

Error Sensitivity of the GBT Subreflector Positioning Mechanism

FREDERIC R. SCHWAB

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1. INTRODUCTION

This memorandum addresses a question raised at the third GBT Design Review Meeting, held in Sterling, Va. on Oct. 20–22, 1992: What is the nature of the subreflector positioning errors that would result from imperfect behavior of the U-joints and the ball screws of the subreflector positioning mechanism? It was stated at the meeting that imperfections in the U-joints would result in joint positioning errors, due to backlash, of $\sim \pm 0.75$ mils (peak) at each of the twelve joints and that imperfections in the ball screws of the actuators (one per actuator), combined with servo-system granularity, would cause additional joint positioning errors of about the same amount. Several of the NRAO attendees have expressed concern that the resulting subreflector positional errors, both translational and rotational, are not sufficiently well understood and that the error analysis presented by the subcontractor at the meeting may have been too simplistic.

2. ERROR ANALYSIS

The three-dimensional Cartesian coordinate system used in describing the operation of the positioning mechanism has its origin located at the vertex of the subreflector. The orientation of the coordinate system is such that the (x, z) -plane is parallel to the subreflector mounting platform when the mechanism is positioned in its "home" configuration (see Loral Technical Memorandum No. 19, *Equations of Motion—Subreflector Positioner*, Revisions 0 and 1). Six linear actuators are attached, via U-joints, between mounting points P_i located on the feed arm and Q_i on the subreflector mounting platform, $i = 1, \dots, 6$. Following the conventions of the Loral memorandum, desired motions of the platform are specified by a column vector $(x_D, y_D, z_D, \theta_x, \theta_z)^T$ of three translational parameters and two rotational ones; and, given an initial configuration $(\mathbf{P}, \mathbf{Q}^{\text{initial}})$ and a desired motion of the platform, the final configuration $(\mathbf{P}, \mathbf{Q}^{\text{final}})$ can be calculated according to

$$Q_i^{\text{final}} = \begin{pmatrix} x_D \\ y_D \\ z_D \end{pmatrix} + \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{pmatrix} \cdot \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix} Q_i^{\text{initial}} \quad (1)$$

(for $i = 1, \dots, 6$), where $\alpha = 36.7$ degrees has been chosen to achieve the orientation of the x' -axis which is appropriate for optical alignment. The changes in actuator lengths which are needed in order to accomplish the desired change of configuration are given by $\Delta \ell_i = \|Q_i^{\text{final}} - Q_i^{\text{initial}}\|$.

Now let us suppose that the subreflector positioning mechanism is in some configuration of interest: maybe its “home” configuration ($\mathbf{P}, \mathbf{Q}^{\text{home}}$); or perhaps an arbitrary configuration ($\mathbf{P}, \mathbf{Q}$) that can be reached from the home position in a manner consistent with Equation 1. We wish now to see how perturbations in the actuator lengths affect the position and orientation of the subreflector mounting platform. These perturbations could, of course, tilt the platform about any of its three axes, so we now need an equation like Equation 1, but incorporating three rotational parameters.

The natural choice for the rotational parameters might seem to be the Euler angles, but associated with this choice would be a problem that would effectively preclude a straightforward error analysis.¹ The rotations that are of interest are small, so in place of Equation 1 we may use

$$Q_i^{\text{final}} = \begin{pmatrix} x_D \\ y_D \\ z_D \end{pmatrix} + \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_z & -\theta_y \\ -\theta_z & 1 & \theta_x \\ \theta_y & -\theta_x & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} Q_i^{\text{initial}}. \quad (2)$$

The Jacobian matrix,

$$J = \begin{pmatrix} \frac{\partial \ell_1}{\partial x_D} & \frac{\partial \ell_1}{\partial y_D} & \frac{\partial \ell_1}{\partial z_D} & \frac{\partial \ell_1}{\partial \theta_x} & \frac{\partial \ell_1}{\partial \theta_y} & \frac{\partial \ell_1}{\partial \theta_z} \\ \frac{\partial \ell_2}{\partial x_D} & \frac{\partial \ell_2}{\partial y_D} & \frac{\partial \ell_2}{\partial z_D} & \frac{\partial \ell_2}{\partial \theta_x} & \frac{\partial \ell_2}{\partial \theta_y} & \frac{\partial \ell_2}{\partial \theta_z} \\ \frac{\partial \ell_3}{\partial x_D} & \frac{\partial \ell_3}{\partial y_D} & \frac{\partial \ell_3}{\partial z_D} & \frac{\partial \ell_3}{\partial \theta_x} & \frac{\partial \ell_3}{\partial \theta_y} & \frac{\partial \ell_3}{\partial \theta_z} \\ \frac{\partial \ell_4}{\partial x_D} & \frac{\partial \ell_4}{\partial y_D} & \frac{\partial \ell_4}{\partial z_D} & \frac{\partial \ell_4}{\partial \theta_x} & \frac{\partial \ell_4}{\partial \theta_y} & \frac{\partial \ell_4}{\partial \theta_z} \\ \frac{\partial \ell_5}{\partial x_D} & \frac{\partial \ell_5}{\partial y_D} & \frac{\partial \ell_5}{\partial z_D} & \frac{\partial \ell_5}{\partial \theta_x} & \frac{\partial \ell_5}{\partial \theta_y} & \frac{\partial \ell_5}{\partial \theta_z} \\ \frac{\partial \ell_6}{\partial x_D} & \frac{\partial \ell_6}{\partial y_D} & \frac{\partial \ell_6}{\partial z_D} & \frac{\partial \ell_6}{\partial \theta_x} & \frac{\partial \ell_6}{\partial \theta_y} & \frac{\partial \ell_6}{\partial \theta_z} \end{pmatrix}, \quad (3)$$

of the transformation from the parameters ($x_D, y_D, z_D, \theta_x, \theta_y, \theta_z$) to actuator lengths ℓ_i (or length changes $\Delta \ell_i$) is straightforward to calculate analytically from Equation 2. Having done so, one may evaluate the Jacobian numerically at any desired

¹The Euler-angle parametrization is non-unique and, in particular, is singular at the identity (i.e., for zero rotation); see [1, p. 135]. In the common definition [2, p. 107 ff.] a rotation by an angle ψ about the z -axis can be described by Euler angles $(0, 0, \psi)$, or by the choice $(\alpha, 0, \psi - \alpha)$ for arbitrary α .

values of these parameters, and then approximate, numerically, the inverse Jacobian,

$$J^{-1} = \begin{pmatrix} \frac{\partial x_D}{\partial \ell_1} & \frac{\partial x_D}{\partial \ell_2} & \frac{\partial x_D}{\partial \ell_3} & \frac{\partial x_D}{\partial \ell_4} & \frac{\partial x_D}{\partial \ell_5} & \frac{\partial x_D}{\partial \ell_6} \\ \frac{\partial y_D}{\partial \ell_1} & \frac{\partial y_D}{\partial \ell_2} & \frac{\partial y_D}{\partial \ell_3} & \frac{\partial y_D}{\partial \ell_4} & \frac{\partial y_D}{\partial \ell_5} & \frac{\partial y_D}{\partial \ell_6} \\ \frac{\partial z_D}{\partial \ell_1} & \frac{\partial z_D}{\partial \ell_2} & \frac{\partial z_D}{\partial \ell_3} & \frac{\partial z_D}{\partial \ell_4} & \frac{\partial z_D}{\partial \ell_5} & \frac{\partial z_D}{\partial \ell_6} \\ \frac{\partial \theta_x}{\partial \ell_1} & \frac{\partial \theta_x}{\partial \ell_2} & \frac{\partial \theta_x}{\partial \ell_3} & \frac{\partial \theta_x}{\partial \ell_4} & \frac{\partial \theta_x}{\partial \ell_5} & \frac{\partial \theta_x}{\partial \ell_6} \\ \frac{\partial \theta_y}{\partial \ell_1} & \frac{\partial \theta_y}{\partial \ell_2} & \frac{\partial \theta_y}{\partial \ell_3} & \frac{\partial \theta_y}{\partial \ell_4} & \frac{\partial \theta_y}{\partial \ell_5} & \frac{\partial \theta_y}{\partial \ell_6} \\ \frac{\partial \theta_z}{\partial \ell_1} & \frac{\partial \theta_z}{\partial \ell_2} & \frac{\partial \theta_z}{\partial \ell_3} & \frac{\partial \theta_z}{\partial \ell_4} & \frac{\partial \theta_z}{\partial \ell_5} & \frac{\partial \theta_z}{\partial \ell_6} \end{pmatrix}. \quad (4)$$

This matrix comprises the partial derivatives that would be required in order to apply the standard formulas for error propagation.

For a continuously differentiable function f of n variables $x_1, \dots, x_n$, the absolute error in f , given small perturbations Δx_i in the x_i , can be approximated by

$$\Delta f(x_1, \dots, x_n) \approx \frac{\partial f}{\partial x_1} \Delta x_1 + \dots + \frac{\partial f}{\partial x_n} \Delta x_n. \quad (5)$$

If the errors are zero-mean random variables, the variance of f can be expressed in terms of the variances and covariances of the x_i (see, e.g., [3, p. 118]) by

$$\begin{aligned} V\{f(x_1, \dots, x_n)\} &\approx \left(\frac{\partial f}{\partial x_1}\right)^2 V\{x_1\} + \dots + \left(\frac{\partial f}{\partial x_n}\right)^2 V\{x_n\} \\ &+ 2\frac{\partial f}{\partial x_1} \frac{\partial f}{\partial x_2} V\{x_1, x_2\} + \dots + 2\frac{\partial f}{\partial x_{n-1}} \frac{\partial f}{\partial x_n} V\{x_{n-1}, x_n\}. \end{aligned} \quad (6)$$

From Equations 4 and 5, we see that absolute errors in the actuator lengths would propagate according to the formula

$$\begin{pmatrix} \Delta x_D \\ \Delta y_D \\ \Delta z_D \\ \Delta \theta_x \\ \Delta \theta_y \\ \Delta \theta_z \end{pmatrix} \approx J^{-1} \begin{pmatrix} \Delta \ell_1 \\ \Delta \ell_2 \\ \Delta \ell_3 \\ \Delta \ell_4 \\ \Delta \ell_5 \\ \Delta \ell_6 \end{pmatrix}. \quad (7)$$

Also, from Equations 4 and 6 we see that uncorrelated random errors in the actuator lengths, with standard deviations σ_{ℓ_i} , would propagate according to

$$\sigma_{x_D} \approx \sqrt{\sum_{i=1}^6 (J_{1,i}^{-1})^2 \sigma_{\ell_i}^2}, \quad \sigma_{y_D} \approx \sqrt{\sum_{i=1}^6 (J_{2,i}^{-1})^2 \sigma_{\ell_i}^2}, \quad \text{etc.} \quad (8)$$

For correlated random errors, the full form of Equation 6 would apply: in matrix terms, the variance/covariance matrix of the errors in $\{x_D, y_D, z_D, \theta_x, \theta_y, \theta_z\}$ is given by the matrix product $(J^{-1})\mathcal{V}(J^{-1})^T$, where $\mathcal{V}$ denotes the variance/covariance matrix of the actuator length errors.

3. NUMERICAL RESULTS

I have written a *Mathematica* program which can perform the calculations described in Section 2 and have used it to carry out some analysis of the positioning mechanism of the current GBT structural model. Figure 1 shows the “home” configuration of the subreflector actuators. The coordinates (measured in inches) of the connection points to the feed arm are

$$\mathbf{P} = \{(58.391, 150.161, 0), (-29.753, 166.683, 51.123), \\ (-29.753, 166.683, -51.123), (-163.08, 82.478, 51.123), \\ (-163.08, 82.478, -51.123), (-17.798, 70.290, -12.25)\}.$$

The connection points to the subreflector mounting platform are

$$\mathbf{Q}^{\text{home}} = \{(57.225, 39.219, 0), (-30.15, 55.703, 51.123), \\ (-30.15, 55.703, -51.123), (-40.493, 48.375, 51.123), \\ (-40.493, 48.375, -51.123), (-18.495, 48.856, 45.473)\}.$$

The first three coordinates in each list pertain to the y -actuators, the next two to the x -actuators, and the final one to the z -actuator.

The r.m.s. errors in the actuator lengths are expected to be around 0.75 mils. I have computed (from Eq. 8), for two configurations of the mechanism, the r.m.s. translational and rotational errors that would result from (zero-mean) independent, identically distributed, actuator length errors with $\sigma_{\ell_i} \equiv 0.75$ mils.² In the neighborhood of the “home” configuration, $(\sigma_{x_D}, \sigma_{y_D}, \sigma_{z_D}) \approx (0.68, 0.43, 1.05)$ mils and $(\sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) \approx (2.5, 2.0, 2.2)$ arc seconds. In the neighborhood of an “extreme” configuration—namely $(x_D, y_D, z_D, \theta_x, \theta_z) = (17.95 \text{ in}, -24.34 \text{ in}, 1.25 \text{ in}, 0, 0)$ —the errors are not very different: here, $(\sigma_{x_D}, \sigma_{y_D}, \sigma_{z_D}) \approx (0.72, 0.47, 1.33)$ mils and $(\sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) \approx (2.9, 1.9, 2.2)$ arc seconds. I selected this particular “extreme” configuration because it showed up in Loral’s analysis as a worst case. I have explored the parameter space a bit further and have not found much larger errors in other parts of the configuration space. The errors have very little dependence on the tilt angles θ_x and θ_z when the latter are varied over the narrow range to which they will be restricted ($\pm 1^\circ 5$ and $\pm 0^\circ 5$, respectively).

However, the errors are not likely to be independent. We can expect, since the y -actuators are roughly parallel, that they will be similarly loaded and that their length errors will be positively correlated. Similarly for the x -actuator errors. Assuming no other correlations, the variance/covariance matrix would be of the form

$$\mathbf{V} = \sigma^2 \begin{pmatrix} 1 & \gamma_{12} & \gamma_{13} & 0 & 0 & 0 \\ \gamma_{12} & 1 & \gamma_{23} & 0 & 0 & 0 \\ \gamma_{13} & \gamma_{23} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \gamma_{45} & 0 \\ 0 & 0 & 0 & \gamma_{45} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

²The results scale directly with σ_{ℓ} .

with all of these $\gamma_{ij} > 0$. I have performed a very simple-minded analysis of the error propagation, by assuming, for the moment, that $\gamma_{12} = \gamma_{13} = \gamma_{23} = \gamma_{45} \equiv \gamma$ and, again, that $\sigma = 0.75$ mils. The results, for $0 \leq \gamma < 1$, are summarized in Figure 2 (for the “home” position) and Figure 3 (for the “extreme” position). The translational errors remain of order 0.75 mils: σ_{z_D} , the largest, decreases with increasing γ , while σ_{x_D} and σ_{y_D} increase significantly but remain less than σ_{z_D} . However, as $\gamma \rightarrow 1$ the rotational errors tend to zero. (Again there is not a dramatic difference in behavior between the “home” and the “extreme” position.)

For correlation coefficients equal to one-half, the rotational errors are reduced by a factor of about two-thirds. But very strong, $\geq 90\%$, correlations would be required to reduce the rotational errors by a factor of one-third or better. Such high correlations are probably unlikely, because of many possible effects: differences in the machining of the joints; differences in their orientations; differences in their wear; somewhat different gravitational loadings among the grouped actuators (e.g., actuators 2 and 3 among the y -actuators will be loaded differently than actuator 1);³ differing amounts of relaxation, with time, of the pre-load conditions; etc. The contributions from the servo system (expected to be $\sim \pm 0.5$ mils, peak) would, I think, be uncorrelated.

It is expected that the maximum actuator length errors will be $\sim \pm 2.25$ mils. The absolutely worst-case errors can be approximated from Equation 7, by now taking the absolute values of each element of the inverse Jacobian and of each Δl_i ; i.e.,

$$\begin{pmatrix} \Delta x_D \\ \Delta y_D \\ \Delta z_D \\ \Delta \theta_x \\ \Delta \theta_y \\ \Delta \theta_z \end{pmatrix}_{\max} \approx \begin{pmatrix} \left| \frac{\partial x_D}{\partial l_1} \right| & \left| \frac{\partial x_D}{\partial l_2} \right| & \left| \frac{\partial x_D}{\partial l_3} \right| & \left| \frac{\partial x_D}{\partial l_4} \right| & \left| \frac{\partial x_D}{\partial l_5} \right| & \left| \frac{\partial x_D}{\partial l_6} \right| \\ \left| \frac{\partial y_D}{\partial l_1} \right| & \left| \frac{\partial y_D}{\partial l_2} \right| & \left| \frac{\partial y_D}{\partial l_3} \right| & \left| \frac{\partial y_D}{\partial l_4} \right| & \left| \frac{\partial y_D}{\partial l_5} \right| & \left| \frac{\partial y_D}{\partial l_6} \right| \\ \left| \frac{\partial z_D}{\partial l_1} \right| & \left| \frac{\partial z_D}{\partial l_2} \right| & \left| \frac{\partial z_D}{\partial l_3} \right| & \left| \frac{\partial z_D}{\partial l_4} \right| & \left| \frac{\partial z_D}{\partial l_5} \right| & \left| \frac{\partial z_D}{\partial l_6} \right| \\ \left| \frac{\partial \theta_x}{\partial l_1} \right| & \left| \frac{\partial \theta_x}{\partial l_2} \right| & \left| \frac{\partial \theta_x}{\partial l_3} \right| & \left| \frac{\partial \theta_x}{\partial l_4} \right| & \left| \frac{\partial \theta_x}{\partial l_5} \right| & \left| \frac{\partial \theta_x}{\partial l_6} \right| \\ \left| \frac{\partial \theta_y}{\partial l_1} \right| & \left| \frac{\partial \theta_y}{\partial l_2} \right| & \left| \frac{\partial \theta_y}{\partial l_3} \right| & \left| \frac{\partial \theta_y}{\partial l_4} \right| & \left| \frac{\partial \theta_y}{\partial l_5} \right| & \left| \frac{\partial \theta_y}{\partial l_6} \right| \\ \left| \frac{\partial \theta_z}{\partial l_1} \right| & \left| \frac{\partial \theta_z}{\partial l_2} \right| & \left| \frac{\partial \theta_z}{\partial l_3} \right| & \left| \frac{\partial \theta_z}{\partial l_4} \right| & \left| \frac{\partial \theta_z}{\partial l_5} \right| & \left| \frac{\partial \theta_z}{\partial l_6} \right| \end{pmatrix} \begin{pmatrix} |\Delta l_1| \\ |\Delta l_2| \\ |\Delta l_3| \\ |\Delta l_4| \\ |\Delta l_5| \\ |\Delta l_6| \end{pmatrix}. \quad (10)$$

In the neighborhood of the “home” configuration the calculated worst-case errors are $(\Delta x_D, \Delta y_D, \Delta z_D)_{\max} = (4.04, 2.26, 6.01)$ mils and $(\Delta \theta_x, \Delta \theta_y, \Delta \theta_z)_{\max} = (14.4, 11.0, 10.7)$ arc seconds. In the neighborhood of the “extreme” configuration, the corresponding errors are $(3.91, 2.76, 7.59)$ mils and $(17.5, 10.1, 11.1)$ arc seconds, respectively.

³From geometrical considerations alone (see Fig. 1) one would expect that $\gamma_{23} > \gamma_{12}$, $\gamma_{23} > \gamma_{13}$, and $\gamma_{12} \approx \gamma_{13}$.

By examining the individual elements of the inverse Jacobian we can discover which patterns in sign of the actuator errors would cause each of the worst-case errors to be realized. At the “home” configuration, J^{-1} is equal to

$$\begin{pmatrix} -0.520251 & 0.121654 & 0.121654 & 0.516602 & 0.516602 & 0 \\ -0.345096 & -0.326634 & -0.326634 & -0.00309505 & -0.00309505 & 0 \\ -0.0491685 & -0.756083 & 0.43103 & -0.178991 & 0.189781 & 1.06678 \\ 0 & -0.00945832 & 0.00945832 & 0.00603182 & -0.00603182 & 0 \\ 0 & -0.00365986 & 0.00365986 & -0.00815301 & 0.00815301 & 0 \\ 0.0114321 & -0.0057268 & -0.0057268 & 0.0000410899 & 0.0000410899 & 0 \end{pmatrix}. \quad (11)$$

Thus, in this configuration, the sign pattern $(-, +, +, +, +, \pm)$ (or its negative) in the actuator length errors would lead to the worst-case error in Δx_D , the pattern $(-, -, -, -, -, \pm)$ (or its negative) would lead to the worst-case error in Δy_D , similarly for the pattern $(-, -, +, -, +, +)$ and Δz_D , the pattern $(\pm, -, +, +, -, \pm)$ and $\Delta \theta_x$, the pattern $(\pm, -, +, -, +, \pm)$ and $\Delta \theta_y$, and the pattern $(+, -, -, +, +, \pm)$ and $\Delta \theta_z$. At the “extreme” configuration, J^{-1} is equal to

$$\begin{pmatrix} -0.512483 & 0.0409619 & 0.0336339 & 0.565557 & 0.577145 & -0.00697503 \\ -0.338461 & -0.367908 & -0.364487 & 0.0693016 & 0.0751308 & 0.0112076 \\ -0.112041 & -1.03735 & 0.446309 & -0.345475 & 0.12328 & 1.30999 \\ 0 & -0.0109494 & 0.0109494 & 0.00785587 & -0.00785587 & 0 \\ 0 & -0.00275432 & 0.00275432 & -0.00810922 & 0.00810922 & 0 \\ 0.0118192 & -0.00595057 & -0.00590609 & 0.000141302 & -0.0000653794 & -5.03368 \times 10^{-7} \end{pmatrix}. \quad (12)$$

Here, the corresponding sign patterns are $(-, +, +, +, +, -)$, $(-, -, -, +, +, +)$, and $(-, -, +, -, +, +)$ for the translational errors, and $(\pm, -, +, +, -, \pm)$, $(\pm, -, +, -, +, \pm)$, and $(+, -, -, +, -, -)$ for the three respective rotational errors.

With regard to pointing, the rotational errors are the ones which are of chief concern. The worst-case rotational errors (with one exception) all occur when the three y -actuator errors are of different parity and—simultaneously—the two x -actuator errors are of different parity. (The exceptional case is that of $\Delta \theta_z$ in the neighborhood of the “home” configuration; this is a bit immaterial, since the two matrix elements equal to $\sim 4.11 \times 10^{-5}$ in the sixth row of J^{-1} are much smaller in magnitude than the other nonzero elements of that row.)

By the same arguments which led, in the analysis of random errors, to Equation 9, the *typical* length errors of the three y -actuators (especially actuators 2 and 3) would not be of differing parity, nor would those of the two x -actuators; and I doubt that simultaneous conditions of non-parity would be frequent. At certain elevations where actuators switch from tension to compression, or vice-versa, wind buffeting may be a problem.

Figure 4 shows the *Mathematica* commands that can be used to do the error calculations and which were used to generate Figures 2 and 3; they take about thirty seconds to run.

4. DISCUSSION

According to [8], subreflector rotational errors do not translate directly into errors in the direction of the main telescope beam, but rather there is around a 7 : 1 demagnification effect. (A tilt of 1.0 arc second about the x - or the z -axis causes a

main-beam pointing error of 0.13 or 0.15 arc second, respectively.) The translational errors also affect pointing; translational motions $\Delta x_D = 1$ mil, $\Delta y_D = 1$ mil, or $\Delta z_D = 1$ mil would cause pointing errors of 0.074, 0.054, or -0.096 arc seconds, respectively.

The subreflector positional errors, according to the above analysis, will be $< \sim 1.3$ mil, r.m.s., in the translational components and $< \sim 3$ arc seconds, r.m.s., in the rotations. The worst possible errors, of 5 to 6 times these r.m.s. values, are frighteningly large, but perhaps improbable. 2σ actuator length errors should cause smaller than 1-arc-second main-beam pointing errors, but 3σ errors would not.

My numerical results show that the rotational errors are significantly reduced if there are strong positive correlations among the errors of the three y -actuators and strong positive correlation between the errors of the two x -actuators. One possible design implication is that it might be desirable to choose matched pairs of U-joints (for the x -actuators) and matched triples (for the y -actuators), in order to strengthen the correlation. Similarly, it might be helpful to position the grouped U-joints in identical spatial orientations if this possibility is not precluded by other design considerations.

Lee King has suggested that the best-performing actuator mechanism be assigned to actuator six. From the sixth column of the inverse Jacobian matrices it is evident that this choice would help to minimize the largest translational error component, Δz_D , but we see that actuator six is the one with the least effect on the rotational errors. According to [8], Δz_D is the translational error component which has the largest effect on pointing, so this still might be a good choice.

Additional Remarks. A kinematic analysis of the subreflector positioning mechanism was carried out last year, by Johann Schraml and myself, in order to determine the actuator speed and acceleration requirements. I see, in retrospect, that we could have addressed the problem more directly, via the Jacobian matrix formulation of Section 2 above. The complete forward and inverse instantaneous kinematics could have been derived fairly straightforwardly. See [4], [5], and [6]. I mention this because I believe that some related work is continuing at Loral.

Reference [7] addresses the configuration of in-parallel-actuated manipulators in great generality, with particular emphasis on the connectivity (i.e., which spherical joints coalesce). It is pointed out there that a configuration with one triply-coalesced joint and one doubly-coalesced joint—essentially like our configuration—is more easily controlled than a configuration like the Stewart platform, with three doubly-coalesced joints. Specific restrictions which ought to be applied when controlling a mechanism such as ours are discussed in detail in the concluding section of the paper. I would recommend this paper to the structural and mechanical engineers involved with the GBT project.

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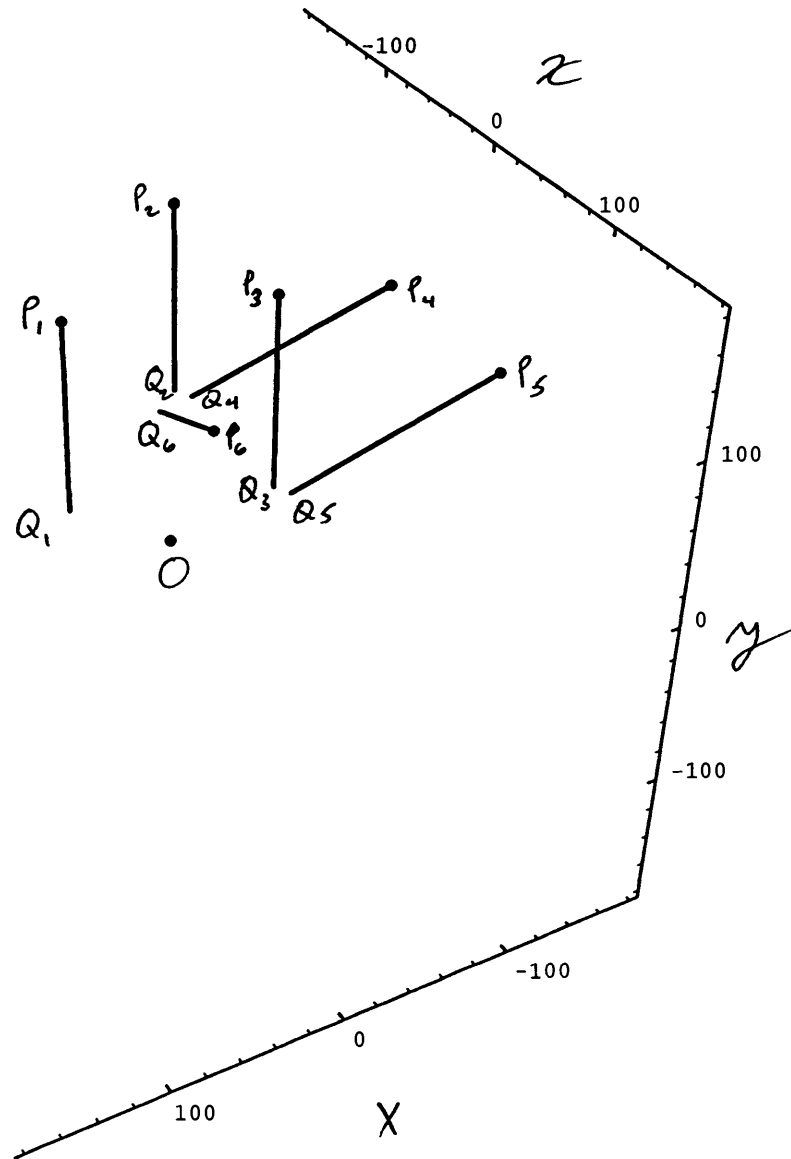


Figure 1. The GBT subreflector positioning mechanism in its “home” configuration. The connection points to the feed arm are represented by the P_i , and the connection points to the subreflector mounting platform by the Q_i .

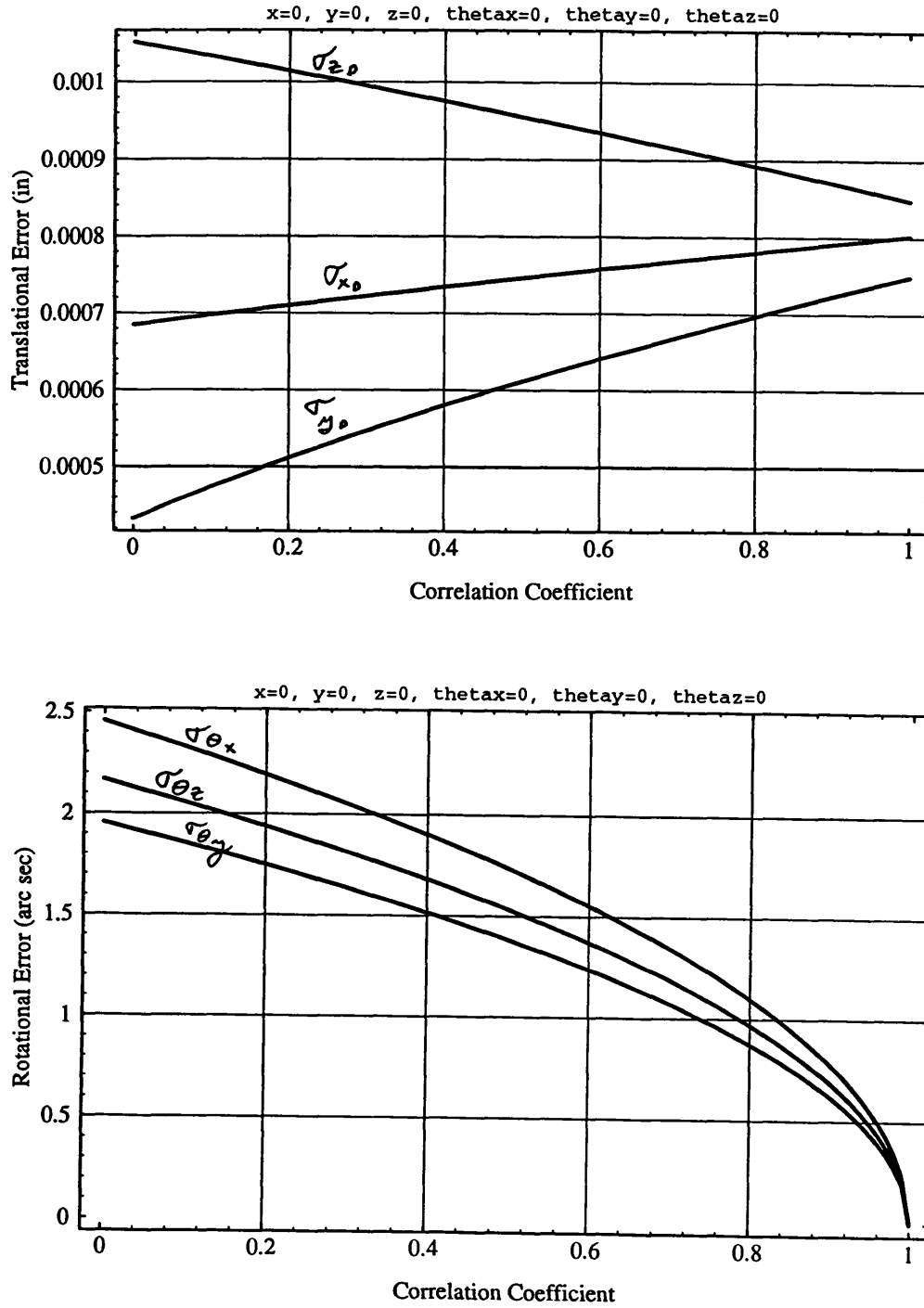


Figure 2. Subreflector r.m.s. translational errors ($\sigma_{xD}, \sigma_{yD}, \sigma_{zD}$) (*Top*) and rotational errors ($\sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}$) (*Bottom*) corresponding to 0.75×10^{-3} inch r.m.s. errors in the actuator lengths, with the mechanism in its nominal "home" configuration. The values at the leftmost abscissa (correlation coefficient = 0) correspond to uncorrelated errors in the actuator lengths. The abscissa represents the (positive) coefficient of correlation between the three y -actuator errors, as well as the coefficient of correlation between the two x -actuator errors.

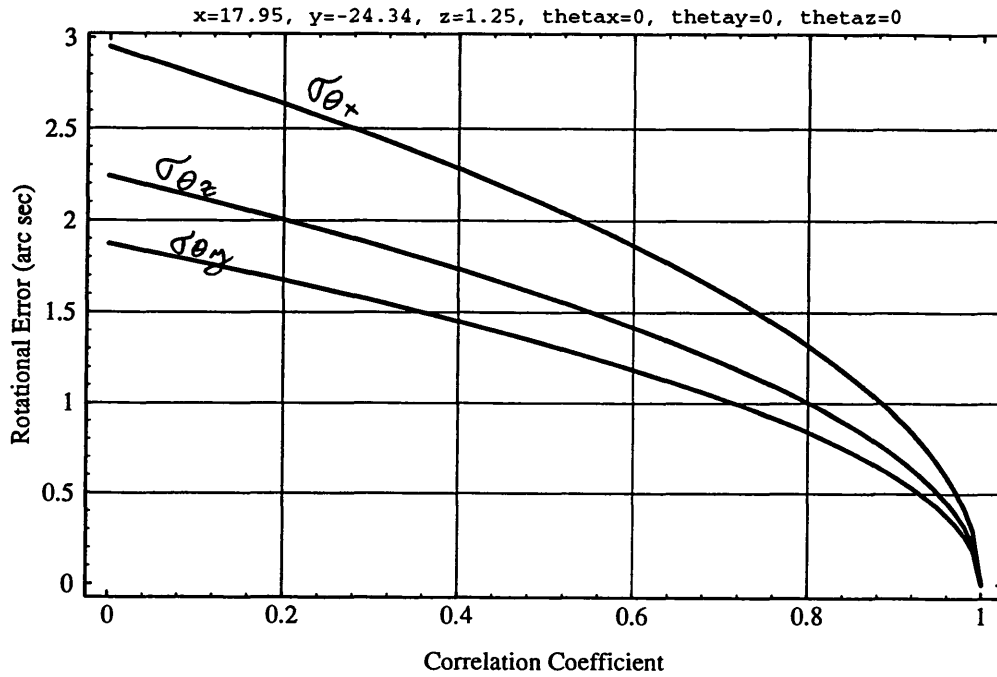
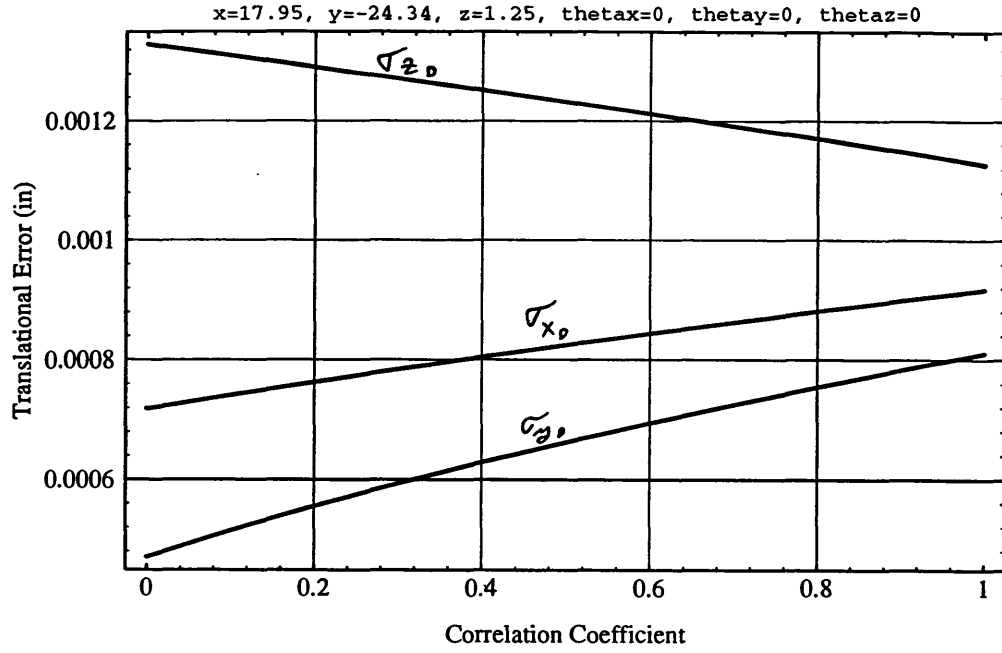


Figure 3. Subreflector r.m.s. translational errors (*Top*) and rotational errors (*Bottom*) corresponding to 0.75×10^{-3} inch r.m.s. errors in the actuator lengths, with the mechanism in an extreme configuration, far from the “home” configuration. The errors shown here are not much larger than those shown in Figure 2.

```

(*m={0,0,0,0,0,0}*)
m={17.95,-24.34,1.25,0,0,0}
sd=.00075
emax=.00225
$DefaultFont={"Times-Roman",12}
sigmal=Table[sd,{i,6}]

V={{1,cc,cc,0,0,0},{cc,1,cc,0,0,0},{cc,cc,1,0,0,0},{0,0,0,1,cc,0},
{0,0,0,cc,1,0},{0,0,0,0,0,1}} sd^2

const=3600/Degree//N
alpha=36.7 Degree
papex={{58.391,150.161,0},{-29.753,166.683,51.123},{-29.753,166.683,-51.123},
{-163.08,82.478,51.123},{-163.08,82.478,-51.123},{-17.798,70.290,-12.25}}
p0={{57.225,39.219,0},{-30.15,55.703,51.123},{-30.15,55.703,-51.123},
{-40.493,48.375,51.123},{-40.493,48.375,-51.123},{-18.495,48.856,45.473}}

Z[a_]:={{Cos[a],-Sin[a],0},{Sin[a],Cos[a],0},{0,0,1}}
g[{xs_,ys_,zs_}]:={xd,yd,zd}+
Z[-alpha].{{1,Sin[thz],-Sin[thy]},{-Sin[thz],1,Sin[thx]},{
Sin[thy],-Sin[thx],1}}.Z[alpha].{xs,ys,zs}

p1=Table[g[p0[[i]]],{i,1,6}]
l0=Table[Sqrt[(papex[[i]]-p0[[i]])*(papex[[i]]-p0[[i]])],{i,6}]
l1=Table[Sqrt[(papex[[i]]-p1[[i]])*(papex[[i]]-p1[[i]])],{i,6}]
deltal=l1-l0

J={{D[deltal[[1]],xd],D[deltal[[1]],yd],D[deltal[[1]],zd],D[deltal[[1]],thx],
D[deltal[[1]],thy],D[deltal[[1]],thz]],
{D[deltal[[2]],xd],D[deltal[[2]],yd],D[deltal[[2]],zd],D[deltal[[2]],thx],
D[deltal[[2]],thy],D[deltal[[2]],thz]],
{D[deltal[[3]],xd],D[deltal[[3]],yd],D[deltal[[3]],zd],D[deltal[[3]],thx],
D[deltal[[3]],thy],D[deltal[[3]],thz]],
{D[deltal[[4]],xd],D[deltal[[4]],yd],D[deltal[[4]],zd],D[deltal[[4]],thx],
D[deltal[[4]],thy],D[deltal[[4]],thz]],
{D[deltal[[5]],xd],D[deltal[[5]],yd],D[deltal[[5]],zd],D[deltal[[5]],thx],
D[deltal[[5]],thy],D[deltal[[5]],thz]],
{D[deltal[[6]],xd],D[deltal[[6]],yd],D[deltal[[6]],zd],D[deltal[[6]],thx],
D[deltal[[6]],thy],D[deltal[[6]],thz]] }/.
{xd->m[[1]],yd->m[[2]],zd->m[[3]],thx->m[[4]]Degree,thy->m[[5]]Degree,
thz->m[[6]]Degree)//N

JI=PseudoInverse[J]
sigma=Table[Sqrt[Sum[JI[[i,j]]^2 sigmal[[j]]^2,{j,6}]] If[i<=3,1,const],{i,6}]
COV=JI.V.Transpose[JI]
sigmac=Table[Sqrt[Abs[COV[[i,i]]]] If[i<=3,1,const],{i,6}]

plt[i_]:=ListPlot[Table[{cc,sigmac[[i]]},{cc,0,1,.01}],PlotJoined->True]
lab=FontForm[StringForm["x='', y='', z='', thetax='', thetay='', thetaz=''",
m[[1]],m[[2]],m[[3]],m[[4]],m[[5]],m[[6]]],{"Courier",10}]
plt1=Show[plt[1],plt[2],plt[3],PlotLabel->lab,Frame->True,GridLines->Automatic,
FrameLabel->{"Correlation Coefficient","Translational Error (in)"}]
plt2=Show[plt[4],plt[5],plt[6],PlotLabel->lab,Frame->True,GridLines->Automatic,
FrameLabel->{"Correlation Coefficient","Rotational Error (arc sec)"}]

f[{x1_,x2_,x3_,x4_,x5_,x6_}]:={x1,x2,x3,const x4,const x5,const x6}
maxerr=f[Map[Abs,JI].Transpose[{1,1,1,1,1,1} emax]];

```

Figure 4. The *Mathematica* commands which were used to generate Figures 2 and 3 and to perform most of the related computations.