<u>The Value of B_1 for Interferometer Baseline 2</u>

C. M. Wade
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All of the interferometer reductions to date for baseline 2 have used B_1 = 5030.00, a value estimated from the survey data. Unfortunately, the calculation which led to this value contained an arithmetical error. The correct solution gives B_1 = 5080.10. This is about 1 per cent greater than the value used in the reductions. As a result, the derived phases contain a large and strongly declination-dependent error. The present report describes an attempt to find the correct value of B_1 from existing observations of standard small-diameter calibration sources.

I. Procedure

The method used to find B_1 rests on the assumption that B_3 remains nearly constant for fairly long periods of time. As it happens, there are occasionally periods of several days when B_3 remains practically the same. Most of the observations used in this report were made during these periods. Furthermore, we have used only observations made near the instrumental equator, since we then deal with fringes which are nearly vertical and hence are not shifted by possible changes in atmospheric refraction.

Suppose that we have observations of two calibration sources <u>a</u> and <u>b</u> at different declinations. The computed phases Φ_a and Φ_b will differ by an amount which depends on the error in the assumed value of B_1. It is easy to show that the values of B_1 which are consistent with $\Phi_b - \Phi_a$

are given by

$$B_1' = B_1 + \frac{(\Phi_b - \Phi_a)/360 + B_2\Delta h(\cos\delta_b - \cos\delta_a) + N}{\sin\delta_b - \sin\delta_a} \qquad (1)$$

where B_1 is the value assumed in the reduction and $N = 0, \pm 1, \pm 2, \ldots$. The phases are in degrees and Δh is in radians.

Successive values of B_1' differ by

$$\frac{1}{\sin\delta_b - \sin\delta_a} .$$

Thus there should be only one value of B_1' which is consistent with all pairs of calibration sources. Our method is therefore to find as many sequences of possible values of B_1' as we can, using different pairs of calibrators, and then to see where the solutions most nearly coincide.

II. Data

With single exception noted in Table 1, all of the data used here were reduced with $h = 4^h\ 49^m\ 41^s.36$, which is too high by $0^s.60$.

Table 1 lists the observed phase differences for various pairs of calibrators. The subscripts on the Φ's are the 3C numbers of the sources. The mean values of the first three differences in the table have their estimated uncertainties shown in parentheses.

TABLE 1

Quantity	Tape	Value	Mean
$\Phi_{48} - \Phi_{147}$	20	$143\overset{\circ}{.}00$	
	21	143.14	
	22	146.23	
	46	149.93	$145\overset{\circ}{.}58$ ($\pm$ 2°)
$\Phi_{48} - \Phi_{196}$	33	$74\overset{\circ}{.}66$	
	34	70.42	
	36	59.14	
	39	111.45	$78\overset{\circ}{.}92$ ($\pm$ 10°)
$\Phi_{48} - \Phi_{286}$	33	$171\overset{\circ}{.}16$	
	34	201.12	
	61*	238.16	$203\overset{\circ}{.}48$ ($\pm$ 40°)
$\Phi_{295} - \Phi_{147}$	40	$42\overset{\circ}{.}76$	-
$\Phi_{48} - \Phi_{216}$	45	$92\overset{\circ}{.}23$	-

*Reduced with $h = 4^h\ 49^m\ 40\overset{s}{.}76$. The phase difference given in the table has been adjusted to the value corresponding to $h = 4^h\ 49^m\ 41\overset{s}{.}36$.

We have no direct values of $\Phi_{196} - \Phi_{147}$, but the first two entries in Table 1 imply that

$$\Phi_{196} - \Phi_{147} = 66\overset{\circ}{.}66 \pm 12^\circ .$$

Table 2 gives further data involving 3C 380, which is not a calibration source. These imply that

$$\Phi_{196} - \Phi_{147} = 79\overset{\circ}{.}97 \pm 15^\circ$$

The two values are of comparable accuracy. Adopting their mean but being conservative with regard to its accuracy, we have

$$\Phi_{196} - \Phi_{147} = 73\overset{\circ}{.}32 \pm 15^\circ$$

which we shall adopt.

TABLE 2

Quantity	Tape	Value	Mean
$\Phi_{147} - \Phi_{380}$	22 31	$56^{\circ}\!.88$ 58.20	$57^{\circ}\!.54$ ($\pm$ 5°)
$\Phi_{196} - \Phi_{380}$	30 43	$142^{\circ}\!.97$ 132.04	137.51 ($\pm$ 8°)

The last two differences in Table 1 are single values and must be given low weight in the discussion.

IV. Solution

Using equation (1) and the above data, we find the possible solutions for B_1' given in Table 3.

TABLE 3

Sources	B_1'
48,147	5027.68 + 4.549 N $\pm$ 0.03
48,196	5028.45 + 4.938 N $\pm$ 0.14
48,286	5046.30 + 29.44 N $\pm$ 3.26
295,147	5033.60 + 36.04 N $\pm$?
48,216*	5027.41 + 7.242 N $\pm$?
196,147	5017.59 + 57.84 N $\pm$ 2.41

*The right ascension used in the reduction was too high by $0^{s}\!.4$, according to D. E. Hogg. This error has been taken into account in the above solution.

Figure 1 shows how the possible solutions are distributed in the vicinity of the survey value B_1 = 5080.10. There are possible solutions near 5073 and 5078. The latter appears to be more consistent with the data, and we tentatively adopt it. The solutions nearest 5078 are listed in Table 4.

TABLE 4

Sources	B_1'
48,147	5077.72 $\pm$ 0.03
48,196	5077.83 $\pm$ 0.14
48,286	5075.74 $\pm$ 3.26
295,147	5069.64 $\pm$?
48,216	5078.10 $\pm$?
196,147	5075.43 $\pm$ 2.41

Since the first solution in Table 4 has by far the smallest uncertainty, we adopt it as the result:

$$B_1 = 5077.72 \pm 0.03$$

It is still possible that the correct solution is 5073.17. It is very desirable to check the value used for $\Phi_{48} - \Phi_{216}$, since this would provide a critical test between the alternatives.

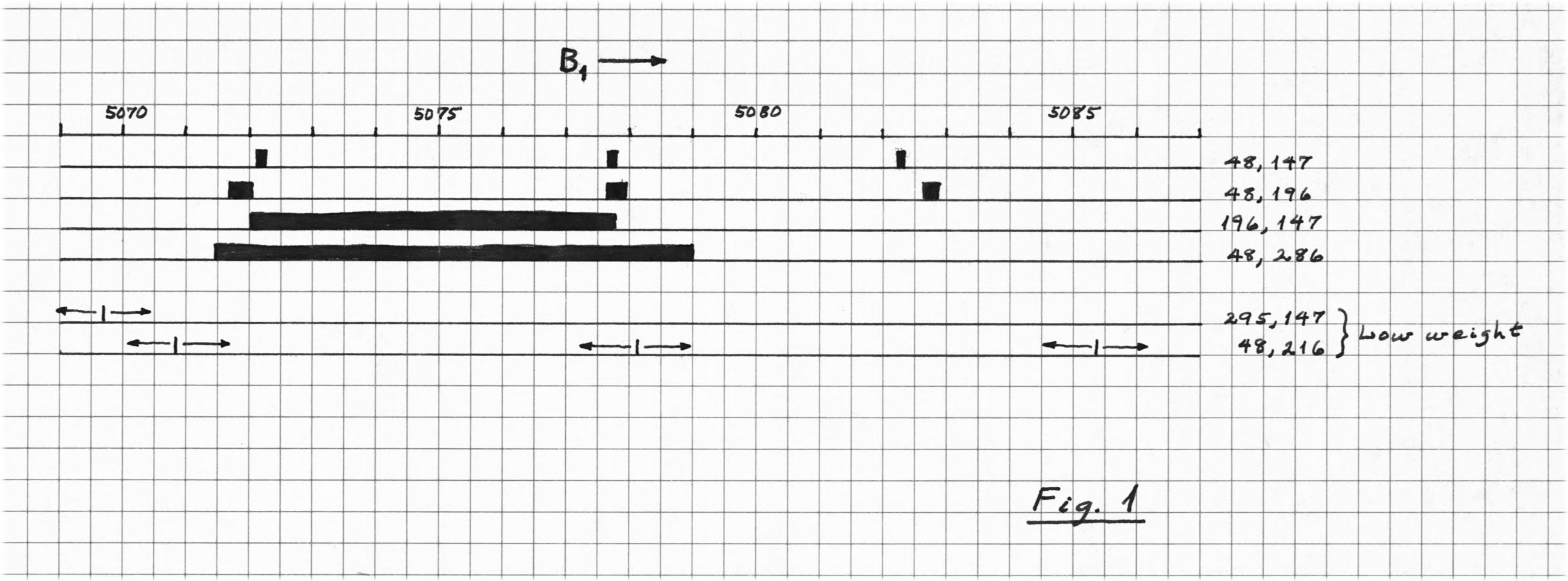
B_1
5070
5075
5080
5085
48,147
48,196
196,147
48,286
295,147
48,216
Low weight

Fig. 1

<u>Refraction</u>

1. <u>The Altitude Difference Effect</u>. A plane parallel layer of atmosphere inserted in front of the interferometer introduces the same phase change in one element as in the other. However, if the telescopes are not at the same height above sea level, there is a layer of air in front of the lower telescope which is not in front of the upper. If the southern telescope is H meters <u>lower</u> than the northern, an additional delay of $(n-1)\,\frac{H}{\lambda}$ sec Z is introduced into the southern arm of the interferometer. The interferometer function becomes

$$(f(H,\delta)= \cos \{2\pi\frac{D}{\lambda}(\sin d \sin \delta + \cos d \cos \delta \cos(H-h) + a+2\pi\frac{H}{\lambda}(n-1) \quad \sec Z\}$$

then, for a point source with d, δ, h, α well known

$$\Phi = a + 2\pi\,\frac{H}{\lambda}(n-1) \sec Z$$

so the calculated Φ's should be corrected by $-2\pi\,(n-1)\,\frac{H}{\lambda}$ sec Z

for $H = 30$ m, $\lambda = 10$ cm, $p = 920$ mb, $T = 280°$K and relative humidity = 50%, this term is 30° sec Z. The variation of this coeff from a hot and muggy summer day, T = 310°, relative humidity = 80% to a cold dry winter day T = 270°, relative humidity = 10% is from $46°\frac{1}{2}$ to $28°\frac{1}{2}$, a factor of more than $1\frac{1}{2}$, most of which is due to the amount of water vapor in the air. To sufficient accuracy,

$$n-1 = 10^{-4}\,(2.55 + 0.30\ e^{0.064\ T_W})$$

where T_W is the wet bulb temperature in °C. It would be nice if a wet bulb thermometer could be installed somewhere high on the structure of 85-2, as that is the air layer in question. For reference, the machine may wish to generate internally

$$H/\lambda = \sin \Phi\, B_1 + \cos \varphi\, B_2 \cos h$$

where φ is the latitude.

Of course

$$\cos Z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H.$$

2. <u>The Spherical Term</u>.

To sufficient accuracy, the total phase delay in the atmosphere is

$$\Delta\varphi = \Delta\varphi_o \sec Z$$

where
$$\Delta\varphi_o = \frac{2\pi}{\lambda} \int_o^\infty (n-1)\, dh$$

$$\sec Z_{85-2} = \sec Z_{85-1} + \sec^2 Z[(\cos \varphi \sin \delta - \sin \varphi \cos \delta \cos H)$$

$$(\varphi_{85-2} - \varphi_{85-1}) - \cos \varphi \cos \delta \sin H(\ell_{85-2} - \ell_{85-1})]$$

where ℓ is longitude. If A is the azimuth of the baseline, and R the radius of the earth, an additional delay of

$$\Delta\varphi = \Delta\varphi_o \sec^2 Z \frac{D}{R}[-(\cos \varphi \sin \delta - \sin \varphi \cos \delta \cos H) \cos A$$

$$- \cos \delta \sin H \sin A]$$

is introduced in 85-2.

For dry air

$$\Delta\varphi_o \frac{D}{R} = 1^\circ\!.15 \text{ D for D in Km.}$$

Barry Clark
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