GBT Memo No. 52

Azimuth and Parallactic Angle Tracking near the Zenith

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Introduction.

When tracking a celestial source across the sky with an alt-az-mounted telescope, there is a zone of avoidance near the zenith set by the maximum azimuth tracking rate. A similar effect occurs if a multiple feed system is rotated while tracking a source to keep each feed pointed to the same spot on the sky. Formulas are given here so that these effects can be calculated. Some cases are illustrated.

For a multiple feed system, the correction angle required to keep a constant orientation with respect to the equatorial coordinate system is known as the parallactic angle, as defined in Figure 1, below.

Within a few degrees of the zenith, the azimuth tracking rate is nearly equal to the rate of change of the parallactic angle. In the case of the GBT, the proposed slew rate in azimuth is 40° per minute. By equation 7, the radius of the zone of avoidance is 17 arc-minutes. To match this sky coverage, a multiple feed system would also need to rotate at 40°/min.

Note that the azimuth and parallactic angle rates must be continuously variable from zero to the maximum rate in order to track a source across the sky.

Formulas.

Figure 1 shows the relation between the various angles in the equatorial and horizon systems. The symbols are defined as follows:

S = the source position,	NP = the celestial north pole,
Z = the zenith,	ϕ = latitude (38° 26' for Green Bank),
δ = declination,	H = hour angle
z = zenith distance,	a = altitude or elevation (= 90° - z)
A = azimuth,	p = parallactic angle,

Given ϕ, δ, and H, it is possible to calculate z and A from the following:

(1) $\cos z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H$

(2) $\cos A = (\sin\delta - \sin\phi \cos z)/(\cos\phi \sin z)$

(3) $\sin A = \sin H \cos\delta/\sin z$

The parallactic angle (p) is given by:

(4) $p = \tan^{-1}[\, \sin H / (\cos\delta \tan\phi - \sin\delta \cos H)\,]$

Equation 4 is illustrated in Figure 3. Also, one may refer to "An Introduction to the NRAO Very Large Array" by Hjellming (1983), pages 2-17 and 2-18.

For a celestial source, δ and ϕ are constants, and H increases at the sidereal rate of 0.25 °/minute. The rate of change of the parallactic angle is given by:

(5) $dp/dH = 0.25(x \cos H - \sin^2 H \sin\delta)/(x^2 + \sin^2 H)$ (in °/min)

where $x \equiv \cos\delta \tan\phi - \sin\delta \cos H$

See Figure 4 for plots of Equation 5.

The rate of change of azimuth is given by:

(6) $dA/dH = 0.25(\cot z \cos A \cos\phi - \sin\phi)$ (in °/min)

To find the radius of the zone of avoidance for a given azimuth tracking speed (dA/dH), set A=0 in equation 6, and solve for z:

(7) $z = \tan^{-1}[\cos\phi / (4(dA/dH) + \sin\phi)]$ (dA/dH in degrees per minute).

Equation 7 is illustrated in Figure 2.

Graphs.

Figure 1. The relation between zenith-horizon and pole-equator coordinate systems.

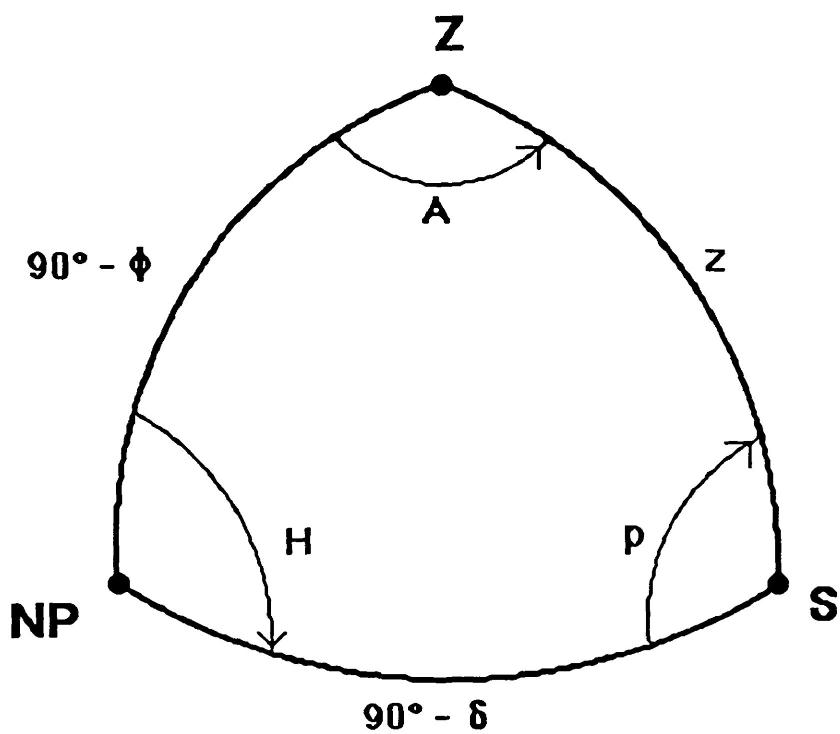

Figure 2. The azimuth tracking rate (dA/dH) versus zenith distance (z) within 2.5° of the zenith. The magnitude of dp/dH is very close to dA/dH over this range of z.

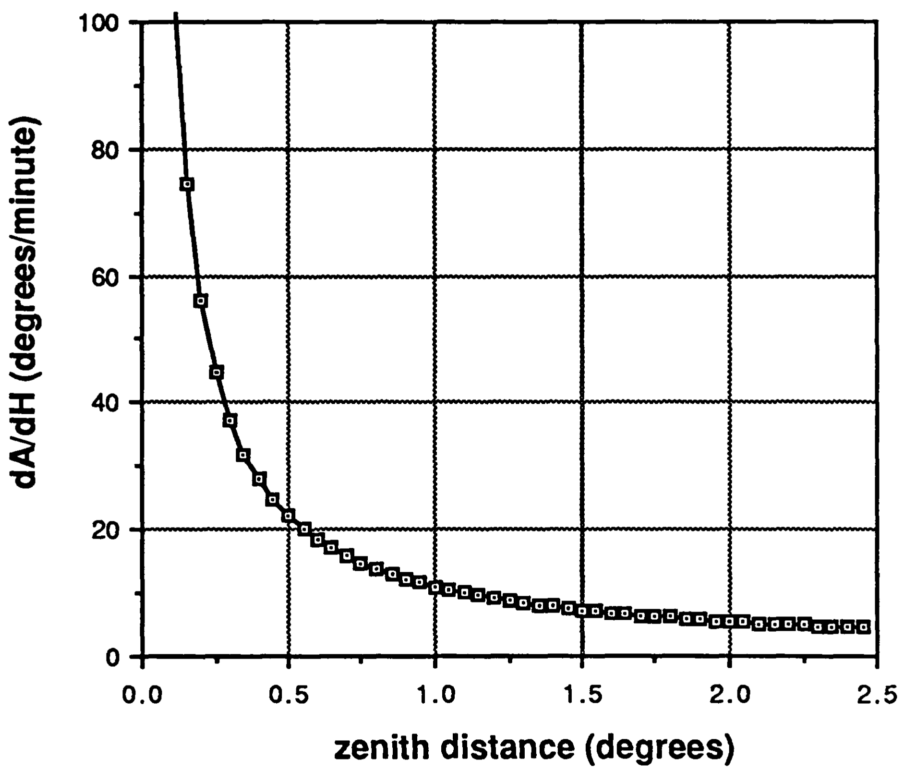

Figure 3. The parallactic angle ϕ is plotted versus Hour Angle for four declinations.

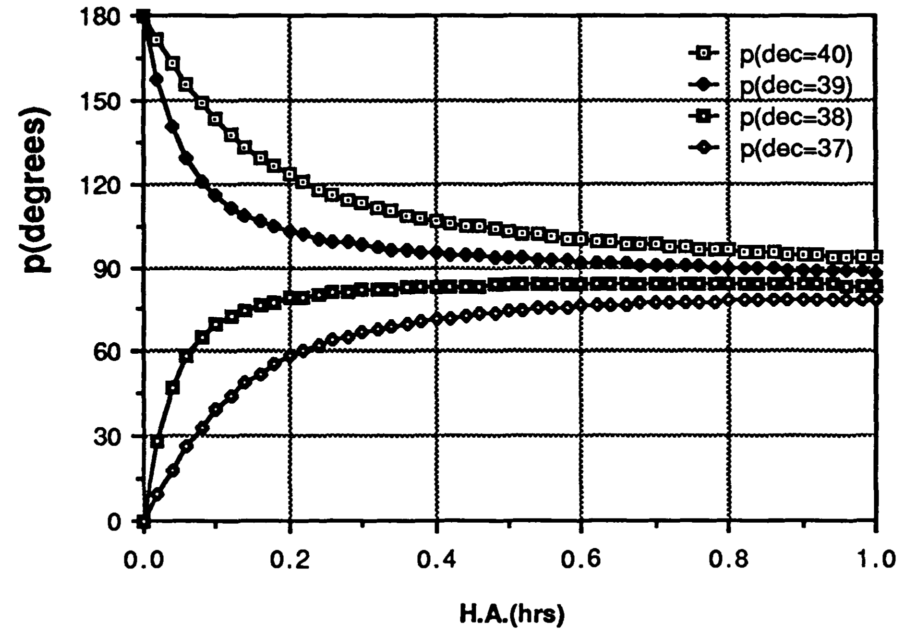

Figure 4. The rate of change of parallactic angle for 4 different declinations versus H.A. within one hour of the meridian.