One of the surprises in developing the VLBA holography system was that the satellite beacons being used for the purpose are actually very good oscillators. Their coherence time clearly exceeds the two millisecond size of the New Mexico array, and thus they are good enough to provide a local oscillator for that array, in lieu of having a maser or fiber optic connection at each station. They may even be good enough to provide LO signals for VLBI systems. The problem is that the satellite signal is affected by the differential Doppler as the satellite pursues its not quite exactly circular and geostationary orbit around the earth. To take correct for this effect requires knowledge of the satellite’s position to an accuracy of a few centimeters. I briefly describe below a technique for using what amounts to a round-trip phase measurement scheme to, in effect, measure the satellite’s position to the required accuracy. This scheme involves leasing a narrow (few tens of kHz) band in a transponder on the satellite carrying the beacon.

The analysis below ignores the dispersive effects of the ionosphere, which may well contribute appreciably to the final phase stability.

Let us consider two stations, A and B, looking at the satellite beacon. The beacon is radiating at frequency (angular) $f$:

$$\exp(i ft)$$

This is received at station A as

$$\exp[i f(t - L_{a1})]$$

where $L_{a1}$ is the path length from the satellite to station A. Similarly, station B receives

$$\exp[i f(t - L_{b1})]$$

At each station, we have a synthesizer (here assumed perfect) which uses this signal to synthesize a signal in the transponder uplink band. We synthesize slightly different frequencies at each station, so that the frequencies differ by a few kilohertz. The synthesizers take the received signals and multiply by factors of $a$ and $b$ respectively, making at station A

$$\exp(i f a t - i f a L_{a1})$$

and at station B

$$\exp(i f b t - i f b L_{b1})$$

These synthesized signals are sent back to the satellite, arriving there as

$$\exp(i f a t - i f a L_{a1} - i f a L_{a2})$$

and

$$\exp(i f b t - i f b L_{b1} - i f b L_{b2})$$

Respectively. The distances back to the satellite, $L_{a2}$ and $L_{b2}$, differ from $L_{a1}$ and $L_{b1}$ by the amount the satellite has moved in the round trip time from the beacon and back again.

These signals are translated by the transponder by a frequency $g$, to

$$\exp[i (f a + g) t - i f a L_{a1} - i f a L_{a2}]$$

and

$$\exp[i (f b + g) t - i f b L_{b1} - i f b L_{b2}]$$

respectively. These are then retransmitted, and are received at station A as

$$\exp[i (f a + g) t - i f a L_{a1} - i f a L_{a2} - i f a L_{a3} - i g L_{a3}]$$

and

$$\exp[i (f b + g) t - i f b L_{b1} - i f b L_{b2} - i f b L_{b3} - i g L_{b3}]$$
(L_{a3} \text{ is again different from } L_{a2} \text{ by the motion of the satellite during a transit time}).

The difference between these signals is

\[ \exp[if(a - b)t - ifa(L_{a1} + L_{a2} + L_{a3}) + ifb(L_{b1} + L_{b2} + L_{a3})] \]

which can be rearranged to

\[ \exp[if(a - b)t - ifa(L_{a1} + L_{a2} - L_{b1} - L_{b2}) - if(a - b)(L_{b1} + L_{b2} + L_{a3})] \]

This is now a low frequency signal; we can digitize it and have our way with it. In particular, we can beat to baseband (by subtracting \((a - b)\) times our received signal), and dividing by the synthesizer setup \(b\), and an additional factor of two, giving, after some rearrangement,

\[ \exp[-if(L_{a1} - L_{b1}) - if(L_{a2} - L_{b1} - L_{b2} + L_{b1})a/2b - if(L_{b2} - L_{b1} + L_{a1} - L_{a3})(a - b)/b] \]

If we then subtract this signal from the original received signal, we get

\[ \exp(if - ifL_{b1} + \phi) \]

where \(\phi\) is

\[ \phi = if(L_{a2} - L_{a1} - L_{b2} + L_{b1})a/2b + if(L_{b2} - L_{b1} + L_{a1} - L_{a3})(a - b)/b \]

That is, the signal at station A differs from that at station B only by the phase \(\phi\). Of this phase, the second term is small and apparently negligible, and the first term, involving the velocity of the satellite, is reasonably small and calculable, except possibly during station keeping procedures.

Note that the LOs at the two ends have phases \(ft - fL_{b1}\), which may differ appreciably from the desired frequency of operation of the interferometer. This is probably undesirable. It is easily correctable by comparing this signal with the desired master local oscillator and, after suitable filtering, to send the difference frequency back to station A, perhaps in the space between the tones on the satellite link. Alternately, it could be used to make a HALCA type deltaT file (for VLBI purposes) or fed into the WIDAR correlator as part of the station offset frequency (for New Mexico array purposes).