Determination of Relative Coordinates of the Interferometer Stations from Measured Baseline Components

C. M. Wade

July 26, 1972

1. Baseline Constants: We shall use the following values, assuming that one of the antennas is 85-1 and the other is on the indicated station:

Station	B_x	B_y	B_z
12	-3009.730	- 9528.510	4066.190
15	-3774.070	-11910.240	5072.740
18	-4533.301	-14292.733	6083.561
19	-4785.520	-15086.942	6421.548
21	-5280.870	-16675.190	7104.060
24	-6048.500	-19056.590	8107.530
27	-6812.280	-21439.990	9115.650

Values tabulated by M. Ewing, July 1972.

2. The North, East, and Zenith Components of the Baselines: We have the elementary transformation

$$\begin{pmatrix} B_N \\ B_E \\ B_Z \end{pmatrix} = \begin{pmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & -1 & 0 \\ \cos\phi & 0 & \sin\phi \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

where ϕ is the latitude. The precise choice of ϕ is not critical; we use the mid-points of the lines joining 85-1 to the various stations, as follows (data from S. C. Smith, July 1972):

Station	$\bar{\phi}$	Station	$\bar{\phi}$
12	$38\overset{\circ}{.}4332$	21	$38\overset{\circ}{.}4314$
15	$38\overset{\circ}{.}4326$	24	$38\overset{\circ}{.}4307$
18	$38\overset{\circ}{.}4320$	27	$38\overset{\circ}{.}4301$
19	$38^\circ 4318$		

B_x, B_y, and B_z are given above in wavelengths at 2695 MHz. For the present discussion it is more convenient to have B_N, B_E, and B_Z in meters. The conversion factor from wavelengths to meters is

$$\frac{2.997925\text{x}10^8}{2695\text{x}10^6} = 0.1112402597.$$

We find:

Station	B_N	B_E	B_Z
12	562.435 m	1059.954 m	18.903 m
15	702.996	1324.898	21.892
18	843.576	1589.927	25.619
19	890.468	1678.275	27.008
21	984.193	1854.952	31.029
24	1124.714	2119.860	33.508
27	1265.373	2384.990	36.643

These are based on the stations; i.e., 85-1 is north, east, and above each station.

3. <u>Conversion to Latitude and Longitude Differences</u>: According to the American Ephemeris and Nautical Almanac, one has for the adopted spheroid at sea level (in meters):

1° of latitude $= 111133.35 - 559.84 \cos 2\phi + 1.17 \cos 4\phi$

1° of longitude $= 111413.28 \cos \phi - 93.51 \cos 3\phi + 0.12 \cos 5\phi$

Radius vector $= 6367489.8 + 10692.6 \cos 2\phi - 22.4 \cos 4\phi$

Adopting $\phi = 38^\circ\!.432$, we have for Green Bank:

$1''$ of latitude $= 30.83474$ m

$1''$ of longitude $= 24.25418$ m

Radius vector $= 6369939.9$ m

These are sea level values. The latitude and longitude arcs still need correction by the factor

$$\frac{\text{Radius vector + 840 m}}{\text{Radius vector}} = 1.000131869$$

to make them suitable for the elevation of the interferometer (assumed to be 840 m).

We obtain finally for Green Bank:

$1''$ of latitude $= 30.83881$ m

$1''$ of longitude $= 24.25738$ m.

Now we can convert B_N and B_E to the equivalent differences of latitude and longitude, respectively:

Station	$\Delta\phi$	$\Delta\lambda$
12	$-18''\!.238$	$+43''\!.696$
15	$-22''\!.796$	$+54''\!.618$
18	$-27''\!.354$	$+65''\!.544$
19	$-28''\!.875$	$+69''\!.186$
21	$-31''\!.914$	$+76''\!.470$
24	$-36''\!.471$	$+87''\!.390$
27	$-41''\!.032$	$+98''\!.320$

Here we have adjusted the signs so that $\Delta\phi$ and $\Delta\lambda$ are in the sense [Station - (85-1)].

4. <u>Comparison with the 1964 Survey</u>: A ground survey of the interferometer baseline was made eight years ago by Fagerlin, Smith, and Bream (NRAO Report, August 1964). We compare here the baseline lengths and azimuths they found with the ones we compute from B_N, B_E, and B_Z:

$$\text{Azimuth from 85-1 to station} = 180° + \arctan\left(\frac{B_E}{B_N}\right)$$

$$\text{Length in meters} = (B_N^{\,2} + B_E^{\,2} + B_Z^{\,2})^{1/2}.$$

Station*	Present Report		Fagerlin et al.		Differences	
	Azimuth	Length	Azimuth	Length	ΔA	ΔL
12	242°02'55"	1200.080 m	242°02'46"	1200.122 m	9"	-0.042 m
15	02'58"	1500.012	02'49"	1500.067	9"	-0.055
18	03'02"	1800.042	02'57"	1800.097	5"	-0.055
19	03'01"	1900.071	--	--	--	--
21	03'02"	2100.106	02'58"	2100.144	4"	-0.038
24	03'05"	2399.981	02'52"	2400.043	13"	-0.062
27	03'05"	2700.128	--	2700.206	--	-0.078

*Station 19 did not exist in 1964.

The differences are clearly systematic, and of pretty moderate magnitude.

It should be noted that the Fagerlin azimuths given above are the values deduced directly from the field observations. The so-called final values given on page 2 of the Fagerlin report are 30"-60" smaller; they include an effort to adjust for meridian convergence which evidently went sour.

5. <u>Comparison with Sidney Smith's Tabulation of Geodetic Positions of NRAO Telescopes (12 July 72)</u>: S. C. Smith has derived latitudes and longitudes for the NRAO telescopes from the Geonautics survey of a few years ago, tied into the first-order monuments on site which were established by the U.S. Air Force.

Smith's results for the latitude and longitude differences of the interferometer stations from 85-1 are tabulated below, together with the values implied by Smith's data for B_N, B_E, and B_Z, and the corresponding azimuths and baseline lengths.

Station	$\Delta\phi$	$\Delta\lambda$	B_N	B_E	B_Z	Azimuth	Length
12	-17".818	+43".996	549.486	1067.228	19.3	242°45'26"	1200.5 m
15	-22".274	+54".989	686.904	1333.889	22.3	45'11"	1500.6
18	-26".732	+65".986	824.383	1600.647	26.0	45'00"	1800.7
19	-28".216	+69".647	870.148	1689.454	27.3	44'58"	1900.6
21	-31".190	+76".982	961.862	1867.382	31.3	44'52"	2100.8
24	-35".644	+87".974	1099.219	2134.019	33.7	44'50"	2400.7
27	-40".101	+98".963	1236.667	2400.583	36.8	44'41"	2700.6

The azimuths and baseline lengths are badly at variance with both the radio measurements and the 1964 survey (see section 3 above), which agreed with each other fairly well. The differences from the radio measurements are as follows (SCS-Radio):

Station	ΔA	ΔL	$\Delta A/L$ ("/m)
12	42'31"	0.4 m	2.126
15	42'13"	0.6	1.689
18	41'58"	0.7	1.399
19	41'57"	0.5	1.325
21	41'50"	0.7	1.195
24	41'55"	0.7	1.048
27	41'36"	0.5	0.924

The baseline lengths are different by a constant 0.6 $\pm$ 0.1 meters. The run of ΔA and $\Delta A/L$ suggests that a "line of best fit" through the stations would not pass through 85-1, and further that the line is wrong in azimuth by about 2/3 of a degree. You could get results this accurate with a good Boy Scout compass.

Let us see how the differences ΔB_N, ΔB_E, ΔB_Z (SCS-Radio) run:

Station	ΔB_N	ΔB_E	ΔB_Z
12	-12.949 m	7.274 m	0.4m
15	-16.092	8.991	0.4
18	-19.193	10.720	0.4
19	-20.320	11.179	0.3
21	-22.331	12.430	0.3
24	-25.495	14.159	0.2
27	-28.706	15.593	0.2

There is clearly a difference which increases progressively with baseline length. For the radio measurements,

$$B_E = -L \sin A$$

$$B_N = -L \cos A$$

where L is the baseline length in meters and A is the azimuth from 85-1 to the station. Now, assume that Smith's data are made from a different origin, displaced by (b_E, b_N) from the actual position of 85-1. Assume further that his azimuths are consistently too high by an amount ΔA. Then his values for B_E and B_N are given by

$$B_E' = b_E - L \sin (A+\Delta A)$$

$$B_N' = b_N - L \cos (A+\Delta A).$$

We have then

$$B_E = B_E' - B_E = b_E - L [\sin (A+\Delta A) - \sin A]$$

$$B_N = B_N' - B_N = b_N - L [\cos (A+\Delta A) - \cos A]$$

We already know, from the above discussion, that ΔA is small, of the order of a degree or less. Hence these relations reduce to

$$\Delta B_E = b_E - L \cos A \cdot \Delta A$$

$$\Delta B_N = b_N + L \sin A \cdot \Delta A$$

where ΔA is in radians.

A least square solution with the data in the table above gives:

$$b_E = +0.593 \pm 0.147 \text{ meters}$$

$$b_N = -0.351 \pm 0.137 \text{ meters}$$

$$\Delta A \cos A = -0.0056027 \pm 0.0000735 \text{ radians}$$

$$\Delta A \sin A = -0.0104874 \pm 0.0000687 \text{ radians.}$$

The magnitude of ΔA is

$$|\Delta A| = \sqrt{(\Delta A \cos A)^2 + (\Delta A \sin A)^2}\ ,$$

and its sign is evident from the sign of $\Delta A \sin A$ (remembering that A is in the third quadrant). We have then

$$\Delta A = +0°40'53'' \pm \sim 20''.$$

Thus, if the true azimuths are about 242°03', Smith's positions as given would lie along a line on an azimuth of about 242°44'.

To summarize: The discrepancy between Smith's data and the radio measurements can be reconciled if

(a) his adopted position of 85-1 is 0.593 m east and 0.351 m south of the true position relative to the observing stations; and

(b) his azimuth frame is rotated 0°40!9 clockwise with respect to true north.

We will not speculate on the sources of error. Perhaps the Geonautics survey is suspect. It is clear that some checking is needed.

Finally, the trend of ΔB_Z with baseline length needs comment. The values given in the table on page 6 suggest that there is a progressive error. This actually is not the case, because

(a) Smith's values of B_Z are referred to sea level, whereas

(b) the radio values of B_Z are referred to the horizontal tangent planes passing through the individual stations. These planes are not parallel, owing to the curvature of the earth.

In order to remove the curvature effect, and thereby to make legitimate a comparison of the two determinations, a correction $\underline{\delta}$ should be applied to the radio values of B_Z. In other words, the elevations should be referred to sea level instead of to the different tangent planes. It is easy to show that

$$\delta = L^2/2R$$

where $L << R$ (δ, L, and R in meters).

The correct expression for ΔB_Z is then

$$\Delta B_Z = B_Z' - B_Z + \delta$$

where B_Z' is the Smith value and B_Z is the radio value derived above. The following table gives the correct values:

Station	δ	ΔB_Z
12	0.11 m	0.5 m
15	0.18	0.6
18	0.25	0.7
19	0.28	0.6
21	0.35	0.7
24	0.45	0.7
27	0.57	0.8

There is clearly no systematic change with baseline length. Instead, there is a constant difference of about 2/3 of a meter, in the sense that 85-1 is high with respect to the stations. Perhaps the origin of this effect lies in the differences between the mountings of the fixed and movable antennas.