Multidirectional retroreflector assembly with a common virtual reflection point using four-mirror retroreflectors

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Abstract

A brief survey of retroreflector designs and applications is presented. A novel multidirectional (as opposed to omnidirectional) retroreflector concept, which uses a four-mirror retroreflector subassembly with a common virtual reflection point (thus eliminating the Abbe error), is described. Applications include multilateration with interferometers, laser trackers, and electronic distance measurement surveying instruments—as well as other radiation sources, e.g., microwaves and acoustics. Example configurations are given.

Keywords: retroreflector; multilateration; laser tracker; large-scale metrology; coordinate measurement machine calibration; four-mirror retroreflector; electronic distance measurement

1 Introduction

A number of laser interferometer and electronic distance measurement (EDM) applications desire wide angle of acceptance, or multiple retroreflectors[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. The primary problem using multiple retroreflectors is due to the measurement axes not passing through the measurement point, and thus making the measurements sensitive to rotations of the object and/or angle between the instrument and object, i.e., the Abbe error[15, 16].

Surveying equipment manufacturers have assembled solid glass retroreflectors, such as the Leica[17] GRZ4 360 degree prism, but the glass offset is a function of the incident angle[2, 18, 19], and coverage overlaps between adjacent retroreflectors, so there is a significant Abbe error (several mm for the GRZ4).

Goldman designed a "triplet" assembly consisting of a cat's-eye retroreflector midway between two solid glass corner cubes directed to the rear of the cat's-eye[20]. The center of the cat's-eye and the optimal "optical center" of the corner cubes are colinearly mounted on a rigid beam assembley. One group of EDM instruments ranging on the cat's-eye and another group of EDMs ranging on the pair of corner cubes can be tied together through the triplet bench mark. In order to avoid crosstalk between the two corner cubes, it is necessary to physically space them apart by several beam diameters. While the angle to the cat's-eye can in general be anywhere in the field of view, the angle to the corner cubes must be limited to minimize the Abbe error.

Laser interferometers are typically calibrated in a back-to-back retroreflector configuration, where the rotation of the retroreflectors is constrained. For example, NIST has built a Laser Rail Calibration System (Larcs) for calibrating laser trackers[21, 22] against an interferometer on a linear rail[23]. Larcs uses two spherically mounted retroreflectors (SMRs) (described in more detail below), in a back-to-back configuration on a carriage, to build a bidirectional retroreflector assembly, i.e., one direction fixed for the reference interferometer parallel to the rail, and the other free to rotate in a nest to accommodate the laser tracker under test.

The Abbe error is minimized by mounting the two retroreflectors as close as practical and constraining the carriage to a rail system to minimize rotations of the assembly. However, for portable rails, the uncertainty due to the Abbe error is estimated to be a significant part of the total error budget. We have also used a similar technique at NRAO, but used an additional mirror on the carriage for an autocollimator target and tweek the carriage mechanical alignment before taking readings.

NASA has built custom hollow retroreflector as-

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semblies with a common physical reflection point[24, 25], and thus eliminated the Abbe error. These solve the Abbe error problem for some classes of measurements. However, since these retroreflectors share a common physical point, they sacrifice part of the center aperture, are difficult to build, are difficult to reference to an outside mechanical point, are expensive for routine applications, and the directions are not adjustable.

Gelbart and Laberge describe an "omnidirectional retroreflector" pair combined with a fixed probe[9, 10]. By multilaterating on the pair of retroreflectors, the probe coordinate is calculated. The omnidirectional retroreflector, described in the '091 patent, "consist of two concentric spheres made of transparent material and having the refractive index of the inner sphere higher than the refractive index of the outer sphere, the outside sphere coated with a partially reflective coating." A prototype of this design was built by CREO Products Inc., Burnaby, B.C., Canada; but is not commercially available. Gelbart suggested that an even better design could be achieved by using three concentric spheres[26].

Recently, ideal omnidirectional spherical retroreflectors have been built from high index of refraction N=2 glass[13, 14, 27]. Unfortunately, the glass is difficult to work, expensive, and the return power is low–due to the spherical aberation and small working aperture, as well as the low reflection coefficient of the glass/air interface on the back side of the sphere, i.e., most of the power is transmitted through the sphere. While these problems will hopefully be overcome by advances in materials and manufacturing techniques, only a few of these highly coveted retroreflectors have been built since being introduced in 1994.

1.1 Multidirectional applications

Laser trackers incorporate a laser interferometer with an automated mirror system to track a retroreflector. The interferometer measures differential range very accurately, with the fundamental limitation being the uncertainty of the index of refraction—which is typically in the 1 ppm range.

The angle measurements are somewhat less accurate. The fundamental limitation is atmospheric turbulence and temperature gradients bending the beam. There are also practical limitations with the encoders, mechanical system, beam quality, gravitational reference, etc. Nakamura et al[13] points out that for a distance measurement uncertainty of δr, in an ideal orthogonal trilateration measurement, the uncertainty volume is $(\delta r)^3$. For two angles and a distance measurement, the uncertainty volume is $(r\delta\theta)^2\delta r$. For example, for a typical distance measurement uncertainty of 1 ppm ($\delta r/r = 10^{-6}$) and an angle uncertainty of one arc second ($\approx 5x10^{-6}$ radians), the trilateration uncertainty volume would be

$$\delta v = r^3 10^{-18} \tag{1}$$

whereas the uncertainty volume for two angles and a distance would be

$$\delta v = 25 r^3 10^{-18}, \tag{2}$$

or 25 times greater than the trilateration uncertainty volume—hence the inherent potential improvement in accuracy by using multiple distance measurements. In practice, there are two primary obstacles to achieving this huge improvement. Conventional retroreflectors do not support simultaneous measurements in the three orthogonal directions, and in actual field conditions it can be hard to mount an instrument on a stable tower or structure.

Laser trackers typically use spherically mounted retroreflector (SMR) targets. These are typically hollow or cat's-eye type[28, 29, 30] retroreflectors, with the optical centers carefully located in the center of the spherical mounting—thus allowing the optical measurements to be related to the physical center of the sphere. Hollow SMRs, such as those built by PLX Inc.[31] are more economical than cat's-eye SMRs, but have a reduced angle of acceptance and thus are more susceptible to dropping the laser interferometer beam while tracking.

An obvious improvement in the accuracy of the laser tracker (or EDM) is to use multiple instruments and/or augment with additional information, e.g., known artifacts, stable bench marks, hydrostatic leveling, or other constraints. For three or more instruments, oriented in the proper baselines, the less accurate angle measurements can be neglected or weighted less in a least squares, or more sophisticated, reduction. While crosstalk is not a problem using multiple laser trackers on a common SMR, the relatively small angle limitation of even the cat's-eyes makes the instrument baselines unfavorable for high accuracy multilateration measurements, and of course the Abbe error is the limitation for conventional assemblies of SMRs.

The Robert C. Byrd Green Bank Telescope large-scale metrology system was designed to operate as a multilateration system employing 18 laser ranging instruments measuring ranges to cardinal points on the moving telescope[5]. While some paths are physically blocked by the structure, it behooves the designers to use multidirectional retroreflectors in order to maximize the number of independent measurements, and thus strengthen the calculation of cardinal point coordinates.

The increasing interest in multilateration, using laser trackers or other EDMs, has created a need for less expensive and more practical multidirectional retroreflectors with zero Abbe error.

2 The four-mirror retroreflector

Laser tracker manufacturers use a clever four-mirror retroreflector design disclosed by Brown in US Patent 5,530,549[32, 33], and a variation of the same concept in US Patent 5,861,956[34]. See the Faro Corp. "RetroProbe" [35], and the Leica Corp. "Surface Retroreflector" [17] for commercial examples. Brown describes a system of a retroreflector (built of three mirrors), probe tip, and additional first surface mirror.

The principle is simple and elegant. If the line connecting a hollow retroreflector and probe tip is bisected by a first surface mirror, as shown in Figure 1, then a beam directed at the probe tip which intersects the "forth mirror" is reflected by the mirror to the retroreflector, i.e., the virtual point of the probe is at the retroreflector. The optical path length, and angle, to the image of the retroreflector, is identical to the optical path length, and angle, to the probe tip. A similar system is used to focus photographic enlargers by reflecting the projected image from a mirror, at a fixed offset from the photo paper plane, onto a reticle which is viewed through a magnifier.

By using a hollow retroreflector, it is insensitive to the orientation of the four-mirror retroreflector, i.e., no glass offset. Using this system, laser trackers are used to probe locations inaccessible to the more conventional SRMs.

2.1 Cat's-eyes

It should be pointed out that a cat's-eye retroreflector could also be used in the same way as a hollow retroreflector, for applications that need larger acceptance angles. The center of the cat's-eye would replace the apex of the hollow retroreflector. The maximum aperture of the fourth mirror is determined by the retroreflector acceptance angle and distance to the mirror. For EDM measurements, the glass offset(s) would have to be corrected, but it would be a constant and independent of the viewing angle. For interferometer measurements, the glass offset would be absorbed into the interferometer initialization.

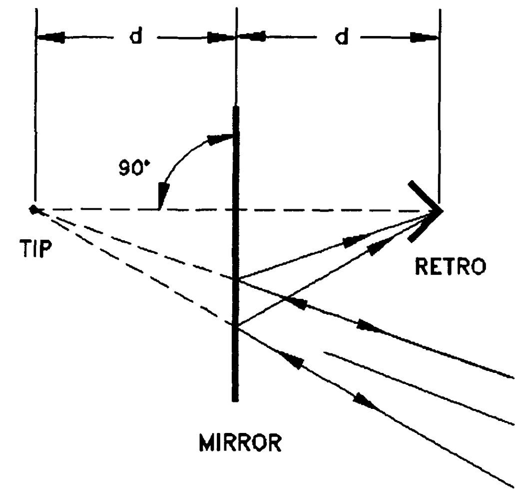

Figure 1: Principles of the four-mirror retroreflector.

3 Extensions of the four-mirror retroreflector

There are a number of practical extensions of the principles used in the four-mirror retroreflector for bidirectional and multidirectional retroreflectors—both with and without a probe; and with fixed or adjustable directions[1].

3.1 Bidirectional retroreflector with virtual point at one retroreflector

By replacing the probe tip with a second retroreflector, with the apex at the center of the former probe tip location—as shown in Figure 2—a system is constructed whereby measurements to both retroreflectors are made to the same virtual point, i.e., the former probe center.

Note that the probe tip replacement retroreflector can be oriented in any direction, e.g., 180 degrees to the four-mirror retroreflector path (for back-to-back measurements) or orthogonal (including into and out of the paper) to the four-mirror retroreflector path for X-Y measurements, etc. Moreover, the probe tip replacement retroreflector direction can be fixed or adjustable, and captive or separable (such as a SMR/nest configuration). For example, this could be used to make simultaneous EDM measurements, sequential EDM measurements from different directions without turning the retroreflector, bring two laser trackers into coincidence, etc.

[1]Patent pending

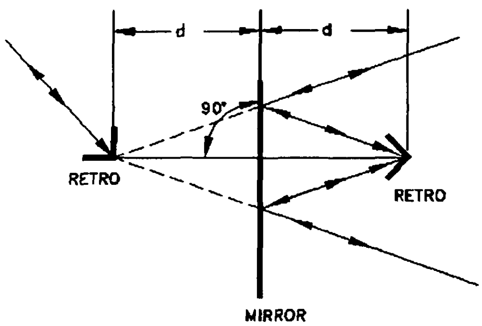

Figure 2: Bidirectional retroreflector.

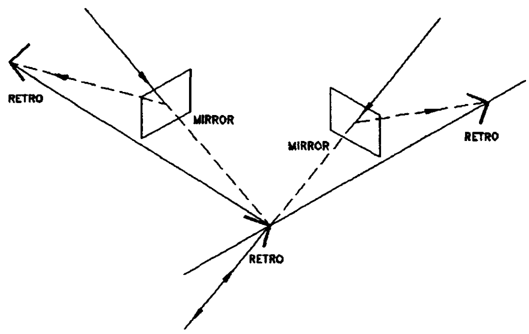

Figure 3: Multidirectional retroreflectors.

3.2 Multidirectional retroreflectors with virtual point at one retroreflector

Since the four-mirror retroreflector does not physically intersect the virtual reflection point, the bidirectional assembly can also be extended to multiple mirrored retroreflectors, as shown in Figure 3. The only restriction is that the mirrored retroreflectors can't block the visibility of other retroreflectors.

3.3 Multidirectional retroreflectors using all mirrored retroreflectors

Of course, the multidirectional retroreflectors, with the virtual point at one retroreflector, can be extended to a cluster of four-mirror retroreflectors without a directly illuminated retroreflector, or the probe tip could be reintroduced.

3.4 Other applications

While the immediate application is directed at laser beams, the same concepts could easily be adapted for other radiation sources, e.g., microwaves and acoustics.

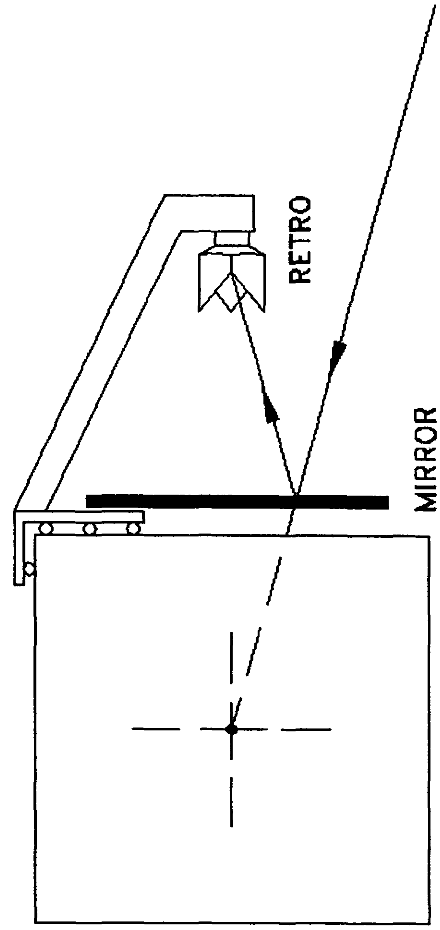

Figure 4: Cubic manifold.

4 Example configurations

In US Patent 5,335,111[28], Bleier describes how to construct hollow retroreflectors in a hard mount assembly. One-piece replicated retroreflectors are also available from manufacturers such as Opticon, which the author has used with success in hostile outdoor environments.

In a practical application of the four-mirror system, the problem is to mechanically bisect the line between the retroreflector and virtual point with a first surface mirror. Mechanically locating the apex of a hollow retroreflector is traditionally accomplished by carefully inserting a sphere into the retroreflector. The center of the sphere is located at a height

$$h = r\sqrt{3} \tag{3}$$

from the apex. Of course, the Abbe error, due to the distance between the center of the sphere and the apex, is a problem. The orientation of the retroreflector can be established by autocollimating on each of the retroreflector mirrors, or by building a fixture to fix the orientation. At NRAO, we have used a solid glass retroreflector mounted in a fixture to orient hollow retroreflectors, i.e., gently drop the hollow retroreflector over the inverted glass retroreflector. Differential optical measurements could be made by EDM or an interferometer using a SMR.

While fixturing could be built to construct every desired configuration, there may be merit to simply building the four-mirror retroreflector assembly and then configuring the assemblies as needed. For example, by building four-mirror retroreflectors, with a uniform offset and reference footing, any number of configurations would be practical. The foot could be a fixed attachment or magnetic base to facilitate rapid reconfiguration and adjustment.

4.1 Flat foot

By constructing a four-mirror retroreflector with a flat, three ball, or Kelvin mount foot, one could construct any number of configurations, in an orthogonal coordinate system, around a simple (and inexpensive) cubic manifold, as shown in Figure 4

4.2 "V" foot

By constructing a four-mirror retroreflector with a "V" foot, one could construct an array that would cover a circular pattern around a simple cylinder manifold, as shown in Figure 5.

4.3 Nest foot

Three-dimensional coverage could easily be achieved by building four-mirror retroreflectors with a three-point, or conical, nest foot that would attach to a spherical manifold, as shown in Figure 6.

4.4 Articulated assemblies

Due to the fact that the Abbe error is eliminated, any of the suggested examples could be articulated. For example, an assembly could be mounted on a coordinate measurement machine, rotated about an axis by a motor; or maintained in a fixed orientation, with respect to gravity, by a pendulum counterweight, e.g., to compensate for the rotation in elevation of a radio telescope.

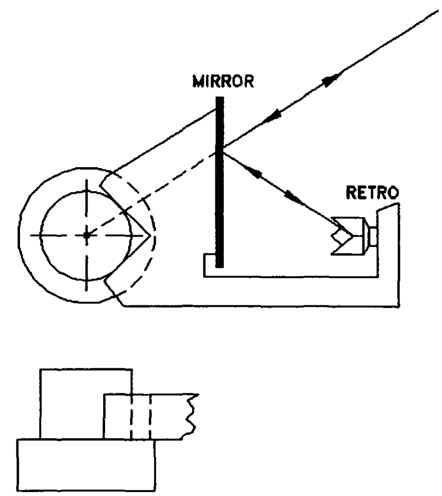

Figure 5: Cylinder manifold.

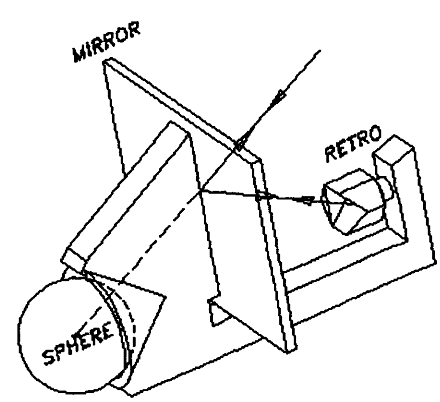

Figure 6: Spherical manifold.

5 Summary

Until true omnidirectional retroreflectors become economical and commercially available, the four-mirror retroreflector offers a partial solution by building multidirectional retroreflectors.

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