

ACTIVE DAMPING FOR THE GBT ARM

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Abstract

A method of reducing the amplitude of the oscillation in the GBT arm is outlined in this report. It is shown that a simple, inexpensive device may be used to increase the damping coefficient from 0.01 to 0.04. This results in a significant increase in the rate of decay of the oscillation after position switching. In the simulations given in GBT Memo No. 129 (Ben Parvin and Wodek Gawronski) the telescope is commanded to step 1 degree in azimuth. After 10 seconds the telescope reaches the nominal commanded position, but there is an oscillation with a peak-to-peak amplitude of 36 arcseconds. With a damping factor of 0.01, after a further 10 seconds the oscillation has decayed to 23 arcseconds. If the damping is increased to 0.04, that oscillation decays to 5 arcseconds in the same time.

Introduction

GBT Memo #127 outlines the problem with the vibration induced in the GBT arm by position switching the telescope in azimuth. The result of the position switching is that the diffraction beam oscillates around the commanded position with a lightly damped oscillation that lasts for many tens of seconds. Although it seems likely that position command preprocessing may reduce the amplitude of this oscillation in the case of position switching, it is possible that other mechanisms may excite the oscillation. At low tracking rates, limit cycling is shown in the simulations to excite the oscillation, as does the wind.

It was recommended in Memo #127 that a study be made of methods of increasing the damping of this oscillation. This report outlines a method that increases the damping of the arm from 0.01 to 0.04 resulting in a significant improvement.

General Considerations for Damping

Over the past few years, the damping of structures has received a great deal of attention. Structures in space, aircraft, and tall buildings all suffer to some degree from unwanted vibration and the study of how to suppress these vibrations has resulted in many new techniques.

The GBT requirement differs from many of the more usual requirements. Quite frequently it is required only to prevent an oscillation of a certain frequency from building up slowly. In this case, either a tuned passive or active system may be used. A passive damper consists of a mass/spring/damper system and an active system of this type has a power injector to increase the

reaction forces needed for damping. These tuned systems, however, are inevitably high "Q" systems. In our application, we need the damping to take place in a short period of time, so implying a low "Q" system.

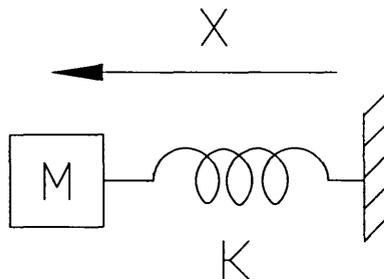
The system chosen here uses a moveable mass in the vicinity of the receiver cabin. The arm vibration is sensed and the mass moved so that the reaction force generated damps out the arm motion.

Analysis

This analysis should be considered very preliminary as obtaining the exact numbers concerning the structure will require more analysis. Factors of two or three may be fairly easily accommodated, however. It is the main purpose of this report to demonstrate the feasibility of such a method. In this analysis, we make the following assumptions:

1. All the pointing jitter discussed in GBT #127 results from motion of the subreflector with respect to the main reflector, *i.e.*, arm vibration.
2. The mass of the arm is considered to be concentrated at the end of the arm. In making this assumption, we tacitly ignore any motion in the supporting structure. Considering the arm mass to be concentrated at the end of the arm is such a pessimistic assumption that it is felt that ignoring the rest of the structure is warranted. This needs to be investigated further.

The vibration in the GBT arm may be approximated to a simple spring mass system. Although the vibration is obviously the sum of two modes of oscillation, they are so close in frequency that a simple spring mass model is sufficient to describe the oscillation.



The oscillation at the end of the feed arm is described in the report by Gawronski and Parvin as an oscillation of 6mm peak at a frequency of 0.75 Hz.

The mass of the feed arm is $2.26 \times 10^5 \text{ Kg} = M$ and the

Angular frequency $\omega = 2 \pi f = 4.71 \text{ rad/sec}$

Instantaneous displacement $x = a \sin \omega t = 6 \sin (4.71t) \text{ mm}$

Velocity $\frac{dx}{dt} = a\omega \cos (\omega t) = 28.3 \cos (4.71t) \text{ mm/s}$

Acceleration $\frac{d^2x}{dt^2} = -a\omega^2 \sin (\omega t) = -133.3 \sin (4.71t) \text{ mm/s}^2$

The energy in the arm changes from all kinetic at $x = 0$, to all potential at $x = 6 \text{ mm}$.

At $x = 0$

$$E = \frac{1}{2} M V^2$$

$$V = 0.0283 \text{ m/s}$$

$$E = \frac{2.26 \times 10^5 \times (0.0283)^2}{2} = 90.5 \text{ Joules}$$

or in terms of peak power, $P = \omega E = 4.71 \times 90.5 = 426 \text{ W}$.

The spring constant K may also be calculated

$$E = \int F \times dx = \int K \times dx = \frac{1}{2} Kx^2$$

$$\text{At } x = 6 \text{ mm, } E = 90.5 \text{ Joules}$$

$$K = \frac{90.5 \times 2}{(0.006)^2} = 5 \times 10^6 \text{ N/m}$$

Damping

The damping predicted for the structure as delivered is $Z = 0.01$ where Z is defined as:

$$\frac{x_n}{x_{n+1}} = e^{2\pi Z}$$

where x_n and x_{n+1} are the amplitudes of successive cycles. So, as delivered

$$\frac{x_n}{x_{n+1}} = 1.065$$

or each cycle is 0.94 the amplitude of the previous cycle.

If we want a damping of 0.04, each cycle must be 0.77 the amplitude of the previous cycle. So, we must add active damping to reduce each cycle by 0.83 by applying a reaction force to the arm to reduce the amplitude of the oscillation. This is done by accelerating a mass to generate the opposing force. For any given force, one can use a small mass with a large acceleration or a large mass with a smaller acceleration. The movement of the compensating mass will mimic the movement of the end of the arm — with only a difference in scale.

There are several approaches to take in calculating the mass and accelerations required. The following is a simple one.

We know the spring constant K for the arm so may calculate the restoring force at the peak initial displacement of 6 mm.

$$\text{Force} = 5 \times 10^6 \text{ N/m} \times 0.006 \text{ m} = 3 \times 10^4 \text{ N}$$

We wish to generate an opposing force of 0.17 of this = $5.1 \times 10^3 \text{N}$. If we say that the compensating mass is 1500 Kg, then the acceleration required is

$$\frac{5.1 \times 10^3}{1.5 \times 10^3} = 3.4 \text{m/s}^2$$

This compares with the maximum arm acceleration of 0.133 m/s^2 and velocity, amplitude, and acceleration of the compensating mass all scale by this amount, *i.e.*,

$$\frac{3.4}{0.133} = 25.6$$

The compensating mass then has movements as follow:

$$\text{Displacement } x_c = 153 \sin (4.71t) \text{ mm}$$

$$\text{Velocity } \frac{d x_c}{dt} = 724 \cos (4.71t) \text{ mm/s}$$

$$\text{Acceleration } \frac{d^2 x_c}{dt^2} = -3404 \sin (4.71t) \text{ mm/s}^2$$

The power required to move the mass may be calculated as follows:

$$\begin{aligned} \text{Peak power} &= \text{peak of (Force} \times \text{velocity)} \\ &= \text{peak of (mass} \times \text{acceleration} \times \text{velocity)} \end{aligned}$$

From the above, the peak power is the peak of

$$1500 \times 0.724 \cos (4.71t) \times 3.4 \sin 4.71t$$

$$\text{but } \sin (x) \times \cos (x) = 0.5 \sin (2x)$$

so the peak value of the sin term times the cosine term is 0.5. The peak power becomes:

$$\frac{1500 \times 0.724 \times 3.4}{2} = 1846 \text{ W} = 2.5 \text{ Hp}$$

Note that the peak instantaneous power occurs when the arm is half way between its peak displacement and the central zero displacement position (in time) and that the power fluctuates sinusoidally at twice the arm vibrational frequency. The power changes sign through the cycle so the mean power is zero, but the rms is around 1.3 KW.

It is interesting to note that the input power to the compensator is many times -- in fact, 150 times -- the power expended on the arm itself. The ratio of (energy put into the compensator)/(energy put into the mass of the arm) is simply the reciprocal ratio of their respective masses, or $2.26 \times 10^5 / 1500 = 150.7$. This is analagous to the energy imparted to a rocket by a stream of gas; the energy in the gas emitted is greater than the energy imparted to the rocket.

Another interesting point is that as the mass increases, the required power for the compensator decreases. The power required is proportional to mass \times velocity \times acceleration. If the mass is increased by n , the velocity required decreases by $1/n$ as does the acceleration, so the total power required is reduced by $1/n$.

A Practical System

A block diagram of a practical system for implementing the proposed damping scheme is shown in Figure 1.

The system is designed only to correct arm oscillations, of course, so any dc term must be removed from the arm position error signal. In fact this gives more flexibility to the arm position error sensing instrumentation. The quadrant detector is suitable but also the differenced output of two laser rangefinders mounted on the arm looking at retroreflectors mounted on the rim of the dish would give a suitable output. For this application atmospheric effects and instrumental drifts are unimportant, so the rangefinder could be very simple with no switching mirror required.

The position error signal passes through a high-pass filter with a low frequency cut-off point around 0.5 Hz. The output from the filter then serves as the input to a simple position servo that positions the 1500 Kg mass in the X-EL direction. The components needed to realize this simple system are "off-the-shelf" hydraulic components that are very similar to those used in Green Bank on the 140-ft nutator. Various strategies may be used to implement the compensator. The simplest, when

position switching, is to activate the compensator as soon as the desired position is reached. The compensating mass will then begin to move to damp out the first overshoot. Due to the ac coupling on the input to the position loop, the average position of the mass will be in the center of its travel.

The waveforms of the various relevant movements, etc., are shown in Figure 2. Several additional points about this system should be made.

1) The bandwidth of the compensator position servo may be made sufficient to damp out any vibration of less than 5 Hz in the cross-elevation direction. This should be effective in damping out wind-induced vibration.

2) The moving mass would be supported on linear bearings and would not be affected by movement in elevation.

Performance

The improvement obtained by the damping system is illustrated in Figure 3. The implementation of the system as shown in Figure 1 is a linear system; that is the movement of the mass is proportional to the movement of the arm. This means that the damping is increased to 0.04 with the improvement shown in Figure 3.

If we arrange so that the full compensator movement is maintained for the first 6 seconds; however, the oscillation decreases linearly instead of exponentially with the result shown in the third curve in Figure 3, a substantial further improvement.

In connection with these performance figures, it should be noted that a serious error was made in Memo 127. The improvement obtained by increasing the damping was greatly overestimated. The error shows most clearly in Figure 1 of that report in which the curve for damping = 0.02 starts at a peak error of 18 arc secs, while the curve for 0.01 damping starts at 36 arc secs. This is incorrect. Both curves should start at virtually the same place, so the improvement illustrated in both Figure 1 and Table 1 of Memo 127 is far too optimistic.

Design Parameters

There is freedom in the design parameters. This initial design uses the following parameters.

	Metric	British
Compensating mass	1500 Kg	3300 LB
Maximum movement of mass	± 153 mm	± 6 in
Maximum pressure in hydraulic system	—	1500 psi
Maximum flow rate	—	10 GPM
Maximum force on structure	5.1×10^3 N	1150 LB
Maximum horsepower required	—	2.5

The mass may be reduced, but the maximum movement will increase; so will the flow rate and so will the required horsepower, all for the same increase in damping.

Cost Estimate

The costs of such as system are very low. The two major components in the hydraulic system are the hydraulic power supply and the servo valve. A quote has been obtained on a suitable supply — \$1600 — and the servo valve is made by Moog at a cost of \$600. These valves are used on the 140-ft nutator and have a life of around 10 years.

Acknowledgements

We wish to thank Dave Parker, Bob Wilson and Ed Wollack for many helpful discussions.

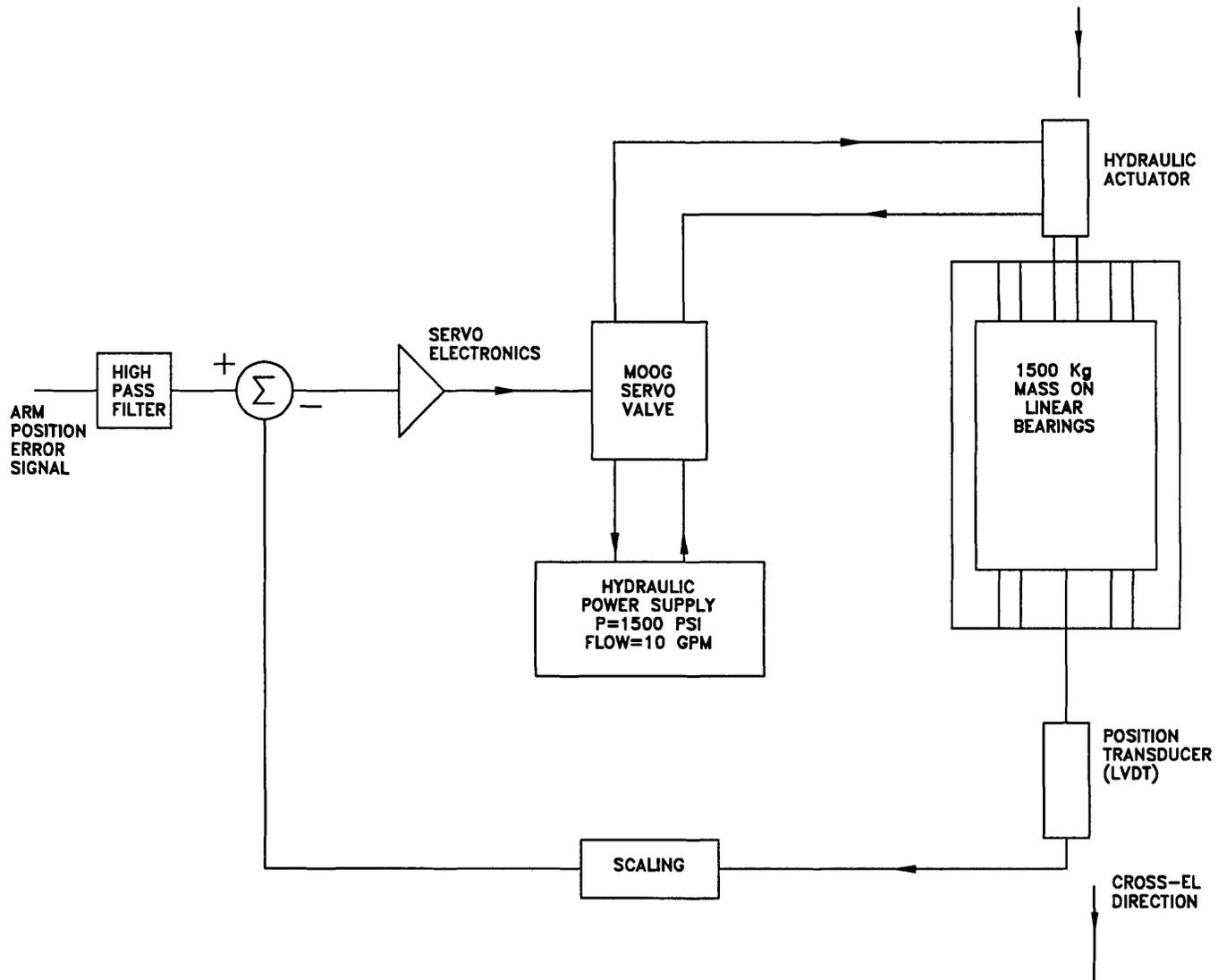


FIGURE 1 - BLOCK DIAGRAM OF ACTIVE DAMPER

Mass at end of arm:	$M = 2.26 \cdot 10^5$	kgm
Frequency of oscillation	$f = 0.75$	Hz
	$\omega = 2 \cdot \pi \cdot f$	
Peak amplitude of oscillation of the arm	$a = 0.006$	m
Compensation mass:	$m := 1500$	kgm

For the mass of the arm:

Instantaneous arm displacement:	$x(t) := a \cdot \sin(\omega \cdot t)$
Instantaneous velocity	$v(t) := a \cdot \cos(\omega \cdot t) \cdot \omega$
Instantaneous acceleration	$accn(t) := -a \cdot \sin(\omega \cdot t) \cdot \omega^2$

For the compensating weight:

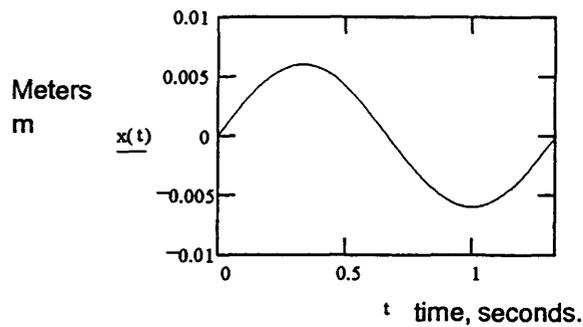
Fraction of restoring force of the arm spring which is being compensated.

$$f_m := 0.17$$

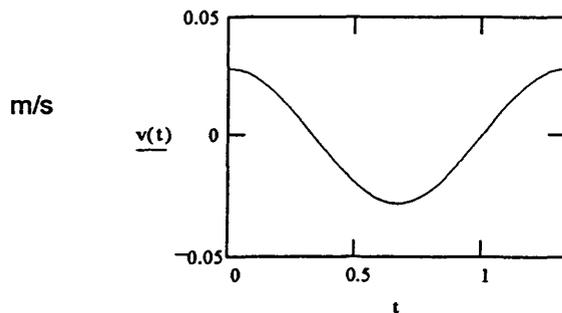
$$x_m(t) = f_m \cdot \left(\frac{M}{m}\right) \cdot (a \cdot \sin(\omega \cdot t))$$

$$v_m(t) = f_m \cdot \left(\frac{M}{m}\right) \cdot (a \cdot \cos(\omega \cdot t) \cdot \omega)$$

$$accn_m(t) := f_m \cdot \left(\frac{M}{m}\right) \cdot (-a \cdot \sin(\omega \cdot t) \cdot \omega^2)$$

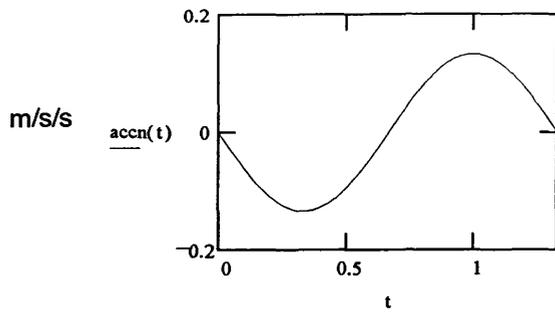


Displacement at end of arm

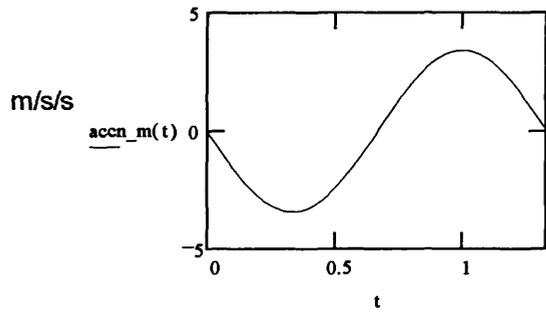


Velocity at end of arm

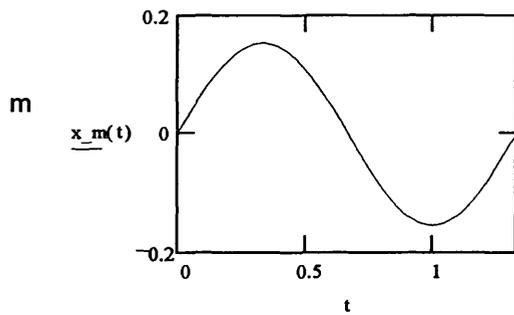
Fig. 2. Practical waveforms in the GBT active damping system.



Acceleration of end of arm



Acceleration of compensating mass



Displacement of compensating mass

Work expended on compensating mass

$$W(t) = m \cdot \text{accn}_m(t) \cdot v_m(t)$$

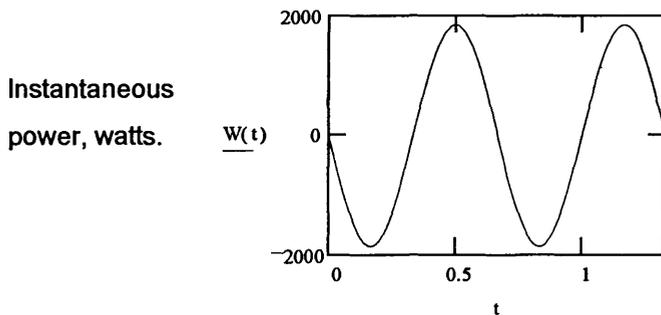


Fig. 2. (continued)

Amplitude of oscillation with 1%, and with 4% damping factors

Initial peak-peak displacement at $t=10$ seconds is 36 arc seconds. $ppa := 36$ arc sec

Count of oscillation cycles is "n".

Elapsed Time $t(n) := 10 + \frac{n}{f}$ seconds

Amplitude with 1% damping $a1(n) := ppa \cdot e^{-(2 \cdot \pi \cdot n \cdot 0.01)}$

Amplitude with 4% damping $a4(n) := ppa \cdot e^{-(2 \cdot \pi \cdot n \cdot 0.04)}$

Linear regime: $aln(n) := ppa - ppa \cdot (1 - e^{-2 \cdot \pi \cdot 0.04}) \cdot n$

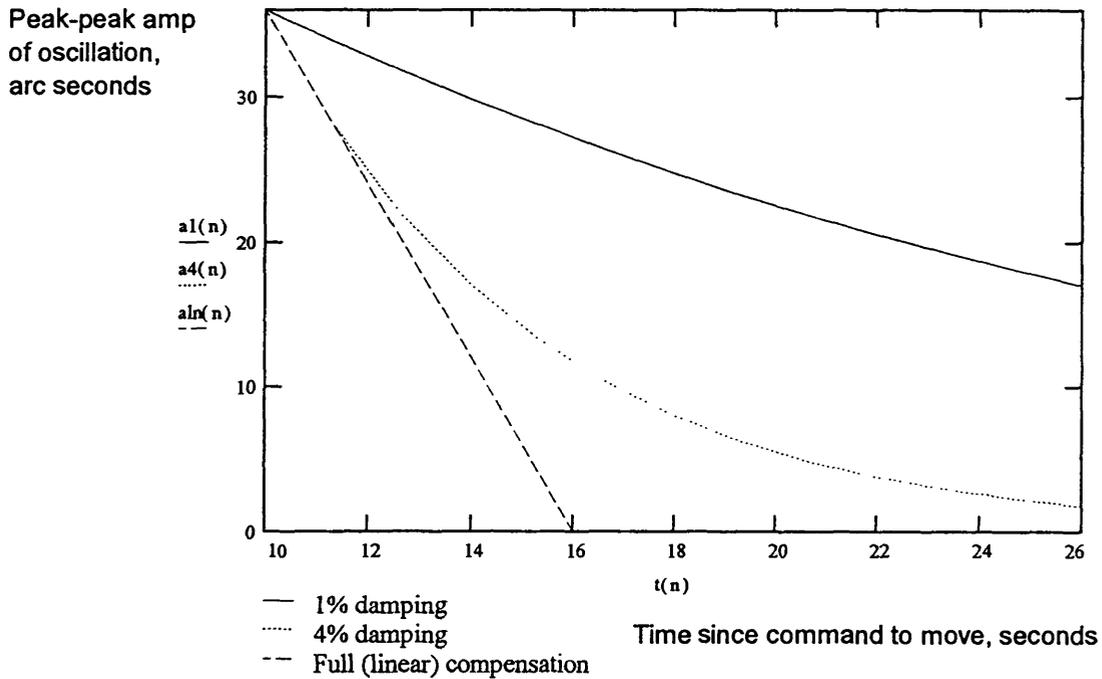


Fig. 3. Improvements in performance.