Engineering Memo #102

SUGGESTIONS FOR A 100 METER HIGH-GAIN LOW-NOISE RADIO TELESCOPE

(Tentative Draft)

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I. Goals

1. High Gain

a. Two shaped reflectors, transforming any given feed pattern into any wanted aperture illumination, either constant for maximum gain, or slightly tapered for sidelobe trade-off. Constant illumination gives 17.6 db for first sidelobe, 23.8 db for second one.

b. Large secondary reflector and narrow feed pattern, for reducing gain loss from spillover.

2. Low Noise

a. Completely unobstructed view, from feed to secondary to primary to sky.

b. Remaining feed spillover at secondary shall always go into sky, high enough above ground.

c. Primary reflector has small reflecting collar beyond optical rim, to keep diffraction spillover from secondary off ground. No accuracy for collar.

3. Procedure

First: Conceptual structural design needed, for fixing dimensions, angles, parameters, and coordinate systems.

To be based on (unshaped) parabola-hyperbola system.

Second: Detailed derivation of shaped surfaces for primary and secondary mirrors, giving wanted illumination and constant path length. Available degrees of freedom to be used for smallest structural changes from unshaped design.

Third: Detailed structural design.

4. Previous Calculations

First shaping methods suggested by V. Galindo (IEEE, AP 12, 1964, p. 403) and B. Ye. Kinber (Radio Engineering + Electronic Physics, 7, 1962, p. 914). Solutions exist for reflectors with rotational symmetry but maybe not for others; some solutions are calculated using geometrical optics. A. C. Ludwig + R. E. Cormack (JPL Space Programs Summary No. 37-35, Vol. IV, 1965, p. 266) calculate a symmetric solution with geometrical optics, and then investigate the loss resulting from diffraction. W. F. Williams (Microwave Journal 5, 1965, p. 79) calculates another similar example.

A case with symmetric primary and slightly asymmetric secondary is investigated by P. P. Potter (JPL Deep Space Network Progress Report 42-20 (1975 ?) p. 92), and an approximate solution is given; the shaping near the reflector rims then is improved using wave optics.

For our present goals, however, we demand completely unobstructed view and a spillover at the secondary which always is above ground, and this demands a highly asymmetric shape for both reflectors. Thus, a new approach seems to be needed.

II. Conceptual Structural Design

1. Reflectors and Feed

Start in Fig. 1 with a paraboloid of revolution, pointed at zenith (z direction), focal length F = 50 m. Select a circle of D = 100 m diameter in (x,y) plane for the aperture, off-axis, and leaving enough clearance, 15 m, for secondary and collar. The primary rim then lies in a plane and forms an ellipse, short axis = 100 m, long axis = 119.3 m. The collar, omitted in picture, should be about 4 m wide.

The feed must be outside the primary rim, Fig. 2b. For having its spill-over at secondary well above ground, for both zenith and horizon position of telescope, the feed should be about at the 45° line in Fig. 1.

For a secondary larger than usual, we select d = 15 m diameter (D/d = 6.67, and $d/\lambda = 75$ for $\lambda = 20$ cm). The secondary then is seen from the feed with 14° diameter, and its lower rim is always seen at least 26° above ground, in all telescope positions.

2. Telescope Mounting

Best choice seems: alt-azimuth mount, elevation axis held on top of two tetrahedral towers. One leg of each tower rests on central pintle bearing, the other two legs on trucks moving on a circular rail in azimuth (Fig. 2a and 3).

The elevation drive (ED) is connected tangentially to the pintle bearing (P) for maximum dynamical stiffness. The height of the elevation axis (47 m) gives enough clearance for point K when pointing at horizon. The (redundant) part D-DE of the wheel allows pointing 16° beyond zenith for convenience.

The telescope backup structure is held from the elevation bearings with heavy suspension members, Figs. 2b and 3.

For avoiding heavy counterweights, the elevation axis should go through the center of gravity or close to it. Considering the asymmetric weight of all supports for secondary reflector and for feed-receiver cabin, we placed the axis 10 m off the dish center (Fig. 2a), as a rough guess to be improved when actually designing.

3. Backup Structure

For avoiding surface degradation from gravitational deformations under varying elevation tilt, demand homologous deformations (from one paraboloid to another one). For D = 100 m diameter, this will be needed if observation at $\lambda \leq 10$ cm is wanted. Homologous deformations have been achieved for NRAO designs of D = 65 m (λ = 3.5 mm) and D = 25 m (λ = 1.2 mm), both for parabolic primaries. Slightly different approach will be needed for shaped primary; may achieve λ = 1 cm for design of Fig. 1.

General features of conceptual design: a) Support from point H of several points (12 points in Fig. 2b) of basic plane. From basic plane, go up in several layers of decreasing mesh size until fine enough at surface. Goal: all surface points must have "equal softness."

b) Anything above basic plane shall not be touched from outside (telescope suspension, supports for feed and secondary).

4. Supports for Secondary and Feed

As suggested and explained in Fig. 2b. For reducing weight, all longer members should be "built-up members" (like slender tower structures, not just single pipes). The feed-receiver cabin will contain heavy equipment, but it rests on three relatively short supports. The secondary support legs have a clearance of about 4 m from the aperture beam.

The secondary will need some elevation-dependent movement against its supports, to allow for change of focal length and axial direction during homologous deformation of primary, and to allow for gravitational deformations of support legs, too.

5. Coordinate Systems

Basic system is Cartesian, centered at phase center of feed horn; z is vertical, x is North, y is East, for telescope pointing of Fig. 1. Both shaped reflector surfaces to be calculated in this system. Call:

$$\left.\begin{array}{ll} x,\ y,\ z & \text{for primary surface} \\ \xi,\ \eta,\ \zeta & \text{for secondary surface} \end{array}\right\} \text{in basic system,} \tag{1}$$
$$u,\ v \quad \text{for feed pattern} \qquad \text{in } (u,v) \text{ plane.}$$

The (u,v) plane is centered at the feed axis and perpendicular to it, u is horizontal and v is up. Its distance from the feed is arbitrary, most conveniently chosen at distance of secondary, with

$$a,\ b,\ c \ = \ \text{coordinates of } (u,v) \text{ plane center, in basic system.} \tag{2}$$

III. Shaping Procedure, General Features

1. Projection of Secondary into (u,v) Plane

A point (ξ, η, ζ) of the secondary yields in the (u,v) plane the point

$$\begin{aligned} u &= \frac{L^2}{\ell}\,\frac{b\xi - a\eta}{a\xi + b\eta + c\zeta} \\ v &= \frac{L^2}{\ell}\left\{\frac{L\zeta}{a\xi + b\eta + c\zeta} - \frac{c}{L}\right\} \end{aligned} \tag{3}$$

where

$$L^2 = a^2 + b^2 + c^2 \qquad \text{and} \qquad \ell^2 = a^2 + b^2.$$

2. Illumination Demand

We call:

$$\left.\begin{array}{lll} I(x,y) & = & \text{wanted aperture illumination} \\ i(u,v) & = & \text{given feed pattern} \end{array}\right\} \ \Omega = I/i. \tag{4}$$

As explained in Fig. 4a, the illumination demand reads A $i(u,v) = \Delta x\, \Delta y\, I(x,y)$. Or, written in terms of differentials:

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} = \Omega. \tag{5}$$

For numerical treatment, we consider points 1, 2, 3 of Fig. 4a as having been calculated previously, and point 4 as the one to be calculated next. Equation (5) then is written in terms of differences as:

$$(v_4 - v_1)\ (u_3 - u_2) - (u_4 - u_1)\ (v_3 - v_2) = 2\ \Omega\ \Delta x\ \Delta y. \tag{6}$$

This is a linear equation to be fulfilled for the two unknowns of the new point, u_4 and v_4. Both $i(u,v)$ and $I(x,y)$ should be taken as the average of points 2 and 3.

3. Relation between Primary and Secondary

In a recent paper (IEEE, AP, May 1976) I have described a program called DERIVE which calculates a proper secondary for any numerically given primary, yielding constant pathlengths to any chosen feed location. The only condition is that rays must not cross each other between both reflectors. With $z_x = \partial z/\partial x$ and $z_y = \partial z/\partial y$ of the primary surface, the equations of DERIVE read, somewhat rewritten for present use:

$$\begin{aligned} \xi &= x - z_x W \\ \eta &= y - z_y W \\ \zeta &= z + \tfrac{1}{2}(1 - z_x{}^2 - z_y{}^2)\ W \end{aligned} \tag{7}$$

where

$$W = \frac{(K + z)^2 - (x^2 + y^2 + z^2)}{K\,(1 + z_x{}^2 + z_y{}^2) + 2\,(z - x\,z_x - y\,z_y)} \tag{8}$$

and

$$\begin{aligned} K &= \text{constant, given by system parameters, such that} \\ K + z &= \text{part of pathlength from primary to secondary to feed.} \end{aligned} \tag{9}$$

4. Analytical Features

Equation (5) is the partial differential equation of our problem, with u and v to be replaced by ξ, η, ζ using (3), and these again to be replaced by x, y, z using (7) and (8).

If carried out, the result would be a partial differential equation of second order, quadratic in the second derivatives and unwieldly nonlinear in the first derivatives. This shows that two different types of solution exist (quadratic) and that we may start with a boundary line with two degrees of freedom along it (second order). But if we start with a closed boundary and demand that no singularities are inside, we have only one degree of freedom along it, and each interior point depends on the whole boundary, which could be solved by a matrix inversion for a linear differential equation but would need a very complicated iteration procedure for a nonlinear one. For obtaining a convenient two-mirror system, for example, it would be nice to impose the boundary condition that the rim of the circular aperture is projected by the optical rays into an exact circle on the (u,v) plane; but, as we just found, the procedure for integrating the interior then may be hopelessly complicated.

5. Suggested Procedure

A different approach seems more promising, following a statement of Williams (Microwave Journal, 5, 1965, p. 79) that the main task of the secondary is to spread the given feed pattern into the wanted aperture illumination, while the primary then must deviate from a paraboloid in such a way as to yield constant pathlengths.

We may start out with a parabola-hyperbola system as the one of Fig. 1, for fixing the over-all geometry. Then proceed with several iteration steps, each consisting of two half-steps:

a. Design new secondary, using previous primary, for giving wanted illumination in (x,y) plane. Will give path length errors.

b. Change primary, using new secondary, for giving constant path lengths. Will give small illumination changes.

Maybe two such iteration steps would already be good enough for all practical purposes.

In order to facilitate the problem (existence of solutions?) and in accordance with the practical application, it may be best to use rotational symmetry for both the aperture illumination and the feed pattern (the resulting reflector shapes still being asymmetric).

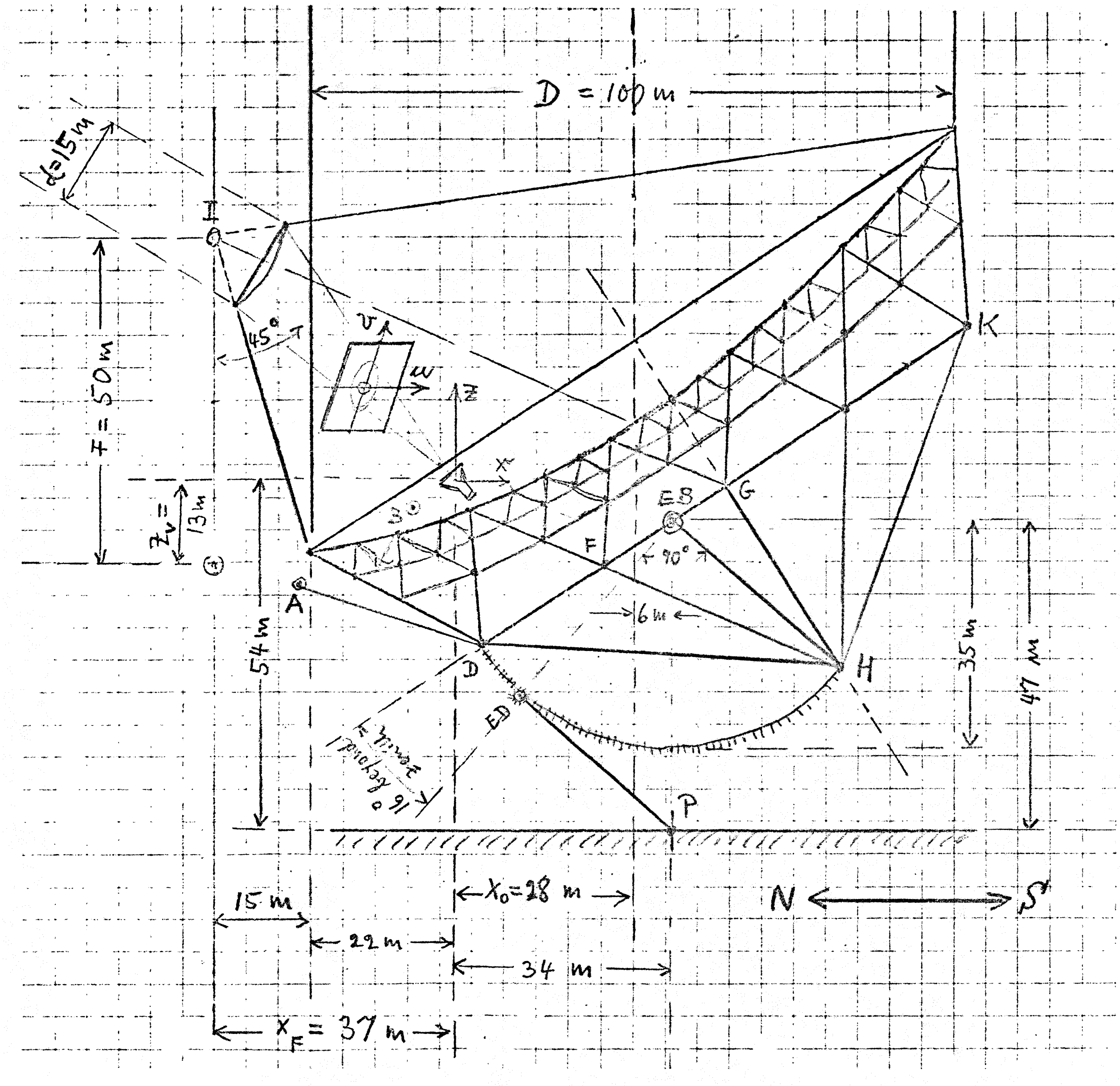
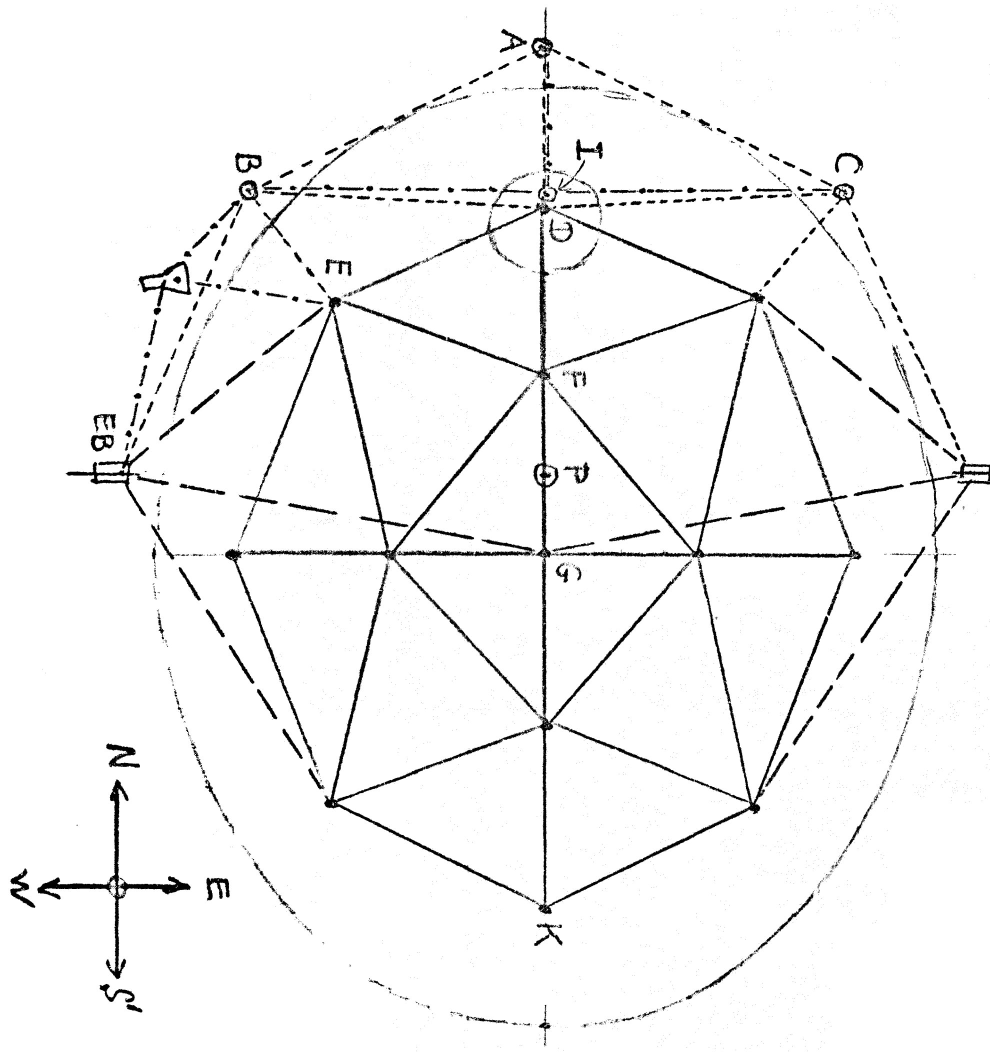


Fig. 1. Telescope pointing at zenith, zero azimuth. View from West.

The elevation bearings (EB) are mounted on two tetrahedral towers (Fig. 3) moving about pintle bearing (P).

The feed is outside the primary rim (Fig. 2b). As seen from the feed, the lower rim of the secondary is always at least 26° above ground.

The (u,v) plane is perpendicular to feed axis.



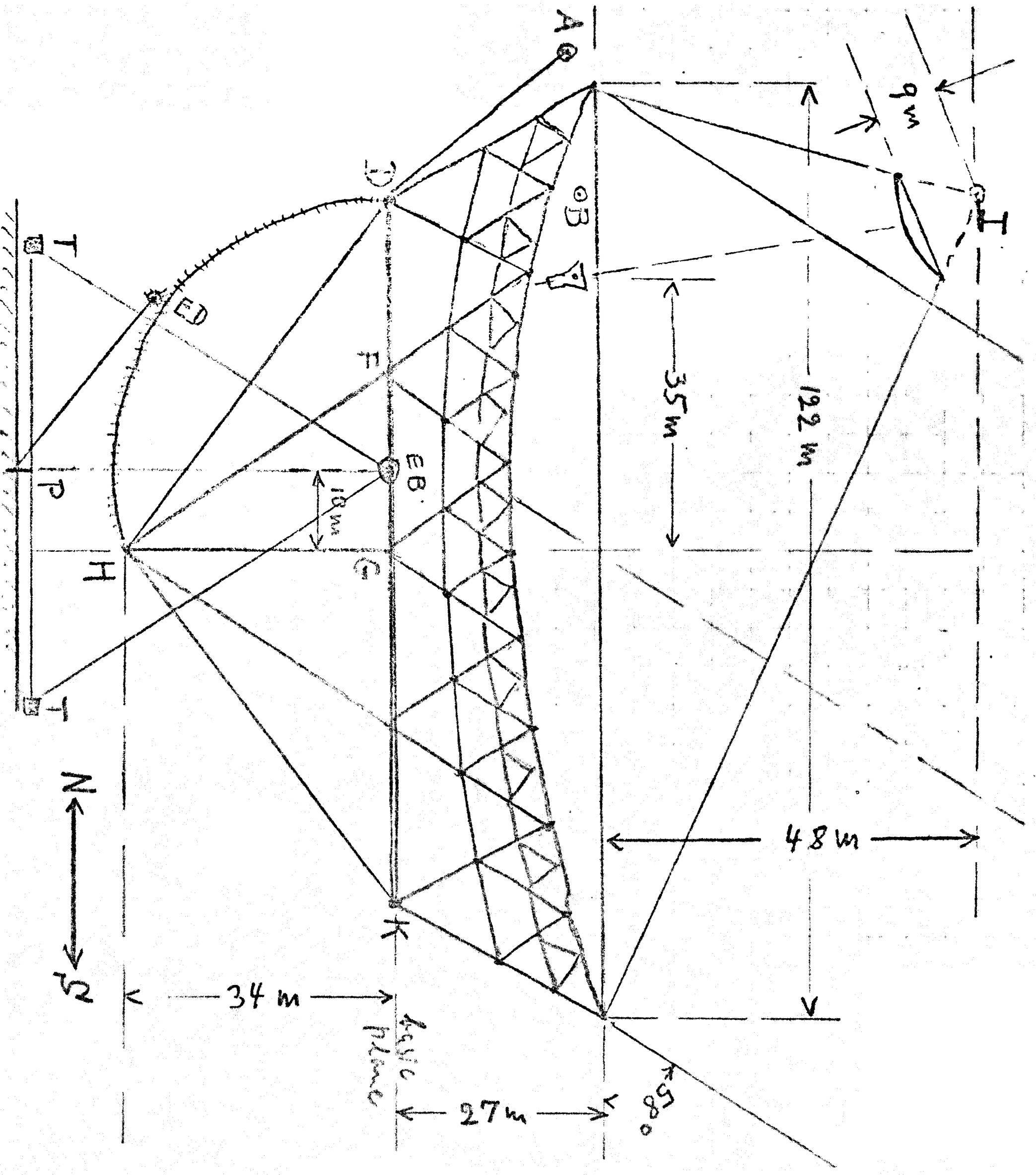


Fig. 2b. Stow position, top view. Showing basic plane (——) and telescope suspension (▬ ▬ ▬). Intermediate points A, B, C are fixed by nine supports (— — —). There are two tripods (—·—·—), one holding the feed-receiver cabin, one holding the secondary.

Fig. 2a. Stow position, pointing at 58° elevation and zero azimuth, view from West. For tower (EB, T, P) see Fig. 3.

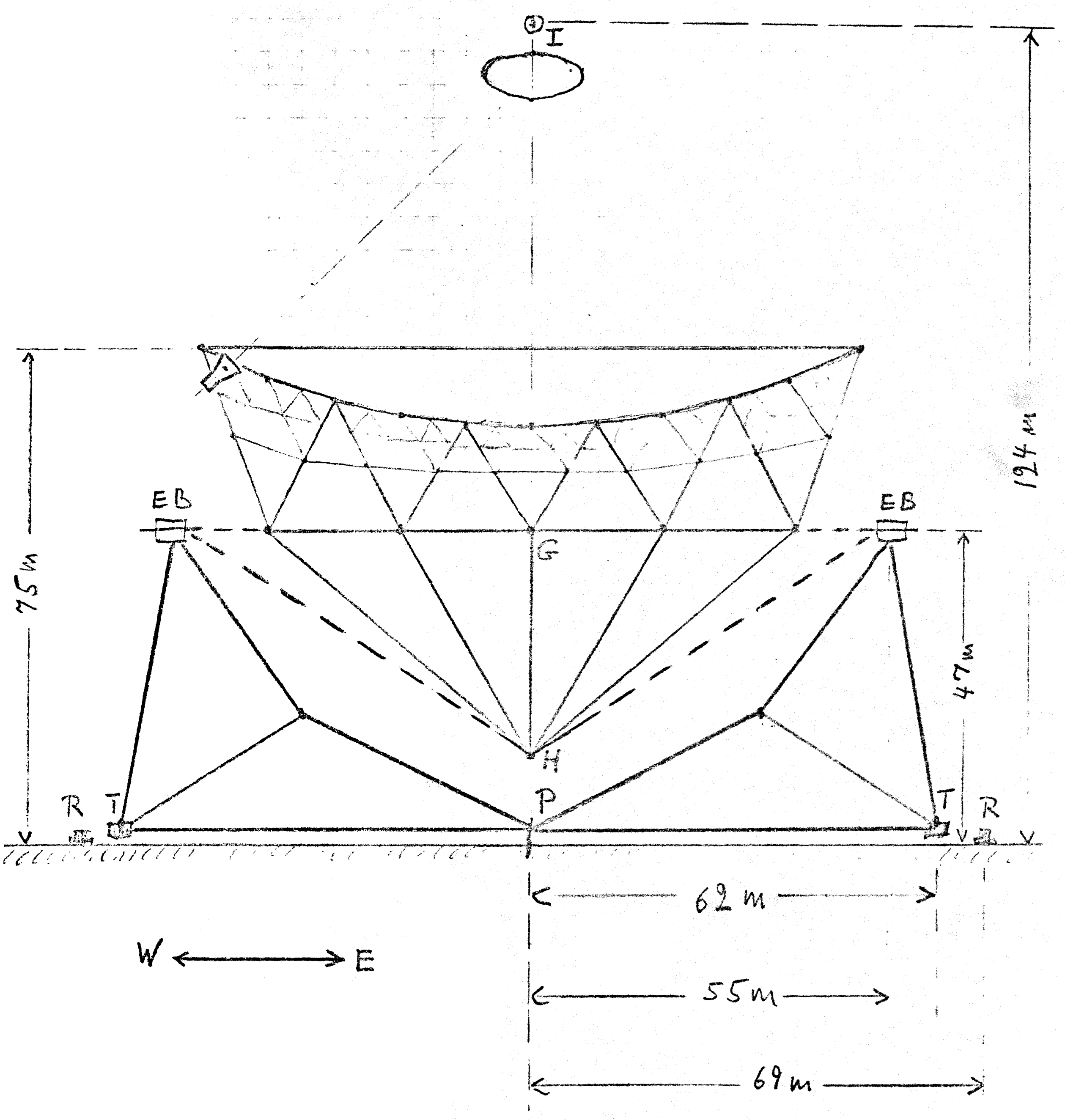


Fig. 3. Stow position, view from South.

Showing the two towers (———) and the suspension members (— — —). The towers move about the pintle bearing (P) on a total of four trucks (T), on circular rails (R).

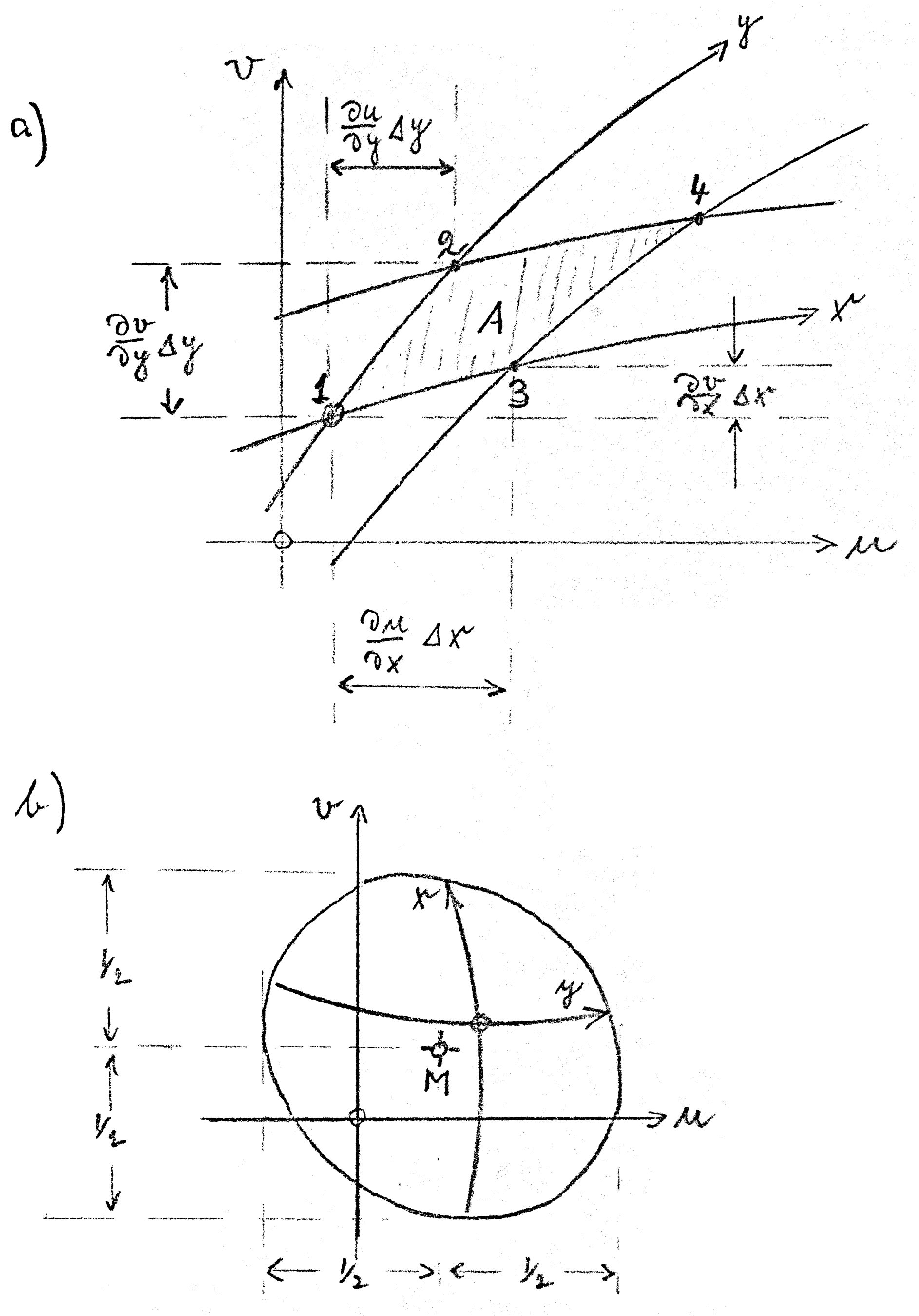


Fig. 4. Mapping of (x,y) plane of aperture, by optical rays, into (u,v) plane of feed pattern.

a) Transformation of a quadratic (x,y) grid.
The shaded area A, times the given feed pattern, must be equal to $\Delta x \Delta y$, times the wanted aperture illumination.

b) Transformation of circular (x,y) boundary into preliminary (u,v) plane. The final (u,v) plane should be centered at point M.