, 50, 125 ft. Now we also investigate the feasibility of the largest possible circular aperture, which does not give any vignetting for the pointing range from 0° to 20° zenith distance. For the 1000-ft Arecibo primary, with 870 ft radius of curvature, this means:

$$\text{aperture} \begin{cases} \text{diameter} = 726 \text{ ft} \\ \text{offset} = 137 \text{ ft} \end{cases} \tag{1}$$

$$\text{limiting rays} \begin{cases} a_1 = -500 \text{ ft} \\ a_2 = +226 \text{ ft} \end{cases} \tag{2}$$

2. Multiple Feeds

Conventional two-mirror systems give mostly a wider field of view at the secondary focus than at the prime focus, allowing simultaneous observations with a cluster of multiple feeds. This is important for fast mapping, and Mike Davis asked about its possibility.

In general, with parabolic primaries, a wide field of view is obtained if the feed sees its next mirror under a narrow angle β. At the prime focus, we mostly have about $\beta = 120°$, and we cannot go beyond one beamwidth off-axis because of increasing coma lobes. But at the Cassegrain focus with a magnification factor M (ratio of focal lengths), we have roughly $\beta = 120°/M$, and we may go about M beamwidths off-axis before coma limitations, which allows roughly M^2 feeds and receivers. And since the feed size increases with M, a closely packed maximum feed cluster will have a radius of roughly $M^2\lambda$.

For most of our calculated three-mirror systems, we have some freedom in the location of the feed and could make β_3 small enough for a wide field of view. But actually we are limited by the size of the feed which increases with λ/β_3, because excessively large feeds would render the whole three-mirror concept undesirable. For our long wavelength specified (λ = 21 or even 30 cm), the two demands, of high multiplicity and small size, are mutually exclusive. I think the small size has priority, and we should not consider feed diameters of about 3 ft or larger. This still allows a moderate amount of multiplicity. For a rough estimate, we generalize the parabolic case, adopting for the number N of beamwidths which we may go off-axis without serious coma lobes

$$N = 120°/\beta_3 \qquad (3)$$

or, using the feed diameter $d_f = 91°/\beta_3$ from the previous report,

$$N = d_f/(0.8 \text{ ft}). \qquad (4)$$

The maximum number of feeds then is about N^2.

3. Shadow

The shadows cast by mirrors and feed should be small. One might argue, for example, that even a shadow of 200 ft diameter in an aperture of 700 ft reduces the area by only 8% which is not so bad. But, first, the signal loss goes with the square of the area loss (that is, as long as the feed still tries to illuminate the shaded area), and a loss of 16% does hurt. Second, shadow means also scatter, and low-noise receivers may be badly hurt by the pickup of thermal noise from ground radiation. For example, if ½ the scatter sees the ground, we add a noise of ½ x 0.08 x 300° = 12°K from a 200-ft shadow.

4. Axial Symmetry

Frank Drake suggested to investigate also those systems where the mirror surfaces have axial symmetry (the rim is still cut asymmetric, for an offset aperture). All surface panels in a circular ring would then be identical, which would reduce the cost. In our previously treated systems, we have only one plane of symmetry, thus only couples of mirror-symmetric panels.

First, individually different panels should not be much more expensive than ringwise identical ones, at least not for $\lambda \geq 4$ cm as specified for the shortest wavelength. All panels in properly selected rings may still have (almost) identical outer dimensions, but would have different internal curvatures. A relatively cheap way of achieving any wanted curvature is the "internal adjustment", where each plate has a number of screws pulling the surface toward its backup frame, to be adjusted on a measuring jig (Findlay and von Hoerner: "A 65-m Telescope for Millimeter Wavelengths", NRAO, 1972). And regarding a backup structure without circular symmetry, we should look for a manufacturer having a computer-controlled pipe cutter (proper length, angles, and bevelling) where individually different lengths and angles make no difference in cost.

Second, symmetry still is an interesting thought. Thus I have added three such systems, by placing F_2 and F_3 exactly on the caustic axis. The unshaped systems then have surfaces of exact axial symmetry. If the feed pattern would have axial symmetry, too, then even the shaping procedure would maintain symmetric surfaces provided we chose, for maximum gain, a uniform aperture illumination (which is symmetrical about any center, offset or not).

But then the asymmetrically cut mirror rims would either demand an impossible feed pattern (axisymmetric intensity, but sharp asymmetric cutoff), or they would yield an excessive spillover. Thus, a tilted or laterally shifted feed is needed, whichever is better for polarization, but this is not symmetric. However, it may well be that one could develop a system such that most of its asymmetry is confined to the feed and the small tertiary, whereas the large secondary is still almost axisymmetric. Keep in mind that shaping changes the surfaces only by small amounts.

5. New Systems

We have calculated four new systems which are shown in Table 1 and Figs. 1-4. Systems #4a and #9a are axisymmetric, and they are meant to replace systems #4 and #9 of the previous report which were already almost symmetric. Systems #12 and #13 are the largest non-vignetting ones, from (1) and (2). System #12 is most compact, by starting from point A in Fig. 3, and a choice of two feed locations is given. Fig. 4 with system #13 is the axisymmetric one.

All axisymmetric systems have a problem in common: the centrally located feed sits in a region of high density of the three-times reflected rays, and thus casts a large shadow. It can be shown in general that the shadow is decreased if we move F_2 up and F_3 down. This we have done, as far as possible, without moving feed or tertiary into a multiple-shadow zone. But even so the shadows S_f of the symmetric systems in Table 1 are still too large.

Maybe this problem will quietly go away during the shaping procedure, where all rays get more evenly distributed. Also, the suggested lateral shift of F_3 (above the center of the tertiary for a convenient feed pattern) may help to some extent. Since it seems difficult to find

Table 1. Some systems, in addition to Table 2 of Report 112. All systems with $\rho = 0$ have surfaces with axial symmetry (but asymmetrically cut rims). The last two systems have the largest possible aperture, without vignetting at 0° and 20° zenith distance. All lengths in feet, angles in degrees.

d_i = largest diameter or length;

z_i = z-coordinate of deepest point;

β_i = angular size of mirror as seen from its focus;

I = illumination ratio;

S_i = length of (additional) shadow in aperture; S_f will be smaller after shaping.

		Parameters			Secondary				Tertiary				Feed	Shadows		
#	Fig.	ν	τ	ρ	d_2	z_2	β_2	I	Type	d_3	z_3	β_3	d_f	S_2	S_3	S_f
50 ft offset, 700 ft aperture																
4a	1	9	-78	0	77.6	-19.4	58	21	C	18	-65	90	1.0	75	120	163
125 ft offset, 700 ft aperture																
9a	2	9	-110	0	106.9	-49.6	59	35	C	18	-98	64	1.4	91	84	225
137 ft offset, 726 ft aperture																
12	3	7	-41	-42	105.5	-82.5	88	26	C	25	-52	{20 {40	4.6} 2.3}	53	18	0
13	4	9	-120	0	123.9	-64.5	64	41	C	18	-109	63	1.4	102	77	256

estimates for these effects, we leave this problem open, waiting for the shaping. And whether or not the large illumination ratios are tolerable, up to I = 41, this must be left waiting for the final investigation of diffraction effects.

6. Rim Calculation

So far, we have calculated the shape of the secondary only along its longest diameter, in the plane of symmetry. But for estimating the wind forces, we need also to know the shape of the rim of the secondary, in three dimensions.

This is no problem, since in Fig. 1 of Report 112, only point 3 (F_2) must now be lifted out of the plane of the drawing, and point 4 by half the amount. Everything else stays still in the plane, and the calculation is rather similar. Again, all was done with the "TI Programmable 59". All equations are given in the Appendix; for definitions see Fig. 10.

II. WEIGHT ESTIMATES

1. Secondary Mirror

The secondaries calculated in the previous and present reports cover a diameter range of $64 \leq d_2 \leq 124$ ft ($20 \leq d_2 \leq 38$ m). For estimating their weights, we start out, first, with three 25-m designs of NRAO telescopes, shown in Table 2. Second, we apply changes reducing the weight. Comparing the surface weight of the very accurate homologous telescope (λ = 1 mm) with that of the others (λ = 1 cm), we may relax a bit more for our present purpose (λ = 4 cm), say by a factor of 0.8. Upper and lower backup structure of a steerable telescope are shown in Fig. 5, and all the lower parts can now be omitted because of much simpler holding conditions without steerability, especially omitting the heavy suspension members from the bearings down to lower center and up again in a cone, also the declination wheel with its bull gear; we use a reduction factor of 0.65, considered conservative. And since weight is crucial as it seems, we use aluminum throughout, factor 0.35. Table 2 then gives a total weight of 33 kip (kilo pound), for a circular aluminum secondary of 82-ft diameter.

Third, we scale to other sizes. In general, for a structure of size D, the weight of the thickest members goes with D^3, of the thinnest members with D because of welding and buckling requirements, and in the average we shall use D^2. We regard both directions: the longest diameter d_2 (in x,z-plane) from the previous calculations, and the widest width w_2 (in y-direction) of the present rim calculations, see Table 3. We thus obtain for the weight of the secondary

$$W_2 = 33 \text{ kip} \frac{d_2 w_2}{(82 \text{ ft})^2} , \qquad (5)$$

Table 2. Weight estimate, for circular secondary aluminum mirror of 25m = 82 ft diameter (based on three NRAO designs).

Item	Weight (1000 lb)				Apply changes	Weight (1000 lb)
	Homologous	VLA	VLBI	Use now		
surface (alum)	22.2	12.6	12.8	12	x 0.8 a)	10
backup (steel)	103.6	101.0	120.4	100	x 0.65 b) x0.35 c)	23
changes:	a) less accurate b) Fig. 5: omit c) Alum./steel				Total	33

1. Legs

Long legs should be built-up structures as shown in Fig. 5, giving less weight and wind area, but for simplicity we adopt a single pipe for each leg; and the optimum is three legs and not four. We assume all legs to be governed by buckling stability, with a slenderness ratio of $L/r = 120$, with circular thin-walled pipes where the radius of gyration is $r = d/\sqrt{8}$, thus the diameter is $d = L/42$; and regarding local buckling we use a wall thickness of $t = d/30$. The legs will have different lengths, with two shorter legs and one longer, but we use only the average length L as given in Table 3, from the tertiary to the backup structure of the secondary since it is the unbraced length which matters. The weight of three legs from aluminum (density ρ_a) then is

$$W_L = 3\ \pi d\ t\ L\ \rho_a = (L/31.9\ \text{ft})^3\ \text{kip}. \qquad (6)$$

2. Tertiary

We proceed similar as with the secondary, now based on our 140-ft deformable subreflector (d = 10 ft), the Kitt Peak telescope itself (d = 12 m = 39 ft), and equation (5) for comparison. Omitting the details, the result is

$$W_3 = 2.0\ \text{kip}\ (d_3/18\ \text{ft})^2. \qquad (7)$$

3. Results

Table 3 shows the single contributions and total weight W, for the 12 systems #2 through #13. The weights of the very long legs could be reduced (factor 2) by built-up structures, which we did not work out at present. For all compact systems where the tertiary is located to the left of the caustic (see the Figures of the previous report), we obtain in the average

Table 3. Weight Estimates.
d = longest diameter,
w = width of mirror,
L = length of legs.

System #	Dimension (ft) Secondary d_2	w_2	Tertiary d_3	L	Weight (1000 lb) Secondary W_2	Tertiary W_3	Legs W_c	Total W
No offset, 700 ft aperture								
2	67.5	67.5	18	52	22.4	2.0	4.3	28.7
50 ft offset, 700 ft aperture								
3	74.4	71.6	18	48	26.1	2.0	3.4	31.6
4a	77.6	73.5	18	65	28.0	2.0	8.5	38.5
5	64.4	57.0	21	30	18.0	2.7	.8	21.6
6	71.8	62.5	20	37	22.0	2.5	1.6	26.1
7	67.1	50.2	23	20	16.5	3.3	.3	20.1
125 ft offset, 700 ft aperture								
8	110.9	94.6	18	71	51.5	2.0	11.0	64.5
9a	106.9	88.9	18	94	46.6	2.0	25.6	74.2
10	92.8	59.6	26	37	27.1	4.2	1.6	32.9
11	88.4	60.4	30	30	26.2	5.6	.8	32.6
137 ft offset, 726 ft aperture								
12	105.5	67.0	25	49	34.7	3.9	3.6	42.2
13	123.9	101.0	18	98	61.4	2.0	29.0	92.4

$$W = \begin{cases} 23 \text{ kip}, & \text{for 50 ft offset,} \\ 33 \text{ kip}, & \text{for 125 ft offset,} \\ 42 \text{ kip}, & \text{for largest aperture.} \end{cases} \qquad (8)$$

This is to be compared with a specification of (including the weight of the carriage house):

$$W = 35 \text{ kip}. \qquad (9)$$

For comparison, the weight of the present focal platform is

$$W_{pl} = 1200 \text{ kip}, \qquad (10)$$

which allows the hope that a detailed structural analysis and subsequent proper beef-up of some weak links would yield a considerable relaxation of constraint (9) without too much cost. This relaxation will especially be needed when wind forces are added, too.

III. WIND FORCE ESTIMATES

A detailed treatment should give both drag and lift forces, and torques. But for the present purpose, we treat the drag only. It is always the larger one, as wind tunnel tests have shown.

1. Wind Areas

For each of the 12 systems, a work sheet was drawn for side and front view of secondary, tertiary, legs and feed. Four examples are shown here as Figs. 6, 7, 8 and 9. Mirror rim and bulge are from the numerical calculations; support legs are added, and a rough shape for a backup structure is suggested.

The projected areas, as seen from the wind, are integrated in steps of 5 ft, see Fig. 6. The diameter of the legs is again taken as d = L/42 for buckling stability, and we call L_o the exposed length from tertiary to rim of secondary, averaged over the three legs. Their wind area then is $A_L = 3\ d\ L_o$. The area of the tertiary and its small structure is just length times width. In some cases, feed and receiver are exposed, too.

The resulting areas are given in Table 4.

2. Shape Factors

The wind force can be written as

$$F = C\ \tfrac{1}{2}\ \rho\ A\ v^2 = 0.00256\ C\ A\ v^2, \quad \left|\begin{array}{l} F \text{ in lb} \\ A \text{ in ft}^2 \\ v \text{ in mph} \end{array}\right. \tag{11}$$

Some general shape factors C are, for example:

$$\begin{array}{lr} \text{sphere} & C = 0.8 \\ \text{circular cylinder} & 1.0 \\ \text{flat plate} & 1.2 \\ \text{concave half sphere} & 1.5 \end{array} \tag{12}$$

Regarding the wind force on parabolic telescopes, there is a large amount of literature, mostly wind tunnel tests plus some theory. The agreement is not very good; for face-on wind (largest force) the results range within $1.3 \leq C \leq 1.8$ for solid surfaces. In the following we use C = 1.70 for the secondary mirror, for both side and front view.

The backup structure of the secondary is an open network of many members, and its wind resistance as compared to a solid surface will be a good deal less but not extremely so. We will adopt C = 0.6 for the backup structure.

The legs are taken to be circular cylinders with C = 1.0. For tertiary plus its structure, and also for feed and receiver if needed, we use C = 1.1. In summary, calculating the forces from the areas in Table 4, we use the following shape factors:

secondary	$C = 1.7$	
backup	0.6	
legs	1.0	(13)
tertiary	1.1	
receiver	1.1	

3. Application

Winds are mostly low at the Arecibo site. I was given the following values:

$$\begin{aligned} \text{median} &= 7.6 \text{ mph}, \\ \text{third quartile} &= 9.5 \text{ mph}, \end{aligned} \tag{14}$$

and the highest winds are expected to be

$$\begin{aligned} &65 \text{ mph once in } 50 \text{ years}, \\ &75 \text{ mph once in } 100 \text{ years}. \end{aligned} \tag{15}$$

For the specifications of the new mirrors system, it was suggested to use

$$\begin{aligned} &80 \text{ mph for survival}, \\ &17 \text{ mph for observation}. \end{aligned} \tag{16}$$

For all our systems, the smallest wind area is the side view. We suggest to declare this as the stow position for high winds. The survival force then is, in pounds,

$$F_{surv} = 0.00256\ (80)^2 \sum_{side} C_i\ A_i . \tag{17}$$

For observation, we use the worst case, the front view:

$$F_{obs} = 0.00256\ (17)^2 \sum_{front} C_i\ A_i . \tag{18}$$

The results are given in Table 4. It is interesting to note that the survival forces are mostly about the same as the total weights.

Table 4. Wind force estimates.

System	projected area, side view (ft^2)				projected area, front view (ft^2)				Force (1000 lb.)	
	second.	backup	tertiary	legs	second.	backup	tertiary	legs	survival	observation
#	A_2	A_b	A_3	A_L	A_2	A_b	A_3	A_L	F_s	F_o
No offset, 700 ft aperture										
2	840	270	85	149	840	270	85	149	30.0	1.36
50 ft offset, 700 ft aperture										
3	1045	280	85	113	1280	360	85	103	35.2	1.92
4a	835	363	100	90	1618	415	100	76	30.1	2.36
5	880	355	95	38	1870	340	0	0	30.3	2.50
6	775	370	108	77	1980	270	80	0	28.4	2.68
7	840	380	105	15	1870	230	0	0	29.3	2.45
125 ft offset, 700 ft aperture										
8	1755	800	85	255	3820	530	85	153	62.5	5.22
9a	1307	736	85	190	4179	498	85	123	48.3	5.64
10	685	680	140	90	3620	280	0	0	29.8	4.68
11	1035	750	220	43	3560	400	0	0	40.9	4.66
137 ft offset, 726 ft aperture										
12	1080	1130	146	169	4750	430	0	0	46.6	6.17
13	1590	1310	100	685	5750	780	100	405	70.2	7.96

IV. RESULTS AND DISCUSSIONS

1. Feasibility

Table 5 summarizes the main results for all 12 systems. Regarding survival, we added the total force F_t, from vertical weight W plus horizontal wind force F_s:

$$F_t = \sqrt{W^2 + F_s^2}. \tag{19}$$

This is always larger than the specified limit of 35 kip from (9), but not extremely so, and it still is always small as compared to the platform weight of 1200 kip. We hope that a structural analysis will show a way to at least double the specified limit by some inexpensive means, making at least all of the more attractive systems feasible. If a structure weighs 1200 kip but can support only 35, it just begs for improvement.

The pointing error from wind would follow from F_o if the stiffness of cables, platform and azimuth arm were known, which is missing for the latter two, needing a detailed structural analysis. We think that all calculated systems will be feasible, because, first, the specification (16) of 17 mph for observation could certainly be relaxed to the third quartile of the wind distribution, as it is done for most exposed telescopes. At Arecibo, this is only 9.5 mph, see (14), and this relaxation would decrease all values F_o of Table 5 by the factor $(17/9.5)^2 = 3.2$.

Second, should we still have a problem, we may consider on-line pointing corrections. For example, done with a laser on a theodolite at the dish vertex, looking at a reflecting target close to the feed. The light-travel time yields the distance, the theodolite two angles, thus yielding the feed location in three coordinates. In this or some similar way, the steady part of the wind can be corrected, and also its slow gusts,

Table 5. Main results, for all 12 systems.

System #	Weight (1000 lb) W	Wind force (1000 lb) surv. F_s	Wind force (1000 lb) Observ. F_o	Total surv. $F_t = \sqrt{W^2+F_s^2}$	Illum. ratio I	Feed (ft) d_f	Tert. + Feed shadows (ft) $S_3 + S_f$	Tiedown clearance (ft) c
No offset, 700 ft aperture								
2	28.7	30.0	1.36	41.5	15	5.1	185	-2
50 ft offset, 700 ft aperture								
3	31.6	35.2	1.92	47.3	32	4.3	21	-20
4a	38.5	30.1	2.36	48.9	21	1.0	163	-14
5	21.6	30.3	2.50	37.2	14	2.2	17	+5
6	26.1	28.4	2.68	38.6	16	3.4	17	+2
7	20.1	29.3	2.45	35.5	24	2.1	11	+6
125 ft offset, 700 ft aperture								
8	64.5	62.5	5.22	89.8	58	6.5	23	-43
9a	74.2	48.3	5.64	88.5	35	1.4	225	-29
10	32.9	29.8	4.68	44.4	17	2.4	18	+1
11	32.6	40.9	4.66	52.3	15	1.1	14	-3
137 ft offset, 726 ft aperture								
12	42.2	46.6	6.14	62.9	26	2.3	18	-5
13	92.4	70.2	7.96	116.0	41	1.4	256	-37

below the lowest dynamical frequency of the structure (about two seconds at present). Only the faster gusts will give errors, but partly damped out by the inertia of the structure, and also by its size since fast gusts have small cross sections.

How large an illumination ratio can be tolerated will be answered only after we have done the shaping procedure and the diffraction investigation. Intuitively, I would expect no problem up to about I = 30, but maybe one could still go further up.

Large feeds would counteract the main purpose of the multi-mirror systems: flexibility and convenience. But most of our values d_f are less than three feet and well feasible. They still allow a moderate feed multiplicity according to (4).

The large feed shadows of the symmetric systems have already been discussed. It is hoped that they will be decreased by shaping, but we do not yet know by how much.

We also added the clearance, c, between the backup structure as suggested in our work sheets, and the tiedown cables which go down from the edge of the platform to the ground, providing stiffness and stability against rotational motions of the platform. In Table 5, "+" means a true (wanted) clearance, while "-" means a (forbidden) overlap. Frank Drake mentioned this criterion, but he added that a small overlap of 15 ft or more could easily be removed by holding the cable at some special support extended beyond the platform edge.

Finally, it should be mentioned that internal deformations (gravity, thermal, wind) of the secondary mirror, and of the tertiary and its legs, are expected to be no problem for $\lambda \gtrsim 4$ cm. This follows from comparison with the behavior of several exposed 85-ft telescopes and the 25-m VLA antennas. Furthermore, gravitational deformations will even be smaller than usual because of the small tilt angle of only 20° zenith distance, and also the wind at Arecibo is calmer than at other telescope sites. As usual, all structures and mirror surfaces should have white protective paint against heating up in sunshine.

So far, we have not encountered any crucial argument against the feasibility of these three-mirror systems for Arecibo, even up to the largest possible aperture without vignetting. A final judgement, however, must wait for the structural analysis, regarding survival, structural improvements, and pointing errors. I cannot provide cost estimates; just for comparison I want to mention a preliminary NRAO cost estimate of 1.8 M$, for the proposed VLBI antennas of 25 m diameter for λ = 8 mm, with aluminum surface, and steel backup structure and mounting, plus drive gear and motors.

2. Symmetry versus Compactness

Some data are provided in Table 6, to be used for the final choice of the best system, regarding offset and aperture versus cost, and also regarding the question whether or not an axisymmetric system is to be preferred. In each offset group, I have compared its symmetric system with the best one of the other more compact systems, selecting that system as the best one which has the smallest total force F_t from (19).

Table 6 shows that we pay a high price for symmetry, regarding both the weight (cost of backup, and size of surface area increasing in proportion with W) and the force (more structural improvements needed for larger F_t). This may outbalance the savings expected for a symmetric surface which is cheaper in cost/area.

Table 6. Comparison of symmetric versus best compact systems

Dimension (ft)		System #		Weight (1000 lb)		Total Force (1000 lb)		Tert.+Feed Shadow (ft)		Clearance (ft)	
apert.	offset	sym.	comp.	sym.	comp.	sym.	comp.	sym.	comp.	sym.	comp.
700	0	2	-	29	-	42	-	185	-	-2	-
700	50	4a	7	39	20	49	36	163	11	-14	+6
700	125	9a	10	74	33	89	44	225	18	-29	+1
726	137	13	12	92	42	116	63	256	18	-37	-5

3. Weak Points of Present Estimates

Our weight estimates are meant to be actual design goals for the future design of a mirror system, not to be surpassed by 20%, say. However, they could be low by larger factors, if it turns out that the present carriage does not provide a support area long and wide enough for a large secondary (or if it cannot take up a large survival moment) such that large heavy replacements or extra structures are needed. Or, if we replace steel by aluminum in the backup structure, it could be that more of it is needed for survival because it is softer.

A more seriously weak point is the estimate of survival wind forces, based on projected area and adopted shape factor, and neglecting, for example, the leeward half of surface and structure. I do not know how to treat this problem in a realistic way, except by building a model for wind tunnel tests. These tests then should also include some small rotation of the model, representing small changes in the direction of the survival winds.

On the other hand, I am still optimistic, because I know that large telescope structures are mostly defined by Parkinson's Law and not by their purpose: if all members are able to hold each other without buckling, they are (almost) stiff enough to support the surface plus survival winds, too.

4. Structural Analysis

The next step needed for this project is a detailed structural analysis for the combined structure of azimuth arm, platform, cables, and towers. The analysis is needed for: (a) obtaining the present stability constraints for weight and survival wind; (b) finding the weakest link of the combined structure, from secondary to ground, and suggesting some not-too-expensive way of improvement; (c) calculating the wind-induced pointing error for each of our systems; (d) investigating the dynamics, looking again for inexpensive ways to improve the lowest mode; (e) without the ability to do structural analysis, no secondary backup could be designed and evaluated.

Furthermore, we have so far adopted aluminum for the backup structure, done for saving weight, but adding cost. If azimuth arm and platform could be improved a great deal without much cost, then the heavier but cheaper steel may be better.

Unfortunately, I have been told that NRAO cannot provide engineering help for this analysis (and the mirror design), at least not during 1983. But the analysis should not be a great problem. All it needs is some young engineer, reading all structural data off the Arecibo blueprints, plus a programmer (or the same engineer) to convert these data into a proper computer input format. The analysis then is done by one of the usual software packages, STRUDL, NASTRAN or others, which should be available at the Cornell Engineering Department (which also might help with the design). It seems that STRUDL is the easiest regarding input preparation, for both static and dynamic analysis.

The input data are:

1. Coordinates x,y,z of each structural joint;
2. Density ρ and modulus E of the cables, and ρ and E of the structural members;
3. Metallic cross-sectional area A of each cable;
4. For each structural member, 4 items (frame analysis, but neglecting shear):
   a. Area A,
   b. Moments of inertia I_x, I_y
   c. Torsional stiffness J.

   to be given only <u>once</u> for each group of identical members.

APPENDIX: CALCULATION OF (UNSHAPED) SECONDARY MIRROR

We call $r = 870$ ft the radius of the primary curvature; ν, τ, ρ are the parameters defining the secondary, and g is the offset of the aperture center from the caustic axis, see Fig. 10. The pathlength of the axial ray (at $a = 0$) then is

$$L_p = r + \nu + \sqrt{(\nu-\tau)^2 + \rho^2} \qquad (A.1)$$

and it must be the same for all other rays, too. We proceed pointwise in a polar grid centered on the aperture center, with b = distance of the present ray from this center, and θ = angle from the x-axis (plane of symmetry). We define another polar system, centered on the caustic axis, where the rays have a distance ξ and angle ψ, see Fig. 10,a. The distance of the present ray from the caustic axis we call $\xi_1 = a$, with

$$a = \sqrt{b^2 + g^2 - 2bg \cos \theta} \qquad (A.2)$$

and its angle ψ from the x-axis we get from

$$\cos \psi = (b \cos \theta - g)/a. \qquad (A.3)$$

The projected coordinates of the secondary focus of F_2 then are

$$\sigma = \rho \cos \psi, \quad \text{and} \quad \omega = \rho \sin \psi. \qquad (A.4)$$

We obtain angle α and height z_1 of the reflection at the primary sphere from

$$\sin \alpha = a/r, \quad \text{and} \quad z_1 = \frac{r}{2} - \sqrt{r^2 - a^2}. \qquad (A.5)$$

Along this reflected ray we define point 2 at the end of one pathlength, which gives the distance L_{12} between points 1 and 2 as

$$L_{12} = L_p + z_1 \qquad (A.6)$$

and the coordinates of point 2 as

$$\xi_2 = a - L_{12} \sin 2\alpha, \quad \text{and} \quad z_2 = z_1 + L_{12} \cos 2\alpha. \qquad (A.7)$$

We also need the squared distances from point 3 (F_2) to points 1 and 2:

$$L_{13}{}^2 = (\sigma - a)^2 + (\tau - z_1)^2 + \omega^2,$$
$$L_{23}{}^2 = (\sigma - \xi_2)^2 + (\tau - z_2)^2 + \omega^2. \qquad (A.8)$$

This would allow us to calculate γ from $L_{13}{}^2 = L_{12}{}^2 + L_{23}{}^2 - 2L_{12}L_{23}\cos\gamma$. From the requirement of equal pathlengths we must have $L_{23} = L_{53}$, yielding an isoscales triangle 2-5-3, with height L_{54} and with $L_{24} = (1/2)L_{23}$ which is known from (A.8). And in the rectangular triangle 2-4-5, we have $L_{24} = L_{25}\cos\gamma$. Thus, by eliminating γ from the last and the first equation of this paragraph, we obtain

$$L_{25} = \frac{L_{12}\,L_{23}{}^2}{L_{12}{}^2 + L_{23}{}^2 - L_{13}{}^2}. \qquad (A.9)$$

This gives for point 5

$$\xi_5 = \xi_2 + L_{25}\sin 2\alpha. \qquad (A.10)$$

Finally, the three wanted coordinates of the secondary mirror are

$$x_5 = \xi_5\cos\psi,$$
$$y_5 = \xi_5\sin\psi, \qquad (A.11)$$
$$z_5 = z_2 - L_{25}\cos 2\alpha.$$

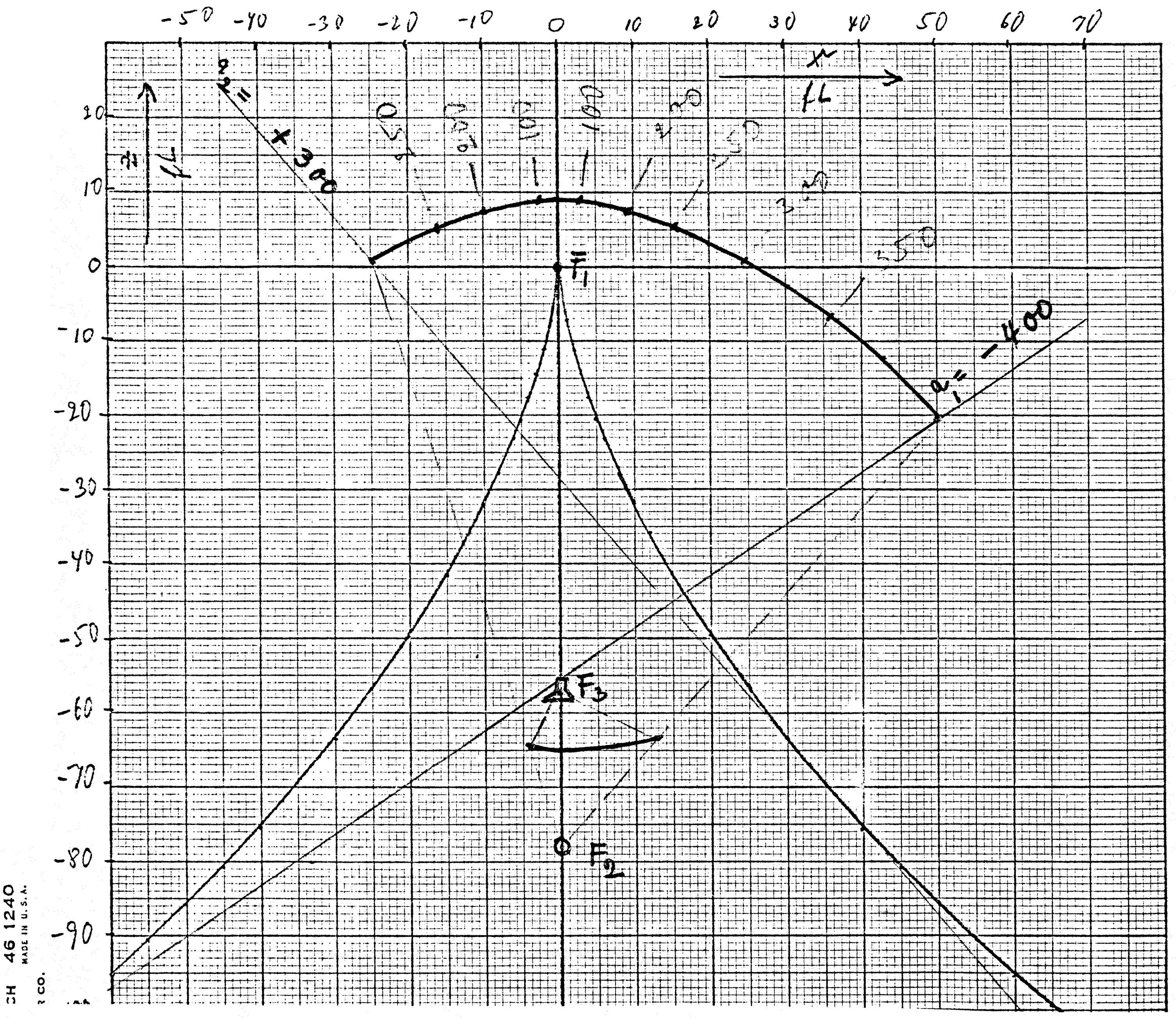


Fig. 1. System # 4a: with 50 ft offset, 700 ft aperture, and axially symmetric surfaces.

Minimum feed shadow, compactness.

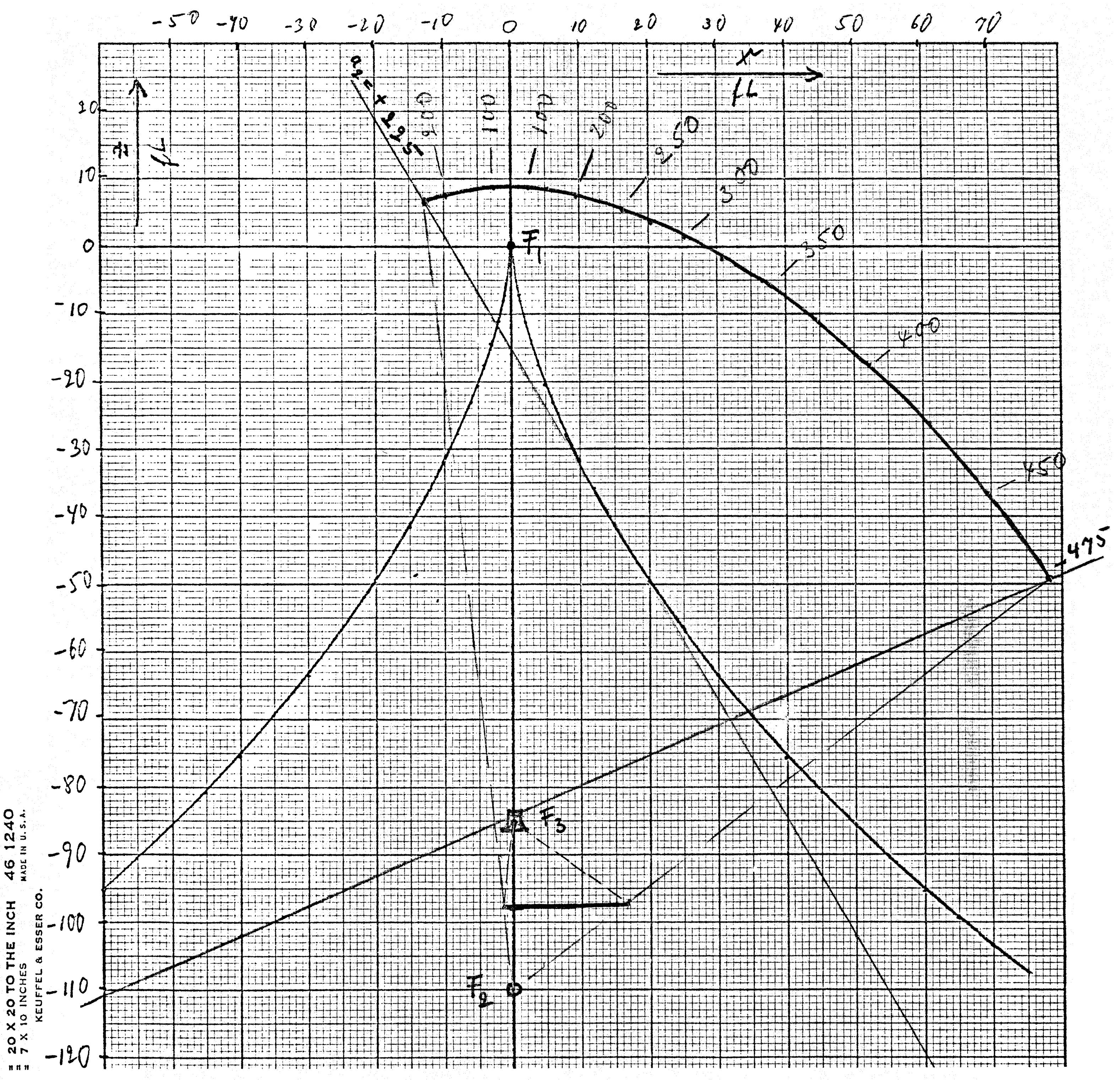


Fig. 2. System # 9a: with 125 ft offset, 700 ft aperture, and axially symmetric surfaces.

Compactness, feed shadow.

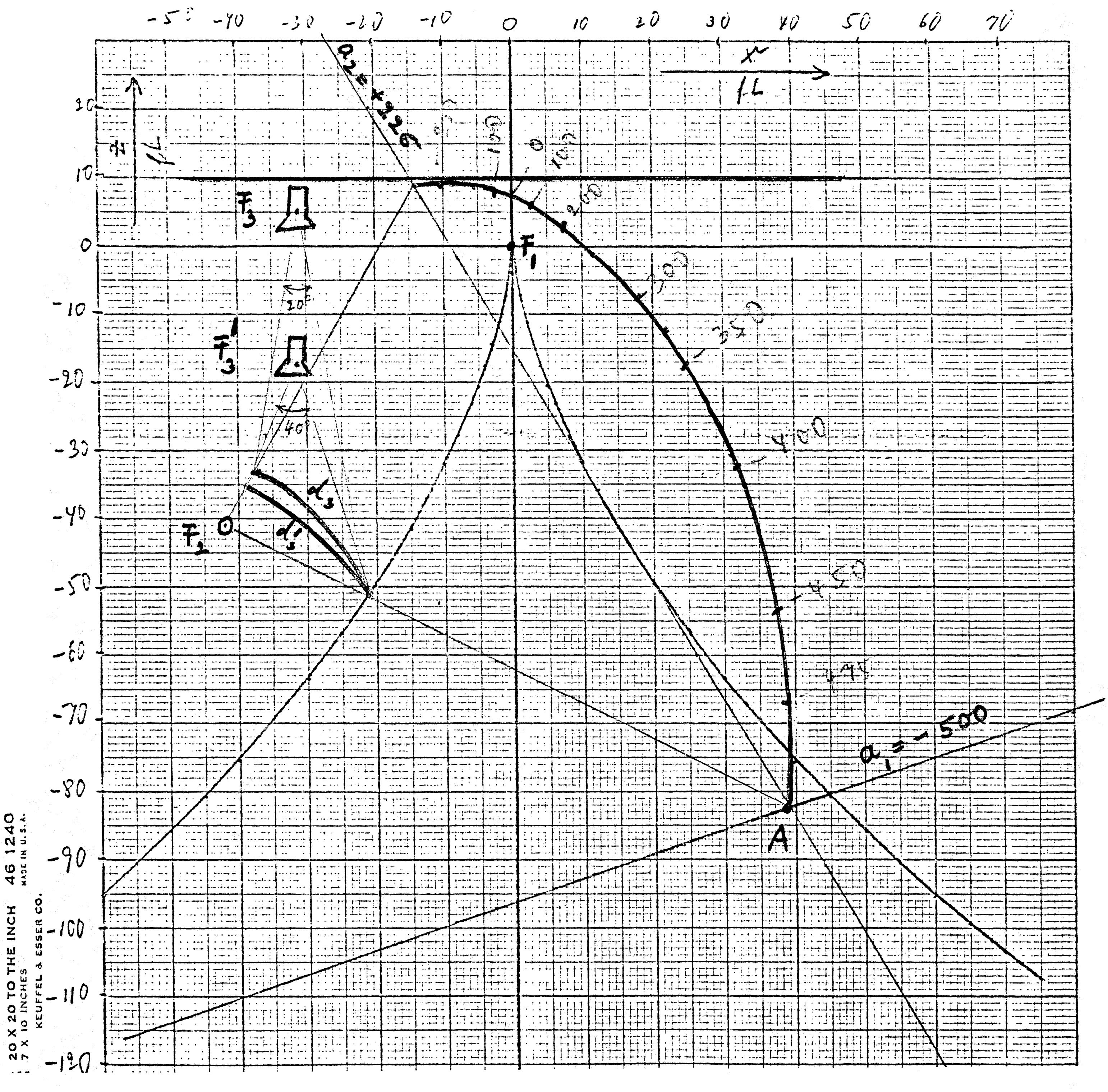


Fig. 3. System # 12, with maximum possible aperture, without vignetting for pointing range from 0° to 20° zenith distance.

Aperture = 726 ft diameter, offset = 137 ft.

Two different feed locations are shown:

F_3' for small feed, F_3 for multiple feeds.

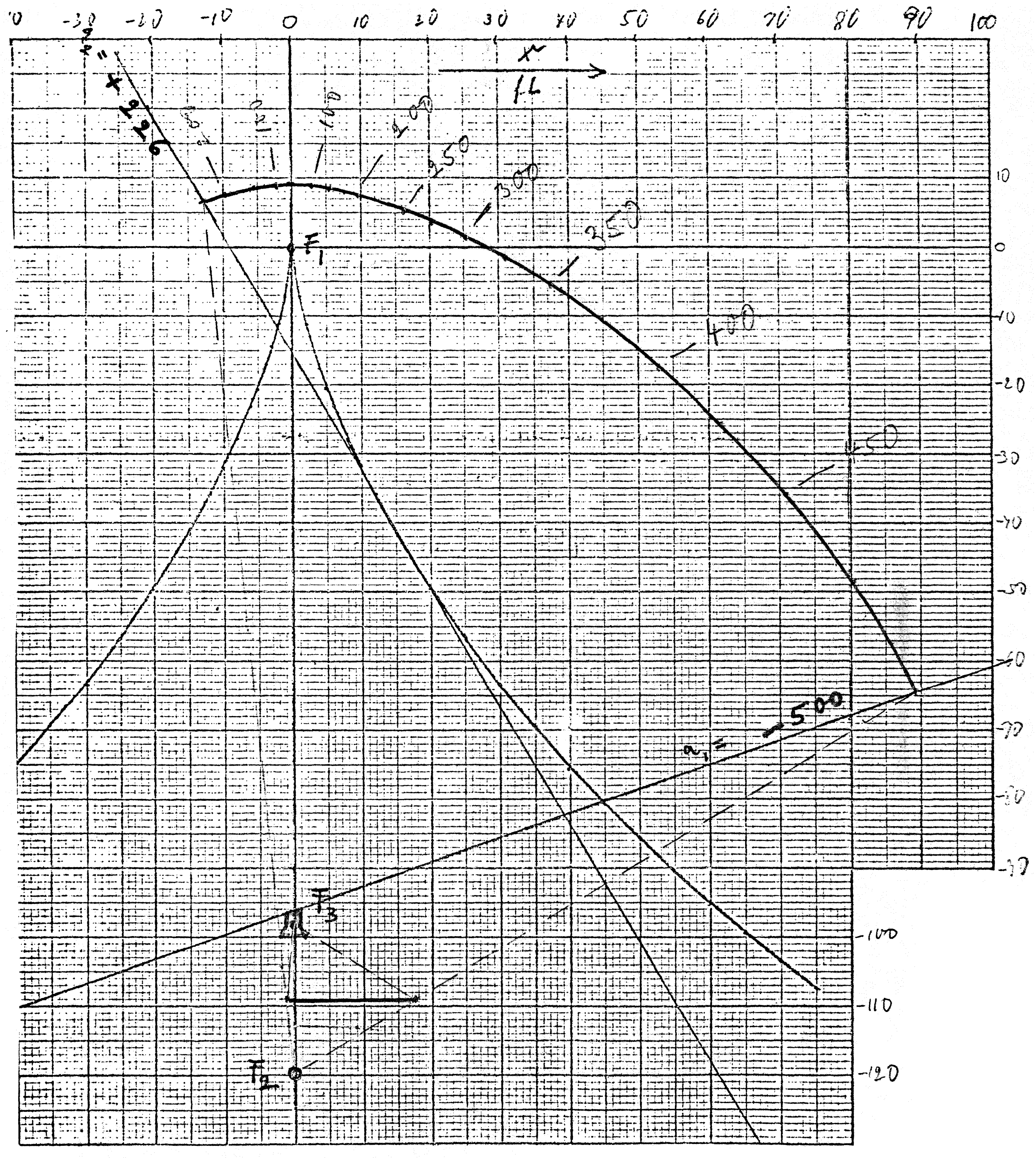
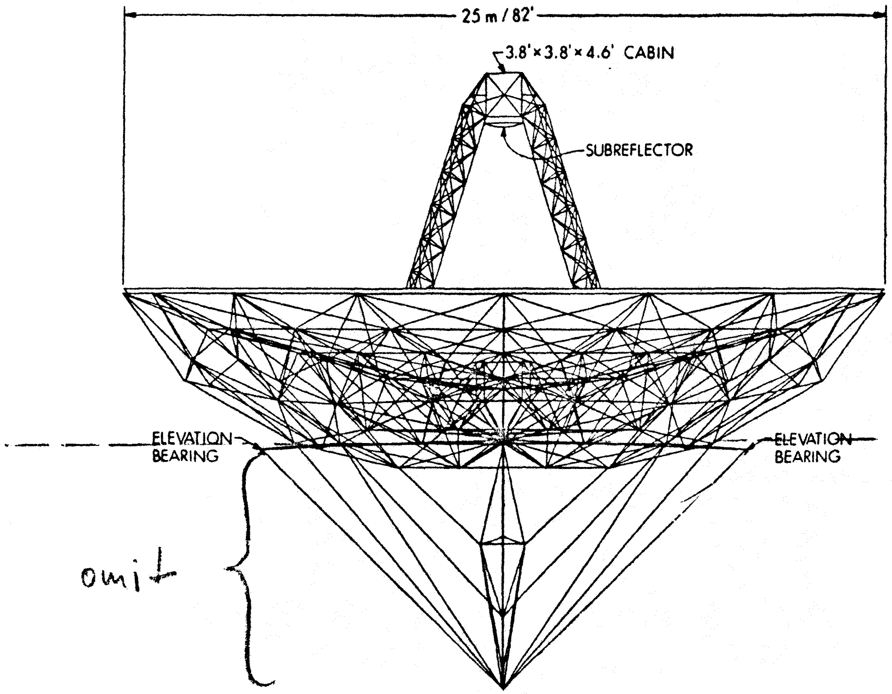


Fig. 4. System # 13: with 137 ft offset, 726 ft aperture, and axially symmetric surfaces.

Optimum compactness and feed shadow.



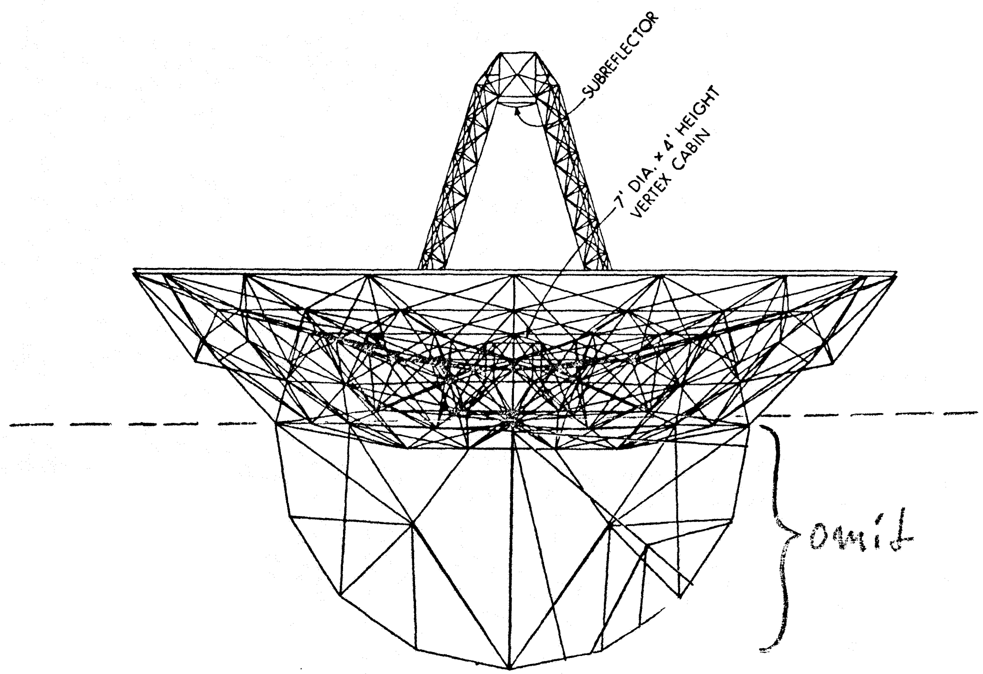


Fig. 5. The 25-m homologous telescope, from proposal of 1975.

To be used for weight estimate, see Table 2.

APPENDIX VI. SUMMARY OF ENGINEERING DATA AND RELATED INFORMATION FOR THE 25-METER DIAMETER RADIO TELESCOPE

Weight

Component	lbs.	Sub-total lbs.
Feed leg structure	2,200	
Panels	39,000	
Surface plates	22,213	
Loading at feed	2,000	
Loading at vertex	2,000	
Backup structure	64,600	
Counterweight	30,000	162,000

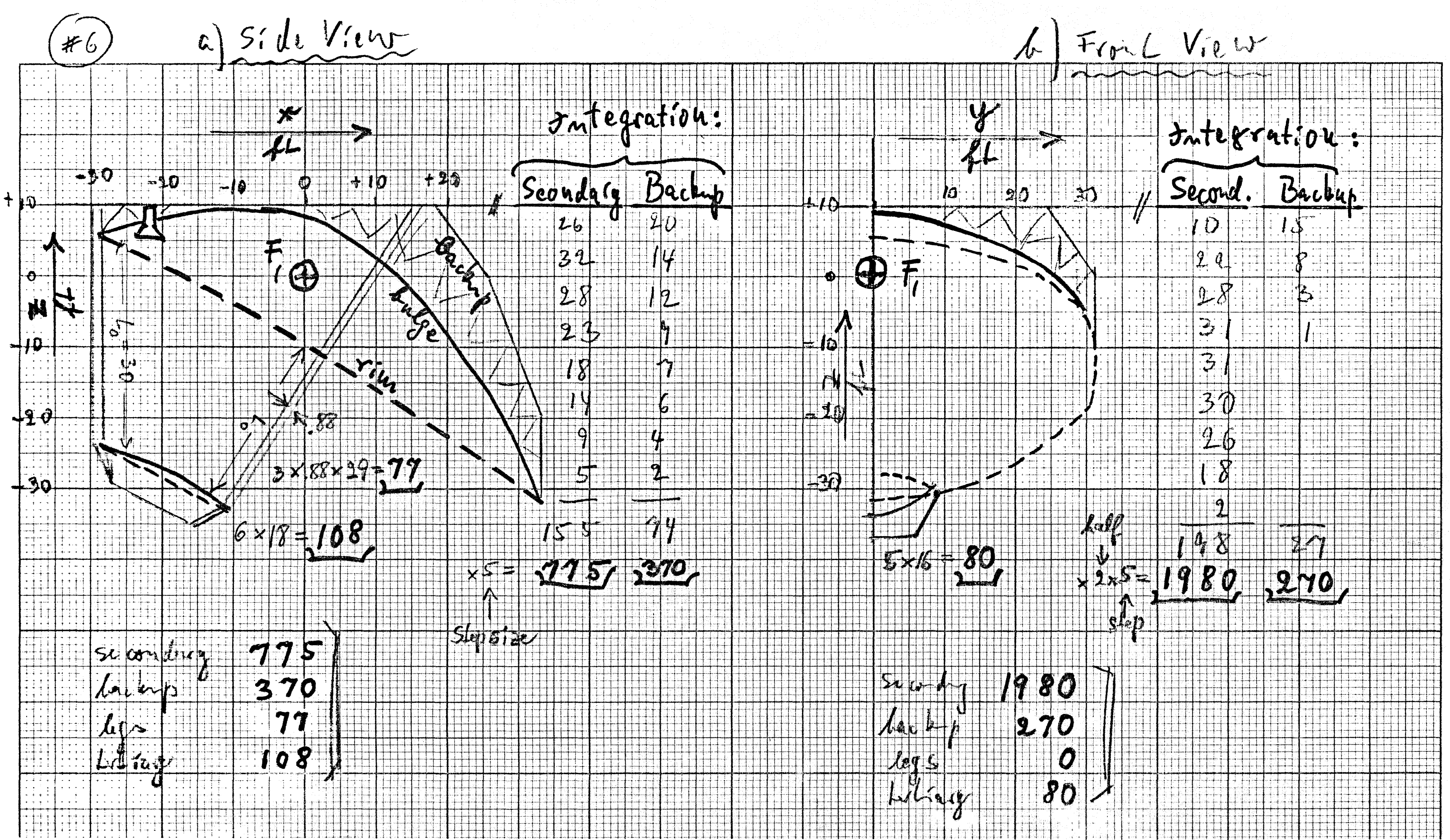


Fig. 6. Work sheet showing integration of wind area, for # 6 system.

a) Side view (smallest area) to be used as stow position for survival winds.

b) Front view (largest area) to be used as worst case for observational winds.

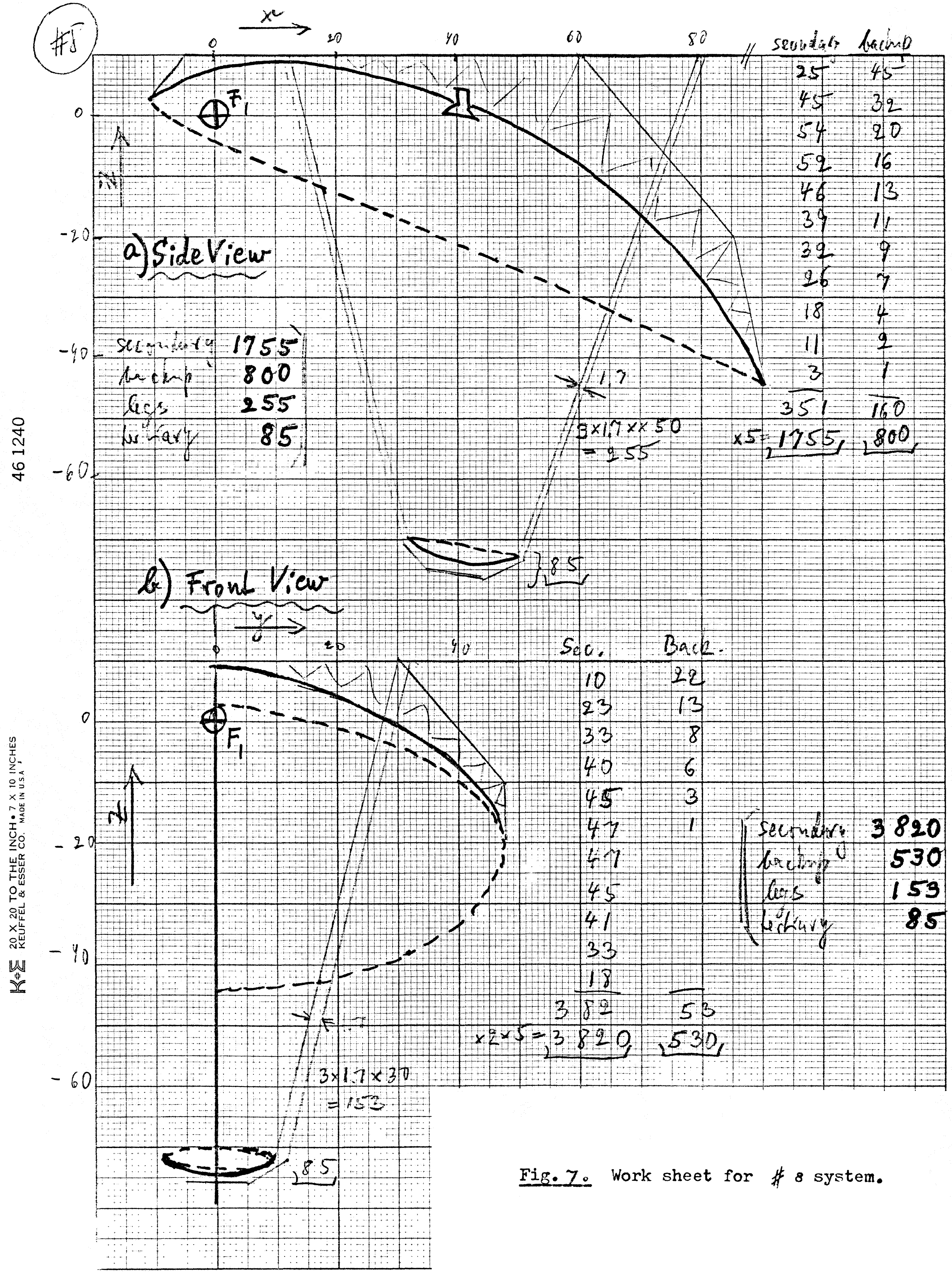


Fig. 7. Work sheet for # 8 system.

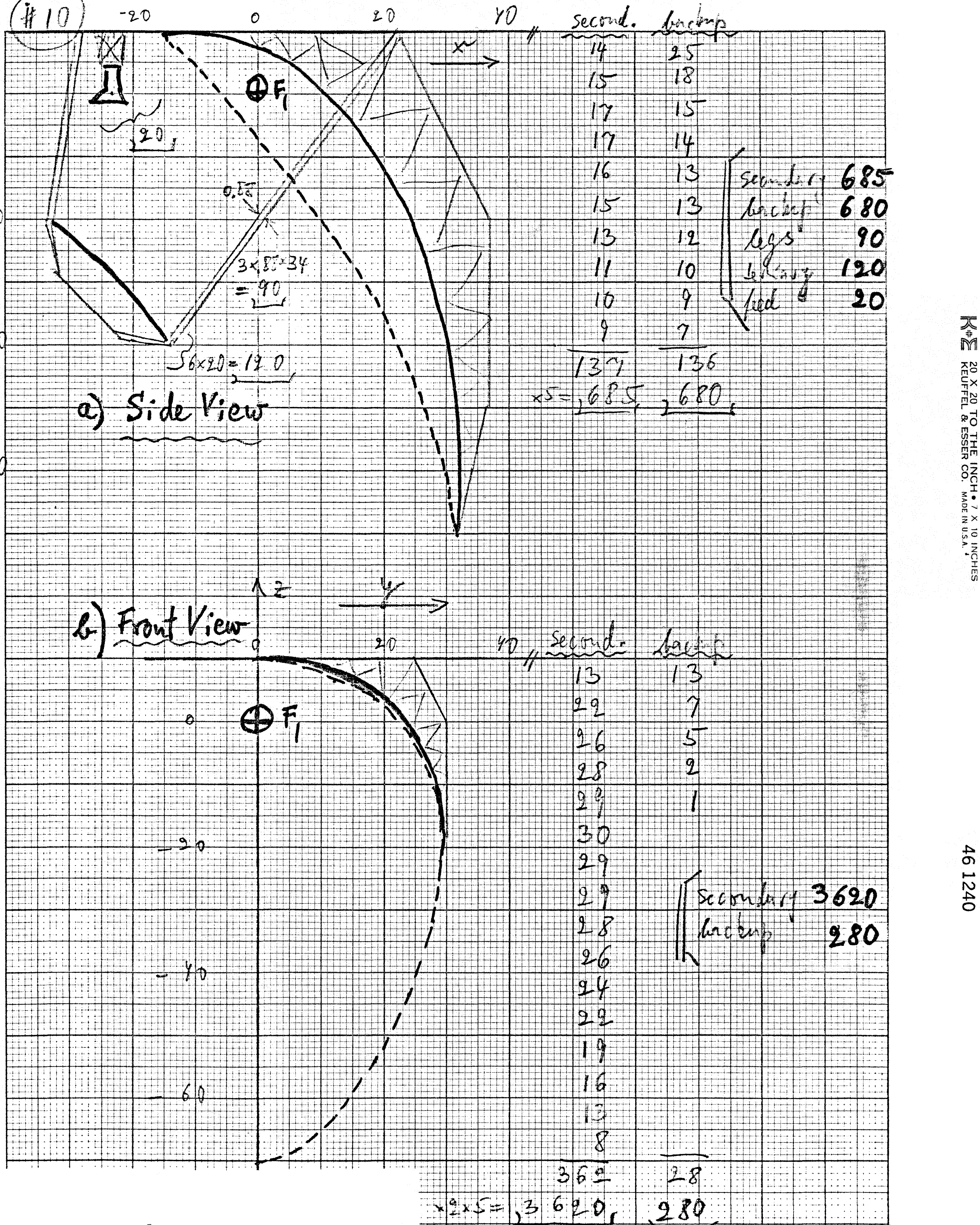


Fig. 8.

Work sheet for # 10 system.

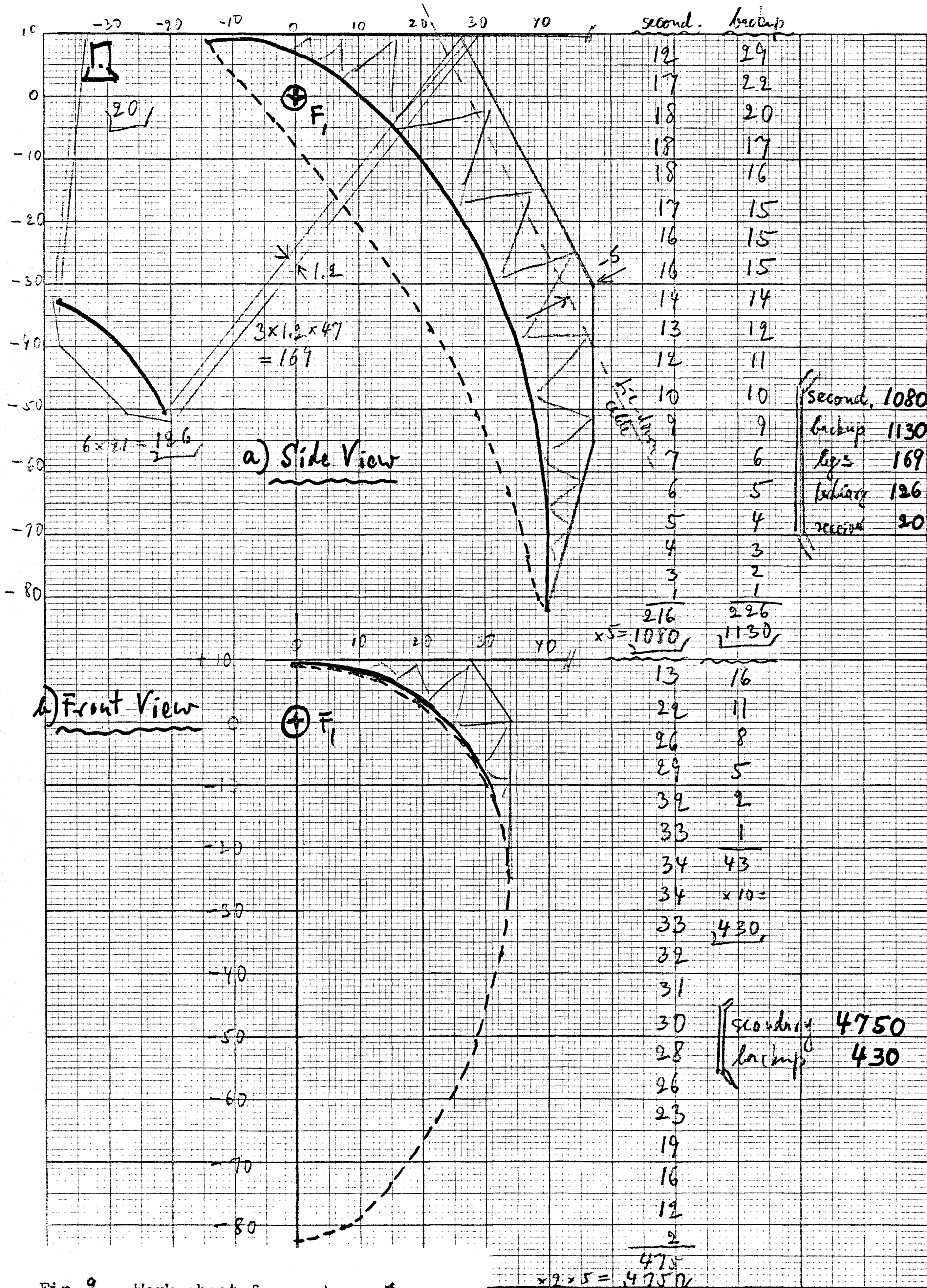
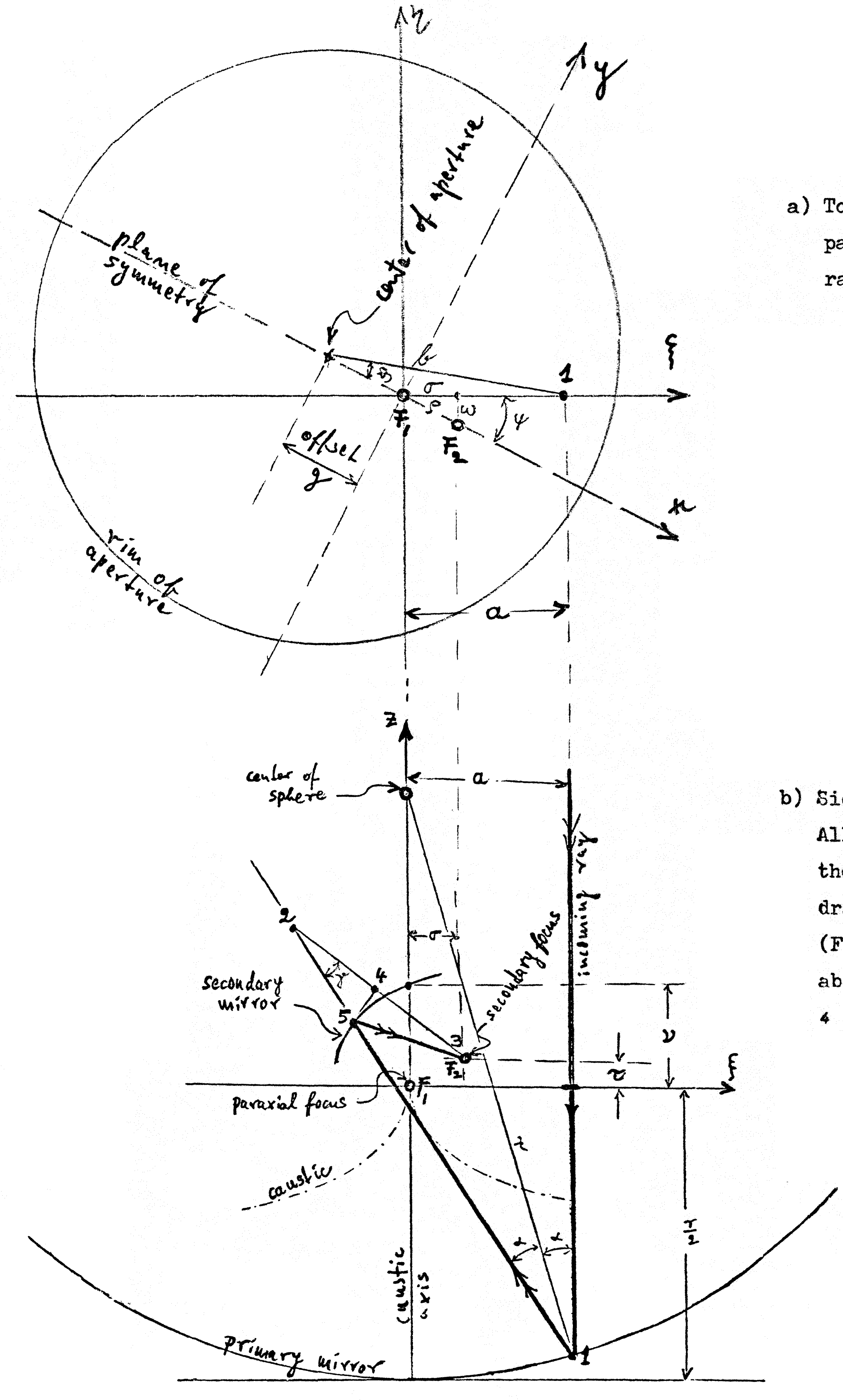


Fig. 9. Work sheet for system # 12.



a) Top view, looking down parallel to incoming rays.

b) Side view.
All points drawn are <u>in</u> the plane(ξ,z) of the drawing; except point 3 (F_2) which is at height ω above the plane, and point 4 at height $\omega/2$.

<u>Fig. 10</u> (Appendix). Geometry and definitions.
The direction of the incoming rays defines the caustic axis, which goes through the center of the primary sphere, parallel to the rays.