GBT Memo 287:
Band-averaged MUSTANG-2 Aperture Illumination and Implications for Out of Focus Holography

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Abstract

I calculate averaged illumination profiles over the full MUSTANG-1.5/MUSTANG-2 bandpass of 75 to 105 GHz and assess the impact of variations on telescope surface quality, target source spectrum, and assumed functional form for approximate descriptions of the illumination profile. I find these all to be comparable, $\sim 10\%$ effects on field profile. A considerably better description of the illumination profile is found by a 3-parameter "cubic" Gaussian fit, the parameters for which are presented. I also analyze an existing GBT 77 GHz OOF observation with two different assumed illumination profiles to get an idea of how sensitive the OOF phase solutions are to the illumination profile that is assumed. I find differences in the phase solutions in the $30-90\,\mu$m range, comparable to other sources of error in the dish surface.

Contents

1 Introduction

Correcting low to medium order aberrations in the GBT primary surface is necessary for useful 3mm science observing. These corrections are typically derived using the technique of phase-retrieval holography, also known as Out of Focus holography or OOF. Deriving the phase corrections at the primary requires knowledge of the telescope illumination pattern. MUSTANG-1.5 is a bolometer camera which will make spectrally-averaged (zero spectral resolution) measurements across a very wide bandwidth (design target: good response over 75 - 105 GHz). Furthermore, in order to facilitate manufacture of a highly scalable large-format camera, smooth-walled feeds have been used; these feeds have an illumination profile which varies considerably across this wide band. I have carried out an analysis to determine what average illumination profile we should expect for MUSTANG-1.5, also factoring in variations in source intensity and surface efficiency with frequency. Note the panel-scale surface accuracy that limit the surface efficiency can vary with the overall thermal environment and telescope elevation (see, e.g., GBT Memo 271).

The Out of Focus (OOF) holography software (Nikolic et al. 2007a, 2007b) implements several illumination profiles. For the current MUSTANG camera, there is a "top hat" illumination profile terminating at a specified radius (for MUSTANG, this is 45 meters). The default illumination profile is a Gaussian with the form

$$E(r) = exp\left(\frac{-r^2}{2\sigma}\right)$$

The radius is in units of the physical aperture maximum radius, so ranges from zero to one. The σ parameter, as defined, does not have the same units as the scaled radius, since it is not squared (that is not a typo). The default value is σ of 0.3, corresponding to a taper of $10 \times log_{10}(e^{-1/\sigma}) = -14.48$ dB in *power*, as opposed to field strength. This is comparable to the standard GBT feed designs (for example, GBT Memo 262) which taper to -12 to -14 dB in power at 14.5 degrees (the angle subtended by the subreflector).

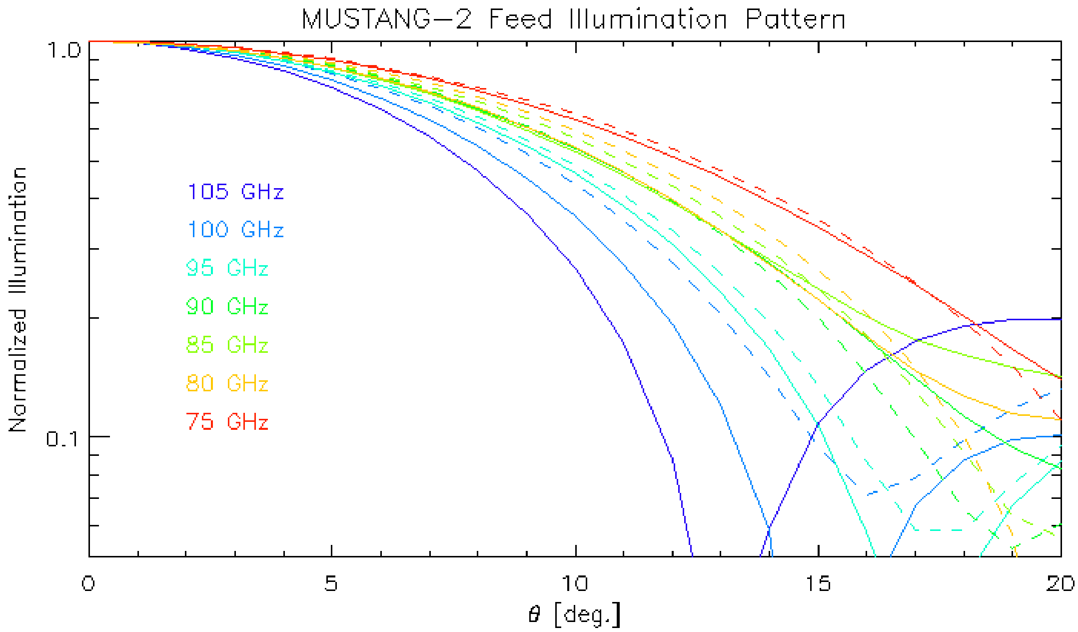

Figure 1: Design field illumination pattern for MUSTANG-2 feeds (V4L).

The goals of this investigation are:

- to determine if a Gaussian profile is a reasonable approximation to the band-averaged MUSTANG-1.5 illumination
- to determine what parameters can be used to describe the profile
- to quantify the dependence of the wide-band average illumination profile on the surface RMS and target OOF source spectrum.

Our method and results are described in § 2 and § 3. In order to place these results in context I also conducted a simple analysis to determine the impact that variations in the assumed illumination profile have on the OOF phase solutions. These results are presented in § 4.

2 Analysis

A wide-band, spectrum-averaging receiver (such as a bolometer) measures the average beam

$$P_N(x,y) = \frac{\int d\nu\, S(\nu)\eta_s(\nu)\ \times\ P_N(\nu,x,y)}{\int d\nu\, S(\nu)\eta_s(\nu)}$$

Here $S(\nu)$ is the (assumed unresolved) target flux density as a function of frequency; η is the surface efficiency; and $P_N(x,y)$ is the normalized telescope beam as a function of sky coordinates x and y. It is straightforward to show that the band-averaged beam response is equivalent to the beam obtained from a similarly weighted field illumination profile:

$$E(x,y) = \frac{\int d\nu\, \sqrt{S(\nu)\eta_s(\nu)w(\nu)}\ \times\ E(\nu,x,y)}{\int d\nu\, \sqrt{S(\nu)\eta_s(\nu)w(\nu)}}$$

I include here an additional receiver-bandpass function $w(\nu)$. The surface efficiency is given by the standard Ruze relation, which we evaluate as a function of frequency for surface RMS values ranging from 200 microns to 300 microns. For the source spectrum I assume $S \sim \nu^\alpha$ and consider alpha values of -1 (steep spectrum synchrotron), 0 (flat, probably typical of bright calibrators at 3mm), and +2 (thermal black body). The receiver bandpass function is taken to be approximated by the OMT power transfer function computed by Jeff MCMahon (private comm.).

The expected illumination profiles have been determined by Jeff MCMahon and Simon Dicker for design V4L for (70, 75, 80, ... 110) GHz[1]. These are shown in Figure 1. In practice I perform a discrete sum of the profiles with their appropriate weights, using the profiles between 75 and 105 GHz, inclusive. Bandpass-averaged illumination profiles were computed for 15 cases by varying the spectral index (-1, 0, +2) and assumed GBT surface RMS (220, 240, 260, 280, 300 um).

3 Results

The main results are as follows:

1. The band-averaged illumination profiles look qualitatively Gaussian, as expected.
2. Variations in the profiles due to source spectrum, assumed GBT surface, and the assumed functional form, are all comparable effects over the range examined. Clearly patterned systematic residuals are evident when a Gaussian is fit to the profile.
3. That said, the differences are relatively small – all of the profiles agree within +/-10%, better at most radii (in E-field; in power, the agreement is +/-20%)
4. A considerably better (qualitative) fit is obtained by assuming a linear and cubic term in the Gaussian.

I take the 240 micron, flat-spectrum source to be most representative of the range of cases likely to be encountered. Allowing sigma (only) to vary and using an implementation of Levenburg-Marquardt in IDL, I find a best fit of $\sigma = 0.317$; the equivalent taper in power at the edge of the primary (R=1) for this model is -13.72 dB. This is an improvement over the default $\sigma = 0.30$, but it is clear that a still better fit is possible. Good results are obtained with a "cubic Gaussian", i.e.:

$$E(R) = exp\left(-\frac{1}{2}(AR + R^2/\sigma + CR^3)\right)$$

Best fit values to the nominal case are $\sigma = 0.431$, $A = 0.083$, $B = 0.976$; the equivalent taper at the edge of the primary (R=1) for his model is -14.66 dB. Arbitrarily-normalized error bars were assigned for the fit and assigned a 1/sqrt(Radius) scaling to give the appropriate weighting as a function of radius, i.e., to weight different radii proportional to aperture area, as would be the case if the fit were performed in two dimensions.

The wideband-averaged MUSTANG-2 illumination profiles for the 15 cases considered are shown in the top panel of Figure 2. The residuals of the best-fit cubic Gaussian (described above) to these band-averaged profiles are shown in the bottom panel of Figure 2. The residuals of a pure Gaussian fit, and of the canonical GBT illumination profile (not fit to the data at all) are presented in Figure 3. A better understanding of these fits and residuals can be obtained by considering Figure 4, which presents the same set of 15 illumination profiles and the standard GBT illumination profile, but this time with a weighting proportional to the amount of dish surface present at each radius. The important range of radii to match is $5^\circ \lesssim \theta \lesssim 12.5^\circ$.

[1] See *https://safe.nrao.edu/wiki/bin/view/GB/Pennarray/FinalFeedAnalysis*

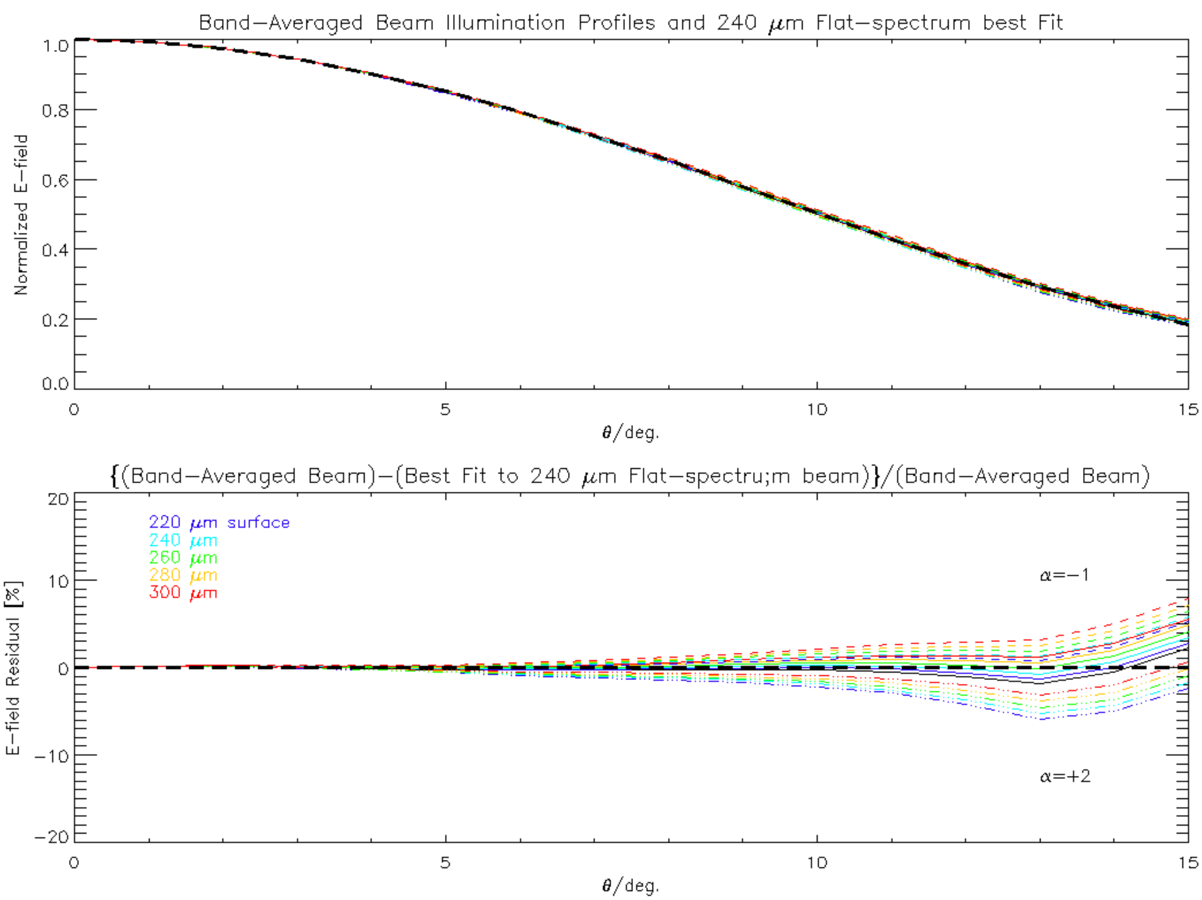

Figure 2: *Top panel*: Band-averaged illumination patterns for three assumed target source spectral indices— $\alpha = 0$ (solid line), $\alpha = -1$ (dashed line), and $\alpha = +2$ (dash-double dot)— and various Ruze-equivalent aperture efficiencies in colors from 200 μm (purple) to 300 μm (red). The best 3-parameter Cubic Gaussian fit to the baseline 240 μm, $\alpha = 0$ case is shown as a thick, black dashed line. *Bottom panel*: Fractional residuals of the individual beams compared to the best-fit baseline case.

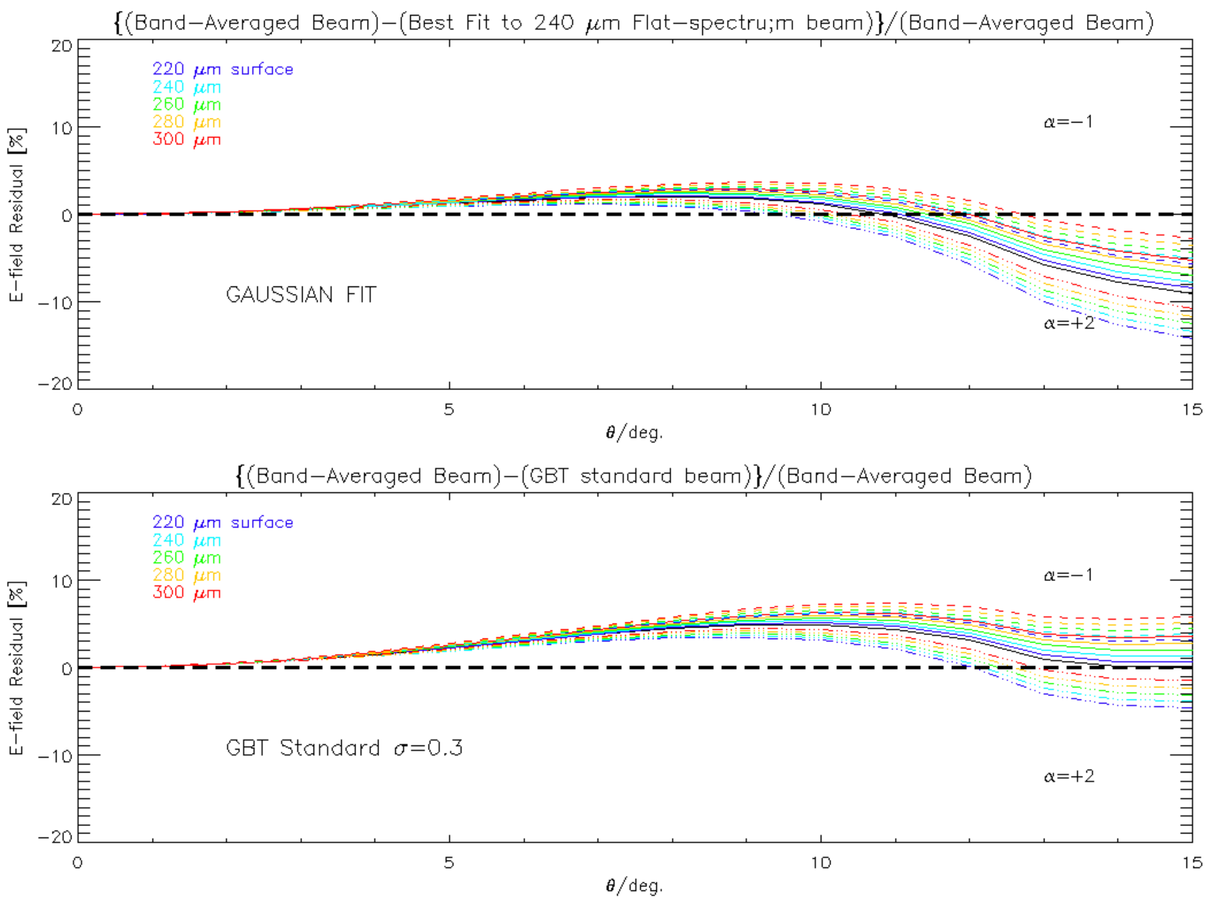

Figure 3: Residuals of calculated beams compared to a Gaussian fit (top panel), and the GBT/OOF default illumination profile (bottom panel), following the scheme of Figure 2.

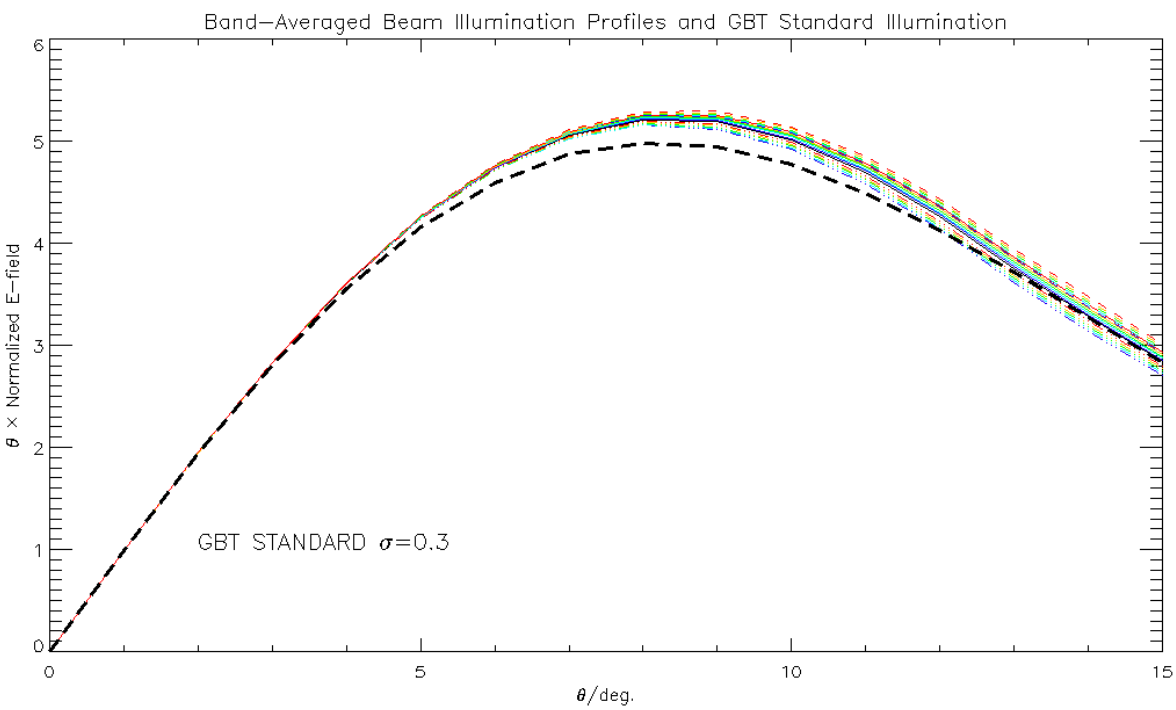

Figure 4: Dish-surface weighted illumination profiles compared to the GBT standard $\sigma = 0.3$ illumination profile.

4 Sensitivity of Phase Solutions to Variations in the Assumed Aperture Illumination Profile

In order to assess the sensitivity of OOF solutions for the surface phase errors to variations in the assumed aperture illumination, I analyzed data from a GBT W-band (77 GHz) OOF observation[2] two different ways: once using an assumed Gaussian illumination profile with the default $\sigma = 0.3$; and again, using an assumed value of $\sigma = 0.33$. This represents a 15 percent variation in E-field at the edge of the primary. Results are shown in Figures 5 - 7. The RMS difference between these solutions is about 0.15 Radians , corresponding to 91 microns RMS error given the observing wavelength of 3.9 mm. As can be seen in Figures 5 and 6 the distribution of differences in the phase solutions is distinctly non-Normal. A more robust estimate of the RMS of the "core" of the distribution– i.e., the *typical* phase difference– is given by $MAD \times 1.4826 = 0.05\,\mathrm{Rad} \sim 32\,\mu\mathrm{m}$ where MAD is the median absolute deviation of the difference in phases between the two solutions. By way of comparison the peak to peak phase error in one of the solutions, after removing the tilt term that corresponds to pointing, is ~ 5 radians. This difference is comparable to other measured, uncorrected, variable effects, such as panel scale thermal and gravitational effects (50 - 100 microns: GBT Memo 271). It is less than the values for the Ruze-equivalent surface quality $\sim 200 - 250\,\mu\mathrm{m}$ given by 3mm aperture efficiency estimates, but not greatly so.

Based on this result, it seems likely but not certain that the effects of errors in the description of the telescope illumination profile (at the level found in § 3) are minor effects. This result in principle depends on the particular dataset used as well as the precise nature of the variation in illumination assumed. It may prove worth while to look at other data sets, and to collect 3-4 mm GBT data with OOF solutions derived from different assumed illumination profiles analagous to the analysis here.

[2] AGBT13B_146_03 scans 1 - 3.

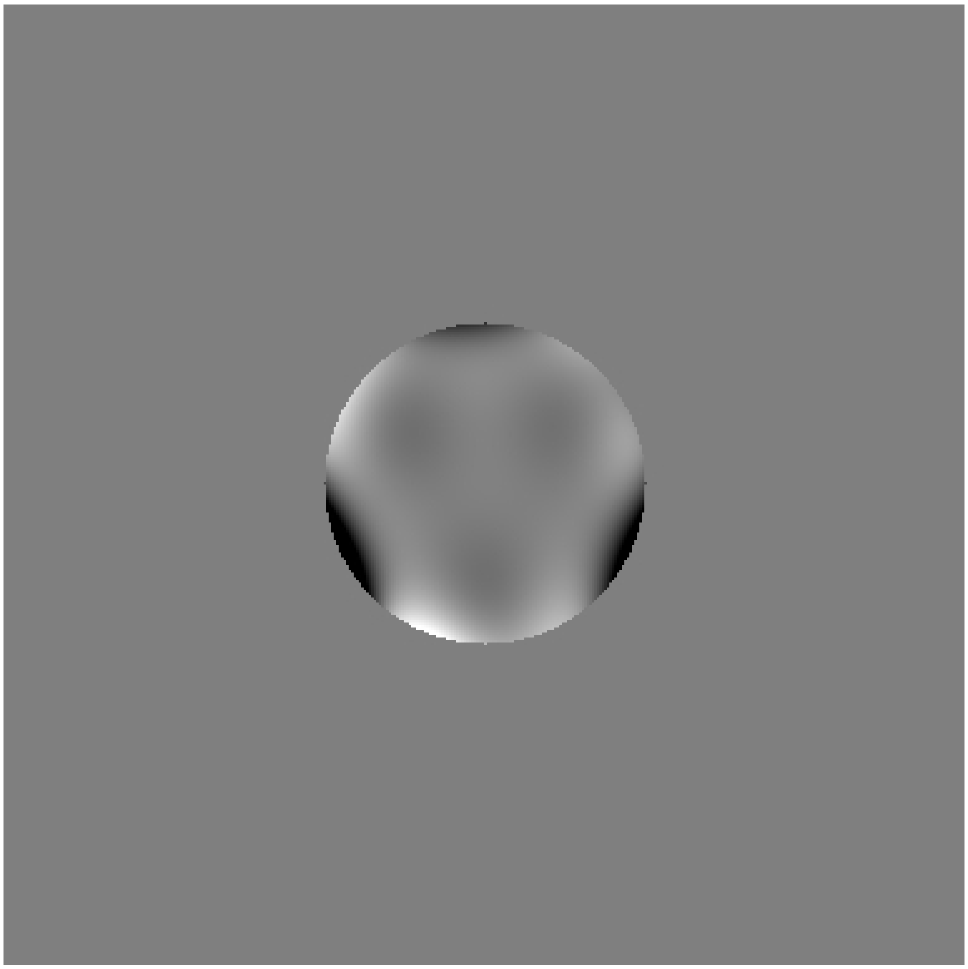

Figure 5: Difference between the OOF phase solutions, measured at a set of fixed points on the primary aperture, for GBT W-band data (AGBT13B_146_03 scans 1 - 3, collected at 77 GHz) for $\sigma = 0.3$ and $\sigma = 0.33$. The color scale runs from -0.5 rad. (black) to $+0.5$ rad. (white).

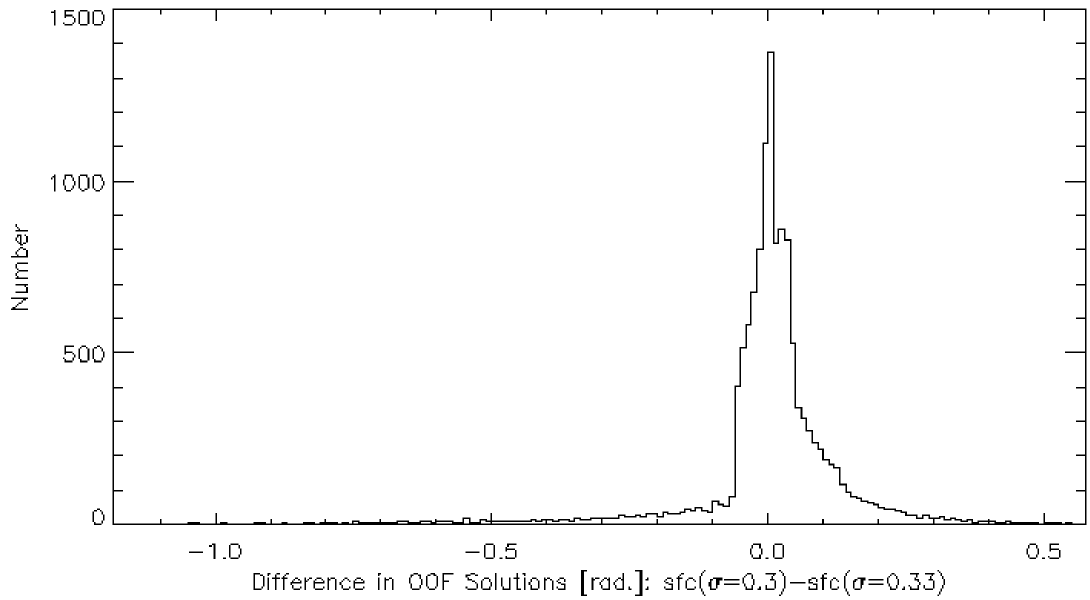

Figure 6: Histograph of the difference between the OOF phase solutions for GBT W-band data for $\sigma = 0.3$ and $\sigma = 0.33$.

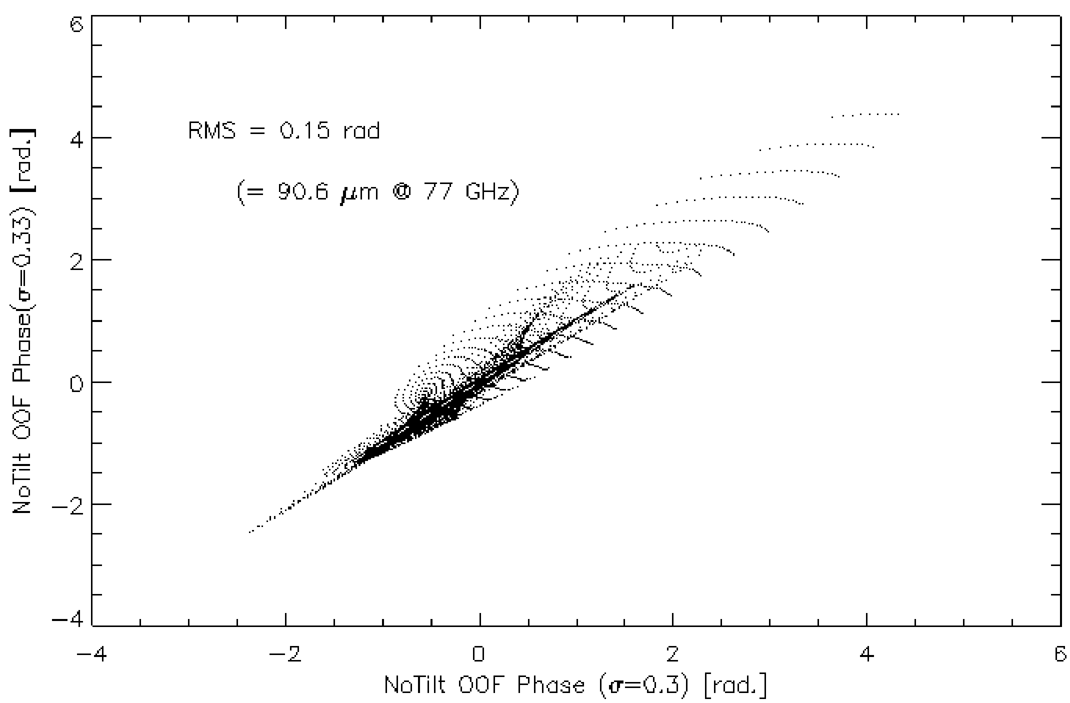

Figure 7: OOF phase solutions for the two cases considered ($\sigma = 0.3$ and $\sigma = 0.33$) directly compared with each other.

References

B. Nikolic et al. 2007a, A&A v465, p685

B. Nikolic, R.E. Hills & J.S. Richer 2007b, A&A v465, p679

F.Schwab & T.Hunter, “Distortions of the GBT Beam Pattern due to Systematic Deformations of the Surface Panels” (GBT Memo 271).

S. Srikanth, “Comparison of the GBT K-Band Feeds Linear Taper Horns and Profile Horn” (GBT memo 262).