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Suggestions for the 140-ft Surface Adjustment

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Summary

We suggest adjusting the 140-ft for a best performance at 45° elevation due south; transformation of the zenith measurements of the stepping method to the 45° pointing is discussed. It is advisable to use weighted averages in all procedures, and an easy approximation for the weight as a radial function is developed.

The paraboloid to which all panels shall be adjusted is partly fixed; its axis must intersect the (deformed) joint of the feed legs, and the (deformed) vertex of the surface, since these are our equipment locations. The focal length F should be obtained from a least-squares fit to the internal curvatures of all panels, completely independent of their present adjustments. A procedure for finding F and its mean error is given.

The panels which have four adjustment screws allow four degrees of freedom for their adjustment: parallel lift, radial tilt, tangential tilt, and internal twist. We suggest to use all four, and a procedure is given for obtaining the adjustments from the measurements.

The internal gravitational deformations of the single panels can be neglected. But their internal thermal deformations would be serious in sunshine, amounting to $\Delta z = 1.8$ mm at the panel center; and the telescope backup structure will deform, too. This means that the measurements should be done only at night (or with completely overcast sky).

I. General Outlines

1. Best-Fit Paraboloid

The surface should be adjusted to a paraboloid of revolution. In general, this has six parameters: the location x_v, y_v, z_v of its vertex, two angles α_a, β_a for its axial direction, and its focal length F. In our case, we will have four structurally fixed parameters, and only two free ones.

Above the middle of the declination shaft is an instrument mount, the center of which defines x_v and y_v. About 20 ft outwards with 90° between each other and about 6° upwards, there are four punchmarks on strong points of the backup structure; these marks yield a best-fit plane, and the axial direction (α_a, β_a) is defined as being perpendicular to that plane. The central telescope axis defined in this way was actually used for fixing (at the prime focus) the axial direction for the Sterling mount and the x,y-location for its center, and for fixing (at the vertex) the x,y-location for the center of the ring of Cassegrain feeds, as well as their axial directions. Thus, any future surface paraboloid must use this same axis.

This leaves two free parameters, to be defined by best-fit procedures from the surface measurements. The focal length F should be found from the internal curvature of each single panel (just its curvature, independent of this panel's present height and tilt), then averaged over all panels. And the vertex height z_v may be chosen for minimum total adjustment: same average surface height before and after adjustment.

This paraboloid, with its four fixed and two optimized parameters, is the one to which each individual panel then must be adjusted for a best fit. The 12 panels of the first (inner) ring have three adjustment screws each; thus their adjustment can be described by three terms: height, radial tilt, and tangential tilt. The 48 panels of the second and third ring have four screws each, which provides one more term: an internal twist. These terms and their resulting screw movements are to be obtained from the surface measurements and the demand of best fit (least squares) to the paraboloid.

2. Surface Measurements

As to the measurements, Findlay's stepping method will be applied in October this year. It will yield, for zenith pointing, the height z and radial distance R for 33 points along each of 72 radii, with a total of 2376 points. First and last points of each radius are measured by conventional means in the structurally defined coordinate system as described above, while the stepping method proceeds along each radius with respect to gravity. The difference between gravity and the structural axis (α_a, β_a) can either be measured at the instrument mount, or it can be reduced to zero by talking from the mount over the intercom to the telescope operator. Either way, it should be repeated in intervals of some few hours, especially during stronger thermal changes. The final data should be given in the structurally defined system.

Independent measurements with a two-feed method are planned for next spring. This will yield the deviation Δz of the surface from a paraboloid of revolution whose axis intersects the phase center of the reference feed and is perpendicular to the plane of movement of the scanning feed. Any difference between this measuring axis and the structural axis (α_a, β_a as well as x_v, y_v) must be carefully measured, for each pointing, and all final data must be reduced to the structural system. This method can be applied for any telescope pointing, but it will be somewhat less accurate than the stepping method and it has much less spatial resolution, yielding only 150-200 independent surface points.

After both methods have been applied, we must check whether they agree within their errors in zenith position. If not, we would have a problem which must be solved before proceeding.

3. 45° Adjustment

The best adjustment angle for the 140-ft is not at zenith but on the south meridian at about 45° elevation, or about zero declination (von Hoerner and Wong, IEEE Trans. AP-23, 689, 1975). We now have two independent means for obtaining the amounts $\Delta z_{a,45}$ of adjustment needed for the 45° pointing. First, we find the adjustments $\Delta z_{a,z}$ needed for the zenith pointing from the stepping method. We then use W. Y. Wong's computer model of the 140-ft, and apply structural analysis for both zenith and 45° pointing under gravitational loads, assuming perfect shape in the absence of gravity. For each of the two pointings we find its best-fit paraboloid (only two free parameters, z_v and F) and get the deviations from it, called $\Delta z_{g,z}$ and $\Delta z_{g,45}$. The needed 45° adjustments then are

$$\Delta z_{a,45} = \Delta z_{a,z} + \Delta z_{g,z} - \Delta z_{g,45}. \tag{1}$$

Second, we obtain the $\Delta z_{a,45}$ directly from the two-feed measurements at this 45° pointing. Again, the results of both methods should agree within their errors. The most reliable result would probably be obtained by using equation (1) with $\Delta z_{a,z}$ from the stepping method, but deriving the gravitational difference between the two pointings, $\Delta z_{g,z} - \Delta z_{g,45}$, from the two-feed measurements.

The 45° adjustment has two more items to be discussed. First, x_v and y_v of the structural axis should be defined by the center of the instrument mount in any case; but its axial direction could be defined in two ways which are different from each other for the 45° pointing, because of the gravitational deformations of the feed support legs; we could either use again the plane of the four punch marks, or use the direction from the instrument center to the center of the Sterling mount. We suggest using the latter, since it is this location of the receiver feed, or of the Cassegrain mirror, which actually matters for the efficiency. This structural vertex-apex axis should be used both for describing the 45° two-feed measurements, and for the 45° computer analysis; it thus will be the axis of the adjustment paraboloid.

The second item is the internal gravitational deformation of the single panels. From equation a) of Fig. 1 we derive, with ℓ = 734 cm, h = 86.4 cm, and for aluminum: Δz_m = 0.18 mm for zenith pointing, and only 0.053 mm for the difference between zenith and 45°, which certainly is negligible. The computer model thus should calculate the deformations of the panel supports, but should regard the panels as completely rigid.

4. Thermal Deformation

The internal thermal deformation of the panels, however, is not negligible. In sunshine and with white paint we have about $\Delta T = 9°F = 5°C$, and for aluminum from Figure 1b:

$$\Delta z_m = 1.82 \text{ mm}. \tag{2}$$

And the backup structure of the whole telescope deforms, too. This means, if we cannot get days with completely overcast sky, then the measurements should be done at night only.

II. Weight Distribution

We must decide whether or not we should use weights in the following averages and best-fit procedures; and if so, which weights. In general, one should use the weights if they are very different from each other, meaning that they vary over a wide range.

After having measured Δz, the paraxial deviation of a surface point from a paraboloid of revolution, the weight to be ascribed to this point is, in general:

weight = area of aperture represented by this point, times aperture illumination at this location, times (pathlength difference)/($2\ \Delta z$). (3)

The pathlength difference is $\Delta p = \Delta z + \Delta L$, see Fig. 2a, where $\Delta L = \Delta z \cos 2\alpha$, and $\tan \alpha = \partial z/\partial R = R/2F$ for a parabola with $z = R^2/4F$. Because $(1 + \cos 2\alpha) = 2 \cos^2\alpha = 2/(1 + \tan^2\alpha)$, we obtain

$$\frac{\Delta p}{2\ \Delta z} = \{1 + (R/2F)^2\}^{-1} . \tag{4}$$

Findlay's stepping method will yield points which are spaced (on the surface) at equal steplengths Δs, along many radii. In Fig. 2b, the projection of Δs into the aperture is $\Delta R = \Delta s \cos \alpha$, and the area represented by a point then is proportional to

$$\text{area} \sim R\ \Delta R \sim R\{1 + (R/2F)^2\}^{-1/2} . \tag{5}$$

Instead of R, we shall mostly use the normalized radius

$$r = \frac{R}{D/2} \tag{6}$$

which is normalized to $r = 1$ at the rim. For equations (4) and (5) this yields, in general and with $F/D = 0.43$ for the 140-ft:

$$(R/2F)^2 = r^2/(4F/D)^2 = 0.3380\ r^2. \tag{7}$$

The aperture illumination depends on the feed pattern and the F/D ratio. We use a typical feed pattern, with 15 db edge taper, as shown in Fig. 3.

We call $V(\phi) = 10^{-a(\phi)/20}$ the voltage amplitude per solid angle, with $a(\phi)$, in db, from Fig. 3. We keep in mind that the power decreases with $1/L^2$ but the voltage only with $1/L$; and we realize from Fig. 2c that the area perpendicular to the beam is the same before and after reflection, because of the symmetry expressed by Snell's law. The voltage of the illumination then is, per aperture area,

$$I(R) = \frac{F}{L} V = \frac{F}{L} 10^{-a(\phi)/20} . \tag{8}$$

From Fig. 2a we derive

$$L = \frac{2F}{1 + \cos\phi} \quad \text{and} \quad R = L \sin\phi. \tag{9}$$

In this way, the illumination $I(r)$ of Fig. 3 was calculated. Looking for some easy approximation, $I_a(r)$, we found to our surprise that the frequently used "parabola on a pedestal" is no good. Calling $I_a = 1 - A r^2$, with the condition $A \leq 1$, it turns out that the best-fitting value is $A = 1$, which leaves a large maximum error of $\max|I_a - I| = 0.184$, at $r = 0.6$. Even if we provide one more free parameter, $I_a = (1 - A r^2)^B$, and if we demand exact fit at $r = 0.5$ and $r = 1.0$, then the solution yields $A \to 0$ and $B \to \infty$ which is no good either. But a very easy and good approximation was found as

$$I_a(r) = e^{-2.10 r^2} \tag{10}$$

with a maximum error of only

$$\max |I_a - I| = 0.018, \quad \text{at} \quad r = 0.30. \tag{11}$$

According to (1), (2) and (3), the weight $w(r)$ then is

$$w(r) = r (1 + 0.338 r^2)^{-3/2} I(r) \tag{12}$$

with I(r) to be calculated from (8) or approximated by (10). But we may as well look for an easy approximation $w_a(r)$ directly. A fairly good one is found as

$$w_a(r) = r\, e^{-2.60\, r^2}, \tag{13}$$

with a maximum relative error of only

$$\frac{\max |w_a - w|}{\max(w)} = 0.040, \quad \text{at} \quad r = 1.0. \tag{14}$$

Fig. 4 shows that the approximation is good enough. It also shows that we should use the weights, since they vary by a large factor (3.70) along the whole surface, and even within the single panels (factors 3.36, 1.24, 2.56). Also the integrated weights for the three rings of panels are quite different from each other and thus should be used:

$$W = \int w(r)\, dr = \begin{array}{ll} 0.0371, & \text{for first ring,} \\ .0844 & \text{second ring,} \\ .0540 & \text{third ring.} \end{array} \tag{15}$$

III. Best-Fit Focal Length for Single Panels

1. Definitions and Goal

We define three different focal lengths

F_o = original design length = 720 inch = 18.288 m;

F_p = present best-fit focal length for the whole telescope, depending mainly on the present adjustment height of the panels;

F = wanted focal length for future adjustment = best-fit for internal curvature of single panel (independent of its present adjustment), averaged over all panels.

We call the difference

$$\Delta F = F - F_o. \tag{16}$$

For describing the surface shape, we call

$$z_o = R^2/4F_o = \text{height of design paraboloid,}$$

$$z = \text{measured height,}$$

and the difference

$$\Delta z = z - z_o . \tag{17}$$

Regarding the internal curvature of a panel, one should in general use its curvatures both in radial and in tangential direction, with the square of its length and of its width as weight factors. Since, in the average of all panels, $(\text{length/width})^2 = 5.5 >> 1$, we may neglect the tangential curvature.

This allows equal and independent treatment of all 72 measuring radii (of the stepping method) within each of the 3 rings of panels. Each radius has 33 measuring points: $n = 8$ points in the first (inner) ring, $n = 11$ in the second, and $n = 14$ points in the third ring. For a given ring, we take these n points of a radius and find the best-fit F to their curvature (independent of height and tilt); doing it for all radii gives 72 values of F, and their average is the best-fit value of F for this ring. We use the weights of equation (15), average over the 3 rings, and obtain the wanted value F for the future adjustment.

2. Procedure

For a given ring of panels, we call P = projected panel length = R (outer edge) - R(inner edge). We consider the n points along some radius within this ring, and we find the weighted average R_a, using the weights $w_i(R_i)$ of equation (13),

$$R_a = \sum_{i=1}^{n} w_i R_i \Big/ \sum_{i=1}^{n} w_i . \tag{18}$$

Within this ring, we further use the normalized radial coordinate

$$\rho_i = \frac{R_i - R_a}{P/2} . \tag{19}$$

We now write the measured height z_i as the sum of four terms

$$\begin{aligned} z_i &= \text{lift} + \text{tilt} + \text{parabola} + \text{deviations} \\ &= u + \frac{P}{2}\,\phi\,\rho_i + R_i^{\,2}/4F + \varepsilon_i \end{aligned} \tag{20}$$

where the deviations are the sum of surface bumps and measuring errors. Equation (20) contains three unknowns (u, ϕ, F) to be determined by a least-squares fit of equation (20) to the actual measurements z_i. We insert equations (16), (17) and (19), and obtain from (20):

$$\Delta z_i = \alpha + \beta\,\rho_i - \gamma\,\rho_i^{\,2} + \varepsilon_i \tag{21}$$

with the three unknowns

$$\alpha = u - \frac{R_a^{\,2}}{4F_o^{\,2}}\,\Delta F$$

$$\beta = \frac{P}{2}\,\phi - \frac{P\,R_a}{4F_o^{\,2}}\,\Delta F$$

$$\gamma = \frac{P^2}{16\,F_o^{\,2}}\,\Delta F . \tag{22}$$

Of these, only γ matters, for finding F of this ring; but α and β are not relevant because panel adjustments (u, ϕ) will be made to a different paraboloid after F has been averaged over all three rings.

The demand of least squares leads to the following three equations for the three unknowns

$$
\begin{aligned}
M_o &= +\alpha & 0 \quad & - q_2\,\gamma \\
M_1 &= 0 & + q_2\,\beta \quad & - q_3\,\gamma \\
M_2 &= + q_2\,\alpha & + q_3\,\beta \quad & - q_4\,\gamma
\end{aligned}
\qquad (23)
$$

where

$$q_k = \overline{\rho^k} = \sum_{i=1}^{n} w_i\,\rho_i^{\,k} \Big/ \sum_{i=1}^{n} w_i \qquad \Big| \; k = 2,\ 3,\ 4; \qquad (24)$$

and

$$M_k = \overline{\rho^k \Delta z} = \sum_{i=1}^{n} w_i\,\rho_i^{\,k}\,\Delta z_i \Big/ \sum_{i=1}^{n} w_i \qquad \Big| \; k = 0,\ 1,\ 2. \qquad (25)$$

The solution of system (23) is

$$\gamma = - \frac{q_2 M_2 - q_3 M_1 - q_2^{\,2} M_o}{q_2(q_4 - q_2^{\,2}) - q_3^{\,2}} \qquad (26)$$

with which we obtain ΔF, the best-fit change of the focal length for the part of this radius within this ring,

$$\Delta F = \left\{ \frac{4\ F_o}{P} \right\}^2 \gamma\ . \qquad (27)$$

Finally, we average over all 72 radii, and we average over the three rings using (15). The result is the wanted focal length for the new panel adjustments. In Section I.3 we have shown that the internal gravitational deformation of the panels can be neglected. This means that F can be obtained from the zenith measurements, and can be applied, unchanged, to the 45° adjustment.

3. Mean Error

It will be interesting to know whether or not the resulting change ΔF is significantly different from zero. We thus want the mean error of our ΔF. In total, we have m = 3 x 72 = 216 single determinations of ΔF, and we call ΔF_j the single one, as obtained from (27). We finally have taken the weighted average, with the W_j from (15).

$$\Delta F = \overline{\Delta F} = \sum_{j=1}^{m} W_j \, \Delta F_j \Big/ \sum_{j=1}^{m} W_j \, . \tag{28}$$

We now calculate the variance

$$V = \sum_{j=1}^{m} W_j \, (\Delta F_j - \overline{\Delta F})^2 \Big/ \sum_{j=1}^{m} W_j \tag{29}$$

and the "effective" number of (equal-weight) cases

$$N = \left\{ \sum_{j=1}^{m} W_j \right\}^2 \Big/ \sum_{j=1}^{m} W_j^{\,2} \tag{30}$$

The mean error of ΔF then is

$$\varepsilon(\overline{\Delta F}) = \sqrt{\frac{V}{N-1}} \, . \tag{31}$$

IV. Panel Adjustment

1. Present Deviations

For each measured surface point, we now calculate $\Delta\zeta$, the deviation of the surface (when pointing at 45° elevation on the south meridian) from a paraboloid of focal length F and vertex height z_v whose axis intersects the structural vertex and the structural joint of the support legs:

$$\Delta\zeta = z - \frac{R^2}{4F} - (\Delta z_{g,z} - \Delta z_{g,45}) - z_v . \tag{32}$$

Here, z and R are given by the stepping method, when pointing at zenith, for 33 points each on 72 radii, in a system whose axis is again structurally defined for zenith pointing; $R^2/4F$ is the wanted paraboloid with F from the panel curvatures; the terms in parentheses are the gravitational difference between zenith and 45° pointings as defined before equation (1); and z_v should be determined such that $\overline{\Delta\zeta} = 0$ in the average over the whole telescope.

It is these deviations $\Delta\zeta$, more precisely the sum of their squares, which we want to minimize by the new panel adjustments.

2. Degrees of Freedom

We consider the 48 panels of the second and third ring, which have four adjustment screws each, yielding the following four degrees of freedom for the adjustment of a panel (see Figure 5 for definitions):

	$\Delta\zeta(R,\alpha)$	if positive, corners are	
1. lift, u	$= u$	all 4 up	
2. radial tilt, ϕ	$\approx (R - R_a)\,\phi$	1,2 up; 3,4 down	(33)
3. tangential tilt, ψ	$\approx R\,\alpha\,\psi$	2,4 up; 1,3 down	
4. internal twist, τ	$= (R - R_a)\,\alpha\,\tau$	2,3 up; 1,4 down	

Similar to equation (20), the local deviation $\Delta\zeta$ as obtained from (32) can now be written as the sum of five terms:

$$\begin{aligned}\Delta\zeta_i &= \text{lift} + \text{radial tilt} + \text{tangent. tilt} + \text{int. twist} + \text{bumps and errors} \\ &= u + (R_i - R_a)\,\phi + R_i\,\alpha_i\psi + (R_i - R_a)\alpha_i\tau + \varepsilon_i \end{aligned} \tag{34}$$

The 12 panels of the first ring have only three screws (in Fig. 5, points 3 and 4 would coincide at the center). This means $\tau = 0$ but no other change.

3. Amounts of Adjustment

For all panels within one ring, we have the weighted average $R_a = \overline{R}$ from equation (18). In addition, we calculate

$$q_R = \overline{(R - R_a)^2} = \sum_{i=1}^{n} w_i (R_i - R_a)^2 \Big/ \sum_{i=1}^{n} w_i \tag{35}$$

which is also the same for all panels in one ring; and we have, for 72 radii and $\alpha_o = 5°$,

$$q_\alpha = \overline{\alpha^2} \begin{cases} = \frac{1}{3} \sum_{i=1}^{3} \alpha_i^2 = \frac{2}{3} \alpha_o^2 = 16.67 \text{ deg}^2, & \text{for rings 2 and 3;} \\ = \frac{1}{6} \sum_{i=1}^{6} \alpha_i^2 = \frac{35}{12} \alpha_o^2 = 72.92 \text{ deg}^2, & \text{for ring 1.} \end{cases} \tag{36}$$

For each individual panel, we then calculate the four averages

$$M_{rs} = \overline{(R - R_a)^r \alpha^s \Delta\zeta} = \sum_{i=1}^{\nu} w_i (R_i - R_a)^r \alpha_i^s \Delta\zeta_i \Big/ \sum_{i=1}^{\nu} w_i \quad \begin{cases} r = 0,1. \\ s = 0,1. \end{cases} \tag{37}$$

where ν = number of measured points/panel, with $\nu = 48$ for the first ring, $\nu = 33$ for the second ring, and $\nu = 42$ for the third ring. From equation (34) we then obtain the following four equations for our four unknowns (u, ϕ, ψ, τ), similar to the system of (23):

$$\begin{array}{llllll} M_{oo} & = & + u & 0 & 0 & 0 \\ M_{10} & = & 0 & + q_R \phi & 0 & 0 \\ M_{01} & = & 0 & 0 & + R_a q_\alpha \psi & 0 \\ M_{11} & = & 0 & 0 & + q_R q_\alpha \psi & + q_R q_\alpha \tau \end{array} \tag{38}$$

The solution is simply

$$
\begin{aligned}
u &= M_{00} \\
\phi &= M_{10}/q_R \\
\psi &= M_{01}/(R_a q_\alpha) \\
\tau &= M_{11}/(q_R q_\alpha) - \psi
\end{aligned}
\tag{39}
$$

The needed amount of adjustment, A, then follows from equation (34) with L_2 and L_4 as explained in Figure 5. As to the sign, we define the adjustment as the opposite of the deviation, $A = -\Delta\zeta$; thus $A > 0$ means that this corner must be moved up, while for $A < 0$ it must be moved down:

$$
\begin{aligned}
A_1 &= -u - L_2\,\phi + R_2\,\alpha_2\,\psi + L_2\,\alpha_2\,\tau \\
A_2 &= -u - L_2\,\phi - R_2\,\alpha_2\,\psi - L_2\,\alpha_2\,\tau \\
A_3 &= -u + L_4\,\phi + R_4\,\alpha_4\,\psi - L_4\,\alpha_4\,\tau \\
A_4 &= -u + L_4\,\phi - R_4\,\alpha_4\,\psi + L_4\,\alpha_4\,\tau
\end{aligned}
\tag{40}
$$

If written in this way, we see the actual signs of all single contributions. But we can also write in a unified way:

$$
A_j = -u - (R_j - R_a)\,\phi - R_j\,\alpha_j\psi - (R_j - R_a)\,\alpha_j\,\tau \tag{41}
$$

with $j = 1,2,3,4$ for the second and third ring; for the first ring, we have only $j = 1,2,3$ and $\alpha_3 = 0$.

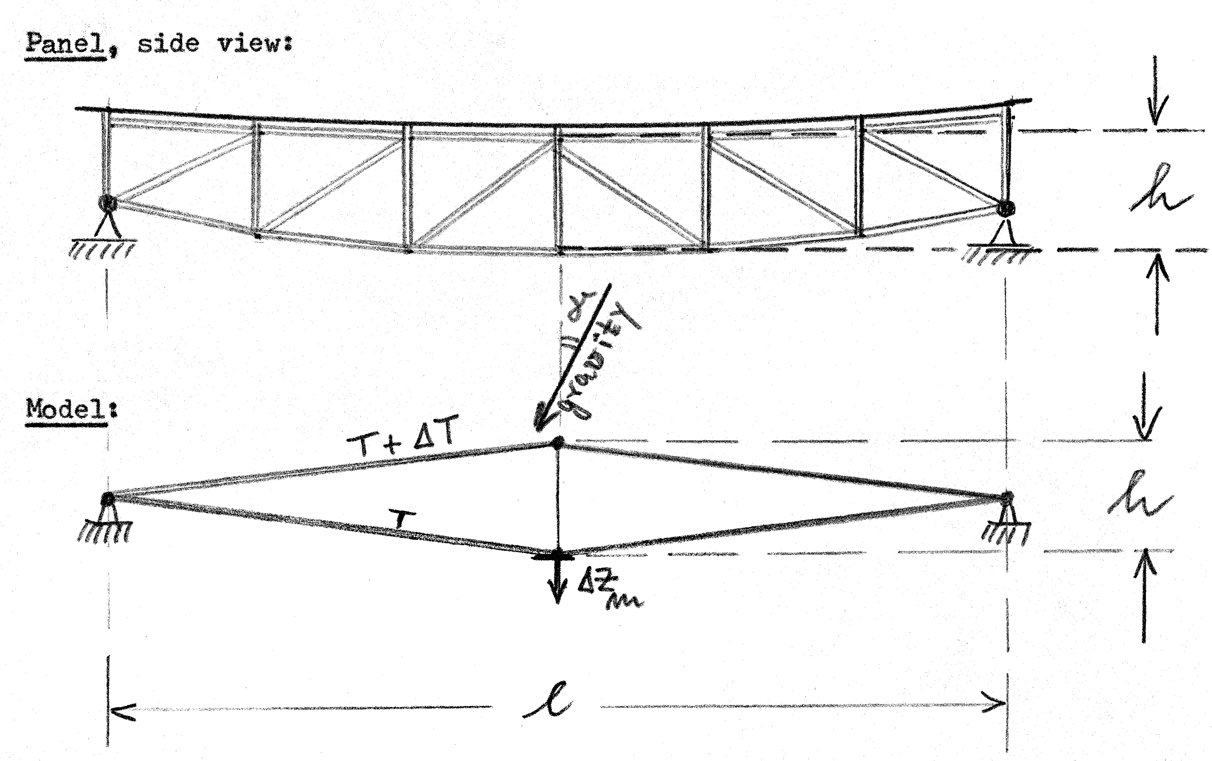


Fig. 1. Simple model for estimating the internal panel deformations.

a) Gravitational: $$\Delta z_m = + \frac{1}{8} \frac{\rho}{E} \frac{\ell^4}{h^2} \cos \gamma$$

b) Thermal: $$\Delta z_m = - \frac{1}{4} C_{th} \, \Delta T \, \frac{\ell^2}{h}$$

where ρ = density

E = modulus of elasticity

C_{th} = coefficient of linear thermal expansion

T = temperature

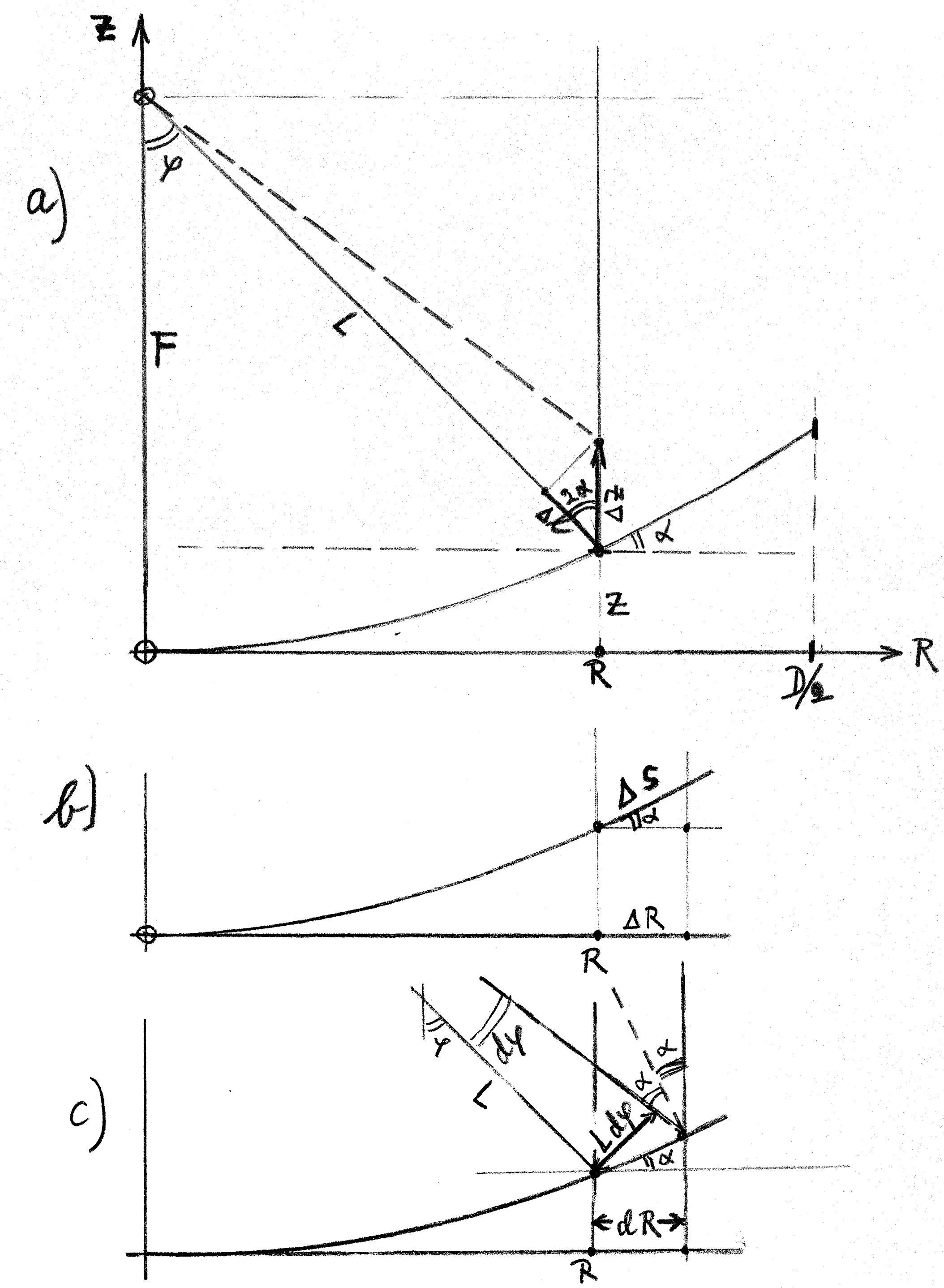


Fig. 2. Geometrical relations.

a) For pathlength difference, $\Delta p = \Delta z + \Delta L$;

b) For projection of surface into aperture, $\Delta R = \Delta s \cos \alpha$;

c) For illumination, $dR = L\, d\varphi$.

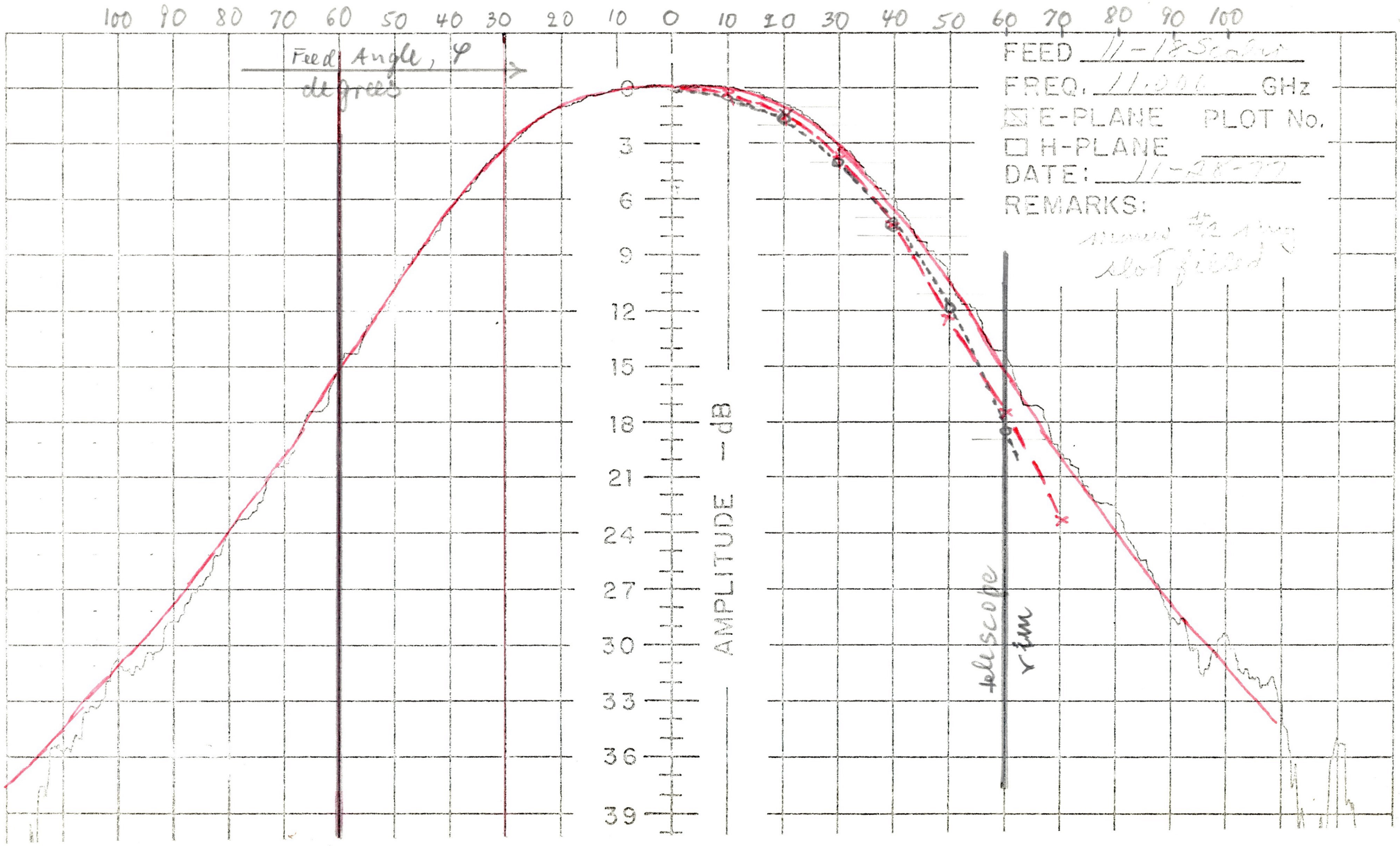


Fig. 3 Typical feed pattern and aperture illumination.

——— average (left and right hand) feed pattern, per solid angle;

x—— ——x resulting aperture illumination, per area, I(r);

o--------o simple approximation: $I(r) = e^{-2.10\, r^2}$.

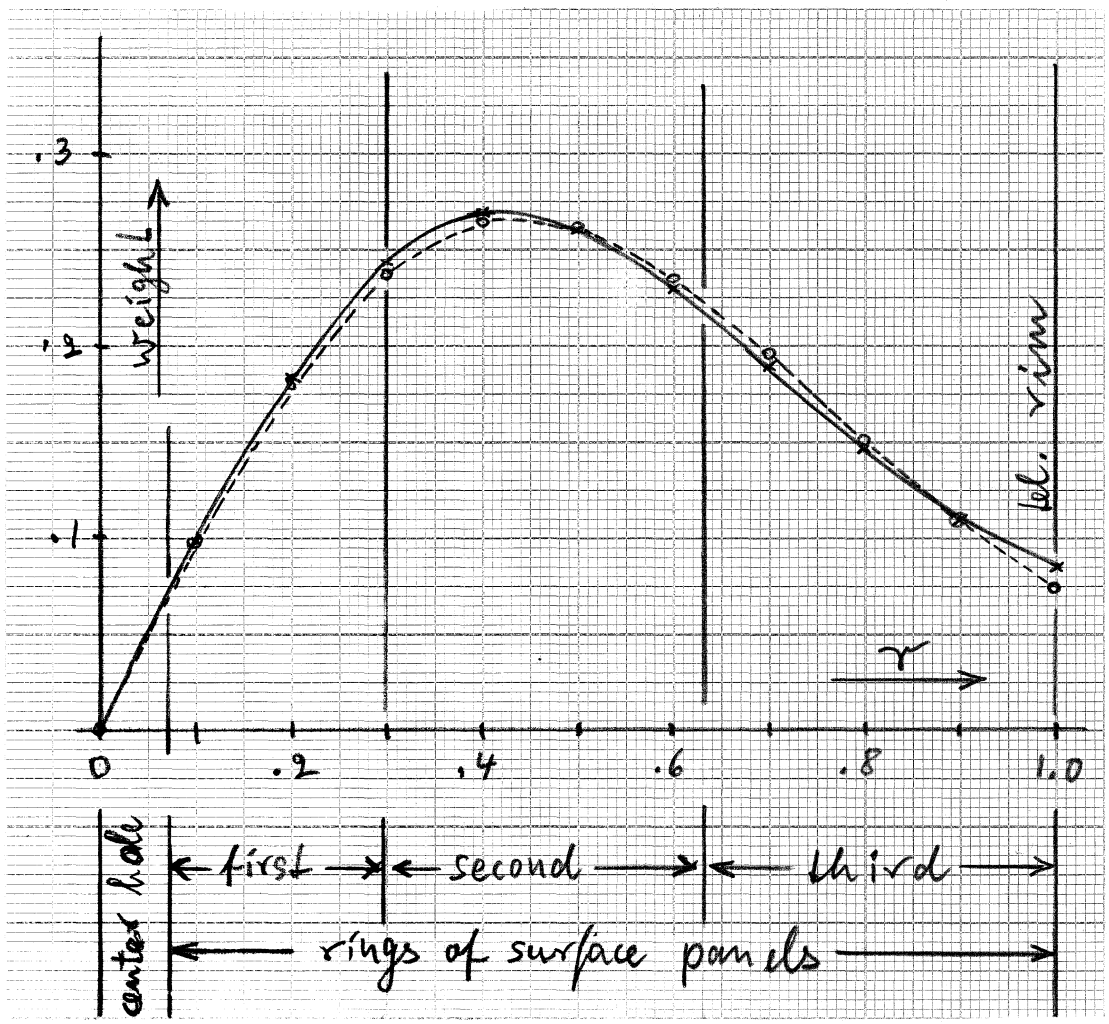
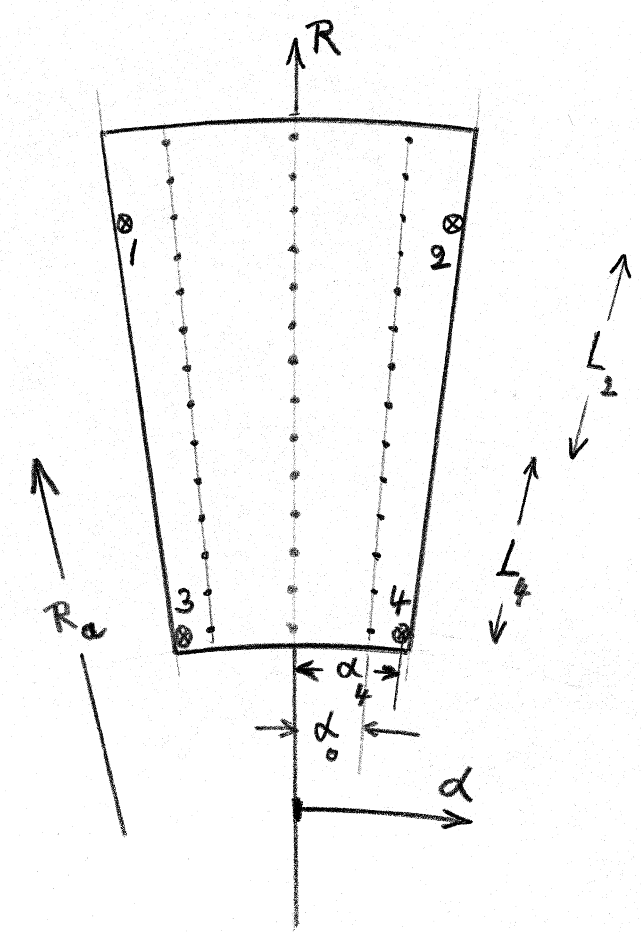


Fig. 4 Weight distribution, for points which are spaced equidistant on the surface along a radius (arbitrary units).

x———x actual distribution, $w(r)$, for feed of Fig. 3 ;

o- - - - - -o simple approximation: $w(r) = r\, e^{-2.60\, r^2}$.



<u>Fig. 5.</u> A panel of the third (outer) ring, projected into aperture plane.

The 42 dots are the measuring points,

the 4 crosses the adjustment screws,

and R_a is the average radius, using weights from (13).