EVLA Memo # 46
RFI Emission Goals for EVLA Electronics

Rick Perley

October 15, 2002

Abstract

The harmful threshold power flux density levels are derived for the Expanded Very Large Array. It is argued that these levels be based on a time average of 9 hours, and a frequency resolution corresponding to a velocity resolution of 1 km/sec. Using these values results in harmful PFD levels about a factor of 7 below published ITU levels. Expressions for estimating the required shielding for EVLA electronics are derived, and a simple methodology for *in situ* measurements of the required shielding is described.

1 The Basics

Consider a transmitter radiating a power P_t Watts, with gain $G_t(\theta, \phi)$ (defined with respect to isotropic as $G_t = 4\pi A_e/\lambda^2$). At distance r, the power flux density, F_h will be

$$F_h = \frac{SP_tG_t}{4\pi r^2}, \tag{1}$$

where S is the shielding factor due, for example, to any intervening material distributed between the emitter and detector. The product P_tG_t is normally referred to as the Effective Isotropic Radiated Power – EIRP.

Now consider an antenna which measures this radiated power flux density through a sidelobe of gain G_r, again defined with respect to isotropic. The available power out of the receiving antenna will be

$$P_r = \frac{\lambda^2}{4\pi} G_r F_h, \tag{2}$$

which with Eq. 1 becomes

$$P_r = \left(\frac{\lambda}{4\pi r}\right)^2 G_t G_r S P_t. \tag{3}$$

The factor $(\lambda/4\pi r)^2$ can be interpreted as a space loss factor. It is the ratio of the collecting area of an isotropic antenna to the area of a sphere at radius r.

This interfering signal power is, by convention, above the harmful level if it exceeds 1/10 of the r.m.s. fluctuations in noise power from the telescope. This condition is met if $P_r < \sigma_P/10$. The noise power fluctuation for an observation of duration τ, with bandwidth $\Delta\nu$, and system temperature T_{sys} is

$$\sigma_P = \frac{kT_{sys}}{\sqrt{\Delta\nu\tau}} \Delta\nu, \tag{4}$$

where k is the Boltzman constant, 1.38×10^{-23} JK^{-1}. From Eq. 2 and 4, and the definition of harmful power, we derive the harmful threshold power flux density to be

$$F_h < \frac{0.4\pi k T_{sys}}{G_r \lambda^2} \sqrt{\frac{\Delta\nu}{\tau}} \tag{5}$$

which is identical to Eq. 15.2 in Thompson, Moran and Swenson, 2nd edition.

Readers might wonder why the forward gain (or effective collecting area) of the receiving antenna is not a factor in setting the maximum tolerable interference level. One could argue that a large antenna with high forward gain will be more sensitive to the desired emission, and thus less susceptible to unwanted signals. In fact, this argument is true, but incomplete. An alternative derivation of the harmful threshold level should help solve this apparent dilemma.

Suppose we observe an unpolarized astronomical source with SPFD S_{obj} $\mathrm{Wm^{-2}Hz^{-1}}$ with an antenna whose forward gain is G. Within the bandwidth $\Delta\nu$, the detected power (to a matched system) is

$$P_{obj} = \frac{S_{obj}}{2} G \frac{\lambda^2}{4\pi} \Delta\nu, \tag{6}$$

where the factor of 1/2 is due to our assumption that the source is unpolarized. At the same time, suppose there is an interfering signal of PFD F_h $\mathrm{Wm^{-2}}$, arriving through a sidelobe of gain G_{sl}. The detected RFI power (to a matched system) is

$$P_h = F_h G_{sl} \frac{\lambda^2}{4\pi}, \tag{7}$$

where we have not included a factor of 1/2 as both interferor and antenna sidelobes can be highly polarized, and we are assuming a worst-case scenario in which these match. Again defining the minimum harmful interference level as being 1/10 of the desired signal, we find that the maximum tolerable PFD for the unwanted signal is

$$F_h < 0.1 S_{obj} \frac{G}{G_{sl}} \Delta\nu. \tag{8}$$

This formulation does include the receiving antenna's forward gain. But in fact this equation is identical to Eq. 5 when the source SPFD S_{obj} is set by the minimum detectable spectral flux density

$$S_{min} = \frac{8\pi k T_{sys}}{G\lambda^2\sqrt{\Delta\nu\tau}}. \tag{9}$$

The explanation for the absence of the antenna forward gain in the generally accepted definition of the minimum harmful level is that the harmful level is defined in terms of the sensitivity of the telescope, not in terms of a defined source spectral flux density. Were the latter definition employed, then Eq. 8 would apply, with the result that the harmful level would be a function of antenna forward gain. Because the harmful level is defined with respect to the telscope's sensitivity, a larger antenna is not more immune to interference than a smaller one, since its sensitivity is greater.

2 Which Bandwidth to Choose?

The limiting power flux density is a function of bandwidth, a fact which causes considerable confusion since astronomers normally think in terms of spectral power flux density, measured in units of Janskys (1 Jy = 10^{-26} $\mathrm{Wm^{-2}Hz^{-1}}$), while engineers often think in terms of radiated power (W), or power flux density ($\mathrm{Wm^{-2}}$), measured within some resolution bandwidth, $\Delta\nu$. Eq. 5 shows that the harmful RFI level goes to zero as the bandwidth utilized goes to zero. Thus the narrow bandwidth useages will determine the tolerable RFI power levels.

The bandwidth chosen for an astronomical observation depends on the experiment at hand. A 'continuum' experiment does not require high frequency resolution images, and thus can utilize a wide bandwidth, giving both higher sensitivity and greater immunity to narrow-band signals. However, for a 'spectral-line' experiment, high frequency resolution is a critical requirement for attaining scientific goals. How much resolution depends on the particular experiment, but the highest spectral resolution which will be *commonly* used will be that corresponding to a velocity resolution of 1 km/sec. For this, the spectral resolution is then $\Delta\nu = 1000/\lambda$ Hz, where λ is the wavelength in meters. Inserting this into Equation 5, and converting to frequency units, gives

$$F_h < 3.52 \times 10^{-43} \frac{T_{sys}\nu^{5/2}}{G_r \tau^{1/2}} \qquad \mathrm{W/m^2}. \tag{10}$$

This expression can be cast into more useful units by defining ν_G = frequency in GHz, and τ_h = integration time in hours, giving

$$F_h < 1.85 \times 10^{-22} \frac{T_{sys}\nu_G^{5/2}}{G_r\sqrt{\tau_h}} \qquad \mathrm{W/m^2}. \tag{11}$$

The harmful threshold interference power flux density allowed for each EVLA band is shown in Table 1. A spectral resolution corresponding to a velocity resolution of 1 km/sec, isotropic VLA antenna gain, and an integration time of 9 hours have been assumed. This time duration is believed representative of the median 'long integration' project likely to be scheduled on the array. Also shown is the harmful level expressed in terms of spectral power flux density (SPFD), using the resolution bandwidth given in column 3, and expressed in Janskys (1 Jy = 10^{-26} $\mathrm{Wm^{-2}Hz^{-1}}$).

Table 1: Harmful Threshold RFI Power Flux Densities for the EVLA

Band	ν_G GHz	$\Delta\nu$ kHz	T_{sys} K	F_h $\mathrm{Wm^{-2}}$	F_h $\mathrm{dBWm^{-2}}$	S_h Jy
L	1.5	5	26	4.4×10^{-21}	-204	88
S	3.0	10	29	2.8×10^{-20}	-196	280
C	6.0	20	31	1.7×10^{-19}	-188	850
X	10	33	34	6.6×10^{-18}	-172	2000
U	15	50	39	2.1×10^{-18}	-167	4200
K	23	77	54	8.4×10^{-18}	-171	10910
A	34	113	45	1.9×10^{-17}	-167	16810
Q	45	150	66	5.5×10^{-17}	-163	36670

The values of F_h listed in the table are the maximum allowable power flux densities within the frequency resolutions listed in column 3, for any frequency within the appropriate band. Note that the tolerable threshold rises rapidly with frequency – this is due mainly to the diminishing effective isotropic collecting area of the antenna, (proportional to ν^{-2}), but also to the decreasing frequency resolution corresponding to fixed velocity resolution (proportional to $\nu^{-1/2}$).

The harmful threshold levels listed in the table are about 8 dB lower than the standard ITU levels – this is because we are assuming a longer time integration (9 hours vs. 2000 seconds) and a narrower frequency resolution (1 km/sec vs. 3 km/sec). The right-hand column, givin the harmful threshold in terms of spectral flux density, demonstrates how sensitive the system will be to interfering signals – at the lower frequencies, some astronomical objects have spectral flux densities above the harmful threshold!

3 Shielding Factors

Given the harmful threshold power flux density F_h at the antenna's input, the required minimum shielding S for a transmitter of known power P_t, distance r, and gain G_t can be calculated directly from Eq. 1:

$$S < 4\pi r^2 \frac{F_h}{P_t G_t}. \tag{12}$$

Using Eq. 11, we then obtain

$$S < 2.33 \times 10^{-21} \frac{T_{sys} \nu_G^{5/2} r^2}{G_r G_t P_t \sqrt{\tau_h}}. \tag{13}$$

In terms of a concrete example, take G_r and G_t to each equal one (isotropic emission and absorption), and P = 1 nanowatt, with r = 100 meters. For $\nu_G = 1.4$, $\tau_h = 9$, and $T_{sys} = 25$, the required shielding is: $S = -63$ dB.

A more physically meaningful form for the shielding equation can be derived from combining Eq. 12 and 4, and rearranging terms to give

$$S < \frac{1}{10} \left(\frac{4\pi r}{\lambda}\right)^2 \frac{k T_{sys} \Delta\nu}{P_t} \frac{1}{G_r G_t} \sqrt{\frac{1}{\Delta\nu\tau}} \tag{14}$$

In this expression the factors are:

- $\frac{1}{10}$ is the ratio of the interfering signal power to the noise power within the desired bandwidth
- $\left(\frac{4\pi r}{\lambda}\right)^2$ is the space attenuation – the fraction of the total radiated power which is intercepted by the receiving antenna at distance r, assuming isotropic emission and reception patterns.

- $\frac{kT_{sys}\Delta\nu}{P_t}$ is the ratio of the system noise power to the interfering signal's total radiated power, within the measurement bandwidth
- $\frac{1}{G_r G_t}$ is an amplification factor due to the directivities of the transmitting and receiving antennas
- $\sqrt{\frac{1}{\Delta\nu\tau}}$ is an enhancement factor due to the number of independent measures of the noise power.

This formulation leads naturally to the following, expressed in the decibel units commonly employed in engineering:

$$S_{dB} = -10 + R_{dB} + P_{dB} - G_{dB} - N_{dB} \tag{15}$$

where $R_{dB} = 20\log(4\pi r/\lambda)$, $P_{dB} = 10\log(kT_{sys}\Delta\nu/P_t)$, $G_{dB} = 10\log(G_r G_t)$, and $N_{dB} = 5\log(\Delta\nu\tau)$. For the system parameters described in the previous example, we find $R_{dB} = 75.4$, $P_{dB} = -87.9$, $G_{dB} = 0$, and $N_{dB} = 40.9$, giving $S_{dB} = -63.4$.

4 Interferometric Attenuation

The preceding analysis is applicable to total power measurements ('single-dish' in the parlance of radio astronomy). In interferometry, calculation of the harmful levels is complicated by the use of multiple antennas which are neither colocated nor have identical sidelobe patterns in the direction of the interferor. Furthermore, in imaging interferometry, the image is not formed from detected total power measurements, but from Fourier transforms of the coherence of the signals from the separated antennas. Analysis must then take into account the effects of the differential phase between the astronomical signals and the interfering signals.

Consider first a situation where a source of local interference is adjacent to two antennas, and ignore the effects of differential phase. (We can justify this by noting that the phase rates are so slow for the short baselines and low frequencies where local interference will be most important that the effects of phase rotation are small). The signal will be received by the two antennas from different azimuths and distances, so the interfering power will not be the same for each. Nevertheless, for the purpose of estimation, it should be sufficient to assume isotropic gains, and to use the geometric mean distance.

The more important effect in general is that due to differential phase. The phase of one of the elements of the interferometer is adjusted continuously to allow coherent integration of the desired astronomical signals (and thus cancel the effect of the rotating earth). Sources of emission from outside the direction of interest will then have a cyclical phase in the correlator product, which upon coherent averaging will reduce the signal's amplitude in the astronomical image. For a stationary source of interference, the frequency of the resulting phase rotation is that of the 'natural' fringe frequency:

$$\nu_F = -\omega_e u \cos\delta \tag{16}$$

where ω_e is the angular frequency of the earth (7.27×10^{-5} rad/sec), and u is the east-west component of the interferometer baseline, measured in wavelengths. For any given baseline, the averaging of the phase of the stationary interference will cause a reduction of the amplitude of the signal by a factor of approximately $1/N_{turns}$, where N_{turns} is the number of turns of phase over the averaging period. This reduction can be considerable (60 dB or more!) for long integrations with long baselines and short wavelengths.

However, it is not sufficient to simply assume the interfering signal will be attenuated in an image by the number of phase turns over the entire length of the observation, as the baselines rotate through the Fourier transform plane, so the details of the loci of these baselines within the transform plane, and the particulars of the Fourier transform process must be considered. Dick Thompson has analyzed this problem (IEEE Trans. Antennas Propag., AP-30, 450 – 456, 1982), and his results are given in Chapter 15 of Thompson, Moran and Swenson, where Fig 15.2 shows that an extra attenuation, varying from a few dB to over 40 dB, is obtained by the fringe-winding over that predicted in our 'total power' analysis, with the higher attenuation being applicable at high frequencies and long baselines, as expected. However, the analysis of TMS is also approximate, and the most realistic estimates of the reduction in the effects of interference to an imaging interferometer will be through actual simulations. Such a study is currently underway, and the results will be reported in a later memo.

The extra attenuation which comes from differential phase rotation in imaging interferometry is small in three important special cases – when observations are made with compact configurations, or at low frequencies, or for objects near the north celestial pole. In each of these cases, the differential phase rotation between the target source and a stationary interfering source is small – and in many future applications, all three will apply simultaneously!. It can therefore be misleading to count on significant attenuation from differential phase effects for the purposes of designing the electronics system, and it is thus appropriate to use the expressions derived for the 'total power' case for the required estimation of necessary shielding from locally generated interference.

5 In Situ Measures of the RFI

There will always be elements of uncertainty in the application of equations 11 and 15, primarily because the emission and reception gain factors, as well as shielding inherent in the present structures, are unknown. Generally the gains are assumed to be unity – meaning that the emission and reception patterns are taken as isotropic, and that there is no shielding already present. This is unlikely to be the case in some practical applications. For example, a transmitting piece of equipment located in the antenna vertex room is very close ($\sim$ 2 meters) to the antenna feeds. Application of the isotropic assumption will result in a very high required shielding. But the presence of the large antenna reflector between the equipment and feed in this case means that considerable shielding must already be present. On the other hand, the shielding between such a transmitting device and the feed of a nearby antenna, viewed when the antennas are pointed slightly off the vertical, will likely be much less.

When judging the potential of RFI from equipment which is to be installed in an antenna or in the vicinity, it seems reasonable that the best judgement of the interfering potential of the equipment will be found by measuring the interfering signal when the equipment is installed in its intended location. Such a measurement can be made in detected power (via an 'autocorrelation spectrum'), and/or in the cross-correlation spectrum. Is this a sufficiently sensitive procedure?

Imagine locating a transmitting device in the vertex room, with a measurement made of its emissions using the VLA's existing system and autocorrelator, using the spectral resolution shown in Table 1. If the signal is observed to be 10 dB above the rms noise in the autocorrelation spectrum (either in the antenna itself, or in an adjacent antenna), then it is immediately known that the device is 20 dB above the harmful level threshold (the extra 10 dB being from the definition of harmful threshold) for the time averaging used in the measurement. If the test measurement was for 10 seconds, then an additional decrease in interfering power of 18 dB will be needed to meet the harmful threshold definition, which is based on a 9 hour integration. In this example, 38 dB shielding of the device will be needed. Adjustments for differing resolution bandwidth or system temperatures are straightforward.

If the total power radiated by a trial transmitter within a bandwidth $\Delta\nu$ is known, then these simple relations can be used to measure the product G_tG_rS. Suppose such a transmitter, known to be emitting a power P_t at wavelength λ is located a distance r from the antenna feed, and an autocorrelation spectrum, with resolution $\Delta\nu$ is taken with equivalent system temperature T_{sys}. If the ratio of the detected signal to that of the total system power is R, (note that we are not utilizing the noise in the autocorrelation spectrum in this argument, as this is generally very small compared to the total power), then the product G_rG_tS can be shown to be

$$G_tG_rS = \frac{kT_{sys}\Delta\nu}{P_t}\left(\frac{\lambda}{4\pi r}\right)^2 R. \tag{17}$$

As an example, suppose a 1 nanowatt transmitter is located in the vertex room, 2 meters below the feed horn. If the observed signal from the transmitter is 1/10 of the power observed in the unaffected channels, and the resolution bandwidth is 3 kHz, the system temperature is 25K, and the wavelength 0.2m, the product G_rG_tS is readily computed to be 1.6×10^{-6} = -58 dB, indicating that very considerable effective shielding is already present. This value can be compared to the estimated shielding required from Eq. 15, to determine the additional shielding.