

# Improvements to the GBT Refraction Model

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## Abstract

The refraction model we have been using for the GBT was first proposed in GBT Memo 112 (Maddalena, 1994). In the model, there is a scaling coefficient that is only roughly known and must be determined empirically. We have used the results of GBT pointing observations to determine the coefficient. There are a few modifications made to the refraction model in the antenna control software that are of the order of 0.5". The antenna software has been reviewed and a few errors eliminated that were of the order of 1".

## Review of Refraction Model

Anyone who is interested in the full details of the refraction model should obtain a copy of GBT Memo 112. Briefly, the GBT model is:

$$\Delta E = E_{obs} - E_{true} = C \cdot (n_o - 1) \cdot f(E_{obs}). \quad (1)$$

$E_{obs}$  and  $E_{true}$  are the observed (apparent) and true (airless) elevations,  $C$  is a constant that has to be determined empirically and is approximately of order  $2 \times 10^5$ ". The model uses local ground-level weather data to calculate the surface index of refraction,  $n_o$ , which is the only weather dependent term in the model. The function  $f$  describes how refraction changes with  $E_{obs}$  and is not dependent upon weather conditions. The GBT model uses for  $f$ :

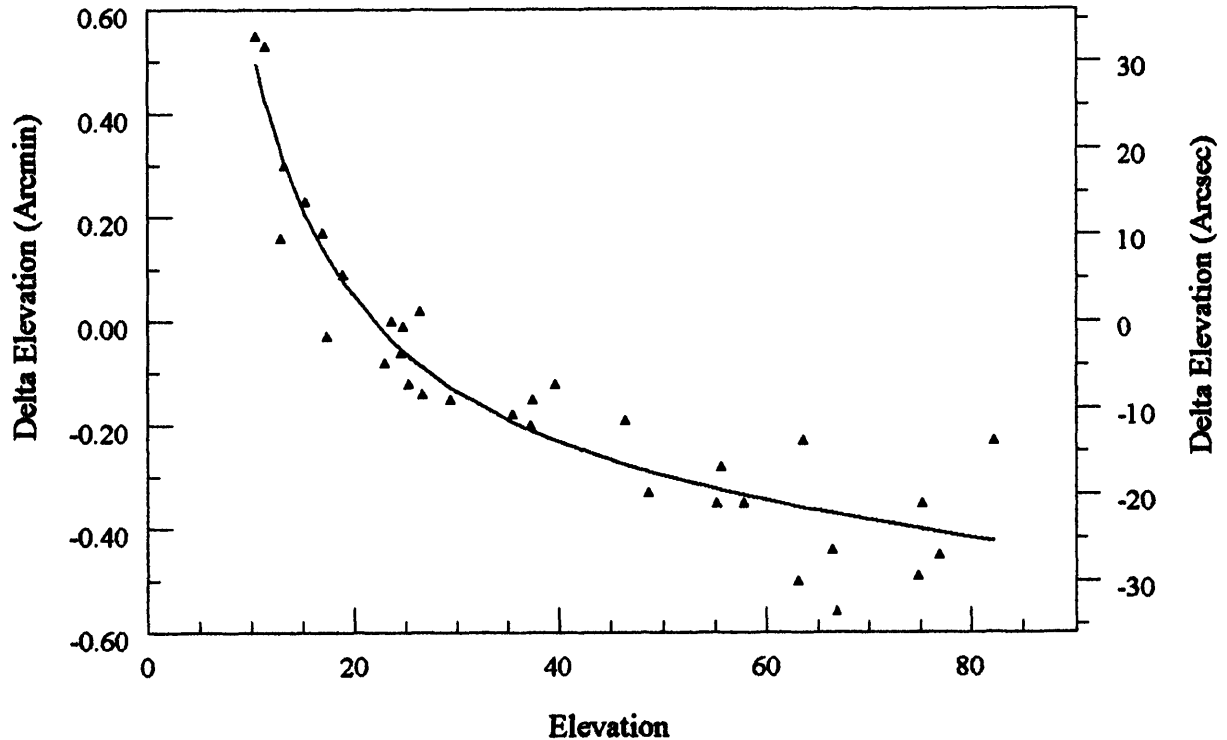
$$R = \cot \left( E_{obs} + \frac{7.31}{4.4 + E_{obs}} \right) \quad (2)$$

$$f(E_{obs}) = R - 0.06 \cdot \sin(14.7R + 13)$$

## Determination of Refraction Constant, C

If we assume that equation 1 and 2 are an adequate model for refraction, then we can use pointing observations with the GBT to determine the refraction constant,  $C$ , the only term in the model that is tunable. We used pointing observations from August 20 to determine  $C$ . The data set, originally taken to determine box offsets, has a significant number of observations at a wide range of elevations taken under almost constant weather conditions. The observations consisted of using in a standard way the "GBT Observe" PEAK procedure which determines azimuth and elevation pointing offsets. We used the 1-2, 2-3, and 8-10 GHz receivers, exchanging receivers

and repeating observations every hour or so. The average weather conditions were overcast with a temperature of 14 C, total barometric pressure of 927 mBar, humidity of 93% and  $(n_0-1) = 0.00031740$ .



**Figure 1: Elevation pointing offsets. Triangles indicate the measured pointing offsets and the smooth curve a fit of the refraction model to the data.**

Figure 1 is the measured elevation pointing offsets as a function of elevation. The telescope was using a pointing model with an rms accuracy of 8" but which used only a rough refraction constant of  $C=2 \times 10^{5''}$ . The shape of the pointing offsets in Figure 1 is almost surely due to not having the correct value for  $C$ . One will note that the pointing offsets don't go to zero at the zenith but that the average value of the offsets is approximately zero. Since the pointing model was determined from observations in March that used the rough value for  $C$ , the all-sky pointing fitting program tried to compensate for the inappropriate value of  $C$  by modifying the  $d_{0,0}$  or zero offset coefficient in the elevation pointing model.

Essentially, the model of Equation 1 can be fit to the data in Figure 1 to determine a more accurate value for  $C$ . The actual form of the fit was:

$$E_{offset} = (n_0 - 1) \cdot \Delta C \cdot f(E_{obs}) + A.$$

Here, the fitted parameter  $\Delta C$  is the difference between the empirically derived (better) value for  $C$  and the value that was in place in the control system on the day the data were taken. The parameter  $A$  is necessary because of the issue with the  $d_{0,0}$  pointing coefficient discussed above.

The result of the fit implies that a better value for  $C$  is  $(2.35 \pm 0.14) \times 10^5$ ". The smooth curve in Figure 1 shows the results of the fit. The residuals of the fit have an rms of 5.3", suggesting that the model of GBT Memo 112 is sufficiently accurate for our current needs.

The new value for  $C$  has been in the control system since December 2001. A preliminary look at 10 GHz pointing measurements made since then suggests that the new value is appropriate.

## Changes to the Refraction Model

Equation 1 is used to calculate  $E_{true}$  from  $E_{obs}$ . But, how does one calculate  $E_{obs}$  from  $E_{true}$ ? GBT Memo 112 suggests:

$$\Delta E = E_{obs} - E_{true} = C \cdot (n_o - 1) \cdot g(E_{true}). \quad (3)$$

The function  $g(E_{true})$  has a similar form to the function  $f(E_{obs})$  but  $g$  describes how refraction changes with  $E_{true}$  and, like  $f$ , is not dependent upon weather conditions. Essentially,  $f(E_{obs}) - g(E_{true})$  should be close to zero. If we use the formulations of  $g$  from GBT memo 112, then  $f(E_{obs}) - g(E_{true})$  can get as large as 1". Using a non-linear, least squares fitting routine, we have been able to refine the coefficients in Equation 7 of GBT Memo 112 so as to reduce  $f(E_{obs}) - g(E_{true})$  to under 0.5" at all elevations. The new formulation of  $g$ , implemented in the antenna control system since early December 2001, is:

$$S = 1.02 \cdot \cot \left( E_{true} + \frac{10.3}{5.11 + E_{true}} \right) \quad (4)$$

$$g(E_{true}) = S - 0.1185 \cdot \sin(14.69S + 7.57)$$

While investigating refraction, we also looked at the antenna control software for refraction and compared its results to an independent implementation of GBT Memo 112 written in Tcl. We found a few minor coding errors in the control system that at most would have altered the pointing by 1". Antenna code changes were made at the same time the new value of  $C$  and new  $g$  function were implemented.

## Summary

Using pointing data from the GBT, we have been able to refine the refraction coefficient  $C$  from GBT Memo 112. Our best estimate for  $C$  is  $(2.35 \pm 0.14) \times 10^5$ ", significantly different from the original estimate of  $2 \times 10^5$ ". All indications are that the refraction model of memo 112 and the new coefficient are adequate for our current needs. We have also derived a better form for the refraction function  $g(E_{true})$  which should increase the self consistency of the refraction model to better than 0.5" at all elevations. We have also reviewed the antenna refraction control software and found a few problems that were of the order of 1". All changes have been made to the antenna control software as of the beginning of December 2001.

