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## Preventing Oscillations of Large Radio Telescopes After a Fast Stop.

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### Summary

Subtraction of background noise is frequently done by fast movements, ON/OFF-source, of the telescope pointing. Time-consuming are either the resulting slowly damped oscillations of the dominant structural mode, or their avoidance by a slow deceleration.- It is shown that these oscillations can be prevented if the acceleration driving the telescope has the form  $A(t) = \sin^n(t)$ , and for a duration measured in multiples of the oscillation wavelength. Four cases are calculated, with  $n = 0, 1, 2, 3$ , (for telescopes of 100 meter diameter). Exact fast solutions are possible for zero structural damping and exact timing, for all four cases. Tolerable solutions (of  $\leq 1$  arcsec amplitude), of somewhat longer but still fast duration, are found only for cases  $n \geq 2$ , for any damping and with small timing deviations; and still somewhat longer durations will require no exact timing at all. The latter will also suppress any higher dynamic modes as well.- And a similar treatment, with similar results, is also given for a quick stop after a fast slew.- A method is suggested to measure the dominant mode of the beam oscillation directly on the telescope, at the half-power point of a strong radio source.

### 1. General Remarks

The paper deals with pointing oscillations from an immediate stop after a fast move. These are especially disturbing during ON-OFF observations, where not much time should get lost by slowly damped oscillations, or by slowly decelerated stops. It also matters for a quick stop after a fast slew.

A fast move consists of two parts: first its acceleration, its speed then being counteracted by its deceleration. It should be possible to let the second part counteract not only the speed, but as well the oscillations done by the first part. Since both parts can be made anti-symmetric, this leaves only one free parameter to be adjusted: the duration, or strength of the force.

All this is most important for large telescopes, which have slow eigenfrequencies and thus slow damping. Fig.1 shows the lowest frequency  $F_r$  and diameter  $D$  of 194 systems (received with thanks from Ralf White of Comsat-RSI). I have also added my old equation

$$F_r(D) = 1.0 \text{ Hz (100m/D)} \quad (1)$$

derived about 1965 as follows. The resonant frequency  $F_r$  of a mass  $M$  and a spring of stiffness  $K$  is in proportion to  $F_r \sim \sqrt{(K/M)}$ . And scaling a structure for different diameters  $D$ , we have  $K = (\text{cross section})/\text{length} \sim D^2/D = D$ , and  $M \sim D^3$ , thus  $F_r \sim \sqrt{(D/D^3)} = 1/D$ . And the constant (1.0 Hz) was obtained for an octahedron hanging between two tetrahedrons. The figure tells: You can do at lot worse than (1), but not much better. This means another "Natural Limit" for radio telescopes.

## 2. The Model

We call:  $Y(t)$  = Telescope Drive Program (to be chosen)  
 $X(t)$  = Telescope Movement (resulting)

and we use the simplified model of Fig.2, with  $K$  = spring constant,  $M$  = moved mass, and  $B$  = internal friction.  $B$  is the friction within the structure which causes the damping (not the external friction of gears and wheels). To obtain the resulting movement  $X(t)$  from the chosen drive  $Y(t)$ , we must integrate the differential equation (where  $\dot{\ } = d/dt$ ):

$$X'' = - (B/M)(X' - Y') - (K/M)(X - Y). \quad (2)$$

The two system constants,  $B/M$  and  $K/M$ , could be obtained by structural dynamical analysis. But what we really want to know is the oscillation of the beam (not of the structure), and its dominant mode, for movements in azimuth and elevation. This should be obtained empirically: move the telescope beam fast to the half-power point of a strong radio source, and stop fast. Record the wiggling receiver output (not the encoders), which will show the dominant beam frequency, depending on the quotient  $K/M$  of (2) as

$$Fr = (1/2\pi) \sqrt{K/M}, \quad (3)$$

and the damping, depending on the product  $K \cdot M$  as

$$Qd = \pi B/\sqrt{K \cdot M} = \text{logarithmic damping decrement} \quad (4)$$

where  $Qd$  is the quotient of one maximum, divided by its next maximum, for small damping. From the measured values (3) and (4) we then obtain the two constants of (2):

$$B/M = 2 Qd Fr \quad \text{and} \quad K/M = (2\pi Fr)^2. \quad (5)$$

The telescope movement must obey two limitations: for acceleration  $A = Y'' \leq Am$ , and velocity  $V = Y' \leq Vm$ . And we call  $G$  = the distance (goal) to be moved for ON/OFF observations. Many examples of  $Am$ ,  $Vm$ ,  $Fr$ ,  $G$  have been calculated, but for the following we use, as a typical set for a large telescope of 100 m diameter, and for a large beam:

$$\begin{aligned} Am &= 0.2 \text{ deg/sec}^2, & Vm &= 0.67 \text{ deg/sec} \\ Fr &= 1.0 \text{ Hz}, & G &= 1.0 \text{ deg}. \end{aligned} \quad (6)$$

We call  $Te$  = duration from start to end of a fast move,  $Nw = Te Fr$  = number of oscillation waves within the duration, and  $Ve$  (arcsec/sec) = velocity at end,  $De$  (arcsec) = deviation from goal at end,  $Dmax$  (arcsec) = the maximum deviation thereafter, during the oscillations. If perfect we have  $Ve = De = Dmax = 0$ . And as tolerable we specify  $Dmax \leq 1.0$  arcsec.

For the application at the telescope, our method shall specify and use only the acceleration of the drive as a function of time, which means the driving voltage or current (not to be changed by any feed back from the decoders). But to calculate and to plot the predicted oscillations now, we must also know the two integrations of  $Y''$ :  $Y'(t)$  and  $Y(t)$  for (2).

After application of this method, the telescope will be at rest, but maybe not exactly at the desired pointing. The drive is then immediately switched to the normal mode (using decoders) to remove any small deviation.

3. Choices for ON/OFF Acceleration A = Y''(t)

The fastest and easiest would be  $A = \pm \text{const} \leq A_m$ . This is possible but it demands a rather accurate timing of  $T_e$ . For a less demanding smoother  $A(t)$ , we could use power series of  $t$ , but integrating twice (from  $A$  to  $Y$ ) would increase the power by two. We prefer powers of  $\sin(t)$ , which are not increased by any integrations, and are more adequate anyway.

We have calculated four examples, D0 - D3 ("D" for one Degree move). Fig.3 shows the accelerations used,  $Y''(t) = \sin^n(t)$ , for  $n = 0, 1, 2, 3$ , and the integrated drives  $Y(t)$ . Whereas the four accelerations of Fig.3a show large essential differences, the four integrated drives  $Y(t)$  of Fig.3b look very similar. But in spite of the latter, the resulting telescope movements  $X(t)$  will be very different again.- We normalize the time by using

$$Z = 2\pi t/T_e. \tag{7}$$

The minimum of  $T_e$  is given by the maximum allowed acceleration. And the proper choice of  $T_e$  shall then minimize the final deviation  $D_{\max}$ .- The following table gives duration  $T_e$ , acceleration  $Y''(t)$ , and drive function  $Y(t)$ . The deceleration is always anti-symmetric.

$T_e \geq$	$Y'' =$	$Y =$	Range of Z
D0) $\sqrt{(4G/A_m)} = 4.47 \text{ sec}$	$+4G/T_e^2$ $-4G/T_e^2$	$(G/2\pi^2) Z^2$ $(G/2\pi^2) (2\pi - Z)^2$	$0 \dots \pi$ $\pi \dots 2\pi$
D1) $\sqrt{(2\pi G/A_m)} = 5.61 \text{ sec}$	$(2\pi G/T_e^2) \sin(Z)$	$(G/2\pi) [Z - \sin(Z)]$	$0 \dots 2\pi$
D2) $\sqrt{(8G/A_m)} = 6.35 \text{ sec}$	$+(8G/T_e^2) \sin^2(Z)$ $-(8G/T_e^2) \sin^2(Z)$	$(G/2\pi^2) [Z^2 - \sin^2(Z)]$ $G - (G/2\pi^2)[(2\pi-Z)^2 - \sin^2(2\pi-Z)]$	$0 \dots \pi$ $\pi \dots 2\pi$
D3) $\sqrt{(3\pi G/A_m)} = 6.87 \text{ sec}$	$(3\pi G/T_e^2) \sin^3(Z)$	$(G/2\pi) [Z - \sin(Z) - (1/6)\sin^3(Z)]$	$0 \dots 2\pi$

**Exact solutions** ( $D_{\max} = 0$ ) exist, with zero damping only, for all integer values of  $N_w = T_e F_r$ , with models D1 and D3; and for all even values of  $N_w$  with models D0 and D2. This is shown in Fig.4 for D0, where  $T_e = 6.0 \text{ sec}$  is the fastest even case. Similar pictures were obtained for D1 with  $T_e = 6.0 \text{ sec}$ , for D2 with  $T_e = 8.0 \text{ sec}$ , and for D3 with  $T_e = 7.0 \text{ sec}$ .- Regarding the similarities of the drives  $Y(t)$  in Fig.3b, the exact solutions are very different.

**Tolerable timed fast solutions** exist, even with rather large damping,  $Q_d = 0.10$  (steel structures have normally about  $Q_d = 0.05$ ), for D2 and D3 only.- Also inaccurate timing,  $T_e \pm 0.2 \text{ sec}$ , is tolerable only for D2 (with  $D_{\max} = 0.14 \text{ arcsec}$ ), and D3 ( $D_{\max} = 0.41 \text{ arcsec}$ ).

**Without special timing** (with small or large damping),  $U_{\max}$  is tolerable for D2 with any  $T_e \geq T_a = 7.6 \text{ sec}$ ; and for D3 with any  $T_e \geq T_a = 6.9 \text{ sec}$ , see Fig.5. It means that above these durations  $T_a$ , all higher dynamical oscillations will be automatically suppressed as well.

If an increase of the acceleration limit, from  $A_m = 0.20 \text{ deg/sec}^2$  to  $A_m = 0.28$ , is technically possible, then the shortest exact solution  $T_e$  (and the important limit  $T_a$ ) are: 4.0 ( $> 2.0$ ) sec for D0; 5.0 (9.6) sec for D1; 6.0 (7.5) sec for D2; and 6.0 (5.9) sec for D3.

#### 4. Quick Stop after Fast Long Slew

This case was handled in the same way as the ON/OFF deceleration, now called S0 to S3. **Exact solutions** exist, with zero damping, for all integer values of  $W_n = T_e F_r$ , with models S0 (below  $A_m$  with  $W_n = 3, 4, \dots$ ) and with model S2 (with  $W_n = 5, 6, \dots$ ); and for all  $W_n$  which are  $\frac{1}{2}$  of odd integers, with model S1 ( $W_n = 5.5, 6.5, \dots$ ) and model D3 ( $W_n = 7.5, 8.5, \dots$ ). The shortest exact solution is shown in Fig.6 with S0, with a slew speed of 0.6 deg/sec. The stop duration is only  $T_e = 3.0$  sec, and the distance moved, between deceleration start and goal, is  $G = 0.90$  degree. But S0 is rather sensitive to timing.

**Tolerable timed solutions** exist, with small or large damping, again only for S2 and S3; and timing deviation,  $T_e \pm 0.2$  sec, is tolerable only for S2 ( $D_{max} = 0.66$  arcsec), and for S3 ( $D_{max} = 0.03$  arcsec).

The best seems to be S2, which is tolerable **without special timing**, for all  $T_e \geq T_a = 6$  sec. See Fig.7 as an example. Again, all higher modes are suppressed as well.

So far, all numerical results hold for values (6). In other cases, limits for  $T_e$  and accelerations  $Y''(t)$  can be obtained from the previous table

#### **Conclusion:**

Fast and good solutions, for ON/OFF observations, as well as for stopping a slew, can be obtained if the telescope is driven as a function of time (no feed back from decoders), with the drive power going with  $\sin^2(t)$  or  $\sin^3(t)$ . Timed exact solutions remove the slowest oscillation, tolerable deviations suppress it sufficiently. Many telescopes have a dominant slowest mode, while the higher modes are faster damped. For a given telescope, its dominant mode of the beam oscillation should be obtained empirically.

For all cases without special timing (whose duration is only slightly longer), all higher oscillation modes will be automatically suppressed as well.

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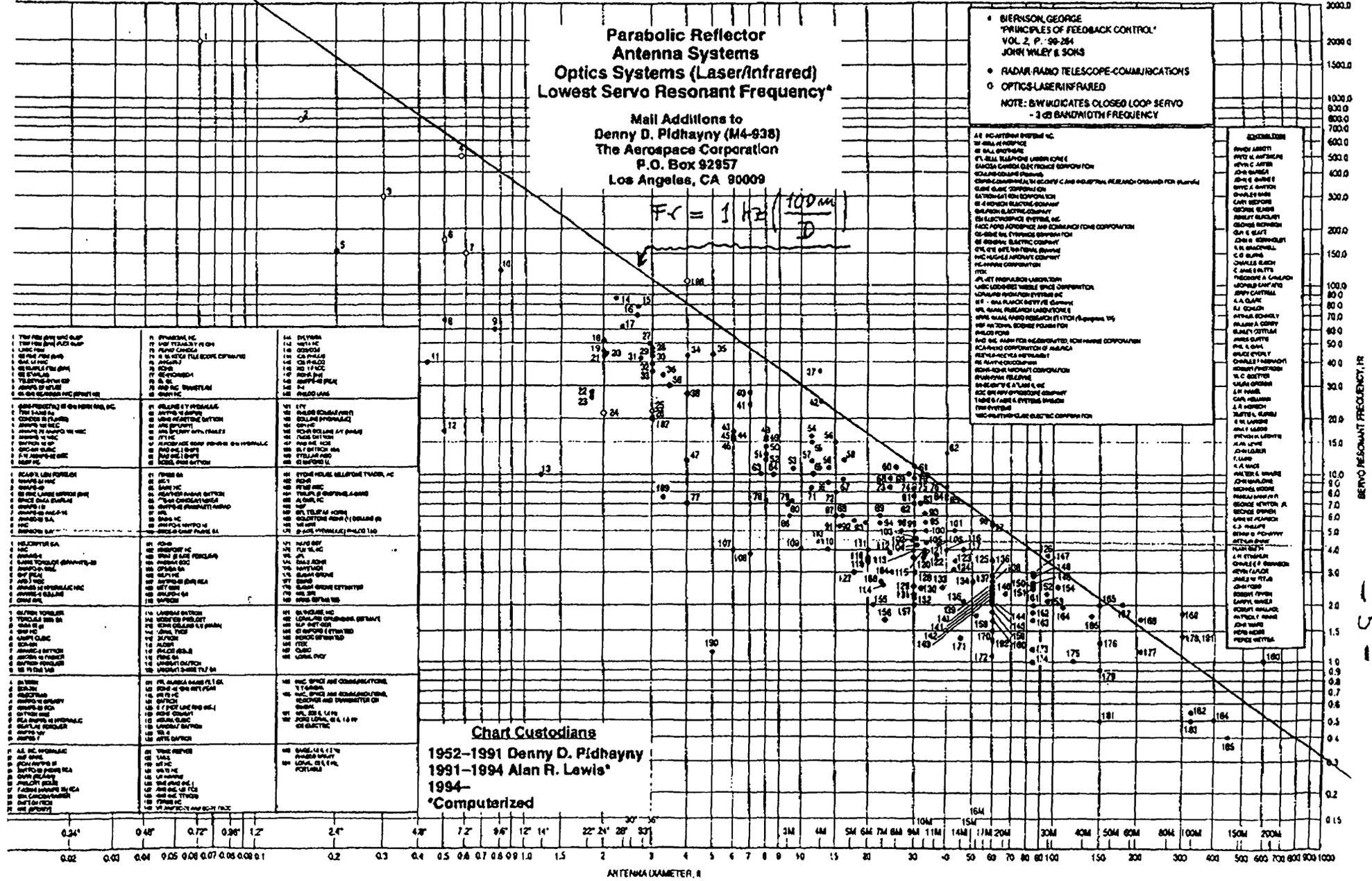


Fig.1 Lowest dynamical mode and telescope diameter, for 194 systems.

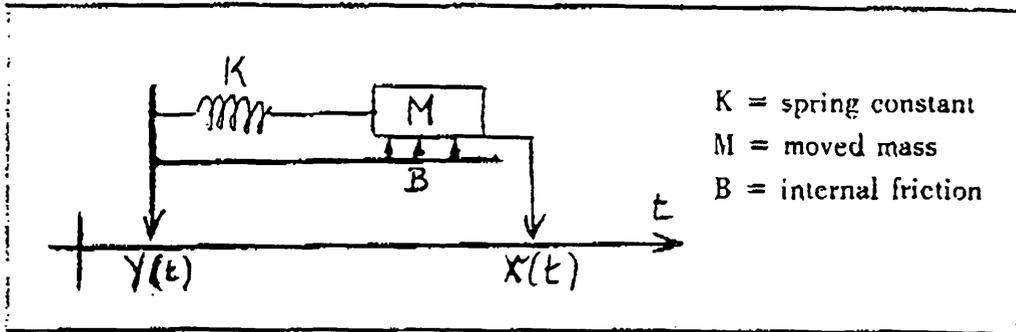
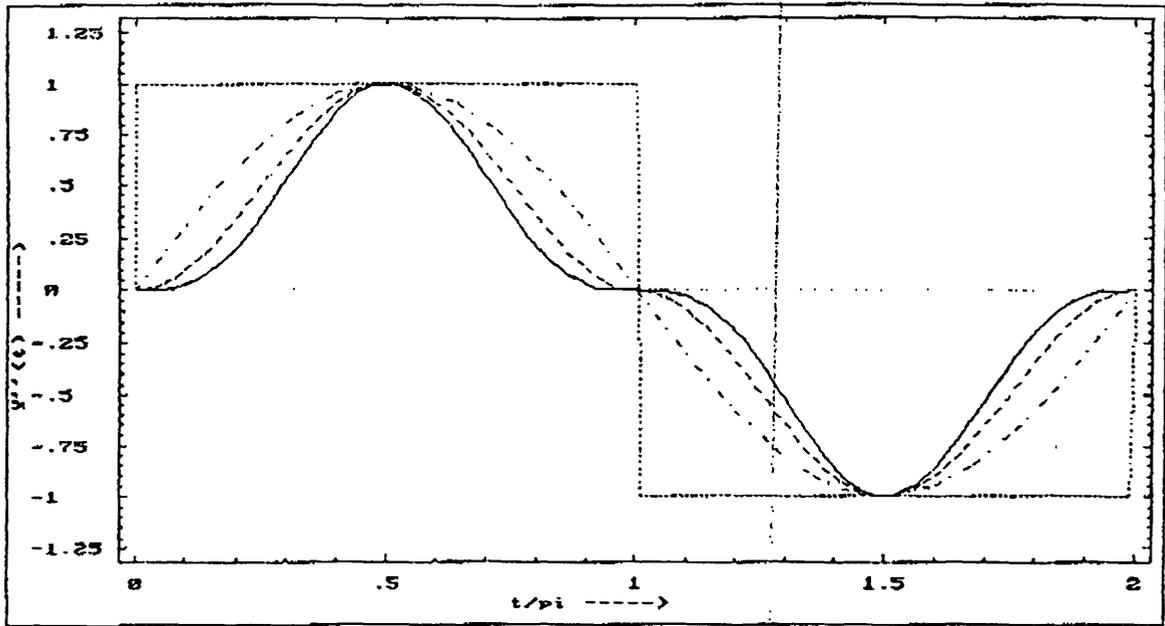


Fig. 2. Simplified model for the telescope drive  $Y(t)$ , and its resulting movement  $X(t)$ .

a)



b)

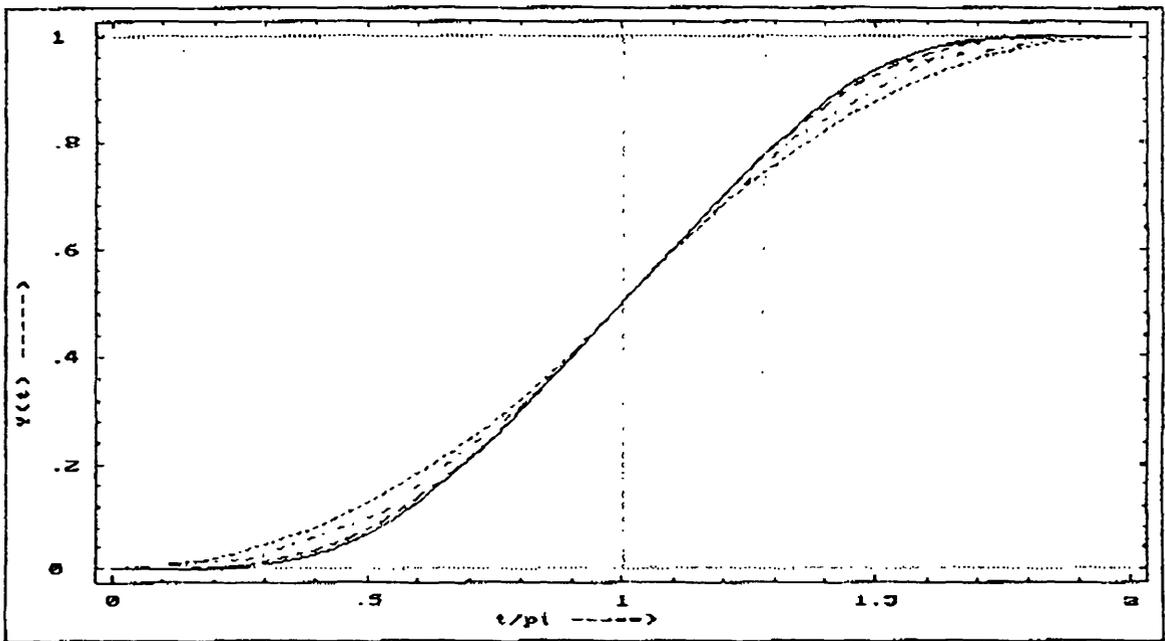


Fig. 3. Four drive functions, from D0 (.....) to D3 (—).

a) The accelerations used:  $Y''(t) = \sin^n(t)$ , from  $n = 0$  to  $n = 3$ .

b) The integrated drive functions:  $Y(t)$ .

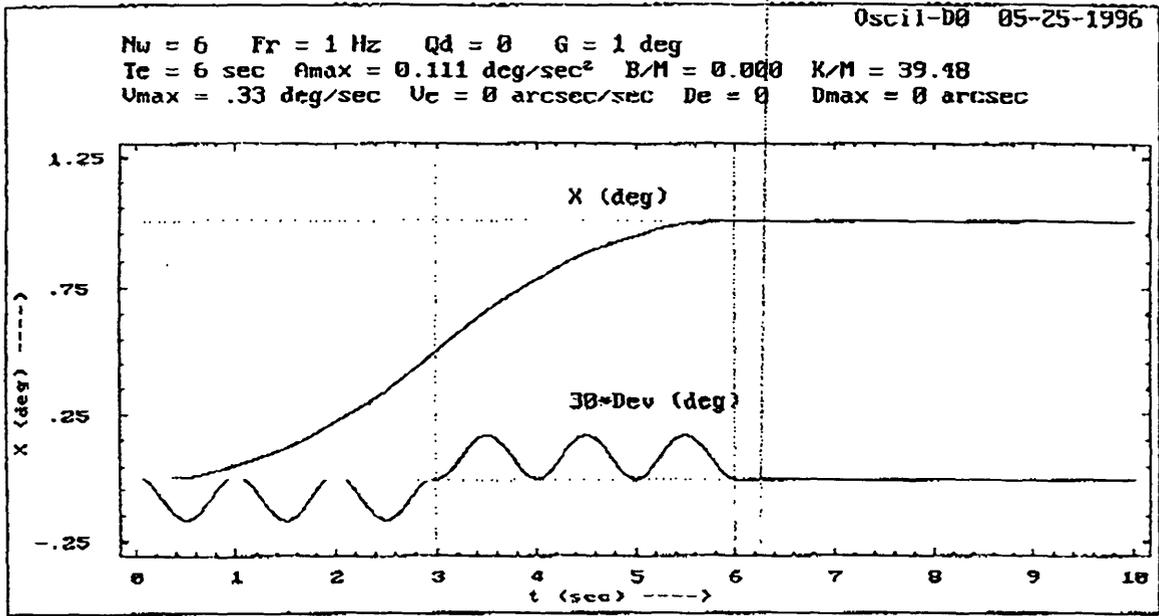


Fig. 4. ON/OFF Drive D0; fastest exact solution, for values (6).  
It would be only  $T_e = 4.0$  sec, if  $A = 0.250$  were permitted .

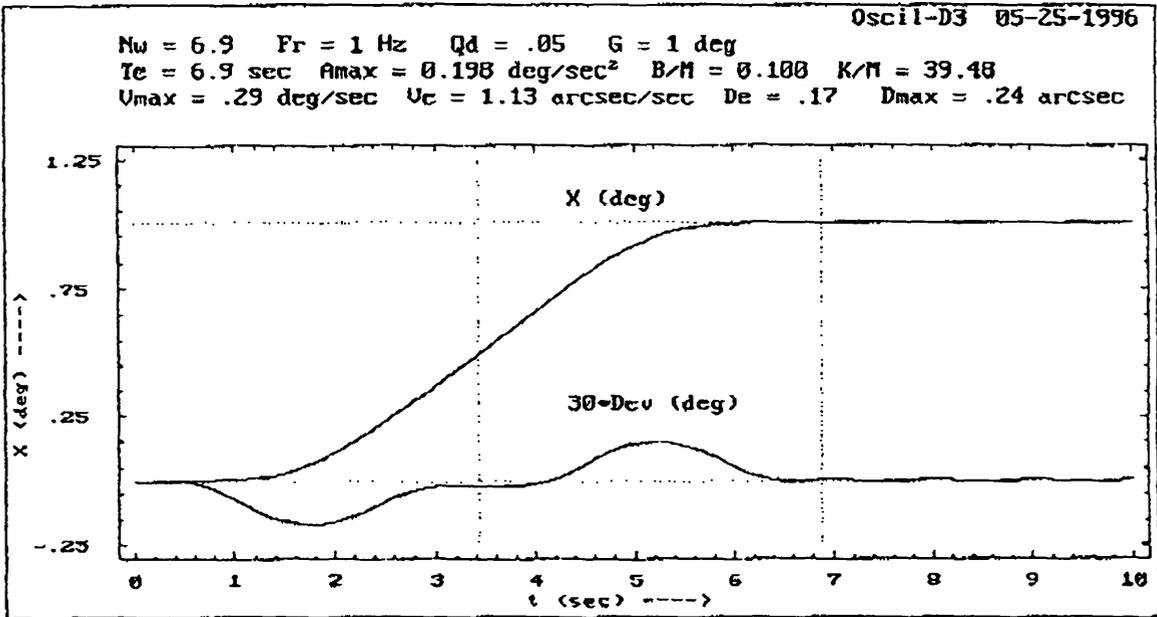


Fig. 5. ON/OFF Drive D3; all solutions are tolerable for any larger Nw.  
If  $A = 0.280$  then  $T_e = 5.9$  sec only.

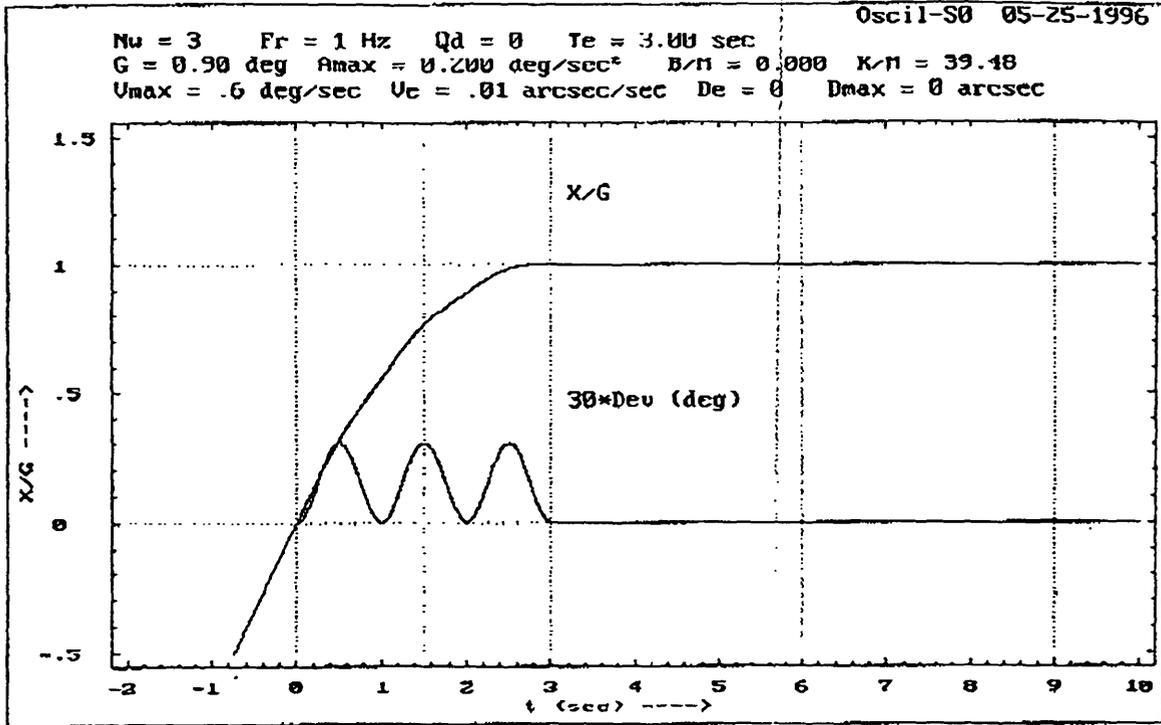


Fig. 6. Drive S0, stop after slew; fastest exact solution for values (6).

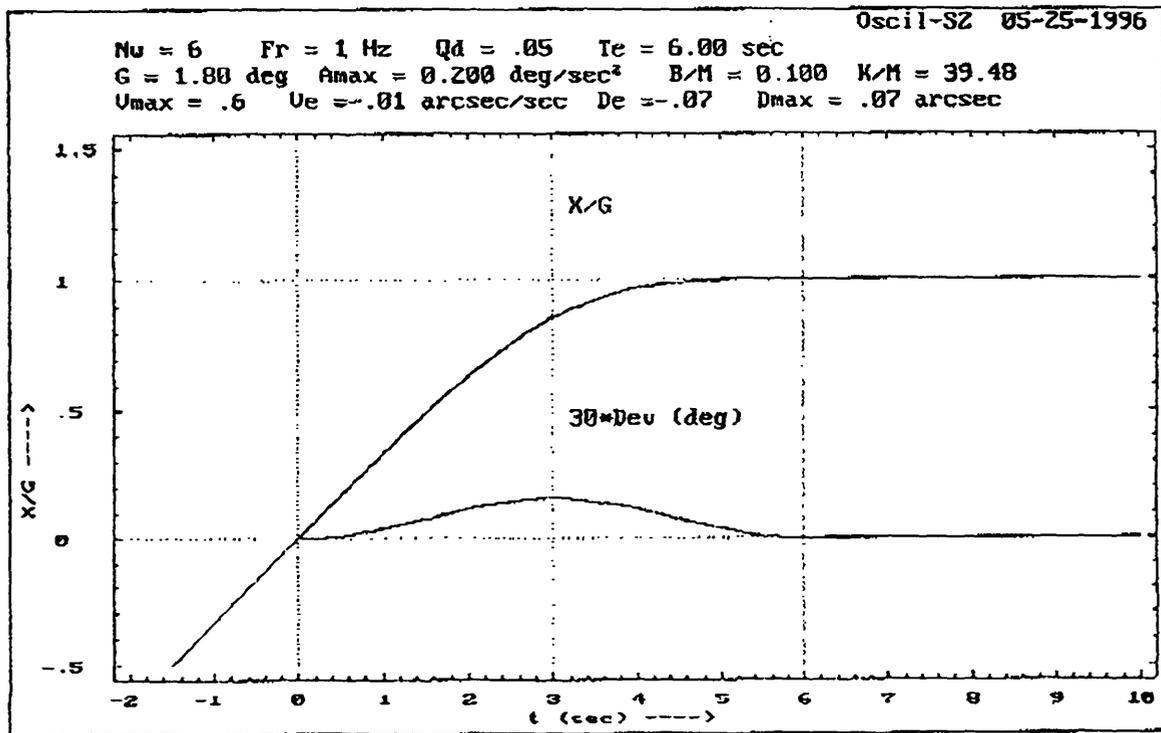


Fig. 7. Drive S2, stop after slew; all solutions are tolerable for any larger Nu.

