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LASER RANGEFINDER LOCATIONS AND ORIENTATIONS

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I. INTRODUCTION

John Payne, in GBT Memorandum No. 36 (Feb. 1990) [1], presented the initial design of the laser rangefinding system which is intended for pointing and surface metrology of the GBT. Today, in early 1996, the conceptual design is not very different, but the details have changed (e.g., the number of rangefinders) and quite a few details which previously had been neglected have now been worked out. The purpose of this memorandum is to fill in some of these details, particularly those which have to do with laser rangefinder locations and orientations.

II. RANGEFINDER COVERAGE

For purposes of the present memorandum, the most essential piece of information about the rangefinders is their directional coverage. A schematic illustration of the present rangefinder mechanical design is shown in Figure 1. In the coordinate system of this figure, where a rangefinder is shown in its "bench" orientation, the direction cosines of the rangefinder beam, as a function of "azimuth" α and "elevation" a , are given by the column vector $\begin{pmatrix} \sin a \\ \sin \alpha \cos a \\ \cos \alpha \cos a \end{pmatrix}$. Azimuthal motion, which here is defined as rotation about the x -axis, is measured clockwise from the z -axis in the (y, z) -plane. Elevation motion, defined as rotation about the y -axis, is measured clockwise from the z -axis in the (x, z) -plane.

Figure 3(a) shows the directional coverage of a rangefinder in its bench configuration. The axes in this plot are the directions cosines (l, m, n) with respect to the (x, y, z) -coordinate system of Figure 1. This plot represents the *nominal* coverage. The actual boundaries of the rangefinder coverage may differ slightly from what is shown here. The effective coverage will be somewhat greater if it turns out that rangefinder performance is not significantly degraded by a small amount of beam blockage. The machining of portions of the base plate and the gimbal drive housing could perhaps be modified to provide slightly greater coverage, if that should turn out to be necessary. The nominal amount of areal coverage, in direction cosine space, is 4.93664 steradians (approximately 79% the area of a hemisphere).

III. RANGEFINDER LOCATIONS ON THE TELESCOPE STRUCTURE

Since the time that Memorandum No. 36 was written, the number of rangefinders to be located on the telescope structure has increased from three to six. Originally, these three were to be devoted solely to measuring the figure of the primary reflector. The consensus of the GBT design group now is that rangefinders should also have some role in positioning the secondary reflector. (They may also be useful in re-calibrating the subreflector positioning mechanism at occasional intervals over the lifetime of the telescope.) Thus, now, four rangefinders are to be mounted near

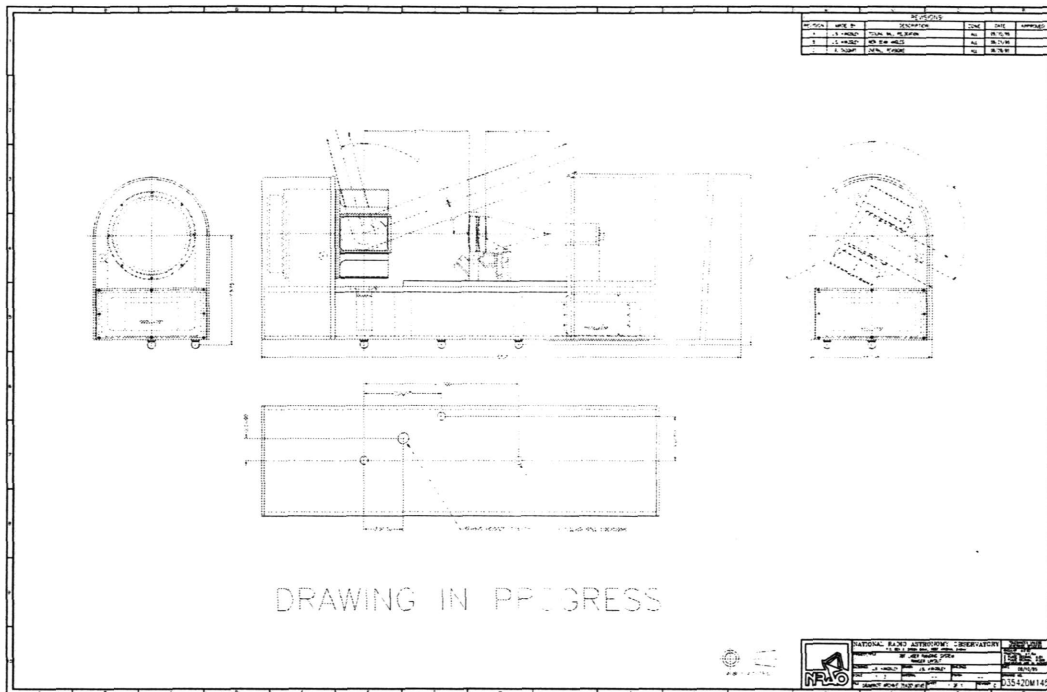


Figure 1. Laser rangefinder schematic illustration, with the rangefinder in its “bench” orientation. The top center view shows the elevation range of the mirror gimbal. The rangefinder directional coverage is described with respect to a Cartesian coordinate system whose origin is at the mirror center, whose positive x -axis is oriented horizontally and to the right in the plane of the top center view, whose positive y -axis is directed into the page, and whose positive z -axis is oriented vertically. The beam can be steered in elevation over the range -15° to $+70^\circ$. The right-hand view shows the range of azimuthal motion, -118° to $+118^\circ$.

the receiver room, and two are to be located lower down on the feed arm. Figure 2 shows the planned locations of these rangefinders.

Quite a number of considerations are involved in selecting the rangefinder locations and orientations. Among these are the following:

- (1) For each point on the primary surface, at least three—and preferably more—rangefinders should be able to see it;
- (2) At least three—preferably more—rangefinders should be able to see the entire surface of the subreflector;
- (3) Rangefinders should be able to aim toward the locations of other rangefinders;
- (4) Their locations should, if possible, be visible from the ground (and perhaps they themselves should be able to view the ground);
- (5) Certain potential mounting locations are precluded because of excessive blockage from substructures such as the receiver room, the prime-focus boom, and the subreflector;
- (6) Wider (and taller) rangefinder baselines are better than narrower (or shorter) ones [2];
- (7) But—the retroreflectors to be mounted on the primary surface have only limited acceptance angles and will be fixed in orientation [3,4];

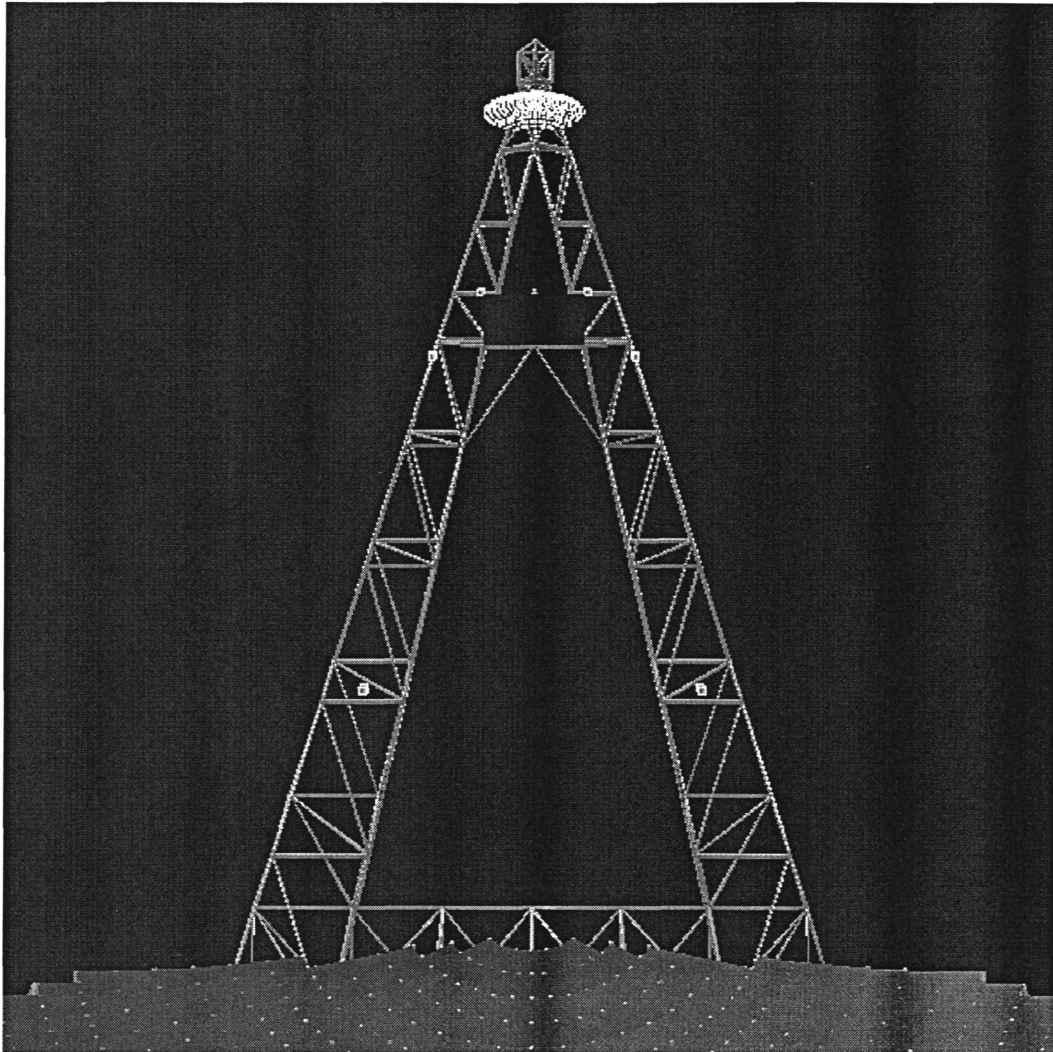


Figure 2. Rangefinder locations on the GBT feed arm. (Rangefinders are represented in this scene by 20-inch cubes.) Four rangefinders are to be located on or near the receiver room, and two are to be located lower down on the feed arm. The rangefinders at the front of the receiver room roof are numbered 1 and 2, the lowermost rangefinders are numbered 3 and 4, and those on the feed arm near the floor of the receiver room are numbered 5 and 6. Odd-numbered rangefinders are rightmost in the figure, and even-numbered ones are to the left. (This figure, generated within the AVS visualization system, was kindly furnished by Don Wells.)

- (8) Rangefinder directional coverage is limited, as was already described in Section II;
- (9) Safe and convenient access to the rangefinders is essential, so that they can easily be serviced. Stairways, walkways, handrails, landings, and mounting platforms are required. These considerations influence the detailed locations of the rangefinders; for example, a mounting location just above large horizontal structural member might be preferred over a location just below such a member, to avoid the need for stooping or prone access.

Memorandum No. 36 showed longer rangefinder baselines than have finally been settled upon, and my Memorandum No. 37, on the error sensitivity of GBT laser metrology, also assumed a more favorable geometry. A wider baseline between the two lower rangefinders is precluded for a couple of reasons: The feed arm has been re-designed since 1990, and the arms now are spaced closer together. Retroreflector acceptance-angle limitations preclude placing the rangefinders much lower down or much further apart. And, if positioned lower down on the arm, the rangefinders would not be able to view the entire primary surface.

More extended vertical baselines are pretty much precluded as well, by the following considerations: Visibility is very much restricted by obstacles such as the receiver room, the prime-focus boom, and the subreflector if rangefinders are located higher than the roof of the receiver room. Retroreflector acceptance angles again become a problem if the vertical components of the baselines are more extended. Finally, locations near the receiver room will be ideal for viewing the subreflector.

In a right-handed Cartesian coordinate system whose origin is at the vertex of the nominal design paraboloid, whose z -axis is directed skyward and is coincident with the central axis of the paraboloid, and whose x -axis is directed toward the far side of the dish, our proposed rangefinder locations, in meters, are

$$\begin{aligned} P_1 &= (0.232, +3.848, 49.336), \\ P_2 &= (0.232, -3.848, 49.336), \\ P_3 &= (0.452, +12.255, 18.062), \\ P_4 &= (0.452, -12.255, 18.062), \\ P_5 &= (-4.717, +7.754, 42.909), \quad \text{and} \\ P_6 &= (-4.717, -7.754, 42.909). \end{aligned}$$

A different coordinate system, with origin near the centroid of the telescope backup structure, has been used for all engineering design drawings of the GBT tipping structure. These drawings are labeled in inches. The proposed rangefinder locations, specified in inches, in this coordinate system are

$$\begin{aligned} P'_1 &= (-151.50, -2149.87, 4039.22), \\ P'_2 &= (+151.50, -2149.87, 4039.22), \\ P'_3 &= (-482.48, -2141.22, 2807.95), \\ P'_4 &= (+482.48, -2141.22, 2807.95), \\ P'_5 &= (-305.28, -2344.74, 3786.17), \quad \text{and} \\ P'_6 &= (+305.28, -2344.74, 3786.17). \end{aligned}$$

(The conversion from the latter coordinate system to the former is $(x, y, z) = 0.0254 (y' + 2159.02, -x', z' - 2096.85)$.)

At this stage in the project, these proposed rangefinder locations do appear to have received final group consensus approval and, therefore, pending the detailed design of mounting platforms, are probably uncertain by at most a meter or so each. Access to Rangefinders 3 and 4 is complicated and will entail the construction of either a truss between the two sides of the feed arm or a network of stairways or ladders. Design work on the mounting platforms was recently begun.

IV. ORIENTATIONS OF RANGEFINDERS ON THE TELESCOPE STRUCTURE

The chief desiderata in deciding upon a suitable rangefinder orientation are the following:

- (1) Ability to view the entire surface of the primary reflector;
- (2) Maximal coverage of the subreflector; and
- (3) The ability to view the locations of as many other rangefinders as possible.

These are ordered roughly by priority. Possibly, the ability to view ground-based monuments should be added to this list. That capability would be very difficult to achieve, however, (except in a haphazard manner) unless one of the other requirements were dropped.

I wrote a set of simple *Mathematica* functions, combining numerical and graphical utilities, that were a great help in my largely “exploratory” (i.e., trial-and-error) investigation of possible orientations. Here is a sketch of the procedure I followed: Since item (1), above, is the most important consideration, I first computed a mean direction to the primary reflector. I then constructed a rotation which would aim the central point of the swath of rangefinder coverage (refer again to Fig. 3(a)) in this mean direction. Then I pivoted the rangefinder coverage about this mean direction, in order to align the principal axis of the swath of rangefinder coverage so that it could be made to cover some of the subreflector as well as the main reflector (here, there were two possibilities to consider). Next, I rotated in rangefinder azimuth by roughly the right amount to maximize coverage of the subreflector. Finally, having chosen easily visualizable rotational parametrizations, I diddled parameters to further maximize the coverage of the two reflectors and to get visibility toward other rangefinders.

Orientation of Rangefinders 1 and 2. Rangefinders 1 and 2, one should recall, are located near the front edge of the roof of the receiver room. Rangefinder 1 is rightmost in the view shown in Figure 2. The results of my optimization are illustrated in Figure 3.

Figure 3(b) shows the direction cosines from Rangefinder 1 to the main reflector, the subreflector, and the other five rangefinders. Figures 3(c–e) are three different 3-D perspective views of Figure 3(b), in combination with the optimally oriented swath of rangefinder coverage. Figure 3(d) shows that the entire main reflector is visible, and Figure 3(e) reveals that the same is true for the subreflector. Figure 3(f) shows the azimuth and elevation angles which are required to view the perimeter of the primary reflector. Rangefinders 5 and 6, just below the receiver room, are not visible. Rangefinder 2 is well within the field of view. Rangefinders 3 and 4, on the lower part of the feed arm, are just outside the rangefinder coverage (at elevation angles of $-16^\circ 9$ and $-16^\circ 6$, respectively); if a small amount of beam blockage is acceptable, then the baselines to these rangefinders can be measured from Rangefinder 1.

The chosen orientation for Rangefinder 1 can be represented by the rotation matrix

$$R_1 = \begin{pmatrix} 0.9488847 & -0.2817400 & 0.1422685 \\ 0.0263096 & -0.3785804 & -0.9251944 \\ 0.3145244 & 0.8816459 & -0.3518167 \end{pmatrix}.$$

(The transformation of unit vectors in each of the x -, y -, and z -directions is given by the first, second, and third columns of R_1 , respectively.) Equivalently, the transformation can be described by three Euler angles, $\phi_1 = 160^\circ 37'$, $\theta_1 = 110^\circ 60'$, and $\psi_1 = 171^\circ 26'$.¹

The orientation derived for Rangefinder 2 is shown in Figure 4. With respect to Figure 3 there is reflection symmetry about the $m = 0$ plane, so no lengthy explanation is required. Besides the main reflector and the subreflector, Rangefinder 2 can see Rangefinder 1; and, if a bit of blockage is acceptable, it can see Rangefinders 3 and 4 as well. The rotation matrix for Rangefinder 2 is

$$R_2 = \begin{pmatrix} 0.9488847 & 0.2817400 & 0.1422685 \\ -0.0263096 & -0.3785804 & 0.9251944 \\ 0.3145244 & -0.8816459 & -0.3518167 \end{pmatrix},$$

and the equivalent Euler angles are $\phi_2 = 19^\circ 63'$, $\theta_2 = 110^\circ 60'$, and $\psi_2 = 8^\circ 74'$. Note that $\theta_2 = \theta_1$ and that ϕ_2 and ψ_2 are the complements, respectively, of ϕ_1 and ψ_1 .

Orientation of Rangefinders 3 and 4. For the locations of Rangefinders 3 and 4, low down on the feed arm, I could not find orientations strictly suitable for viewing the entire primary reflector, under the nominal rangefinder viewing-angle constraints. However, Figure 5 illustrates what I believe, for Rangefinder 3, to be quite a reasonable compromise solution. Over a small region at the near edge of the dish, just below the rangefinder, the minimum elevation limit of -15° is exceeded by a significant amount ($a_{\min} = -18^\circ 16'$). However, beyond a distance of 1 to 2 meters toward the interior of the dish the elevation angle is within the acceptable range, as shown in Figure 6. The nominal elevation-angle upper limit of 70° is approached—but just missed—at the far side of the dish, near $x = 102$ m, $y = -14$ m.

Finding an optimal orientation for Rangefinder 3 required more computational effort than did the other cases. The crucial step involved calculating moments of inertia of the set of directions from the rangefinder to the main dish; my description of the way to do this is borrowed, in part, from a book by Mardia [5, pp. 223 ff.]. The element of surface area on the paraboloid, of focal length $f = 60$ meters, is $dA = \sqrt{1 + \frac{x^2 + y^2}{4f^2}} dx dy$. The area of the dish is $\mathcal{A} = \iint_D dA$, where $D = \{(x, y) \mid (x - 54)^2 + y^2 < 50^2\}$, approximately 8893 square meters. If we let $l(x, y)$, $m(x, y)$, and $n(x, y)$ represent the direction cosines from P_3 to a point $P = (x, y, \frac{x^2 + y^2}{4f})$ on the dish, then the moment of inertia of the set of all such directions, about an arbitrary direction \mathbf{u} , is given by $M = \mathbf{u}^T \mathbf{B} \mathbf{u}$, where $\mathbf{B} = \mathbf{A} \mathbf{I} - \mathbf{T}$, and \mathbf{T} is the matrix of integrated squares and products of l , m , and n , i.e.,

$$\mathbf{T} = \begin{pmatrix} \iint_D l^2(x, y) dA & \iint_D l(x, y)m(x, y) dA & \iint_D l(x, y)n(x, y) dA \\ \iint_D l(x, y)m(x, y) dA & \iint_D m^2(x, y) dA & \iint_D m(x, y)n(x, y) dA \\ \iint_D l(x, y)n(x, y) dA & \iint_D m(x, y)n(x, y) dA & \iint_D n^2(x, y) dA \end{pmatrix}.$$

The integrals were computed numerically, as were the eigenvalues $\beta_1 > \beta_2 > \beta_3$ and corresponding eigenvectors \mathbf{b}_1 , \mathbf{b}_2 , and \mathbf{b}_3 , of \mathbf{B} . The maximum moment of inertia

¹The Euler angles are defined as in Goldstein's *Classical Mechanics* (p. 107 of the 1950 edition).

($M = \beta_1$) is about the direction \mathbf{b}_1 , and the minimum moment of inertia ($M = \beta_3$) is about \mathbf{b}_3 . The principal swath of rangefinder coverage should be oriented in the plane of \mathbf{b}_1 and \mathbf{b}_2 , and the minor axis should lie in the plane of \mathbf{b}_1 and \mathbf{b}_3 . However, since the range of azimuthal coverage is much greater than what is required to view the main dish, there was considerable flexibility in further adjusting the orientation, by means of an azimuthal angle offset, which I did by trial-and-error.

Rangefinder 3 is easily able to view all other feed-arm rangefinders, except Rangefinder 5. And, if a tiny amount of beam blockage is acceptable, then it can also view Rangefinder 5, which is just slightly outside of the nominal azimuth range at $\alpha = -118^\circ 20$

Although Figure 5(e) shows nearly all of the subreflector to be in the field of view, most of that view would be blocked by the receiver room. A narrow portion of the subreflector, overhead of the rangefinder and off to the side of the receiver room, is visible from Rangefinder 3; some of that portion is just outside the azimuth range, however.

This orientation for Rangefinder 3 is represented by the rotation matrix

$$R_3 = \begin{pmatrix} 0.7946583 & -0.5887225 & 0.1480678 \\ -0.2400360 & -0.5287585 & -0.8141235 \\ 0.5575849 & 0.6114084 & -0.5614969 \end{pmatrix},$$

and the equivalent Euler angles $\phi_3 = 137^\circ 64$, $\theta_3 = 124^\circ 16$, and $\psi_3 = 169^\circ 69$. The symmetrically derived orientation for Rangefinder 4 is represented by

$$R_4 = \begin{pmatrix} 0.7946583 & 0.5887225 & 0.1480678 \\ 0.2400360 & -0.5287585 & 0.8141235 \\ 0.5575849 & -0.6114084 & -0.5614969 \end{pmatrix},$$

and the equivalent Euler angles $\phi_4 = 42^\circ 36$, $\theta_4 = 124^\circ 16$, and $\psi_4 = 10^\circ 31$. Orientation plots for Rangefinder 4 are not presented here, since they would simply be mirror images of the plots shown for Rangefinder 3 (cf. Figs. 3 and 4).

Orientation of Rangefinders 5 and 6. The orientation problem for Rangefinders 5 and 6, which are located at opposite edges of the feed arm just below the level of the floor of the receiver room, works out extremely well. These rangefinders can be oriented so that all of the primary reflector and all of the secondary are well within the rangefinder coverage. Additionally, the orientations can be chosen so that, in each case, four of the other rangefinders can be viewed; and all five would be visible were it not for blockage of one by the receiver room.

The proposed orientation for Rangefinder 5 is illustrated in Figure 7. Support beams for the receiver room block a small portion of the view to the main reflector, as shown (for Rangefinder 6) in Figure 8. The lines of sight to Rangefinders 3 and 4 are not blocked by the support beams. Figure 9 shows the view from Rangefinder 6 to the subreflector; there is no blockage by the receiver room.

This orientation for Rangefinder 5 is represented by the rotation matrix

$$R_5 = \begin{pmatrix} 0.9045656 & -0.3361845 & 0.2621852 \\ 0.0861215 & -0.4582093 & -0.8846623 \\ 0.4175454 & 0.8228148 & -0.3855277 \end{pmatrix},$$

and the equivalent Euler angles $\phi_5 = 153^\circ 09$, $\theta_5 = 112^\circ 68$, and $\psi_5 = 163^\circ 49$. The symmetrically derived orientation for Rangefinder 6 is represented by

$$R_6 = \begin{pmatrix} 0.9045656 & 0.3361845 & 0.2621852 \\ -0.0861215 & -0.4582093 & 0.8846623 \\ 0.4175454 & -0.8228148 & -0.3855277 \end{pmatrix},$$

and the equivalent Euler angles $\phi_6 = 26^\circ 91$, $\theta_6 = 112^\circ 68$, and $\psi_6 = 16^\circ 51$.

V. LOCATIONS AND ORIENTATIONS OF THE GROUND-BASED RANGEFINDERS

Some time ago it was decided, by group consensus, that the ground-based laser system should comprise twelve rangefinders approximately equi-spaced around a circle of 120-meter radius, centered on the pintle bearing. The ground-based lasers will be used to measure the distances to retrospheres located around the perimeter of the main dish; to various targets on the alidade structure, especially including points on the elevation gear housings: targets on the azimuth-drive trucks (perhaps); targets on the feed arm; and to the other ground-based rangefinders. Locations for the rangefinder mounting piers have been selected. The precise choices of these locations, in azimuth, were influenced more by logistics, site topography, and the presence of local obstructions than by geometrical requirements.

The main question to consider here is the orientation problem. Initially it was proposed that the rangefinders be mounted with their base plates positioned horizontally. If they were oriented with their positive “ x -axes” (as illustrated in Fig. 2) pointed in the direction of the pintle bearing, then the thirty-degree “zone of avoidance,” which is due to the elevation limit at $a = 70^\circ$, would preclude the possibility of viewing essentially any of the alidade structure, including the elevation shaft (see Fig. 10). There would be good visibility to the other ground-based rangefinders, however. If, instead, they were rotated 90° in the horizontal plane, then the main swath of rangefinder coverage would cover all, or most, of the telescope structure, but neighboring rangefinders could not be seen. The latter orientation was selected for the ongoing 140-Foot Telescope tests.

The optimal solution, I believe, would be to mount the rangefinders with their positive “ z -axes” pointed toward (or a little above) the center of the telescope azimuth ring and their positive “ x -axes” pointed vertically (or tilted back slightly). This would eliminate all the aforementioned viewing restrictions: the entire telescope structure and all neighboring rangefinders could be viewed. From discussions with Dave Parker and John Payne, it does appear that this alternative is feasible. A stable and secure mounting bracket would need to be designed, to attach to the concrete piers.

VI. DISCUSSION

The rangefinder locations and orientations that have been proposed here are, obviously, compromise solutions. It will likely turn out, in the end, that other choices would better have been made, because at this stage in the project it is impossible to know which rangefinding measurements will be of most crucial importance. My main objective has been to avoid any incredibly stupid blunders.

REFERENCES

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- [3] F. Schwab, "Orientation of the retroreflectors on the GBT primary surface", GBT Memo. No. 64, NRAO, May 19, 1991.
- [4] D. Seaman, "Corner cube mounting", GBT Memo. No. 65, NRAO, June 24, 1991.
- [5] K. V. Mardia, *Statistics of Directional Data*, Academic Pr., New York, 1972.

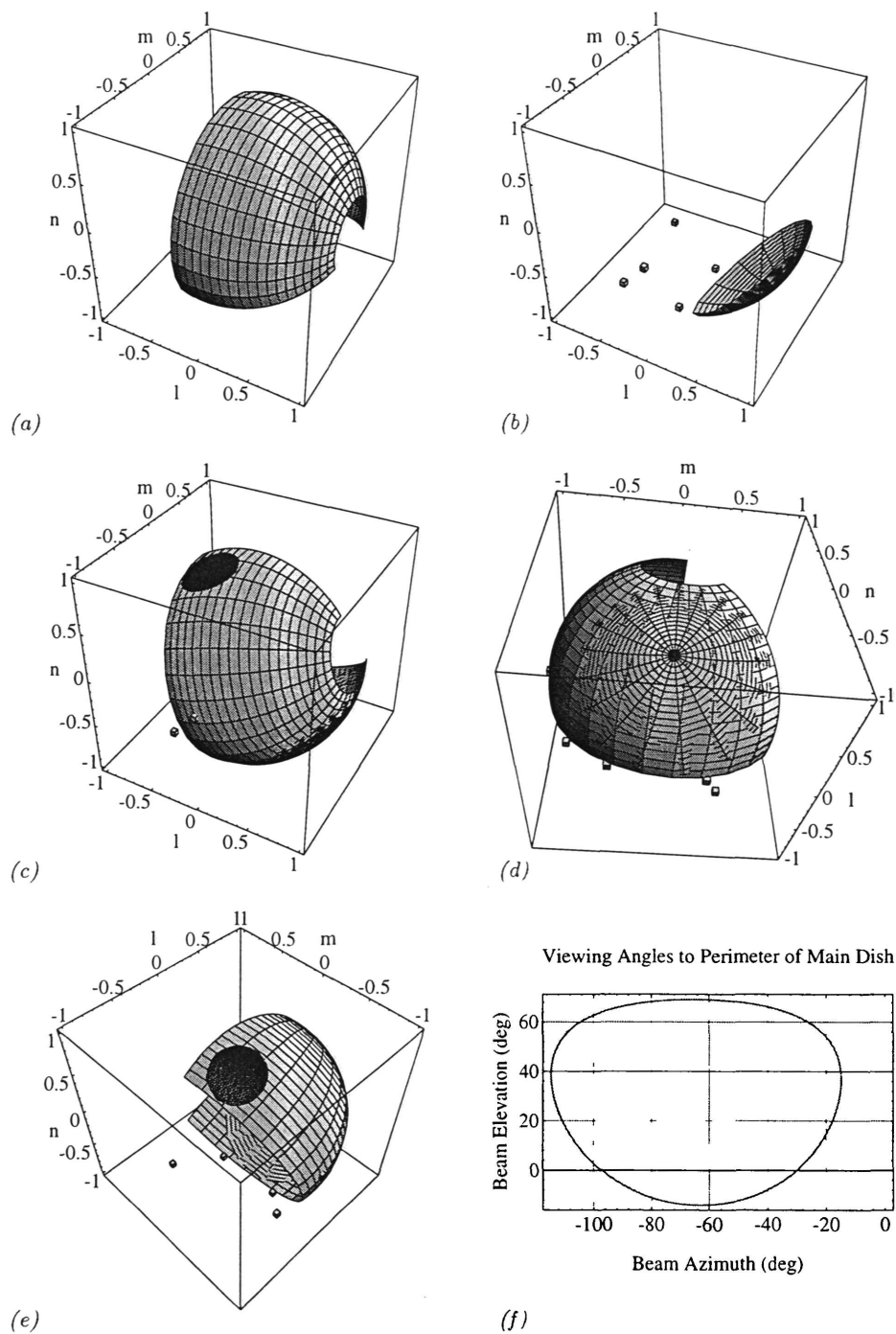


Figure 3. (a) Directional coverage of a rangefinder in its “bench” orientation; (b) direction cosines from the location of feed-arm Rangefinder 1, on the front edge of the receiver room roof, to the main reflector, the subreflector, and the other five rangefinders; (c) directional coverage of Rangefinder 1, when optimally oriented, together with an overlay of plot (b); (d) and (e) perspective views of plot (c), from behind the main reflector and from above; (f) viewing angles to the perimeter of the main reflector.

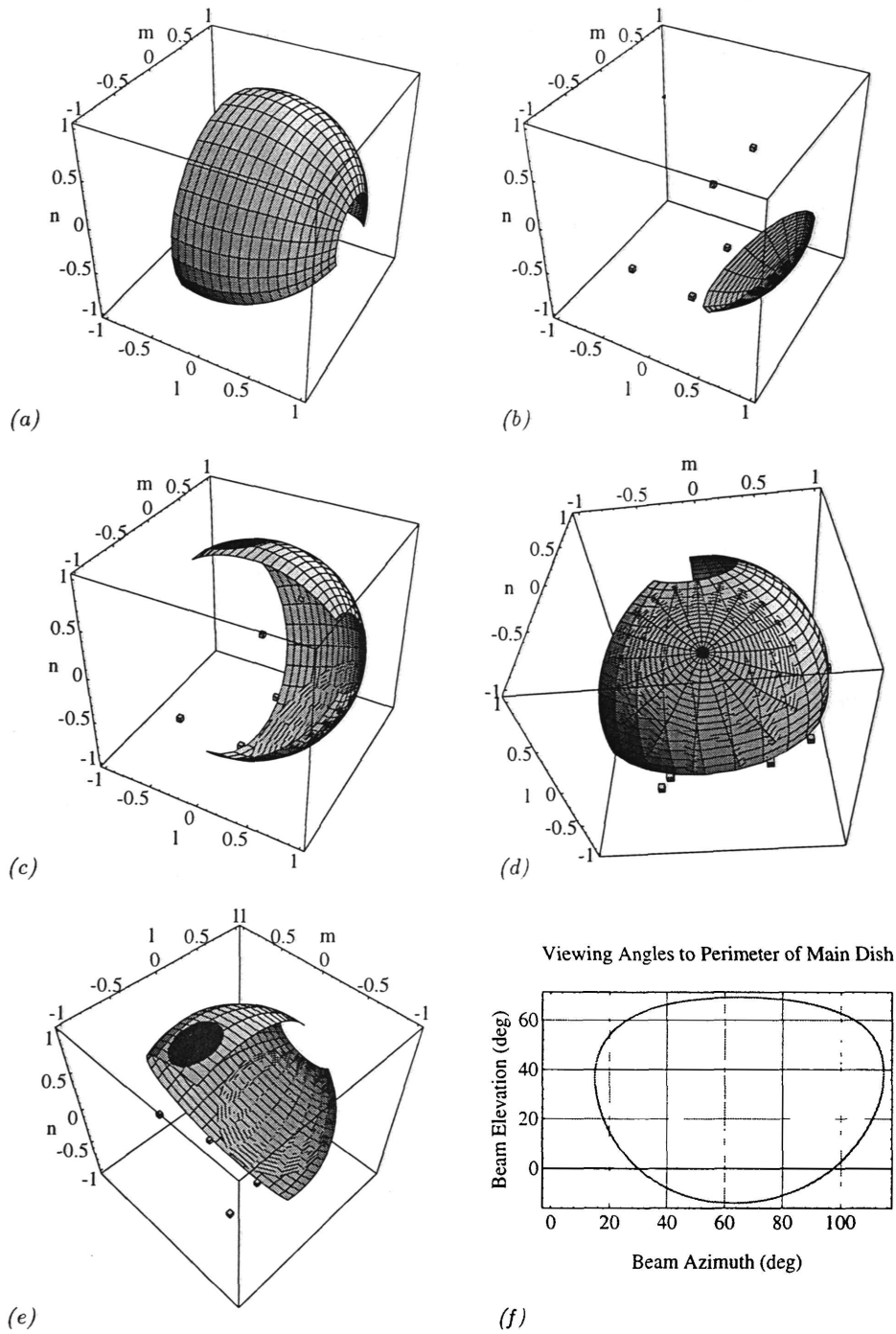


Figure 4. Like Figure 3, but for Rangefinder 2.

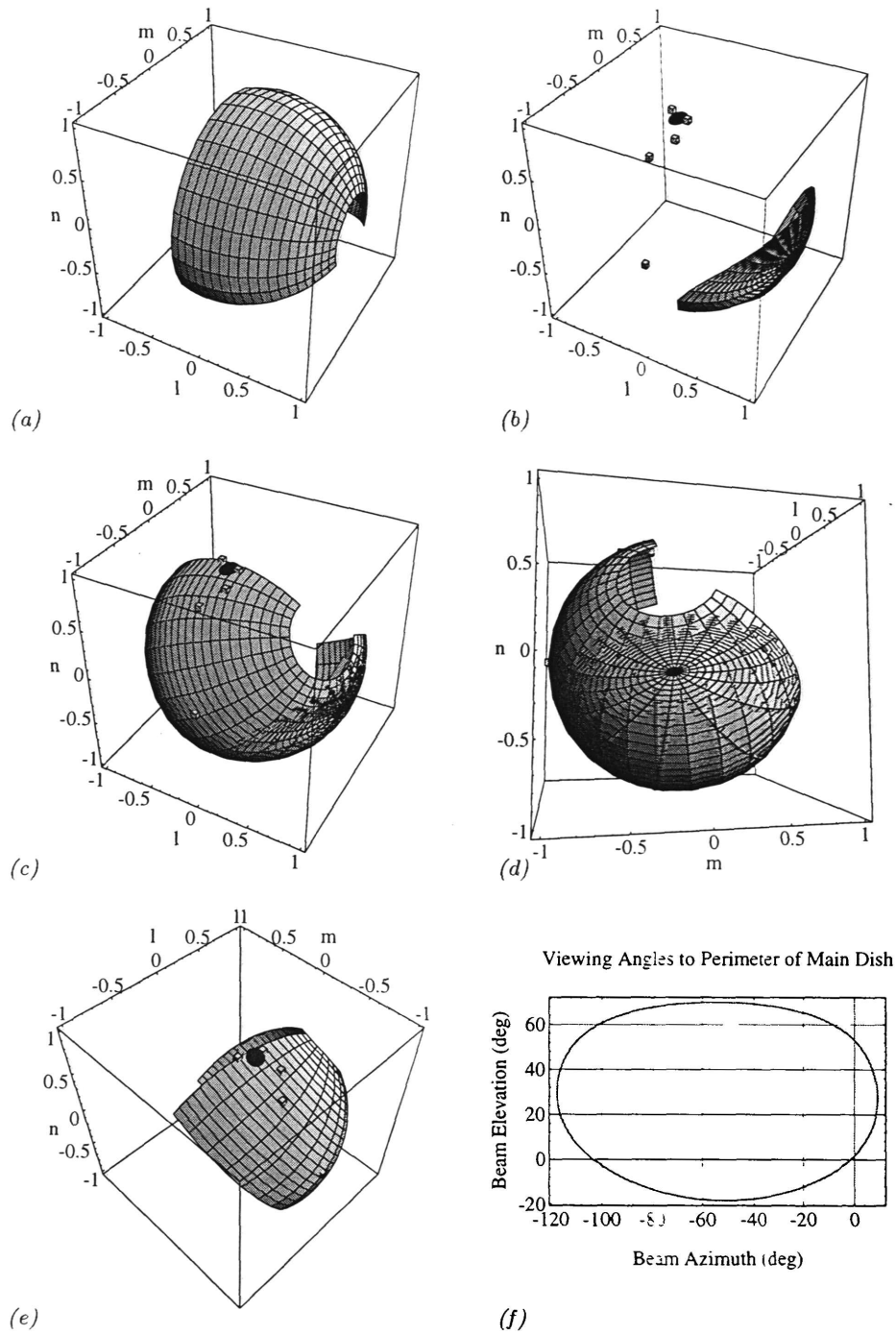


Figure 5. (a) Directional coverage of a rangefinder in its "bench" orientation; (b) direction cosines from the location of Rangefinder 3, on the lower part of the feed arm, to the main reflector, the subreflector, and the other five rangefinders; (c) directional coverage of Rangefinder 3, when optimally oriented, together with an overlay of plot (b); (d) and (e) perspective views of plot (c), from behind the main reflector and from above; (f) viewing angles to the perimeter of the main reflector.

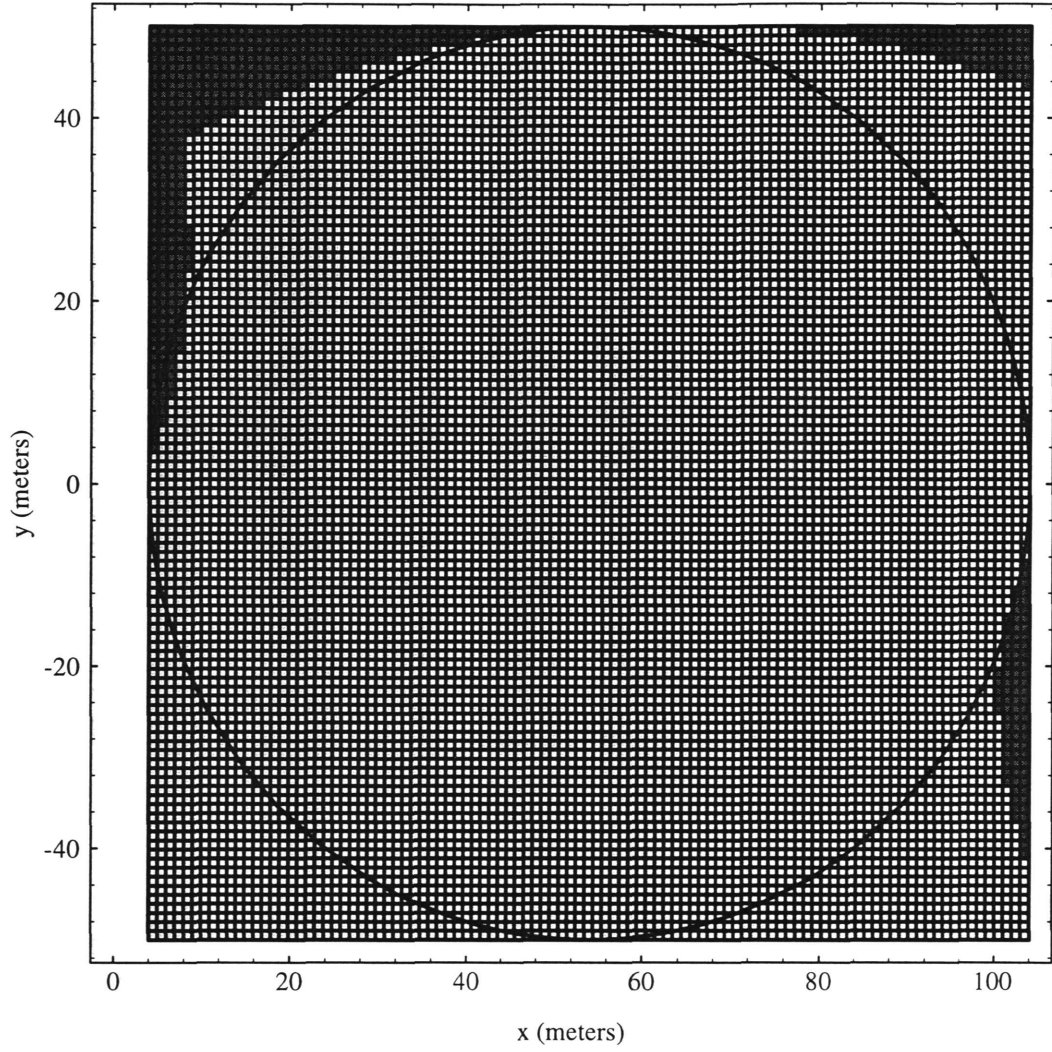


Figure 6. This figure shows the (x, y) -plane projection of the coverage, from Rangefinder 3, of the paraboloidal surface of the primary reflector. The region in which all of the nominal rangefinder viewing-angle constraints are satisfied is shown in yellow; points outside of that region are shown in blue. The mesh spacing is one meter.

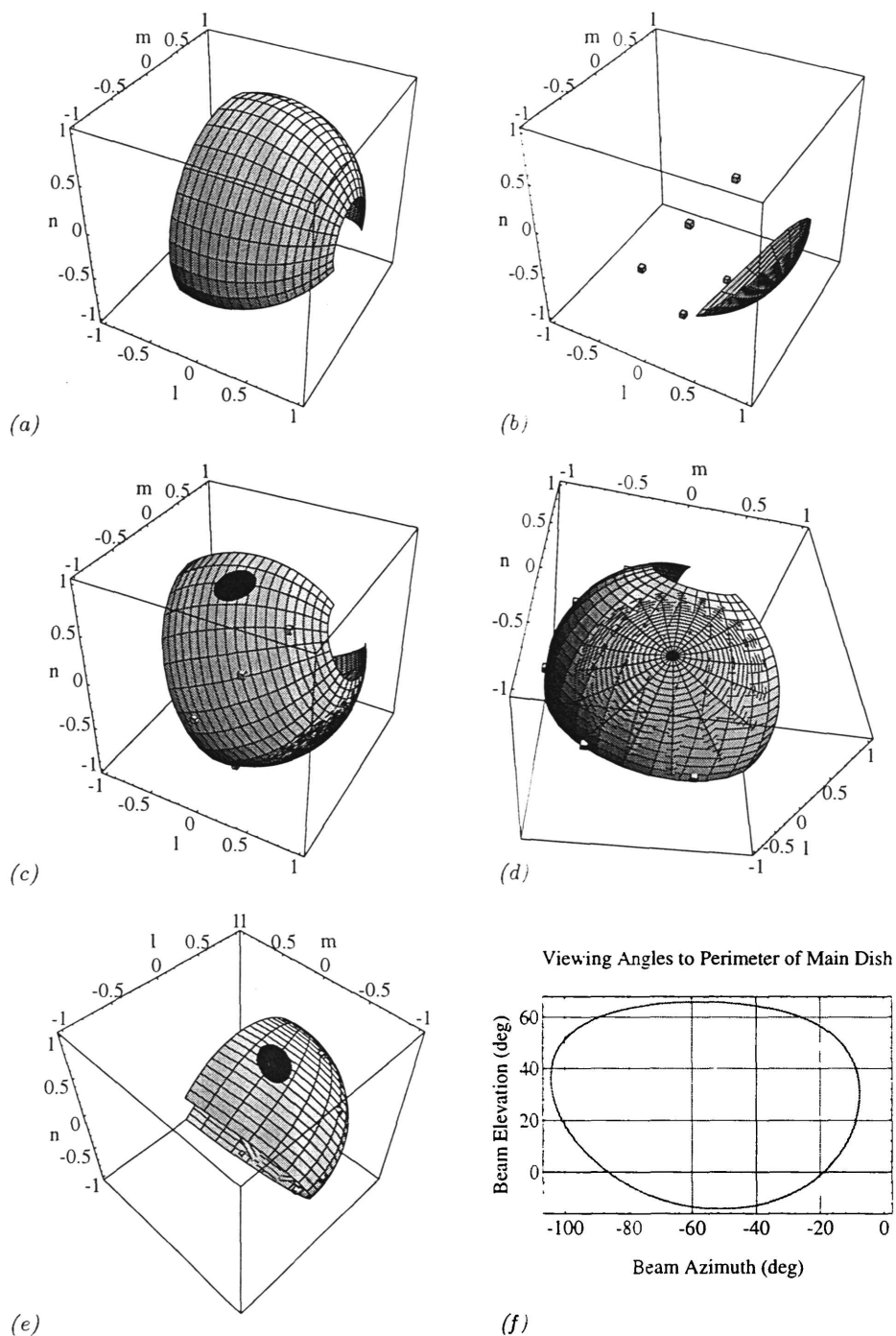


Figure 7. (a) Directional coverage of a rangefinder in its “bench” orientation; (b) direction cosines from the location of Rangefinder 5, which is located near the level of the floor—and toward the side and rear—of the receiver room, to the main reflector, the subreflector, and the other five rangefinders; (c) directional coverage of Rangefinder 3, when optimally oriented, together with an overlay of plot (b); (d) and (e) perspective views of plot (c), from behind the main reflector and from above; (f) viewing angles to the perimeter of the main reflector.

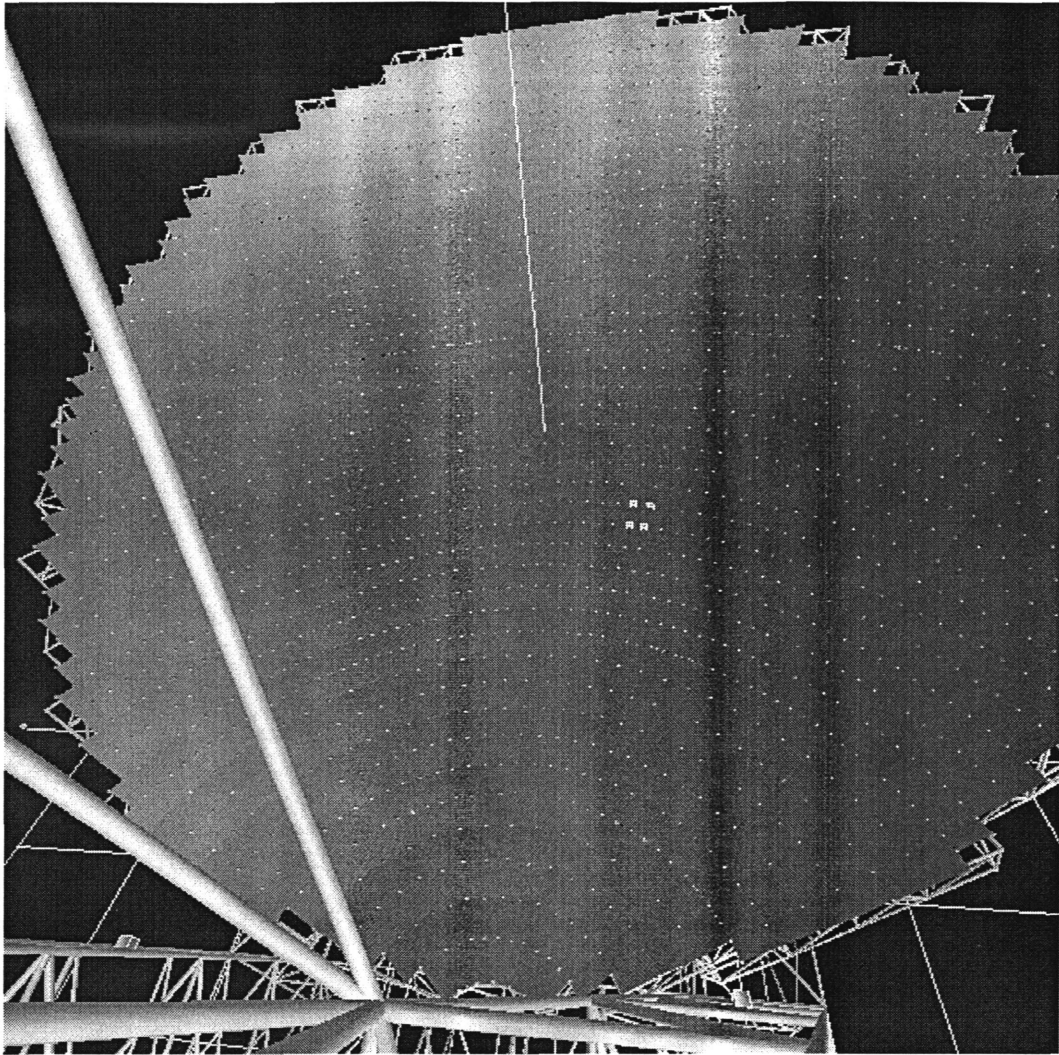


Figure 8. This figure shows the view from Rangefinder 6 to the primary reflector. Rangefinder 6 is located on the feed arm, near the level of the floor and toward the side and rear of the receiver room. The two structural members which partially obscure the view to the primary are shown true-to-scale, as 8-inch-diameter tubes. Rangefinders 3 and 4, on the lower part of the feed arm, are also visible. This scene was generated by Don Wells, using the AVS visualization system.

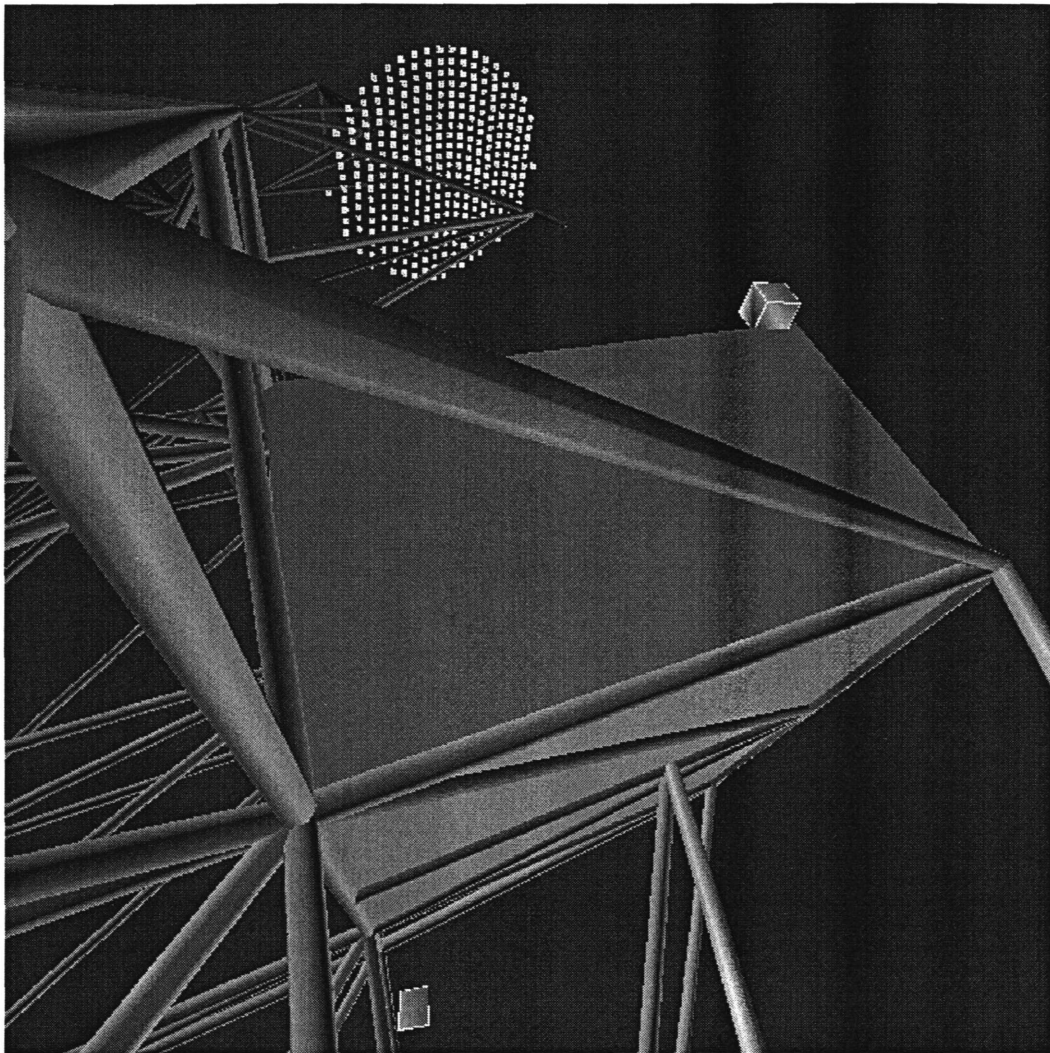


Figure 9. This figure shows the view from Rangefinder 6 to the subreflector. Rangefinders 2 and 5 are also visible. The prime-focus boom, comprising five main structural members, is shown in its extended position; when it is retracted for observations in Gregorian mode it will not block the view to the subreflector. Note that there is no blockage by the receiver room. This scene was generated by Don Wells, using the AVS visualization system.

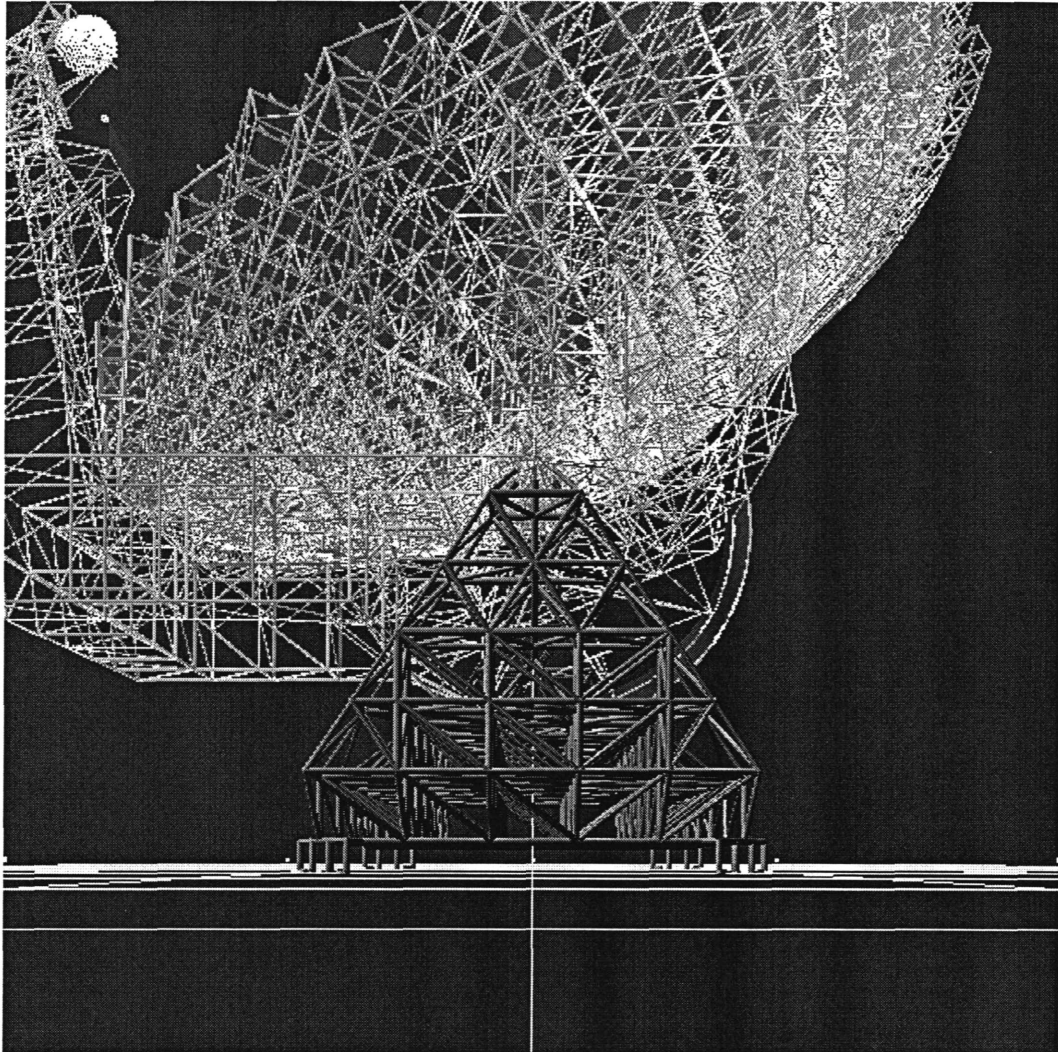


Figure 10. A simulated ground-level view of the GBT, from a distance of 120 meters from the center of the azimuth-track circle. This position corresponds to the location of one of the ground-based laser rangefinders. The 60° field of view is centered on the nearest elevation bearing. The elevation shaft, elevation bearing, and bearing housing could not be represented in this scene, however. This scene was generated by Don Wells, using AVS.