NATIONAL RADIO ASTRONOMY OBSERVATORY

ENGINEERING MEMORANDUM 100
March 22, 1976

TO:	J. Findlay	P. Napier	J. Welch
	M. Gordon	G. Peery	
	D. Hogg	P. Richards	
	H. Hvatum	B. Turner	

FROM: S. von Hoerner

RE: Far Infrared Applications of a Large Millimeter Wave Telescope

In his Memorandum of Feb. 17, 1975, P. L. Richards estimated the performance of our 25-m design at submillimeter wavelengths, and he found a considerable improvement as compared to a 3-m diameter infrared telescope. This was presented by J. Welch at our 25-m meeting in Charlottesville, Jan. 30. Since it seemed that the correlation length of the surface errors was overestimated, I promised to look into this question.

For his estimate, Richards assumes an rms surface error of σ = 0.1 mm, and a radius of the circle of confusion of $\theta = 1.5 \times 10^{-4}$ radians. In order to turn this into a correlation length, we assume for the surface errors a sine wave of amplitude A and length L:

$$\Delta z(x) = A \sin(2\pi\ x/L). \tag{1}$$

The rms error then is

$$\sigma = \mathrm{rms}(\Delta z) = A\ /\ \sqrt{2}, \tag{2}$$

and the rms slope is

$$\phi = \mathrm{rms}(d\ \Delta z/dx) = (2\pi\ A/L)\ /\ \sqrt{2}. \tag{3}$$

The deflected rays deviate from the telescope axis by

$$\theta = 2\phi = (4\pi\ A/L)\ /\ \sqrt{2}. \tag{4}$$

The length of the assumed surface pattern then can be found from the assumed values of σ and θ as

$$L = \frac{4\pi}{\sqrt{2}} \frac{\Lambda}{L} = 4\pi\, \sigma/\theta = 8.4 \text{ m}. \quad (5)$$

We define the correlation length Λ as that distance between two points, where the corrlation has gone down to 1/2. Then, for a sine wave, $\Lambda = L/6$, or

$$\Lambda = \frac{2\pi}{3} \frac{\sigma}{\theta} = 1.4 \text{ m}. \quad (6)$$

As to the actual surface errors of our 25-m design, I enclose Table III.1 of our Proposal of September 1975. Since the telescope will be enclosed in a radome (or astrodome), we use the last column of the table. Of the single items, by far the largest errors are due to the manufacturing error of the surface plates (0.040 mm), and to the offset of the plate corners (measurement 0.040, setting 0.015, combined to a corner error of 0.043 mm). The average plate has a size of 1.50 x 0.85 m, or an average side length of s = 1.2 m. The correlation length is $\Lambda = s/2$ for the corner errors, and smaller than s/3 for the manufacturing errors (depending on their Fourier spectrum). Thermal and gravitational plate deformations are 1/2 of a sine wave in one plate, giving $\Lambda = s/3$. In total, we may assume about $\Lambda = s/3 = 0.40$ m for the surface plates.

The errors from panel structures (side length 3.6 m), from backup structure (D = 25 m), and from subreflector (d = 1.4 m) will all have larger Λ, but they contribute only little because of their much smaller amplitudes. For the whole telescope, we thus may have about

$$\Lambda = 0.45 \text{ m}. \quad (7)$$

This is smaller than the value of (6) by a factor 3.1, thus θ should be increased by this factor. And if I understand Richards' derivations right, then NEF (noise equivalent flux for point sources) of his Table 1 should be multiplied by 3.1 for the "large telescope."

Actually, it would be nice if someone, who knows antenna theory and its literature better than I do, would calculate and plot the gain as a function of wavelength for our 25-m design and for existing or planned infrared telescopes, using formulas which include errors larger than the wavelength, and a correlation length as estimated in (7).

Table III.1

Mechanical Performance of the 25-meter Telescope

	Open			Enclosed
	Noon	Night		
	Calm, Sun	Calm	30 km/h wind	10 km/h wind
Surface Errors	(μm)	(μm)	(μm)	(μm)
Surface Plates	259	73	61	62
Manufacture	40	40	40	40
Gravity	12	12	12	12
Setting	15	15	15	15
Measurements	40	40	40	40
Thermal	252	42	7	16
Wind	-	-	10	1
Panel Structure	29	9	9	7
Manufacture & Gravity	7	7	7	7
Thermal	28	5	1	2
Wind	-	-	5	1
Backup Structure	212	39	20	21
Assembly & Gravity	16	16	16	16
Thermal	211	36	6	13
Wind	-	-	11	1
Subreflector	25	25	25	25
Manufacture & Gravity	25	25	25	25
Total Error 1)	337	87	70	70
Pointing Errors	(Seconds or arc)	(Seconds of arc)	(Seconds of arc)	(Seconds of arc)
Servo and Drive	0.5	0.5	0.5	0.5
Thermal	6.3	1.1	0.0	0.4
Wind	-	-	3.2	0.3
Total Error 1)	6.3	1.2	3.2	0.7

1) Total error is the quadratic sum of the individual, uncorrelated, rms errors.

UNIVERSITY OF CALIFORNIA, BERKELEY

BERKELEY • DAVIS • IRVINE • LOS ANGELES • RIVERSIDE • SAN DIEGO • SAN FRANCISCO   SANTA BARBARA • SANTA CRUZ

DEPARTMENT OF PHYSICS   BERKELEY, CALIFORNIA 94720

February 17, 1976

MEMORANDUM

TO: Dr. Mark Gordon
Dr. David Hogg
Dr. Barry Turner
Dr. John Findlay
Dr. Sebastian von Hoerner

FROM: P. L. Richards

RE: "Far Infrared Applications of a Large Millimeter Wave Telescope" by P. L. Richards.

The copies of my manuscript which Jack Welch delivered to you contained a number of errors. A corrected version is enclosed.

PL Richards

P. L. Richards
Professor of Physics

PLR/bh

LBL-3983

UNIVERSITY OF CALIFORNIA

Lawrence Berkeley Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

FAR INFRARED APPLICATIONS OF A LARGE MILLIMETER WAVE TELESCOPE

P. L. Richards

Department of Physics, University of California
Berkeley, California 94720

July 1975

In this memorandum we consider the benefits to be gained by locating a large mm-wave telescope at a high dry site so as to obtain useful atmospheric transparency in the far infrared (sub-mm) bands $10 \lesssim \nu \lesssim 30 \text{ cm}^{-1}$. The telescope is assumed to have a diameter $D_L = 25\text{m}$, an rms roughness $\sigma = 10^{-2}$ cm, and a circle of confusion of radius $\theta_c = 1.5 \times 10^{-4}$ rad.

Since this telescope will be incoherent in the far infrared bands, we will primarily consider incoherent receivers based on bolometer detectors. The assumption is made that the angular size of the image for $\nu \leqslant 10 \text{ cm}^{-1}$ is the geometrical circle of confusion of the telescope, despite the fact that the telescope is designed to be essentially coherent at $\nu = 7 \text{ cm}^{-1}$ with a much smaller angular resolution $(\nu D_L)^{-1} \approx 6 \times 10^{-5}$ rad. This circle of confusion analysis becomes more nearly justified as ν approaches 30 cm^{-1}.

Three receivers are considered with different spectral resolving powers $R = \nu/\delta\nu$. Each is assumed to have sufficient multimode throughput to accept the full circle of confusion of the telescope $A\Omega = \pi^2 D^2 \theta_c^2/4 = 0.35 \text{ sterad cm}^2$, so that they can accept all of the power from a point source. The receiver parameters represent an estimate of the best that can be built with ^{4}He cooled optics and ^{3}He cooled bolometers.

We assume a dark optical noise equivalent power (NEP) of $\lesssim 10^{-16}$ W/$\sqrt{\mathrm{Hz}}$ at the bolometer. The NEP of such bolometers will be limited by the fluctuations in the background radiation P_B passed by the wider band receivers. In order to compute this effect we must assume values for the atmospheric emission $\varepsilon(=0.5)$, the telescope feed efficiency $\alpha(=0.5)$, and the receiver transmission $\beta(=0.1)$. The background limited NEP_b is then

$$NEP_B = (2kTP_B)^{1/2} = (4\alpha\beta\varepsilon k^2 T^2 \nu^3 cA\Omega/R)^{1/2}. \qquad (1)$$

The optical NEP at the bolometer anticipated for each receiver is listed in Table I. The noise equivalent flux (NEF), as well as the noise equivalent temperature (NET) for an extended source which is large compared with the circle of confusion are also given in Table I. These are the minimum detectable flux (or temperature) in 0.5 sec of observation.

For the case of a source small compared with the diffraction limit of either telescope, the NEP of the detector is converted to a noise equivalent flux as follows

$$NEF = \frac{4\ NEP \cdot R}{2\alpha\beta\varepsilon\pi D^2 c\nu} . \qquad (2)$$

For an extended source, the power accepted by a system with throughput $A\Omega$ is given by the Rayleigh-Jeans law, so the noise equivalent temperature is

$$\mathrm{NET} = \frac{\mathrm{NEP} \cdot R}{2\alpha\beta\epsilon k c \nu^3 A\Omega}. \quad (3)$$

The possibility exists of an improvement by a factor ~ 7 in the assumed performance of the receiver with $R = 10^5$, by cooling the bolometer with a ^{3}He - ^{4}He dilution refrigerator.

The performance of these receiver -telescope combinations is compared in Table I with the same receivers on the next largest available high altitude telescope, which is assumed to have D_S = 3m and be coherent for $\nu < 30\ \mathrm{cm}^{-1}$. For the purposes of the comparison of NET values, the beam diameter on the sky is assumed the same for both telescopes. This need not be quite true in actual use since the diffraction limit of the small telescope varies over $3.3 \times 10^{-4} \geqslant (\nu D_S)^{-1} \geqslant 1.1 \times 10^{-4}$ rad. compared with a circle of confusion of order $2\theta_c = 3\times10^{-4}$ rad. for the large telescope.

It is quite clear from Table I that a spectacular improvement is obtained by using the large telescope, with only a small sacrifice (near $\nu = 30\ cm^{-1}$) of spatial resolution.

Since coherent receivers are being developed for the far infrared band of interest here, it is useful to know the receiver noise temperature T_R which would have equivalent performance to each of the proposed incoherent systems operated on the large telescope. We assume that the coherent receivers are operated on the largest available coherent telescope which is again taken to have $D_S = 3m$.

For the point source

$$T_R = \frac{NEP}{2\beta k}\sqrt{\frac{R}{c\nu}}\left(\frac{D_S}{D_L}\right)^2 . \tag{4}$$

For the extended source

$$T_R = \frac{NEP}{2\beta k\nu^2 A\Omega}\sqrt{\frac{R}{c\nu}} . \tag{5}$$

In each case we measure T_R at the input to the receiver. The atmospheric and telescope feed losses $\alpha\varepsilon$ are assumed to be the same as for the incoherent system.

From the results listed in Table II, we see that the incoherent receiver with $R = 10^5$ is just equivalent (at $\nu = 10\ cm^{-1}$) to a coherent receiver with $T_R = 3{,}000K$ which has a 100 channel multiplex i.f. filter bank. For smaller values of R the incoherent receivers rapidly become more favorable. In fact it seems possible that the uncertainty principle limit $T_R = h\nu/k$ of an ideal coherent receiver which gives $T_R \gtrsim 1.4\nu(°K)$ would prevent any coherent receiver on the small telescope from equalling the performance of the wider band incoherent receivers on the large telescope.

One alternative to the above approach is to use coherent receivers on the large telescope. Because $D_S \ll D_L$, the usefullness of the large telescope for coherent receivers extends beyond $\nu = 10\ cm^{-1}$ even for point sources. Its effective coherent diameter $D_{eff} = D_L e^{-2\pi\sigma\nu} \approx 14m$ at $\nu = 10\ cm^{-1}$ and $D_{eff} = D_S$ at $\nu \approx 21\ cm^{-1}$. Consequently, if the coherent receivers of Table II are used on the large telescope, the values of T_R in the point source column are increased by a factor 25 for $\nu = 10\ cm^{-1}$. This improvement disappears when ν reaches $21\ cm^{-1}$. The usefulness of coherent receivers for observing point sources in the range of $10 \leqslant \nu \leqslant 21\ cm^{-1}$ strengthens the argument for locating the large telescope at a high site.

All of the number listed in Tables I and II were computed for $\nu = 10\ cm^{-1}$. Their frequency dependence is complicated and is quite rapid in some cases. In order to specify this dependence, the exponent of ν obtained from Eqs. (1-5) is given in parenthese in Tables I and II beside the corresponding quantity.

One other size of telescope may be of great interest. It appears to be feasible to construct a telescope with D = 10m which is coherent for $\nu < 30\ cm^{-1}$. Eqs. (1-5) can be used to evaluate the performance of this telescope with either type of receiver. This is most easily done if it is assumed that the solid angle on the sky is kept constant so that $A\Omega$ scales as D^2. For $R = 10^5$ so that background fluctuations are negligible, the values of NEF and NET in Table I scale as D_S^{-2}. In Table II, T_R for a point source scales as D_S^{2} and for an extended source as D_S^{o}. For R = 10 so that the bolometer is background limited, these factors became D_S^{-1}, D_S^{3}, and D_S^{1} respectively.

It appears from the analysis given here that a large mm wave telescope at a high dry site would provide a formidable tool for both spectroscopy and broad band radiometry in the frequency range $10 \leq \nu \lesssim 30\ cm^{-1}$. Only for the highest resolution spectroscopic applications could the projected 3m infrared telescope be competitive, and then only with the best mixers likely to become available.

TABLE I

Noise equivalent flux (for a point source) and noise equivalent temperature (for an extended source) for an incoherent receiver operated on the large D_L = 25 m and the small D_S = 3 m telescopes. The optical NEP at the bolometer which depends on the resolving power R because of background radiation fluctuations is also shown. The numerical values are computed for ν = 10 cm^{-1}. The frequency dependence is given in terms of a power of ν(in parentheses).

	R	NEP ($W/\sqrt{Hz}$)	NEF ($10^{-26} W/M^2 Hz^{3/2}$)	NET ($°K/\sqrt{Hz}$)
LARGE TELESCOPE	10^5	10^{-16} (0)	270 (-1)	1.4 (-3)
	10^3	2×10^{-16} (~3/2)	5.4 (~1/2)	0.03 (~-3/2)
	10^1	1.3×10^{-15} (3/2)	0.35 (1/2)	0.0017 (-3/2)
SMALL TELESCOPE	10^5	10^{-16} (0)	19,000 (-1)	3 (-3)
	10^3	10^{-16} (0)	190 (-1)	1 (-3)
	10^1	2×10^{-16} (~3/2)	3.8 (~1/2)	.02 (~-3/2)

TABLE II

Receiver noise temperature T_R of a coherent receiver operated on a coherent 3m telescope which would have the same performance as the incoherent receivers of Table I operated on the large telescope. Since there is a practical limit of a few GHz to the bandwidth of a coherent receiver the row labeled $R = 10^1$ compares an incoherent receiver with $R = 10^1$ to a coherent receiver with $R = 10^3$. The numerical values are computed for $\nu = 10\ cm^{-1}$. The frequency dependence is given as a power of ν(in parentheses).

R	T_R (°K) Point source		Extended source	
10^5	300	(-1/2)	600	(-5/2)
10^3	60	(~1)	120	(~-1)
10^1	4	(1)	8	(-1)