GBT Memo No. 32

Telescope Consultation

Dr. Sebastian von Hoerner
Krummenackerstr.186
7300 Esslingen
WEST GERMANY
Tel.(0711) 370 1900

TOLERANCE OF PANEL LOCATION ON MOLD

Summary

One of the questions asked at our December meeting is: Must the panels be placed very accurately on the mold, regarding rotation and translation? Or could one mold even be used for more than one ring of panels, if the translation tolerance is large enough?

For the clear aperture GBT (D=100 m, F=60 m), we demand a tolerance of 24 μm for the rms deviation of a panel shape (≈ 1/3 of panel rms of 71 μm). All panels than may be rotated up to ± 9.1°, or may be shifted (radially) up to ± 123 cm. Thus no special care is needed. One mold for two rings (shift of half a panel) would then be possible for all rings, but one mold for three rings only for a few of the innermost rings.

1. Panel Shape

We use coordinates X, Y, Z with the origin at the paraboloid vertex, Fig.1, with

$$Z = (X^2 + Y^2)/4F. \tag{1}$$

We call Po the location of a panel center, at Xo, Yo=0, where

$$Z_o = X_o^2/4F, \qquad Z_o' = \tan\alpha = X_o/2F. \tag{2}$$

We use a second system U, V, W at Po, such that W = 0 is the tangential plane to the paraboloid, and V = Y. The shape of the panel is then described as W(U,V). The transformations are

$$\begin{aligned} X - X_o &= U\cos\alpha - W\sin\alpha, \\ Y &= V, \\ Z - Z_o &= U\sin\alpha + W\cos\alpha. \end{aligned} \tag{3}$$

Inserting (3) into (1), and neglecting W^2, yields after some doing finally the panel shape

$$W(U,V) = \frac{(V^2 + U^2\cos^2\alpha)\cos\alpha}{4F + 2U\cos^2\alpha\sin\alpha} \tag{4}$$

The last term in the denominator gives the slight change of the curvature along the length of the panel and may be neglected in the following. The radius of curvature is in general

$$R = (1 + W'^2)^{3/2}/W'', \tag{5}$$

and at Po, the panel center, we get

$$R = 2F/\cos^3\alpha \quad \text{along U-axis, panel length,} \tag{6}$$
$$R = 2F/\cos\alpha \quad \text{along V-axis, panel width.} \tag{7}$$

2. Shape Tolerance

For the manufacturing tolerance of the panels we derived 71 μm rms (equation (1), GBT-Memo 7). We now demand that any additional deviation, from misplacements of panels on their mold, is negligible, say, 71/3 ≈ 24 μm rms. Where $(1+(1/3)^2)^{1/2} = 1.054 \approx 5\%$.

Panels will be adjusted at the corners. For a parabolic shape, the rms deviation is about 0.73 of the maximum deviation at the center. Since W in (4) is zero at center, the maximum deviation of W to be tolerated, at panel edge, is 24/0.73 = 33 μm:

$$\sigma \le 33\ \mu\text{m} = \text{max deviation of W.} \tag{8}$$

3. Panel Rotation

We define a circle on the U,V plane, of radius R and $\beta=0$ along the U-axis, with $U = R\cos\beta$ and $V = R\sin\beta$. The height above the plane then can be written from (4), with $\sin^2\beta = \frac{1}{2}(1-\cos 2\beta)$, as

$$W = (R^2/8F)\cos\alpha\,(2\cos^2\alpha + \sin^2\alpha\,[1-\cos 2\beta]), \tag{9}$$

and the deviation $\sigma = W(\beta+\delta)-W(\beta)$, after rotation δ, is

$$\sigma = (R^2/8F)\cos\alpha\,\sin^2\alpha\,[\cos 2\beta - \cos(2\beta+2\delta)]. \tag{10}$$

We use F=60m, $\alpha=39.8°$ at the rim where (10) is largest, and use U=1m, V=0, $\beta=0$ at the end of the long center line where it matters most. We demand (8) and have for a panel length of 2 m:

$$\sigma = 655.8\ \mu\text{m}\,(1 - \cos 2\delta) \le 33\ \mu\text{m}, \tag{11}$$

from which

$$\delta \le 9.1°, \tag{12}$$

which shows that no special care is neeed regarding rotation.

The deviation σ is largest at the panel corners, for example σ = 121 μm for a rotation of δ = 5°. But two diagonally opposite corners will go down, the other two up, and this twist (of the manufactured panel) will be removed by any corner adjustment (at plate testing, or after erection) and thus may be tolerated.

Panels have a small bending stiffness. I think I remember that a panel corner sagged down by more than a millimeter by its own weight, if the other three corners were held horizontally (NRAO 65 m design).

4. Radial Shift of Panel

If we shift a panel by S along the surface, the change of Xo is $dXo = S \cos \alpha$; and, since $\tan \alpha = Xo/2F$, the change of α is $d\alpha = (1/2F) \cos^2\alpha \, dXo$. From (4) we get $dW = (3/4F) U^2 \sin \alpha \cos^2\alpha \, d\alpha$. Altogether, the deviation of W, from shift S, is

$$\sigma = (3/8F^2)\, U^2 S \cos^5\alpha \sin \alpha. \tag{13}$$

With F=60 m, U=1 m at panel edge, $\sigma \le 33$ µm, and using $\alpha = 24°$, (where $\cos^5\alpha \sin \alpha = 0.259$ is maximum, at Xo = 53 m) we find for the panel shift S:

$$S \le 1.23 \text{ m}. \tag{14}$$

This is larger than half the panel length. Thus, if the molds are all made exact for the border between two panel rings, then we may have

$$\text{two rings per mold.} \tag{15}$$

If we wanted three rings per mold, then $S \ge 2$ m is needed, and (13) gives

$$\alpha \le 9.8°, \quad \text{or } Xo \le 20.7 \text{ m} \tag{16}$$

which would hold only for a few of the innermost rings.

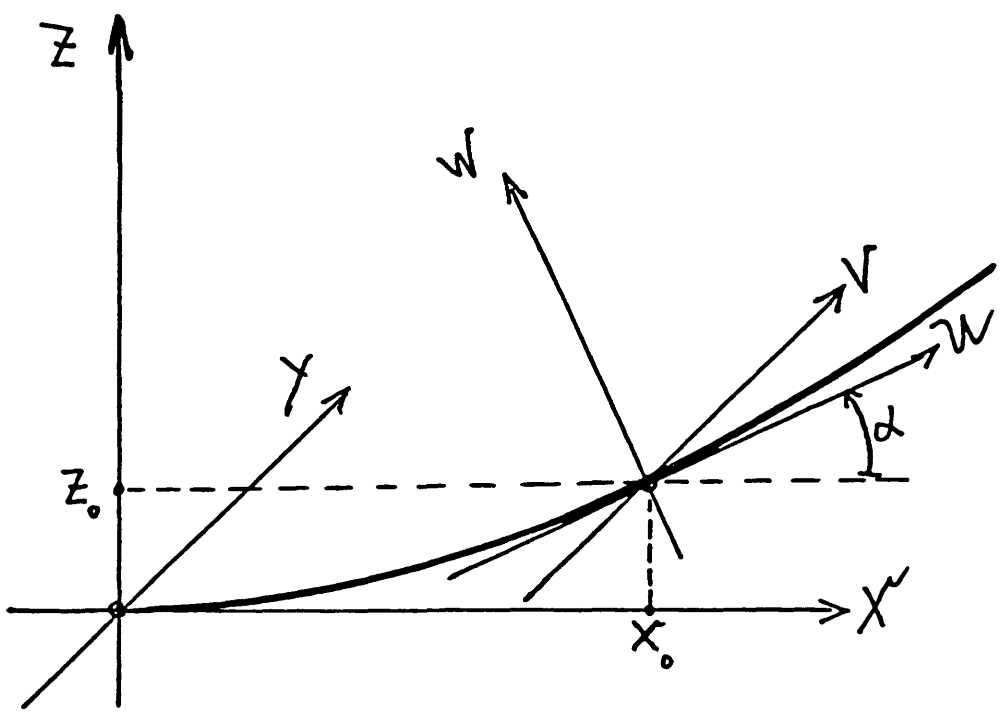

Fig. 1. Telescope coordinates X, Y, Z; and Panel coordinates U, V, W. (W = 0 is the tangential plane)