GBT MEMO 110

Astronomical Pointing Parameters

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1. General Remarks

Memo 105 by Heiles and Maddalena has given ample reasons for having an astronomical pointing system, obtained by observing radio sources of known position. They discuss types of sources and observing methods, and they include the elliptical beam shapes of polarized or off-axis observations. Their final discussion concerns presentation and use of such systems.

The present memo is confined to the **pointing** parameters only, describing the corrections ΔA for azimuth and ΔE for elevation. Their system shall correct the repeating pointing errors (misalignments, gravity, refraction). The least-squares method is described which yields the pointing parameters and their mean errors. Also weighted observations are described, for combining observations of different quality.

I want to emphasize the need for knowing the **mean errors** of these parameters. First, the errors will show whether the observations have been sufficient, and if not, how many more are needed. Second, if observations give different parameter values for different conditions (wavelength, Gregorian/prime, season, after a year, ...), it is important to know whether or not the difference is significant. Third, errors are needed when weights shall be used (weight = $1/\text{error}^2$). In order to minimize the errors and their correlations, one should schedule the observations such that the neighborhood of the **cardinal** angles is emphasized: 0° and 90° for elevation though not too close to the horizon, and 0°, ±90° and ±180° for azimuth. Observations near 45° cannot decide between sine and cosine. And the observations should be done only at calm nights with a constant (and not extreme) temperature.

In cases where the pointing does not depend on wavelength, the highest accuracy will be obtained at the shortest good wavelength. And shortest means here that the beam shape has not yet been deteriorated by the surface rms errors. If the beam is bumpy or very skewed, pointing becomes a matter of definition. Do you use the maximum, or the center of gravity?

It is not always realized that a completely healthy structure cannot have any **hysteresis** at all. Hysteresis can only be produced by friction (gears, bearings), slack (gears, loose bolts, cables), or oil-canning (a joint with all members coplanar). Thus hysteresis, large enough to be measured, should never be tolerated. Even if it takes a long dedicated effort to fight it.

If the telescope is part of an interferometer, one must know two additional repeating errors, the horizontal translation ΔY, and the vertical ΔZ. They do not effect the pointing but the location. They are not discussed in the present memo which deals only with the pointing, but they must be worked out later.

2. Expected Parameters

Parameters, to be solved for, must first be defined. Sometimes this is done (or at least suggested) by just solving for a series of spherical Fourier terms up to a given order. But this I would call "Fishing in the Dark". Results may be used but wouldn'tell you anything.

It is recommended to solve first only for those parameters which are to be expected even for a good healthy structure: small misalignments, deformations from gravity, and atmospheric refraction. Only after a set of good and stable values for these parameters has been obtained, only then should one investigate the residuals (if they give pointing errors larger than specified). Looking at various plots, or doing a Fourier analysis, may indicate some additional correlations. One may then solve again for these unexpected parameters and their errors. And if they are significant, one should try to identify their physical cause and to **understand** it. Sometimes it can be removed or minimized. And if not, also these parameters must be included in the system, after additional tests for their significance and stability.

The following listing of expected parameters and their angular functions is taken from my *VLA Test Memo No. 136, May 1982,* with the convention $E=0$ at horizon, $A=0$ at North, and the sign as Δ = observed − true. We had eight parameters for a symmetrical alt-azimuth mount, where gravity needs only $\Delta E = P \cos(E)$. But in our case of an asymmetrical main dish and heavy feed arm, we need the term with sin(E) as well, with **nine parameters.** The unbalanced turret may even give small terms ΔA, which I assume negligible. The atmospheric refraction term is taken from my *Engineering Division Internal Report No. 101, May 1976,* for the 140-ft.

$P1$ = Azimuth axis offset to North

$$\Delta E = + P_1 \cos(A) \qquad \Delta A \cos(E) = + P_1 \sin(E) \sin(A) \qquad (1)$$

P_2 = Azimuth axis offset to East

$$\Delta E = + P_2 \sin(A) \qquad \Delta A \cos(E) = - P_2 \sin(E) \cos(A) \qquad (2)$$

P_3 = Elevation axis not perpendicular azimuth axis

$$\Delta E = 0 \qquad \Delta A \cos(E) = + P_3 \sin(E) \qquad (3)$$

P_4 = Zero elevation offset, and feed offset to Y

$$\Delta E = + P_4 \qquad \Delta A = 0 \qquad (4)$$

P_5 = Beam not perpendicular elevation axis, and feed offset to X

$$\Delta E = 0 \qquad \Delta A \cos(E) = + P_5 \qquad (5)$$

P_6 = Zero azimuth offset

$$\Delta E = 0 \qquad \Delta A = + P_6 \qquad (6)$$

P_7 = Gravity, symmetrical

$$\Delta E = + P_7 \cos(E) \qquad \Delta A = 0 \qquad (7)$$

P_8 = Gravity, asymmetrical

$$\Delta E = + P_8 \sin(E) \qquad \Delta A = 0 \qquad (8)$$

P_9 = Refraction

$$\Delta E = + P_9 R(E) \qquad \Delta A = 0 \qquad (9)$$

These single contributions add up as:

$$\Delta E = P_1 \cos(A) + P_2 \sin(A) + P_4 + P_7 \cos(E) + P_8 \sin(E) + P_9 R(E) \qquad (10)$$

$$\Delta A \cos(E) = P_1 \sin(E) \sin(A) - P_2 \sin(E) \cos(A) + P_3 \sin(E) + P_5 + P_6 \cos(E) \qquad (11)$$

The **refraction** term R(E) was handled as follows. At the interferometer we measured, and wired to the 140-ft: the barometric pressure P_{bar} in mmHg, the temperature t in °C, and the dew-point temperature D in °C. From these we have the temperature $T = t + 273.15$ in °K; while the water vapor pressure $P_{wv}(D)$ in mmHg was approximated as

$$P_{wv} = 4.58 + 3.369\,(D/10) + 1.029\,(D/10)^2 + 0.2080\,(D/10)^3 + 0.02778\,(D/10)^4 \qquad (12)$$

from which

$$K = 0.354\, P_{bar}/T - 0.0585\, P_{wv}/T + 1701\, P_{wv}/T^2 \qquad (13)$$

where

$$K \approx 1.0 \text{ arcmin, for average days at Green Bank.} \qquad (14)$$

For a flat Earth we would simply have $R(E) = K \cot(E)$.

At low elevations we have two complications. First, the corrections may become so large that we must make a difference between E_{tru} and $E_{obs} = E_{tru} + \Delta E$, where the refraction does depend on the true elevation of the source. Second, for the curvature of the Earth I derived the approximation

$$R(E) = K \frac{\cos(E_{tru})}{\sin(E_{tru}) + 0.00175 \cot(E_{tru} + 2.5°)} \qquad (15)$$

I recommend that someone at NRAO checks these old equations again, with new literature. But please make sure that ΔE does not diverge close to horizon, which some published cases did! Also I would like to mention another complication if we want high accuracy. At low elevations, equation (15) can be used if the true elevation is known, which means for calibrating the pointing parameters, and later for observing sources of known location. However if we discover a new source close to horizon, for example far south, and want to get its true location, then we would have to "iterate backwards": use (15) with E_{obs}, get an approximate ΔE, subtract it from E_{obs} and apply (15) again. But at high elevations we have the opposite case. For obtaining ΔA, the critical term 1/cos(E) should be replaced by **$1/\cos(E_{obs})$**, at least regarding P_1 and P_2 (maybe not P_3 and P_5). But maybe this complication can be neglected if the "zone of avoidance", as defined by the speed limit of the azimuth drive, is large enough, which should be checked.

3. Least Squares Method

After a pointing run, we have n observations, each with the two values ΔE and ΔA cos(E), to be explained by equations (10) and (11) with m unknown pointing parameters, multiplied by given angular functions of E and A. We call these functions F_{ik} and G_{ik} for observation i and parameter k; for example $F_{i2} = \sin(A_i)$ and $G_{i2} = \sin(E_i) \cos(A_i)$ for observation $i = 1 \ldots n$. The number of parameters is $m=9$ for the expected parameters, but larger if we have to add some unexpected ones. We rewrite (10) and (11) as two vectors $U = \Delta E$ and $V = \Delta A \cos(E)$ as

$$U_i = \sum_{k=1}^{m} F_{ik} P_k \quad \text{and} \quad V_i = \sum_{k=1}^{m} G_{ik} P_k \quad \Big| \; i = 1 \ldots n \qquad (16)$$

which would be exact if we had no observing errors, called ξ_i for ΔE_i and η_i for $\Delta A_i \cos(E_i)$.

We now define the **quadratic residual** $R_o = \Sigma\ \xi_i^2 + \Sigma\ \eta_i^2$ as

$$R_o = \sum_i \{ (U_i - \sum_k F_{ik}P_k)^2 + (V_i - \sum_k G_{ik}P_k)^2 \} \tag{17}$$

and "Least Squares" means R_o = minimum, which we obtain by letting all m partial derivatives $\partial R_o/\partial P_k = 0$. This leads to a set of m linear equations

$$\sum_k \sum_i (F_{ij}\ F_{ik} + G_{ij}\ G_{ik})\ P_k = \sum_i (F_{ij}\ U_i + G_{ij}\ V_i) \qquad |\ j=1\ \dots\ m \tag{18}$$

which we write as

$$M\ P = W \tag{19}$$

with matrix M

$$M = F^T\ F + G^T\ G \tag{20}$$

and vector W

$$W = F^T\ U + G^T\ V \tag{21}$$

where F^T is the transpose of matrix F, and G^T the transpose of matrix G. With $I = M^{-1}$ the inverse of matrix M, we finally obtain the wanted parameters, $P_1\ \dots\ P_m$ as the solution of our system (18) as

$$P = I\ W. \tag{22}$$

Next we ask for the **mean errors** ϵ_k of these derived parameters P_k. We insert all P_k into equation (17) and obtain the quadratic residual R_o. It can be shown that the errors then are given, for $P_k \pm \epsilon_k$, as

$$\epsilon_k = \{R_o\ I_{kk}\ /(n-m)\}^{1/2}. \tag{23}$$

One should also obtain the symmetrical error **correlation** matrix, which is

$$C_{kj} = I_{kj}\ /\ (I_{kk}\ I_{jj})^{1/2} \tag{24}$$

with m diagonal members $C_{kk}=1$, and $m(m-1)/2$ independent members, normalized to

$$-1 \le C_{kj} \le +1.$$

We should watch out for bad correlations, where C_{kj} comes close to ± 1. This means most probably that in our pointing run the locations (E, A) of the observations had been badly distributed. Either not close enough to all the cardinal angles, or too crowded at some few locations, not enough at some others. A good schedule for the observations is very important.

4) Weighted Observations

If within one run we have observations with different errors, as estimated from their noise, and assuming $\xi_i = \eta_i$, the weight of observation i then is $w_i = 1/\eta_i^2$. We multiply the right-hand side of (17) and both sides of (18) by $w_i/\Sigma w_i$ and proceed from (19) through (24) as before, except that in (23) the number n of observations is now replaced by the *effective* number $n_e < n$

$$n_e = (\Sigma w_i)^2/\Sigma w_i^2. \tag{25}$$

However, if several runs ($r = 1\dots s$) of different accuracy are combined, we use for each parameter P_{kr} from run r its mean error ϵ_{kr} yielding the weight $w_{kr} = 1/\epsilon_{kr}^2$. The weighted average of P_k and its mean error then are (with $r = 1\dots s$ for all summations):

$$P_k = (\Sigma w_{kr} P_{kr})/\Sigma w_{kr} \pm 1/(\Sigma w_{kr})^{1/2}. \tag{26}$$