The Values of B_1 and B_3 for Interferometer Baseline 2

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A preliminary value for B_1 has been given by Wade (1). This report gives a new determination of this parameter using all the available data for baseline 2. In addition, the value of B_3 during the observing period November 1964 - January 1965 is determined.

I. Procedure.

The procedure has been derived by Wade (1). The phase output of the calibration program is given by

$$K = \Delta B_3 - B_1 \Delta\delta \cos \delta - \Delta B_1 \sin \delta \tag{1}$$

where $\Delta\delta$ is the error in the assumed declination δ of the source

ΔB_1, ΔB_3 are the errors in the assumed values of B_1 and B_3.

The difference in mean phase of two sources of known declinations δ_1 and δ_2 is therefore

$$\Delta K = K_1 - K_2 = \Delta B_1 (\sin \delta_2 - \sin \delta_1)$$

The phase outputs of the calibration program are ambiguous by multiples of 2π; to allow for this, the expression used in the solution of ΔB_1 includes an integer $N = 0, \pm 1, \pm 2$, etc. Thus

$$\Delta B_1 = \frac{\Delta K}{\sin \delta_2 - \sin \delta_1} + \frac{N}{\sin \delta_2 - \sin \delta_1} \tag{2}$$

Finally, for a number of sources an incorrect declination had been assumed in the fringe reduction program. For these, the quantity $B_1 \Delta\delta \cos \delta$ was included in the K term, as indicated in equation (1).

II. The Mean Value of B_1 During the Observing Period.

The best data available for the determination of B_1 are presented in Table I. In general, only those observations which covered at least six hours of hour angles, and whose solutions have errors of less than 10° in phase were used. An exception was made for the solution 3C 119 - 3C 286, where two of the three tapes are of lower quality; the value of B_1 thus found was given lower weight in the final mean.

Column 6 of Table I gives the value of ΔB_1 computed from equation (2) with $N = 0$. Various values of the increment $\frac{N}{\sin \delta_2 - \sin \delta_1}$ were then applied to each source until the value of B_1', which is consistent with all pairs of sources, was found. This solution is given in column 7 of Table I. The pair 3C 147 - 3C 380 was used in the determination of the approximate value of B_1', since an uncertainty of 2π corresponds to an increment of 78 wavelengths. The widely separated sources, e.g. 3C 48 - 3C 147 then give the accurate value of B_1'.

TABLE I

The Determination of B_1 Baseline 2 Nov. 1964 - Jan. 1965

Sources	Tape No.	$K_1 - K_2$ (circles)	$\frac{1}{\sin \delta_1 - \sin \delta_2}$	B_1	ΔB_1	Best Solution B_1'
(1) 3C 147	49	+0.5186	78.446	5030.00	-40.68	5070.7
(2) 3C 380*	50	+0.6942	78.418	5030.00	-54.44	5084.4
	65	+0.5386	78.285	5030.00	-42.16	5072.2
	66	+0.0408	78.269	5077.72	- 3.19	5080.9
	67	-0.1425	78.261	5077.72	+11.15	5066.6
	*Phase output corrected for an error $\Delta\delta = -20''.6$ (-119°)					
	Mean $B_1' = 5074.6 \pm 3.3$ (m.e.)					
(1) 3C 48	34	+0.4383	29.437	5030.00	-12.90	5072.34
(2) 3C 286	60	.4353	29.394	5030.00	-12.80	5072.19
	61	.4714	29.391	5030.00	-13.86	5073.25
	70	+0.8628	29.383	5077.72	-25.35	5073.69
	Mean $B_1' = 5072.87 \pm 0.36$ (m.e.)					
(1) 3C 147	20	+0.5442	4.549	5030.00	- 2.48	5073.42
(2) 3C 48	21	.4703	4.549	5030.00	- 2.14	5073.08
	22	.4158	4.549	5030.00	- 1.89	5072.83
	46	.5589	4.549	5030.00	- 2.54	5073.48
	Mean $B_1' = 5073.20 \pm 0.15$ (m.e.)					
(1) 3C 216*	32	-0.0861	7.241	5030.00	+ 0.62	5072.82
(2) 3C 48	45	-0.0742	7.242	5030.00	+ 0.54	5072.92
	46	+0.0053	7.242	5030.00	- 0.04	5073.49
	Mean $B_1' = 5073.08 \pm 0.21$ (m.e.)					
	*Phase output corrected for an error $\Delta\delta$ $-2''$ (-14°)					
(1) 3C 119*	29	0.6072	6.534	5030.00	- 3.97	5073.17
(2) 3C 286	33	0.4978	6.534	5030.00	- 3.25	5072.46
	34**	0.5567	6.534	5030.00	- 3.64	5072.84
	Mean $B_1' = 5072.83 \pm 0.18$					
	*Phase corrected for $\Delta\delta$ - $8''.7$ (-57°) **Given double weight in mean					
(1) 3C 380*	22	-0.0053	4.828	5030.00	+ 0.03	5073.43
(2) 3C 48	32-33	-0.0222	4.829	5030.00	+ 0.11	5073.35
	33-34	+0.0497	4.829	5030.00	- 0.24	5073.50
	34-35	-0.0694	4.829	5030.00	+ 0.34	5073.13
	35-36	+0.0644	4.829	5030.00	- 0.31	5073.77
	Mean $B_1' = 5073.48 \pm 0.11$					
	*Phase corrected for $\Delta\delta$ $-20''.6$ (-119°)					

Overall mean (weights 0, 1, 2, 2, 1, 2) 5073.15 ± 0.09

The mean value of B_1 for the period is

$$B_1 = 5073.15 \pm 0.09 \text{ (m.e.)}$$

Here the individual determinations have been given weights 0, 1, 2, 2, 1 and 2 respectively. Since the mean value of B_2 was

$$B_2 = 12493.544 \pm 0.005,$$

$$\frac{D}{\lambda} = 1.348427 \times 10^{+4} \quad (\text{Since } B_1^2 + B_2^2 = 1.818254 \times 10^8)$$

$$\cos d = 0.926527_3$$

$$\sin d = 0.376227_3$$

$$d = 22° \; 06' \; 01''$$

It has been shown previously that D/λ varied during the observing period. It is not possible to see the effect on B_1 itself, because of the large uncertainties in the individual determinations. However, an approximate correction, found by scaling the error ΔB_2 by the ratio $\frac{B_1}{B_2} = 0.40606$, can be applied to the mean value of B_1. The adopted values of the error ΔB_1 as a function of tape number are given in Table II.

TABLE II

The Error ΔB_1 as a Function of Tape Number

Tape No.	Error in B_1 Due to LO Drift Along	Total Error ΔB_1
20	-0.02	-43.17
21	-0.02	-43.17
22	-0.03	-43.18
23	-0.02	-43.17
24	-0.02	-43.17
25	-0.02	-43.17
26	-0.02	-43.17
27	-0.02	+ 4.55
28	-0.02	-43.17
29	-0.02	-43.17
30	-0.01	-43.16
31	-0.01	-43.16
32	-0.01	-43.16
33	-0.01	-43.16
34	-0.01	-43.16
35	-0.01	-43.16
36	-0.01	-43.16
37	-0.01	-43.16
38	-0.01	-43.16
39	0.00	-43.15

TABLE II (Continued)

Tape No.	Error in B_1 Due to LO Drift Along	Total Error ΔB_1
40	0.00	-43.15
41	0.00	-43.15
42	0.00	-43.15
43	0.00	-43.15
44	0.00	-43.15
45	0.00	-43.15
46	+0.01	-43.14
47	+0.01	-43.14
48	+0.01	-43.14
49	+0.01	-43.14
50	+0.01	-43.14
51	+0.01	-43.14
52	+0.01	-43.14
56	+0.01	-43.14
57	+0.01	-43.14
58	+0.01	-43.14
59	+0.01	-43.14
60	+0.01	-43.14
61	+0.01	-43.14
62	+0.01	+ 4.58
63	+0.02	+ 4.59
64	+0.02	+ 4.59
65	+0.02	-43.13
66	+0.02	+ 4.59
67	+0.02	+ 4.59
68	+0.02	+ 4.59
69	+0.02	+ 4.59
70	+0.02	+ 4.59
71	+0.02	+ 4.59
72	+0.02	+ 4.59

III. The Value of ΔB_3 During the Observing Period.

The parameter ΔB_3 (equal to $- B_3$ for all tapes) is obtained from equation (1) using the values of ΔB_1 from Table II. Only the observations of the calibrators which covered six hours of hour angle and for which the rms scatter in phase is less than 10° have been included. The values of ΔB_3 are given in Table III, along with the uncertainties in K. The uncertainty in B_1 of 0.09 wavelengths produces an uncertainty in ΔB_3 as given at the bottom of Table III. It should be noted, however, that these uncertainties are coupled for the various sources, and are therefore essentially an uncertainty in the zero point of the plot.

Figure 1 gives the value of ΔB_3 as a function of tape number for all of the calibration sources. This figure is probably the best summary of the long-term phase stability of the system during the observing period November 1964 - January 1965.

TABLE III

Summary of Values of ΔB_3

Tape	3C 48	3C 147	3C 196	3C 286	3C 380
20	+ 51 ± 3	+ 72 ± 1			
21	- 01 ± 2	- 07 ± 3			
22	- 05 ± 1	- 31 ± 3			+ 16 ± 2
23		+ 09 ± 3			
24		- 94 ± 1			
25		-228 ± 1			
26					-222 ± 3
27					
28					
29	-260 ± 6			-197 ± 10	
30			-151 ± 3		-161 ± 2
31		-143 ± 1			
32		-135 ± 2			-144 ± 9
33	-160 ± 4			-156 ± 7	-143 ± 5
34	-173 ± 2		-181 ± 5	-162 ± 6	-159 ± 4
35	-162 ± 2		-157 ± 4		-138 ± 3
36	-184 ± 3		-129 ± 7		-157 ± 4
37					
38			-174 ± 11		
39	-222 ± 2				-113 ± 3
40					-158 ± 3
41	-208 ± 7				
42					
43	- 04 ± 4		- 82 ± 3		- 57 ± 2
44		- 46 ± 1			
45	- 39 ± 1	- 26 ± 3			
46	+ 45 ± 1	+ 73 ± 1			
47		+ 80 ± 1			
48		+ 63 ± 2			
49		+ 71 ± 2			+ 82 ± 2
50		+ 98 ± 4			+ 47 ± 3
51		+ 97 ± 4			+ 57 ± 3
52					
56				+108 ± 2	
57	+ 52 ± 1				
58	+ 54 ± 10			+ 76 ± 3	
59	+ 47 ± 4				
60	+ 24 ± 2			+ 35 ± 1	

TABLE III (Continued)

Tape	3C 48	3C 147	3C 196	3C 286	3C 380
61	+ 44 ± 1			+ 28 ± 3	
62	+ 25 ± 2			+ 30 ± 3	
63	+ 35 ± 2				
64		+ 09 ± 2			+ 22 ± 4
65		+ 10 ± 1			+ 14 ± 4
66		+ 27 ± 1			
67		+ 09 ± 2			+ 40 ± 3
68		+ 15 ± 5			+ 20 ± 3
69					
70	+ 21 ± 2			+ 14 ± 1	
71	+ 26 ± 2				
72		+ 22 ± 1			
Uncertainty due to error in ΔB_1	18°	25°	24°	17°	24°

IV. The determination of Source Declination from the Mean Phase.

The phase drift method for the determination of source declination becomes inaccurate for sources near the equator, since the correction determined is a function of the cosecant and cotangent of the declination. The declination correction can, in principle, be determined much more accurately from the observed phase, since, from equation (1)

$$\Delta\delta = - \frac{K - \Delta B_3 + \Delta B_1 \sin\delta}{B_1 \cos\delta} \qquad (3)$$

Thus, for example, an uncertainty in the quantity $K - \Delta B_3$ of 20° leads to an uncertainty of only 2" in the declination correction for a source within 5° of the equator.

There were three such sources included in the position program run during this observing period. The corrections in declination, as determined using equation (3), are given in Table IV. Table V gives the comparison between the declinations obtained by this method and those of the phase drift program. The errors are smaller, as expected, and moreover, the position of 3C 298 is in much better agreement with the optical position. The position of 3C 459 is very uncertain, since the one observation was poor.

TABLE IV

Source Declination from Mean Phase

Source	Tape	$K - \Delta B_3$ (circles)	$\Delta B_1 \sin \delta$	$\frac{1}{B_1 \cos \delta}$	Primary Value $\Delta\delta$ radians	"Best" Value $\Delta\delta$ (secs)	Mean $\Delta\delta$
273	56	−0.128	−1.692	1.973×10^{-4}	$+1.62 \times 10^{-4}$	− 7.3	
	57	−0.010	−1.692	1.973×10^{-4}	1.38	−12.1	
	58	−0.273	−1.692	1.973×10^{-4}	1.90	− 1.4	-6.9 ± 3.1
298	27	+0.486	+0.526	1.985×10^{-4}	-0.024×10^{-4}	− 0.5	
	33	+0.091	−4.991	1.985	−1.77	− 4.5	
	34	+0.027	−4.991	1.985	−1.91	− 1.5	-2.2 ± 1.2
459	40	+0.331	−2.934	1.976×10^{-4}	+1.19	−16.2	

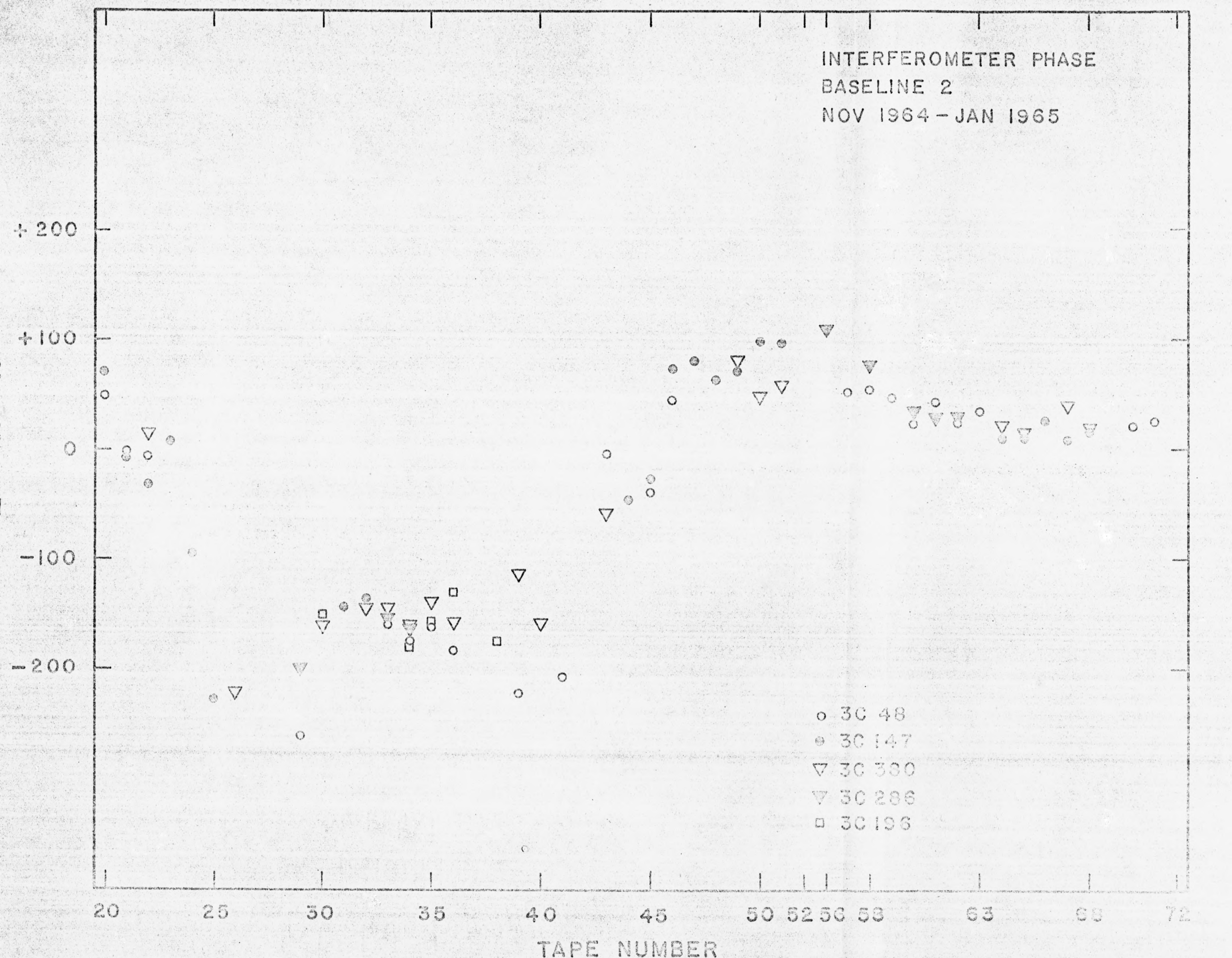
TABLE V

Source	$\delta_R - \delta_o$ Phase Drift	$\delta_R - \delta_o$ Mean Phase
3C 273	$+ 6.6 \pm 7.4$	$+ 6.9 \pm 3.1$
298	-12.1 ± 4.4	$+ 2.2 \pm 1.2$
459	+ 9 ?	+15.5(+25.3)

It is clear that the phase characteristics of the interferometer can be measured accurately enough to allow meaningful positions to be determined from the observed phase of the source. This is most important, since it means that sources near the equator can be measured with the same accuracy as is achieved elsewhere with the phase-drift method.

References

1) Wade, C. M. "The Value of B_1 for Interferometer Baseline 2", NRAP Report January 1965.


INTERFEROMETER PHASE
BASELINE 2
NOV 1964 - JAN 1965
+200
+100
0
-100
-200
3C 48
3C 147
3C 380
3C 286
3C 196
20
25
30
35
40
45
50 52 56 58
63
68
72
TAPE NUMBER