GBT Memo No. 54

Telescope Consultation

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ARM LOCATION AND HOMOLOGY

Introduction and Summary

In our discussions of mid-December at Charlottesville, it was asked whether the location of the arm (top or bottom) could have some influence on the gravitational deformation of the telescope surface, such that the approach to homologous deformation would be for one location easier than for the other.

I have taken our present model, where the arm is detached from the backup of the dish, and where the backup structure is almost symmetrical. Deformations then are composed of those from a force parallel to the plane of the dish, and from a force normal to it. If the parallel force gives a larger deviation from homology, then the bottom location is better; and it is the top location, if the normal force is dominant (which is the case for the 140-ft). The location does not matter if both forces are about equal (which was mostly the case after several homology-iterations).

1. Symmetric Telescope

This case was published in Paper I (v.Hoerner and Wong, 1975), from which we use most definitions and equations, rewritten for the present application. For an alt-azimuth mount, with one plane of symmetry, all problems are only two-dimensional.

A force in any direction can always be described as the vector sum of two orthogonal components. This coordinate system may have any orientation if the deformations are asked for, which add up in a linear way. But if we ask for the rms deviation from a wanted shape (parabola), where contributions add up in quadrature, then the coordinates must be such that their contributions are mutually independent (uncorrelated). For the symmetrical telescope, these are the directions parallel to the plane of the dish (called x), and normal to it (called z).

Let a telescope be adjusted to a paraboloid in the absence of gravity; turn on gravity in x-direction, find the best-fit paraboloid, and call Hx the rms deviation from it. Do the same with gravity in z-direction, find Hz. Then call a "standard" deviation

$$Ho = \sqrt{\tfrac{1}{2}\,(Hx^2 + Hz^2)}\,, \tag{1}$$

and call

$$P = Hz^2/(Hx^2 + Hz^2), \qquad Q = Hx^2/(Hx^2 + Hz^2) = 1 - P, \tag{2}$$

$$g = Hx/Hz = \sqrt{Q/P}\,. \tag{3}$$

If gravity now has the direction Ø below the x-axis, the deviation from homology, HØ, is given by

$$H\text{Ø}^{\,2} = Hz^2 * \sin^2\text{Ø} + Hx^2 * \cos^2\text{Ø}. \tag{4}$$

Actually, the telescope will be adjusted with gravity at a certain angle ß (below x-axis). We then have

$$H\text{Ø}^{\,2} = Hz^2 * (\sin\text{Ø} - \sin\beta)^2 + Hx^2 * (\cos\text{Ø} - \cos\beta)^2. \tag{5}$$

Note: For the symmetrical telescope, Ø and ß are elevation angles, beam above horizon. But not so for the asymmetric case, where x and z are fixed at the dish, with about 45° between z and beam.

Next, we ask for the best adjustment angle ß. Since H increases with the distance of Ø from ß, we define ß such that H is the same at both extremes of the elevation range, $\text{Ø}1 \le \text{Ø} \le \text{Ø}2$, from zero elevation to 90° for the full range, or from 20° (atmosphere) to 80° (not much sky to 90°, and mostly avoidable) as the most useful range.

Demanding HØ1 = HØ2 = Hmax at both extremes, Paper I gave

$$\frac{1 - A * \sin\beta}{1 - B * \cos\beta} = g^2 \tag{6}$$

with

$$A = 2/(\sin\text{Ø}1 + \sin\text{Ø}2), \qquad B = 2/(\cos\text{Ø}1 + \cos\text{Ø}2) \tag{7}$$

where ß(g) was given graphically.

2. Clear Aperture

The two arm locations are sketched in Fig.1, with definitions of the dish directions x and z, and of the angle Ø (clockwise from gravity to x) and best adjustment angle, Ø = ß. The beam elevation at adjustment then is

$$\alpha \approx \begin{cases} \beta - 45°, & \text{if arm on top,} \\ \beta + 45°, & \text{if arm at bottom.} \end{cases} \tag{8}$$

Note: The orientation of x and z with regard to the dish, as used here, would hold if the backup were symmetrical in (x, -x). More in general, H^2 of (4) will be an ellipse, and x and z should be its axes. Also, (8) is only an approximation.

Equations (6) and (7) can be solved for ß:

$$\beta = \arctan\left(\frac{g^2 B}{A}\right) + \arcsin\left(\frac{1 - g^2}{\sqrt{A^2 + (g^2 B)^2}}\right) \tag{9}$$

For an approach to homology, the quantity to be reduced is Ho of (1). Thus, for a given Ho, we ask for the performance Hmax at the extremes of the elevation range, and consider Hmax/Ho as the quality indicator (good if small) for the arm location. With the definitions (1() and (2), equation (5) then can be written

$$(Hmax/Ho)^2 = 2P^*(\sin Ø1 - \sin \beta)^2 + 2Q^*(\cos Ø1 - \cos \beta)^2 \qquad (10)$$

and, as a check, replacing Ø1 by Ø2 must give the same result.

Tables 1 to 6 give adjustment ß and performance Hmax/Ho as functions of P (dominance of Hz) and Q (dominance of Hx), or of g (dominance ratio Hx/Hz). The symmetrical telescope is also added for comparison, followed by the clear aperture with arm at bottom and arm on top. For each case, we have treated both the full range of elevation (0 - 90°) and the most useful range (20°- 80°).

For the clear aperture telescope, we see that, for both ranges, the location does not matter if Hx ≈ Hz, but that

arm at bottom is best, if Hx dominates,
arm on top is best, if Hz dominates. (11)

This result can already be seen from equation (10) and Fig.1. If the arm is at bottom, the range of Ø is from -45° to +45°, thus the adjustment will be about at zero, where the sine is a linear function, but the cosine is only a quadratic one; thus $P < Q$ is of advantage, meaning a dominant Hx. In the same way, the opposite is true if the arm is on top, where the adjustment will be close to 90° with the cosine being the larger linear term.- We also can understand why the symmetrical telescope, full range of Table 1, shows a symmetry in P and Q, since for $P = Q = \frac{1}{2}$ the adjustment is at 45°, where sine and cosine are equal.

Which case is to be expected for the GBT? The 140-ft shows a strong dominance of Hz (Paper I), The 100-m at Effelsberg has a dominant Hx, and our homologous NRAO designs showed no dominance after some iterations. If I try to understand this, I would say that the dish is more extended in x than in z direction, thus its cantelevering outer part deforms more from z-force than x-force. Thus a conventional telescope has Hz dominance as the 140-ft. If a telescope is supported like an umbrella along its z-axis, and homology is approached by trial and error (Effelsberg), then the human intuition has it easier to minimize the axisymmetric Hz, and Hx will remain dominant. Whereas the computerized iterations feel not much difference in either direction.

I would guess that the arm location will not be important for the approach to homology, if the approach is carried sufficiently far, especially so in case of computerized optimization.

Reference

S. von Hoerner and W.Y. Wong, "Gravitational deformation and astigmatism of tiltable radio telescopes", IEEE Trans. A-P 23, 689, 1975.

Table 1. Symmetrical telescope, elev. 0-90°; adjust elev. α = β

Range: Ø1, Ø2 = 0 90 [01-04-1990 #Homol]

P	Q	g	β	Hmax/Ho
0.0200	0.9800	7.0000	59.5104	0.7109
0.1000	0.9000	3.0000	57.4457	0.7253
0.2000	0.8000	2.0000	54.6298	0.7414
0.3000	0.7000	1.5275	51.5763	0.7542
0.4000	0.6000	1.2247	48.3387	0.7625
0.5000	0.5000	1.0000	45.0000	0.7654
0.6000	0.4000	0.8165	41.6612	0.7625
0.7000	0.3000	0.6547	38.4237	0.7542
0.8000	0.2000	0.5000	35.3702	0.7414
0.9000	0.1000	0.3333	32.5543	0.7253
0.9800	0.0200	0.1429	30.4896	0.7109

Table 2. Symmetrical telescope, elev. 20-80°; adjust elev. α = β

Range: Ø1, Ø2 = 20 80 [01-04-1990 #Homol]

P	Q	g	β	Hmax/Ho
0.0200	0.9800	7.0000	55.9786	0.5411
0.1000	0.9000	3.0000	55.1565	0.5386
0.2000	0.8000	2.0000	54.0351	0.5349
0.3000	0.7000	1.5275	52.8033	0.5303
0.4000	0.6000	1.2247	51.4578	0.5246
0.5000	0.5000	1.0000	50.0000	0.5176
0.6000	0.4000	0.8165	48.4378	0.5090
0.7000	0.3000	0.6547	46.7869	0.4985
0.8000	0.2000	0.5000	45.0704	0.4859
0.9000	0.1000	0.3333	43.3177	0.4712
0.9800	0.0200	0.1429	41.9109	0.4580

Table 3. Clear aperture, arm at bottom, el. 0-90°; adj.el.α = β+45°

Range: Ø1, Ø2 =-45 45 [01-04-1990 #Homol]

P	Q	g	β	Hmax/Ho
0.0200	0.9800	7.0000	0.0000	0.4338
0.1000	0.9000	3.0000	0.0000	0.5044
0.2000	0.8000	2.0000	0.0000	0.5807
0.3000	0.7000	1.5275	0.0000	0.6482
0.4000	0.6000	1.2247	0.0000	0.7092
0.5000	0.5000	1.0000	0.0000	0.7654
0.6000	0.4000	0.8165	0.0000	0.8177
0.7000	0.3000	0.6547	0.0000	0.8669
0.8000	0.2000	0.5000	0.0000	0.9134
0.9000	0.1000	0.3333	0.0000	0.9577
0.9800	0.0200	0.1429	0.0000	0.9917

Table 4. Clear aperture, arm at bottom, el.20-80°; adj.el.α = ß+45°

Range: Ø1, Ø2 =-25 35 [01-04-1990 #Homol]

P	Q	g	ß	Hmax/Ho
0.0200	0.9800	7.0000	21.4805	0.1614
0.1000	0.9000	3.0000	9.8955	0.2861
0.2000	0.8000	2.0000	6.9468	0.3607
0.3000	0.7000	1.5275	5.8790	0.4199
0.4000	0.6000	1.2247	5.3319	0.4714
0.5000	0.5000	1.0000	5.0000	0.5176
0.6000	0.4000	0.8165	4.7773	0.5600
0.7000	0.3000	0.6547	4.6176	0.5994
0.8000	0.2000	0.5000	4.4974	0.6363
0.9000	0.1000	0.3333	4.4038	0.6712
0.9800	0.0200	0.1429	4.3425	0.6979

Table 5. Clear aperture, arm on top, elev. 0-90°; adj.el.α = ß-45°

Range: Ø1, Ø2 = 45 135 [01-04-1990 #Homol]

P	Q	g	ß	Hmax/Ho
0.0200	0.9800	7.0000	90.0000	0.9917
0.1000	0.9000	3.0000	90.0000	0.9577
0.2000	0.8000	2.0000	90.0000	0.9134
0.3000	0.7000	1.5275	90.0000	0.8669
0.4000	0.6000	1.2247	90.0000	0.8177
0.5000	0.5000	1.0000	90.0000	0.7654
0.6000	0.4000	0.8165	90.0000	0.7092
0.7000	0.3000	0.6547	90.0000	0.6482
0.8000	0.2000	0.5000	90.0000	0.5807
0.9000	0.1000	0.3333	90.0000	0.5044
0.9800	0.0200	0.1429	90.0000	0.4338

Table 6. Clear aperture, arm on top, elev.20-80°; adj.el.α = ß-45°

Range: Ø1, Ø2 = 55 115 [01-04-1990 #Homol]

P	Q	g	ß	Hmax/Ho
0.0200	0.9800	7.0000	85.6575	0.6979
0.1000	0.9000	3.0000	85.5962	0.6712
0.2000	0.8000	2.0000	85.5026	0.6363
0.3000	0.7000	1.5275	85.3824	0.5994
0.4000	0.6000	1.2247	85.2227	0.5600
0.5000	0.5000	1.0000	85.0000	0.5176
0.6000	0.4000	0.8165	84.6681	0.4714
0.7000	0.3000	0.6547	84.1210	0.4199
0.8000	0.2000	0.5000	83.0532	0.3607
0.9000	0.1000	0.3333	80.1045	0.2861
0.9800	0.0200	0.1429	68.5195	0.1614

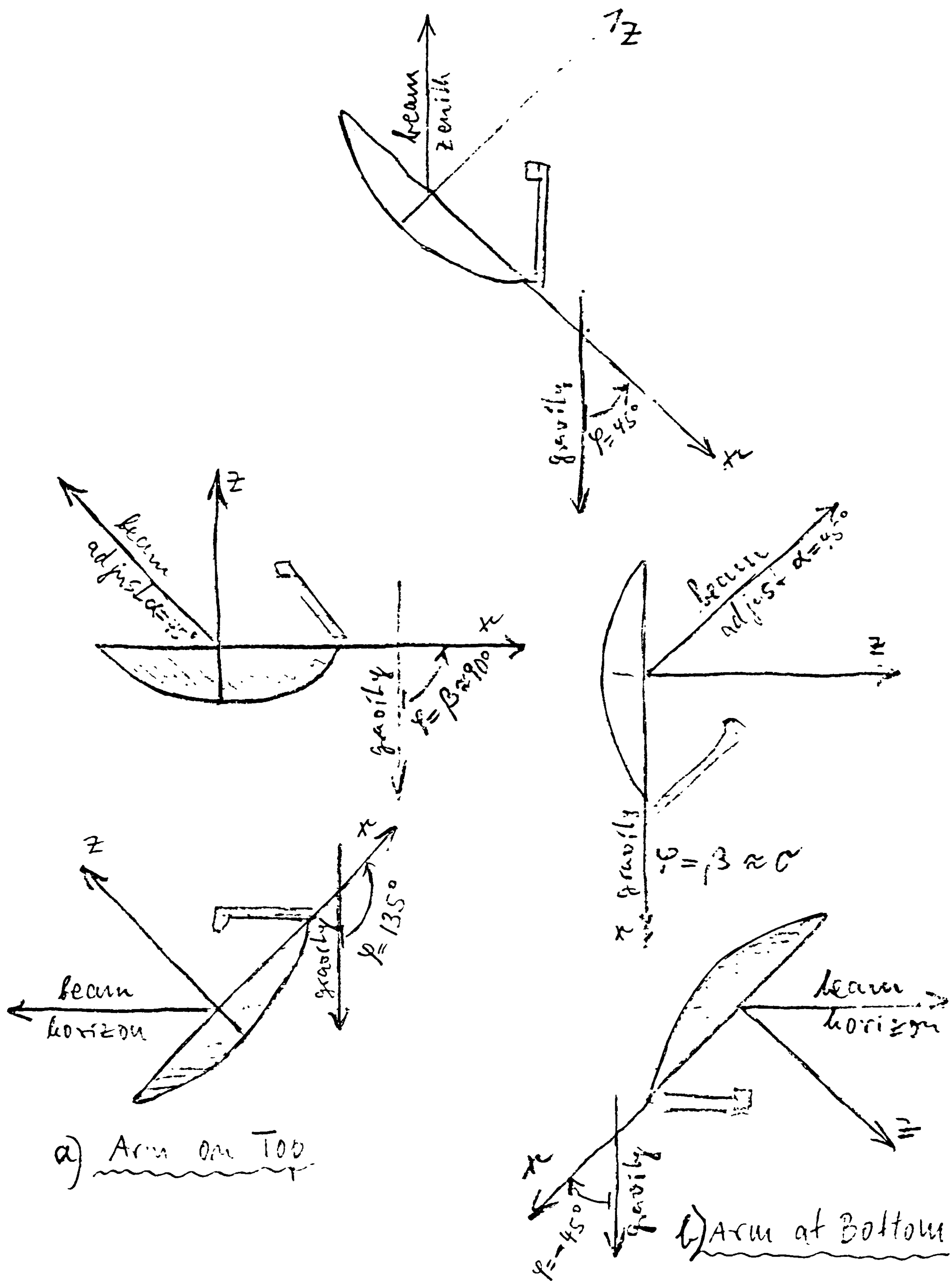

Fig. 1. Arm location, Top and Bottom; and definitions of x, z, Ø, β, α.