

REPORT 2A

**Simulations of the GBT Antenna  
with the Command Preprocessor**

**by**

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## 1. Introduction

In the previous simulations of the slewing (large) motions, see Ref.[1], the excessive vibrations of the structure were observed, and it affected the pointing accuracy. The command preprocessor of PCD, as simulated previously, and reported in Ref.[1] reduces the rigging, but the pointing remains still unsatisfactory due to low-damped vibrations of the subreflector arm. These vibrations are excited during the turn-off of the preprocessor at the distance of 0.044 deg from the acquired trajectory. As an improved solution, we present here the simulation results of the slewing of the GBT antenna with the new command preprocessor (CPP). These CPPs were developed by S. Tyler at JPL, see Ref.[2]. The simulations show significantly reduced pointing error.

The command preprocessor developed at JPL by Steve Tyler has two advantages over the previously simulated one: it turns-off exactly at the target, therefore does not excite the vibrations at the final stage of the acquisition, and its accelerations and decelerations are smooth functions of time, and so that even jerk is smooth, and vibrations are not excited. Thus its performance is considerably improved.

In the Tyler's preprocessor the time of the acquisition,  $T$ , is divided into three regions, see Fig.1: 1. time  $t_1$  of the acceleration (deceleration), 2. time  $t_2$  of constant speed, and 3. time  $t_3$  of the deceleration (acceleration). It is assumed that the acquisition time,  $T$ , is known in advance. This is its main disadvantage of this preprocessor, since this time can be easily under- or overestimated, which causes sharp increase of the servo error in the first case, and unnecessary long acquisition time in the latter case. The determination of the acquisition time is discussed later.

Further we will consider two versions of the Tyler's algorithm. In the first version (called further version A) the maximum allowable acceleration is used in the regions 1 and 3, and the preprocessor matches the initial and final angular position and velocities. In the second version (version B) the sinusoidal acceleration pattern is used in the regions 1 and 3, to limit the

jerk, and the preprocessor matches the initial and final angular position and velocities.

## 2. The estimation of the acquisition time.

This time is composition of the acceleration (deceleration) period ( $t_1$ ), constant rate period ( $t_2$ ), and deceleration (acceleration) period ( $t_3$ )

$$T = t_1 + t_2 + t_3 \quad (1)$$

The acquisition time for the step response of amplitude,  $r$ , with the acceleration limit  $a_{\max}$ , is determined as follows (derivation in the Appendix)

$$T = 2.47 \sqrt{\frac{r}{a_{\max}}} \quad (2)$$

For other commands, it is an iterative procedure or, rather short, trial-and-check process.

## 3. CPP-A: command preprocessor - version A

In this version of the command preprocessor the maximum acceleration is implemented instantaneously, and the target is acquired in the almost minimal time. But the discontinued acceleration at the beginning and the end of the movement can generate unwanted oscillations.

Let  $r_o$ , and  $v_o$  be the initial position and velocity of the antenna, and  $r_f$ , and  $v_f$  be the final position and velocity. The trajectory is determined for each of the three regions. That is, in region 1:

$$r(t) = r_o + v_o t + 0.5 a_1 t^2, \quad 0 \leq t < t_1 \quad (3)$$

In region 2:

$$r(t) = r_{20} + v_2(t - t_1), \quad t_1 \leq t < t_2 \quad (4a)$$

where

$$r_{20} = r_0 + v_0 t_1 + 0.5 a_1 t_1^2 \quad (4b)$$

In region 3:

$$r(t) = r_{30} + v_2(t - t_2) + 0.5 a_3(t - t_2)^2, \quad t_2 \leq t \leq t_3 \quad (5a)$$

where

$$r_{30} = r_{20} + v_2(t_2 - t_1) \quad (5b)$$

In the above equations the accelerations  $a_1$  and  $a_3$  are the maximal accelerations or decelerations, i.e.

$$a_1 = \varepsilon_0 a_{\max}, \quad a_3 = \varepsilon_f a_{\max} \quad (6)$$

where  $\varepsilon_0$  and  $\varepsilon_f$  are the signs of  $a_1$  and  $a_3$ , i.e.,  $\varepsilon_0 = \pm 1$ ,  $\varepsilon_f = \pm 1$ , which will be determined later. Also in the above equations the rate  $v_2$  in the region 2 is not known, and the time periods  $t_1$ ,  $t_2$ , and  $t_3$ .

The signs of the accelerations are determined as follows. Define the dimensionless variables  $x$  and  $y$

$$x = \frac{\Delta r}{a_{\max} T^2} - \frac{v_0}{a_{\max} T}, \quad y = \frac{\Delta v}{a_{\max} T} \quad (7a)$$

where

$$\Delta r = r_f - r_0, \quad \Delta v = v_f - v_0 \quad (7b)$$

then the signs  $\varepsilon_0$  and  $\varepsilon_f$  are

$$\epsilon_o = \epsilon_f = -1 \quad \text{if } y \leq 0 \text{ and } y + 0.5y^2 \leq x \leq -0.5y^2 \quad (8a)$$

$$\epsilon_o = \epsilon_f = 1 \quad \text{if } y < 0 \text{ and } 0.5y^2 \leq x \leq y - 0.5y^2 \quad (8b)$$

$$\epsilon_o = 1, \epsilon_f = -1 \quad \text{if } y > 0 \text{ and } x > y - 0.5y^2 \text{ or if } y \leq 0 \text{ and } x > -0.5y^2 \quad (8c)$$

$$\epsilon_o = -1, \epsilon_f = 1 \quad \text{if } y > 0 \text{ and } x < 0.5y^2 \text{ or if } y \leq 0 \text{ and } x < y + 0.5y^2 \quad (8d)$$

The velocity  $v_2$  in the region 2 is obtained as follows

$$v_2 = a_{\max} T y_2 + v_o \quad (9)$$

where

$$y_2 = \frac{\epsilon_f y^2 - 2x}{2(\epsilon_f y - 1)}, \quad \text{for } \epsilon_f = \epsilon_o \quad (10a)$$

or

$$y_2 = \frac{\epsilon_f y - 1 + \sqrt{-y^2 - 2\epsilon_f y + 2x(\epsilon_f \epsilon_o) + 1}}{\epsilon_f \epsilon_o}, \quad \text{for } \epsilon_f \neq \epsilon_o \quad (10b)$$

Finally, the time intervals are obtained

$$t_1 = \frac{v_2 - v_o}{a_1}, \quad t_3 = \frac{v_f - v_2}{a_3}, \quad t_2 = T - t_1 - t_3 \quad (11)$$

Once the parameters  $a_1$ ,  $a_3$ ,  $\epsilon_o$ ,  $\epsilon_f$ ,  $v_2$ ,  $t_1$ ,  $t_2$ , and  $t_3$  are obtained from Eqs.(6), (8), (9), and (11), respectively, the preprocessed trajectory is obtained from Eqs.(3), (4) and (5). Note that  $v_2$ , the velocity in the region 2, is not necessarily the maximal allowable velocity, and it depends on the parameters  $a_{\max}$ ,  $v_f$ ,  $v_o$ ,  $r_f$ ,  $r_o$ ,  $T$ .

**Preprocessing 1 deg step command.** The command for the 1 deg step input in AZ is preprocessed. For this case:  $r_o=0$ ,  $r_f=1$ ,  $v_o=v_f=0$ ,  $a_{\max}=0.05$  deg/sec<sup>2</sup> (the acceleration was chosen smaller than the allowed 0.2 deg/sec<sup>2</sup>, in order to limit the vibration amplitudes). The estimated time, according to Eq.(2) is  $T=2.47\sqrt{1/0.05}=11$  sec, but we have found that  $T=9$  sec is satisfactory. Next, variables  $x$  and  $y$  are determined from Eq.(10a), obtaining  $x=0.2469$ , and  $y=0$ . For these values one obtains  $\epsilon_o=1$ ,  $\epsilon_f=-1$ , i.e. the command accelerates in the first region and decelerates in the second region. The velocity in the region 2 obtained from the Eq.(9) is  $v_2=0.2$  deg/sec. For these parameters one obtains trajectory from Eqs.(6), (7) and (8) as in Fig.1a. Its rate and accelerations are shown in Figs.1b and c, respectively.

**Preprocessing 10 deg step command.** The command for the 10 deg step input in AZ is preprocessed. For this case the estimated acquisition time is  $T=2.47\sqrt{10/0.05}=31$  sec, and we have chosen  $T=30$  sec. The dimensionless variables are:  $x=0.2222$ , and  $y=0$ , thus  $\epsilon_o=1$ ,  $\epsilon_f=-1$ . The velocity in the region 2, obtained from the Eq.(9), is  $v_2=0.5$  deg/sec. For these parameters one obtains trajectory from Eqs.(6), (7) and (8) as in Fig.2a. Its rate and accelerations are shown in Figs.2b and c, respectively.

#### 4. CPP-B: command preprocessor - version B

In this version of the command preprocessor the acceleration in regions 1 and 3 is of sinusoidal pattern (i.e.  $a=\pm a_{\max}(1-\cos 2\pi\omega)$ ) to avoid its the abrupt changes in acceleration, which cause oscillations of the antenna.

For this case, and for region 1 one obtains

$$a=a_1(1-\cos \frac{2\pi t}{t_1}) \quad (12)$$

Double integration gives

$$r(t) = r_0 + v_0 t + a_1 \left( \frac{t^2}{2} - \frac{t_1^2}{4\pi^2} + \frac{t_1^2}{4\pi^2} \cos \frac{2\pi t}{t_1} \right), \quad \text{for } 0 \leq t < t_1 \quad (13)$$

For region 2, where  $a=0$  one obtains

$$r(t) = r_{20} + v_2(t - t_1), \quad \text{for } t_1 \leq t < t_2 \quad (14a)$$

where

$$r_{20} = r_0 + v_0 t_1 + 0.5 a_1 t_1^2 \quad (14b)$$

and for region 3, ( $t_2 \leq t \leq t_3$ )

$$r(t) = r_{30} + v_2(t - t_2) + 0.5 a_3(t - t_2)^2 + \frac{a_3 t_3^2}{4\pi^2} \cos \frac{2\pi(t - t_2)}{t_3} - \frac{a_3 t_3^2}{4\pi^2}, \quad (15a)$$

where

$$r_{30} = r_{20} + v_2(t_2 - t_1) \quad (15b)$$

In estimating the acquisition time one uses Eq.(2). However, the sinusoidal acceleration is half that effective than the ramp acceleration of the same amplitude. Therefore, in Eq.(2) one replaces  $a_{\max}$  with  $0.5a_{\max}$ .

**Preprocessing 1 deg step command.** The command for the 1 deg step input in AZ is preprocessed. For this case  $r_0=0$ ,  $r_f=1$ ,  $v_0=v_f=0$ . The maximal acceleration is assumed  $a_{\max}=0.1$  deg/sec<sup>2</sup>, twice as much as for CPP-A. The estimated time, according to Eq.(2) is again  $T=2.47\sqrt{1/0.05}=11$  sec, and we have chosen  $T=9$  sec. From Eq.(7a), it follows that  $x=0.2$ ,  $y=0$ , hence  $\epsilon_0=1$ ,  $\epsilon_f=-1$ . The velocity in the region 2 obtained from the Eq.(9) is  $v_2=0.2469$  deg/sec. For these parameters one obtains trajectory from Eqs.(13), (14) and (15) as in Fig.3a. Its rate and accelerations are shown in Figs.3b and c, respectively.

From Figs.1 and 3 one can see that, the acceleration/deceleration time in the

version B is about the same as in version A, despite the maximal acceleration in version B being twice as much as in version A. But, "sharper" accelerations in the version A will actually excite stronger vibrations, as it will be demonstrated later.

**Preprocessing 10 deg step command.** The command for the 10 deg step input in AZ is preprocessed. For this case we have chosen  $T=30$  sec, as for CPP-A. The dimensionless variables are:  $x=0.2222$ , and  $y=0$ , thus  $\epsilon_0=1$ ,  $\epsilon_f=-1$ . The velocity in the region 2 is  $v_2=0.5$  deg/sec. For these parameters one obtains trajectory as in Fig.4a. Its rate and accelerations are shown in Figs.4b and c, respectively.

## 5. Step response simulations

The antenna cross-elevation pointing response to 1 deg step is simulated, using the CPP-A and CPP-B, and compared with the step response with the PCD preprocessor simulated earlier, see Ref.[2]. The XEL response was chosen because it exhibited the most dramatic vibrations, as reported in Ref.[2].

The results are shown in Figs.5, 6, and 7. In Fig.5a,b the step response with the PCD preprocessor is shown. It was simulated earlier, in Ref.[2], Fig.27. The figure shows the overshoot of 10 mdeg, and amplitude of vibrations of 5 mdeg after 12 sec of transient vibrations.

Fig.6a,b shows the the 1 deg step response with the preprocessor A. The overshoot is 1.8 mdeg, and the amplitude after 12 sec of the transient motion is 1 mdeg.

In Fig.7a,b the 1 deg step response with the preprocessor B is shown, The overshoot is 0.5 mdeg, and the amplitude after 12 sec of the transient motion is 0.2 mdeg.



The XEL pointing simulations with the 10 deg step in AZ are also simulated. For the CPP-A the response is shown in Fig.8a,b, and for the CPP-B in Figs.9a,b. CPP-A excites overshoot of 5 mdeg, and amplitude of 0.8 mdeg after 60 sec (Fig.8b), while CPP-B excites 0.3 mdeg overshoot and negligible amplitude of 0.03 mdeg (Fig.9b).

## 6. Conclusions

The command preprocessors A and B show significant improvement in the antenna response to the large offsets, or the commands that exceed the acceleration and rate limits. Their disadvantage is that the preprocessed command is not computed on-line, but has to be determined ahead of time. However, since the computations are actually very simple, it takes a millisecond or less to determine the pre-processed command. The main difficulty lies in the proper determination of the acquisition time  $T$ , but a little experience allows for its quite accurate estimation.

## References

1. W. Gawronski, and B. Parvin, "Modeling and Analysis of the Green Bank Telescope Control System," April 29, 1995.
2. S.R. Tyler, "A Trajectory Preprocessor for Antenna Pointing," *TDA Progress Report*, vol.42-118, August, 1994, pp.139-159.

Appendix. The estimation of the acquisition time.

The acquisition time is composition of the acceleration ( $t_1$ ), deceleration ( $t_3$ ), and constant rate ( $t_2$ ), periods

$$T = t_1 + t_2 + t_3 \quad (\text{A1})$$

The time of acceleration ( $t_1$ ) and deceleration ( $t_3$ ) is obtained as

$$t_1 = t_3 = \frac{v_2}{a_{\max}} \quad (\text{A2})$$

where  $v_2$  is the velocity at the second period, as specified in Eq.(7). The distance covered during the maximal acceleration period is estimated as  $r_a = 0.5a_{\max}T_1^2 = 0.5v_2^2/a_{\max}$ , and about the same as the distance during the deceleration. The remaining is the distance for moving with the maximal speed, which is  $r_v = r - 2r_a$ . The time of moving with the maximal speed is

$$t_2 = \frac{r_v}{v_2} = \frac{r}{v_2} - \frac{v_2}{a_{\max}} \quad (\text{A3})$$

thus the total time for the acquiring target is a sum of the acceleration, constant speed and deceleration

$$T = t_1 + t_2 + t_3 = \frac{v_2}{a_{\max}} + \frac{r}{v_2} \quad (\text{A4})$$

However,  $v_2$  depends on  $T$ , see Eq.(9), thus (5) cannot be readily solved. For the step command of amplitude  $r$  the equations simplifies. In this case, in Eq.(9)  $v_0 = 0$ , and in Eq.(10b)  $y = 0$ , and  $x = r/a_{\max}T^2$ , thus from (10b) one obtains  $y_2 = 0.5(-1 + \sqrt{1+4x})$ . But the region of valid solutions for  $y=0$  is  $x \leq 0.25$ , see Ref.[2], p.146. Thus the maximal  $y_2$  is  $y_2 = 0.5(-1 + \sqrt{2}) = 0.2071$ . In this case, from (9)  $v_2$  is

Introducing (A5) to (A4) one obtains

$$T=2.47\sqrt{\frac{r}{a_{\max}}} \quad (\text{A6})$$

i.e. Eq.(2) is derived.

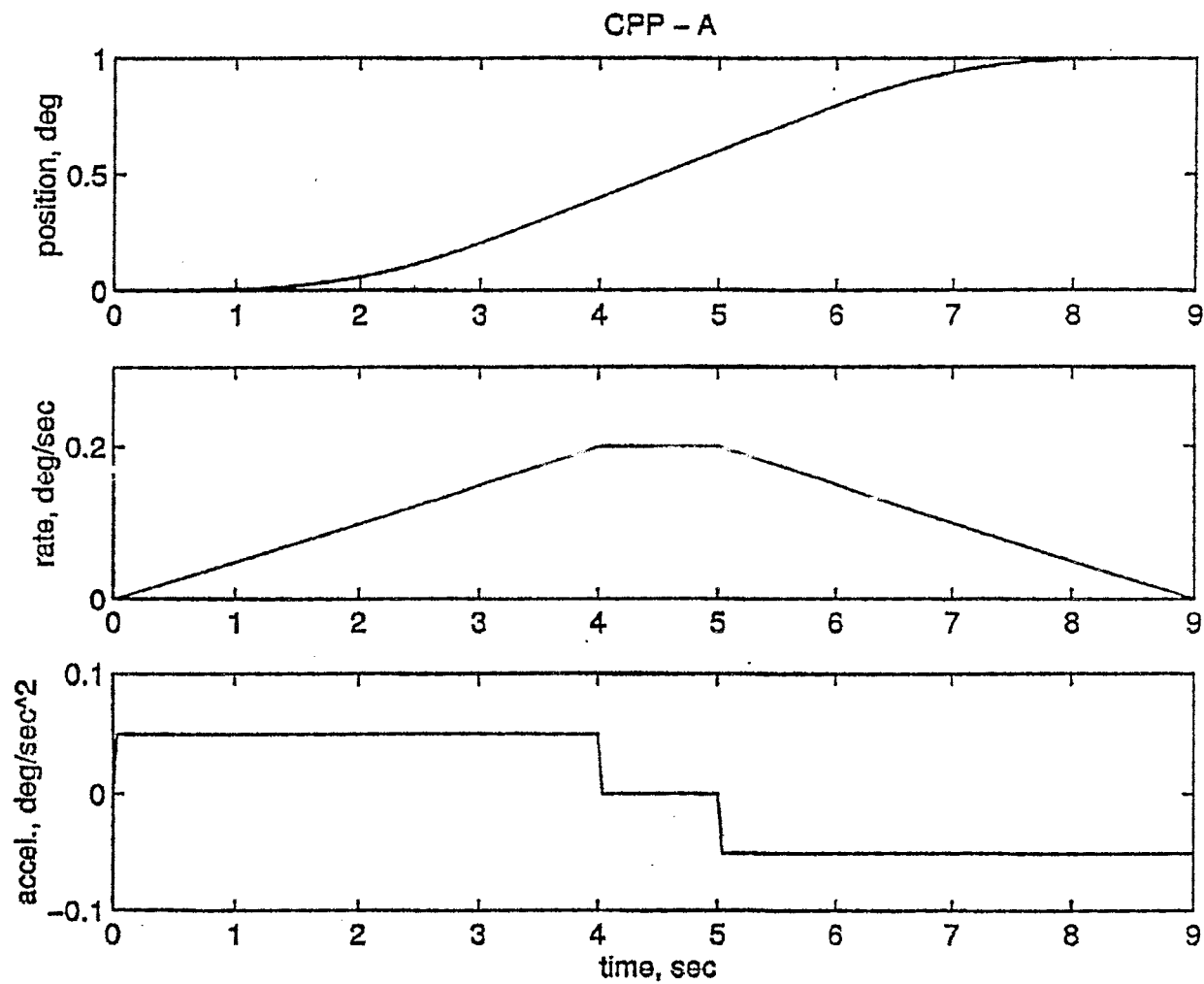


Fig.1. 1 deg step coommand preprocessed by CPP-A: a) the command, b) the command rate, c) the command acceleration.

Fig.1a,b,c

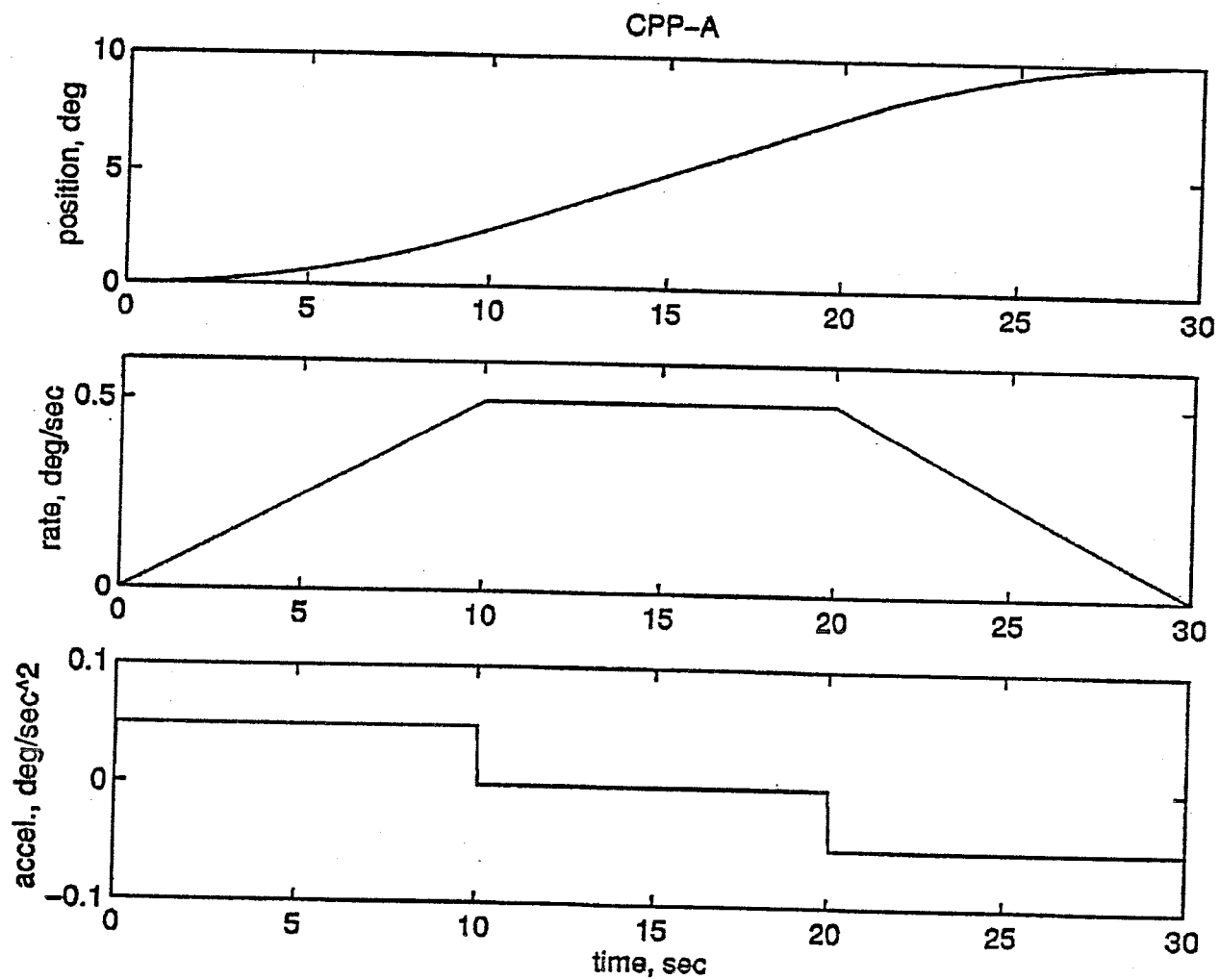


Fig.2. 10 deg step coommand preprocessed by CPP-A: a) the command, b) the command rate, c) the command acceleration.

Fig.2a,b,c

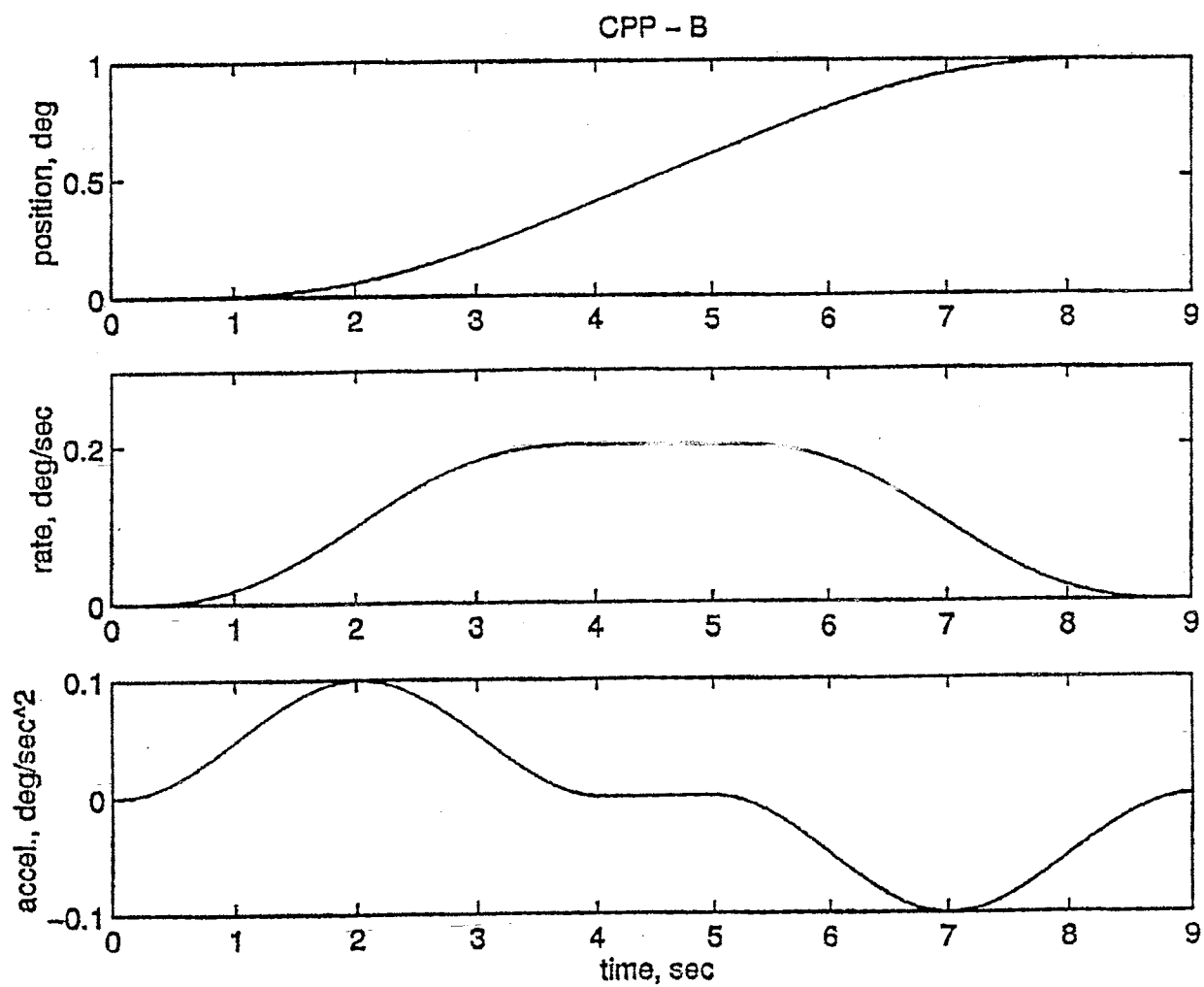


Fig.3. 1 deg step coommand preprocessed by CPP-B: a) the command, b) the command rate, c) the command acceleration.

Fig.3a,b,c

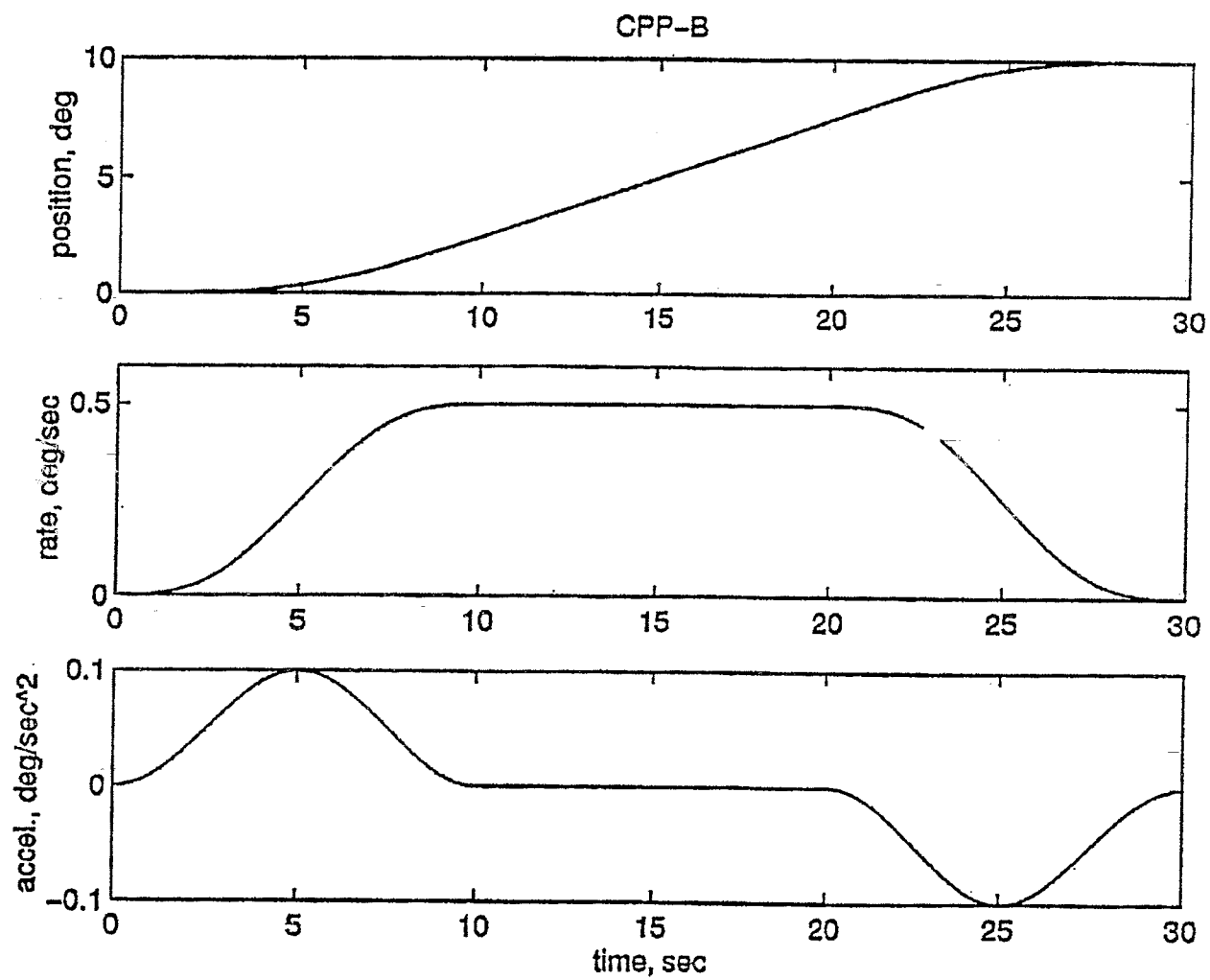


Fig.4. 10 deg step coommand preprocessed by CPP-B: a) the command, b) the command rate, c) the command acceleration.

Fig.4a,b,c

1% damping  
PI+feedforward controller with pre-processor  
Antenna orientation 60 deg in EL  
Acceleration limit 0.2 deg/sec<sup>2</sup>  
Rate limit 0.67 in AZ, 0.33 in EL

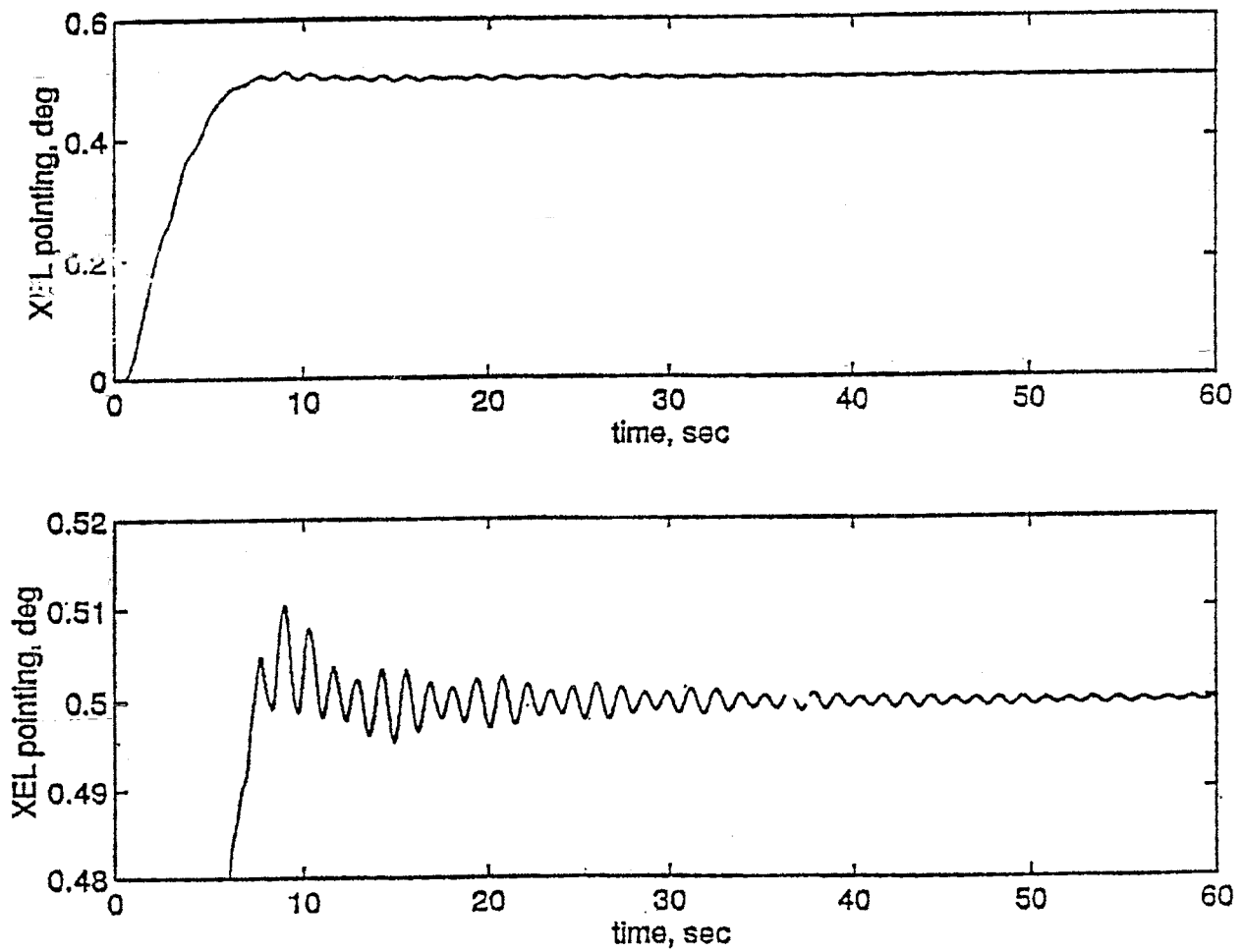


Fig.5. Cross-elevation pointing for 1 deg step command with the PCD preprocessor, a) full figure, b) zoomed figure.



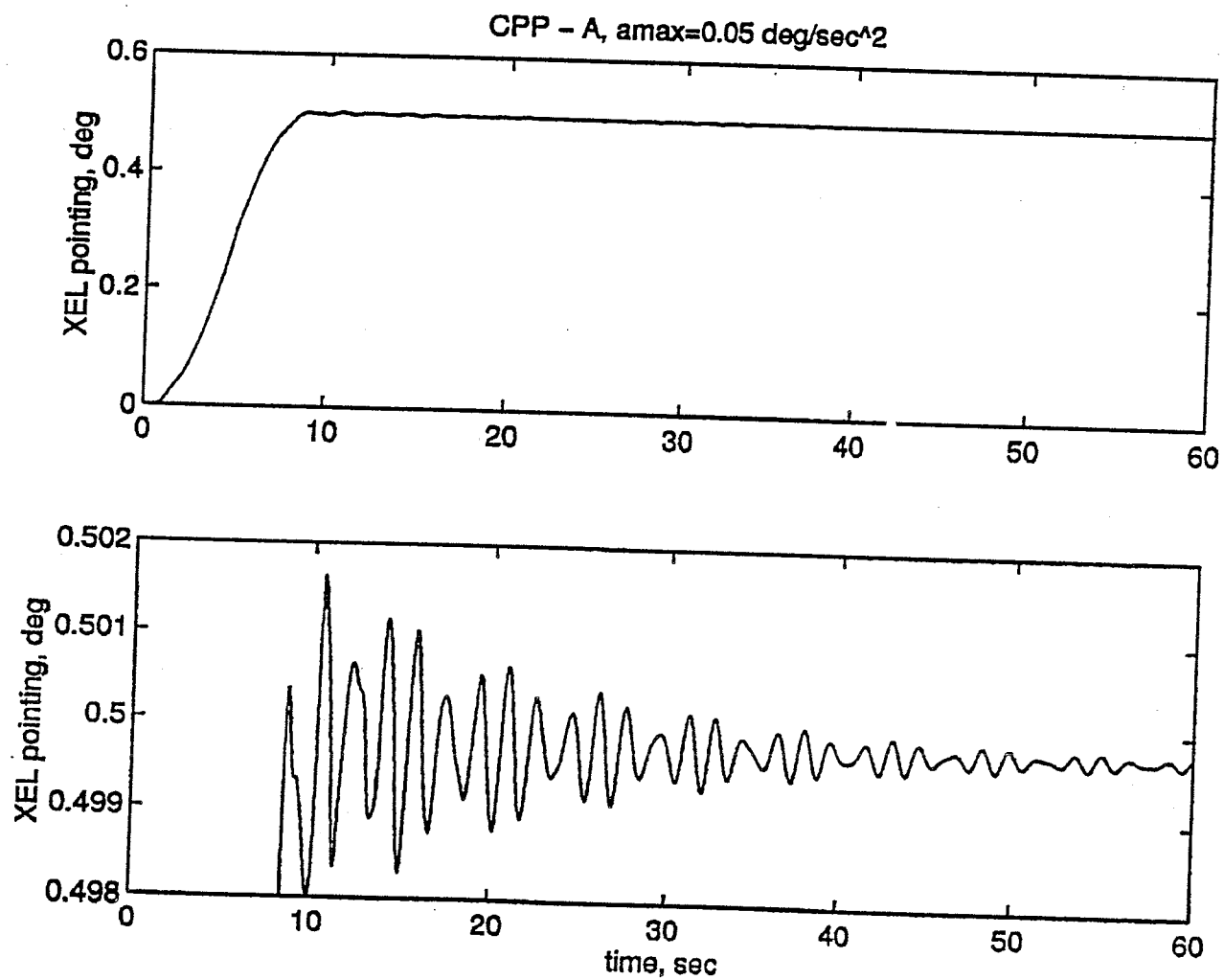


Fig.6. Cross-elevation pointing for 1 deg step command with the CPP-A preprocessor, a) full figure, b) zoomed figure.

Fig.6a,b

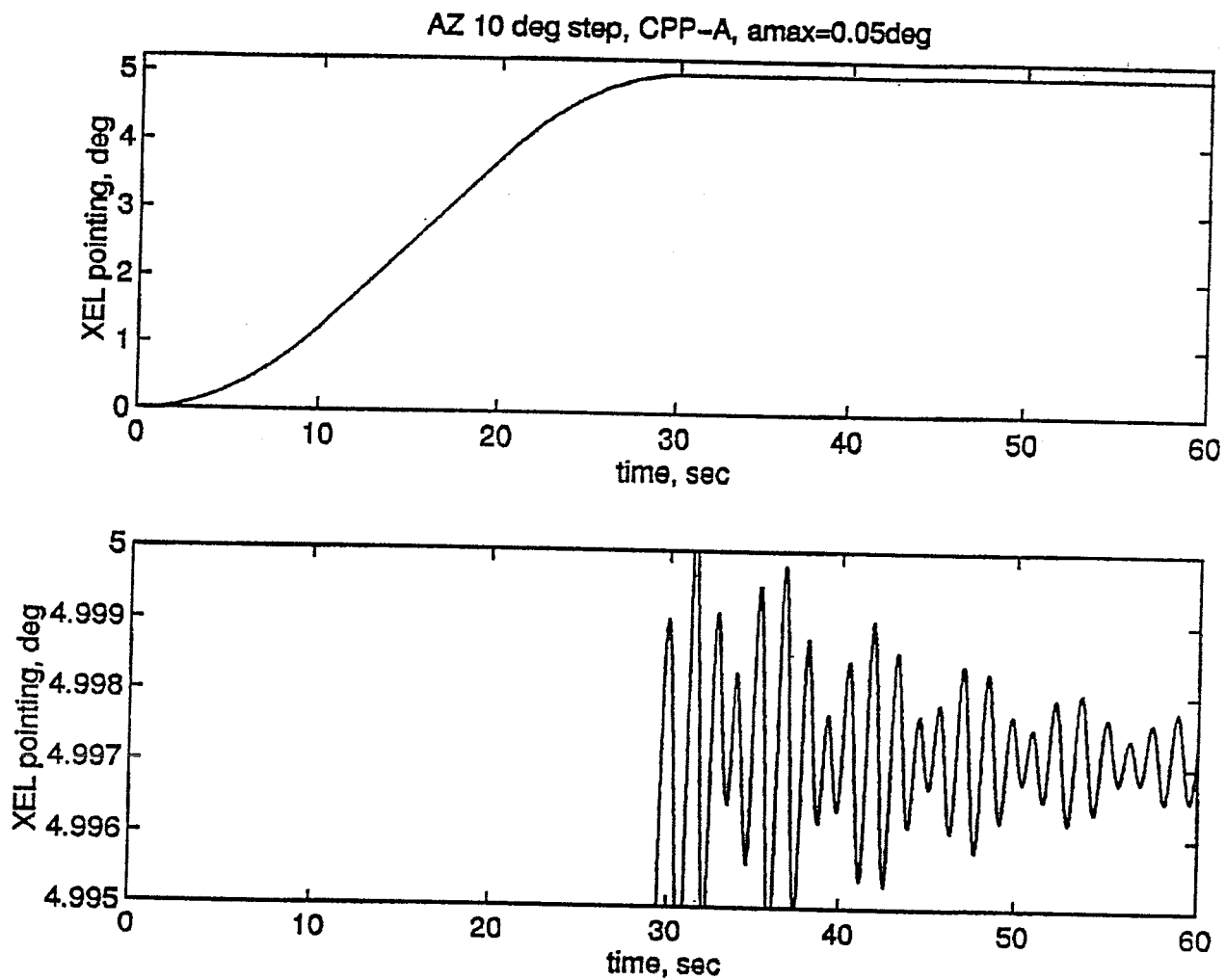


Fig. 7. Cross-elevation pointing for 10 deg. step command with the CPP-A preprocessor, a) full figure, b) zoomed figure.

Figure. 7a, b

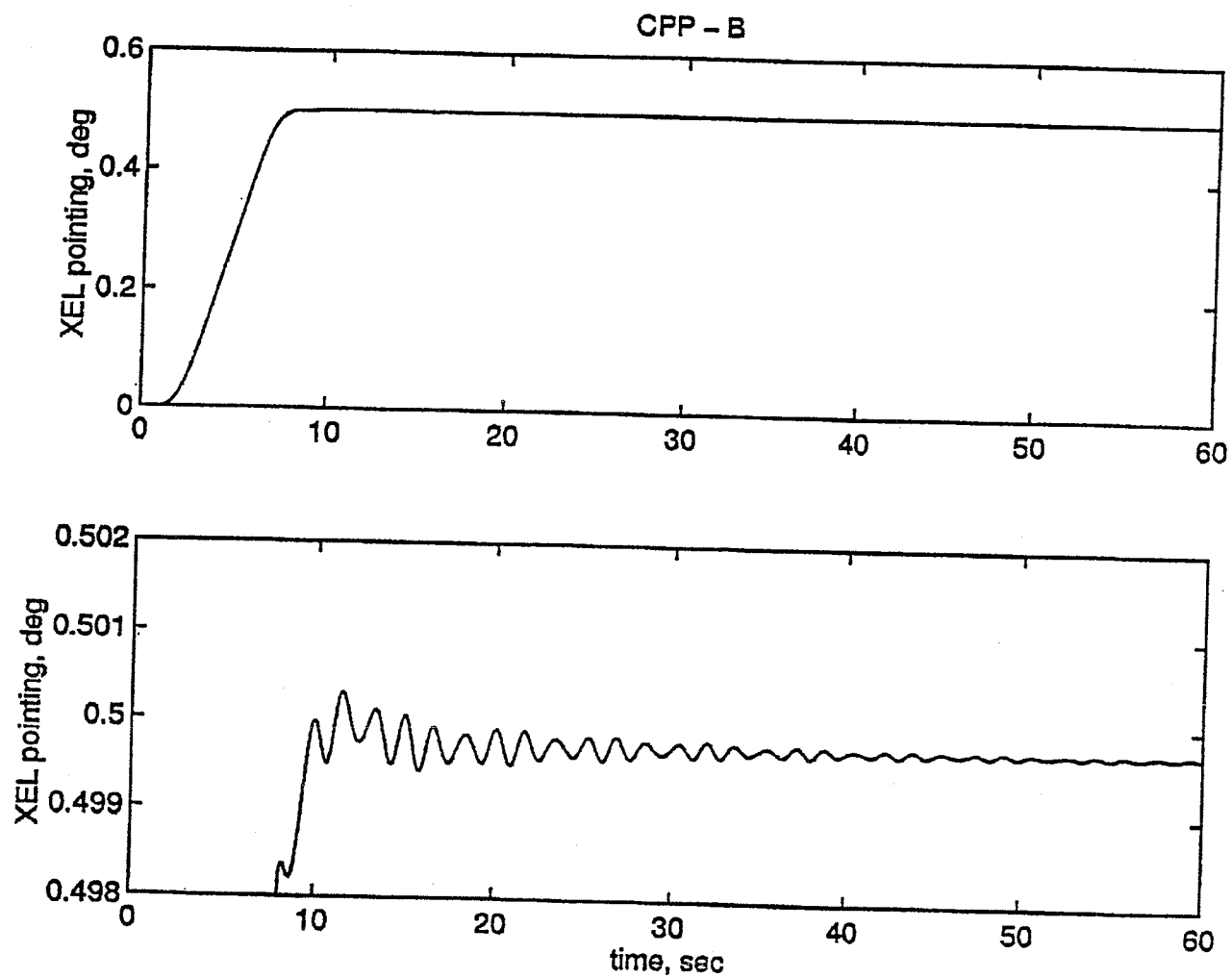


Fig. 8. Cross-elevation pointing for 1 deg. step command with the CPP-B preprocessor, a) full figure, b) zoomed figure.

Figure 8a, b

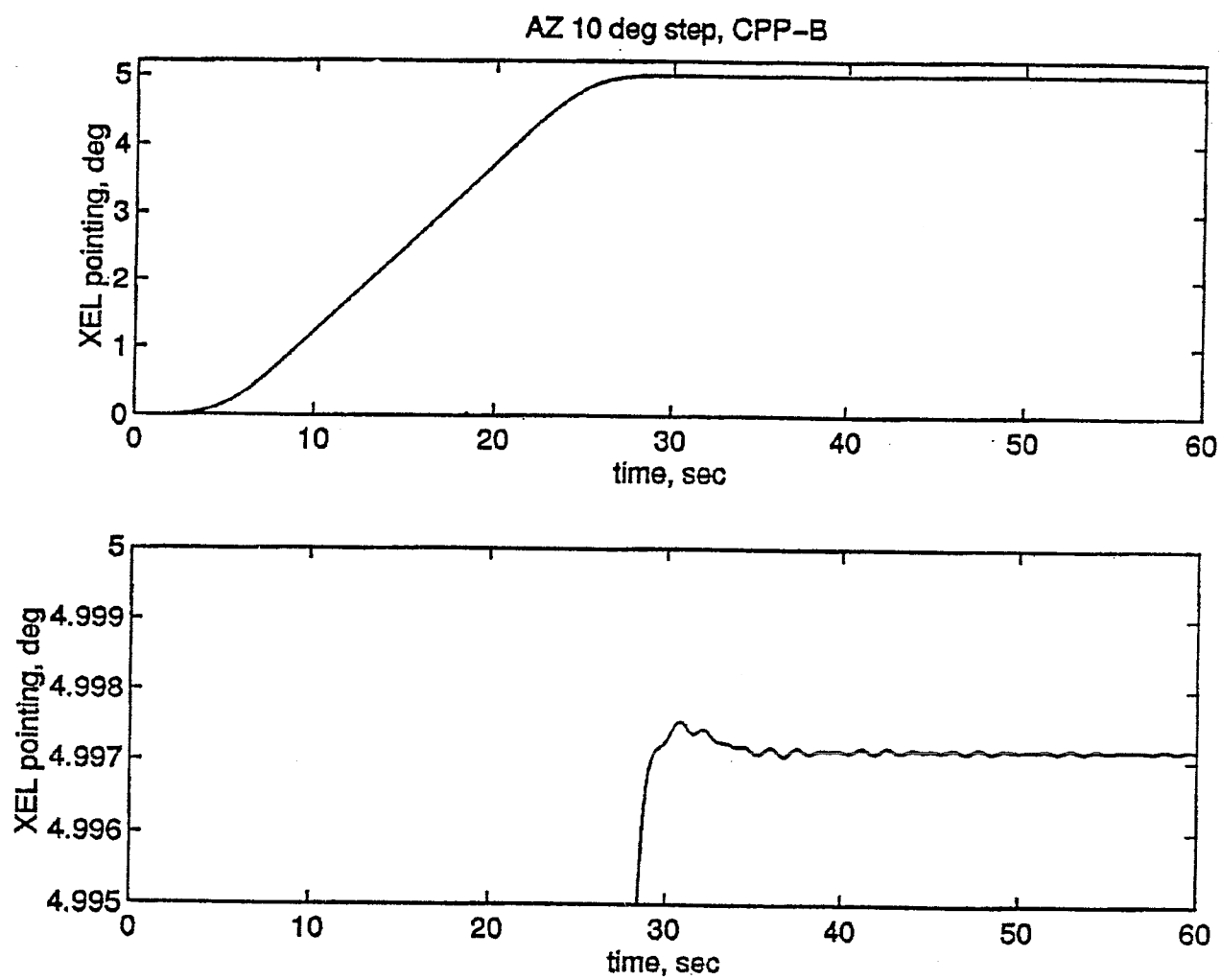


Fig.9. Cross-elevation pointing for 10 deg step command. with the CPP-B preprocessor, a) full figure, b) zoomed figure.

Fig.9a,b

REPORT 2B

**Low Rate Simulations of the GBT Antenna  
with Dry Friction and Sticking**

by

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The GBT antenna will be required to track with rates lower than the sidereal rate (4.4 mdeg/sec). However, the pointing accuracy for these rates might deteriorate due to dry friction and sticking at the driving wheels. It is the purpose of these simulations at what rates one can expect the loss of pointing accuracy due to the dry friction.

The dry friction and sticking model was described in the previous report, Ref.[1]. For this model the command of the constant rate offset (in azimuth) of 4.4 mdeg/sec was introduced. The azimuth wheel angle, shown in Fig.1a, is smooth, and shows no visible friction impact. The cross-elevation position in Fig.1b, shows initial transient motion, and is not influenced by the presence of friction and sticking.

The plot of the azimuth wheel angle for the rate command of 0.5 mdeg/sec is shown in Fig.2a. The sticking of the azimuth wheel is visible at the initial stage of motion (up to 2 sec), but in the later stages the wheel motion does not show the influence of friction. Similar applies to the cross-elevation position, which is shown in Fig.2b.

Finally, the simulations of the azimuth wheel angle for the rate command of 0.3 mdeg/sec show the significant impact of friction on the antenna motion, see Fig.3a. The wheel "stop-and-move" pattern of motion, as shown in Fig.3a, deteriorates significantly the pointing error, as shown in Fig.3b.

In conclusion, the simulations show that one can expect no significant loss of pointing accuracy for rates above 1 mdeg/sec. However, the value of the dry friction and sticking coefficient is not exactly known, therefore, one can expect no loss of accuracy even for rates lower than 1 mdeg/sec.

## References

1. W. Gawronski, and B. Parvin, "Modeling and Analysis of the Green Bank Telescope Control System," April 29, 1995.

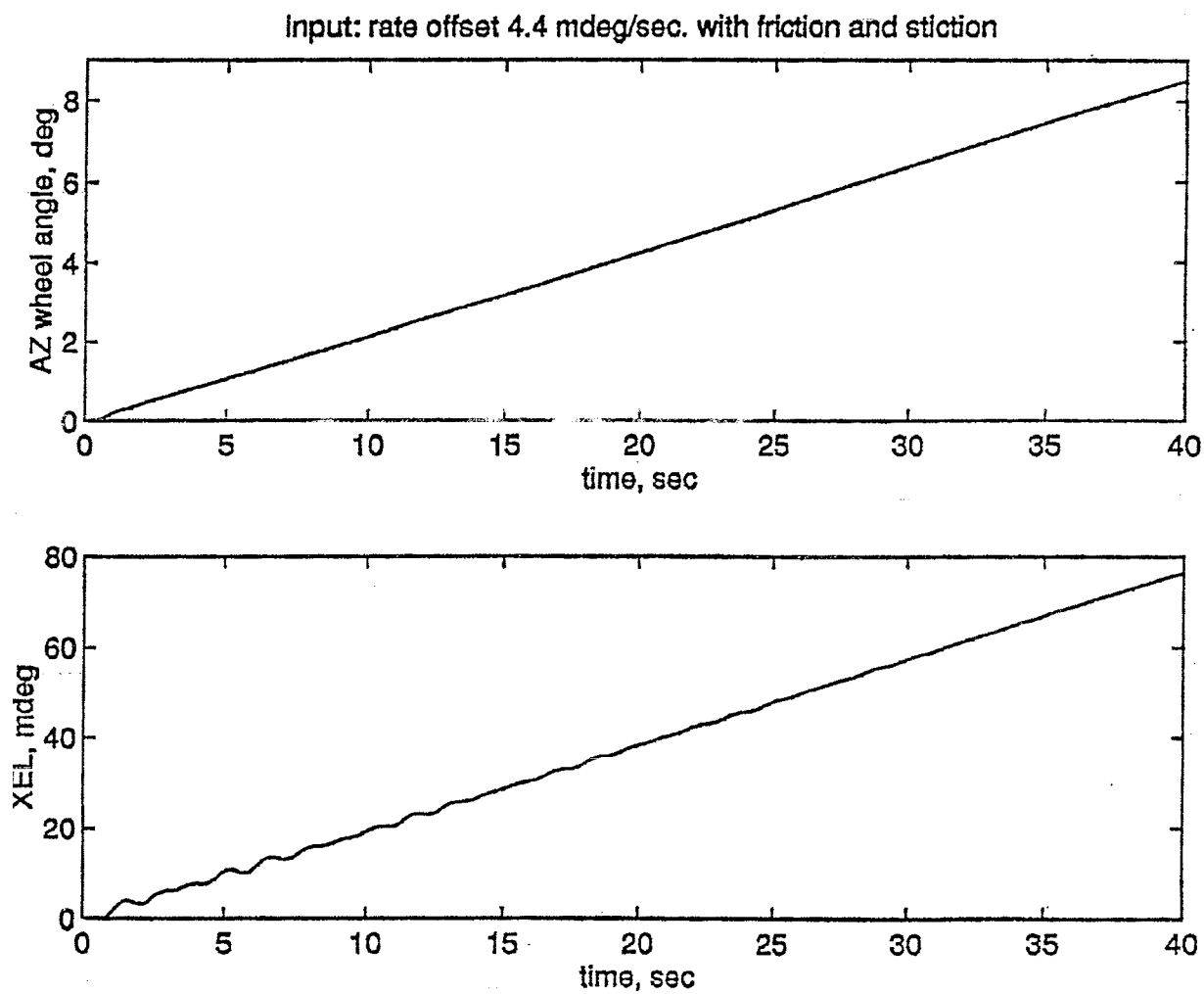


Fig.1. Simulations for the rate offset of 4.4 mdeg/sec, a) azimuth wheel angle, b) cross-elevation position.

Fig.1a,b

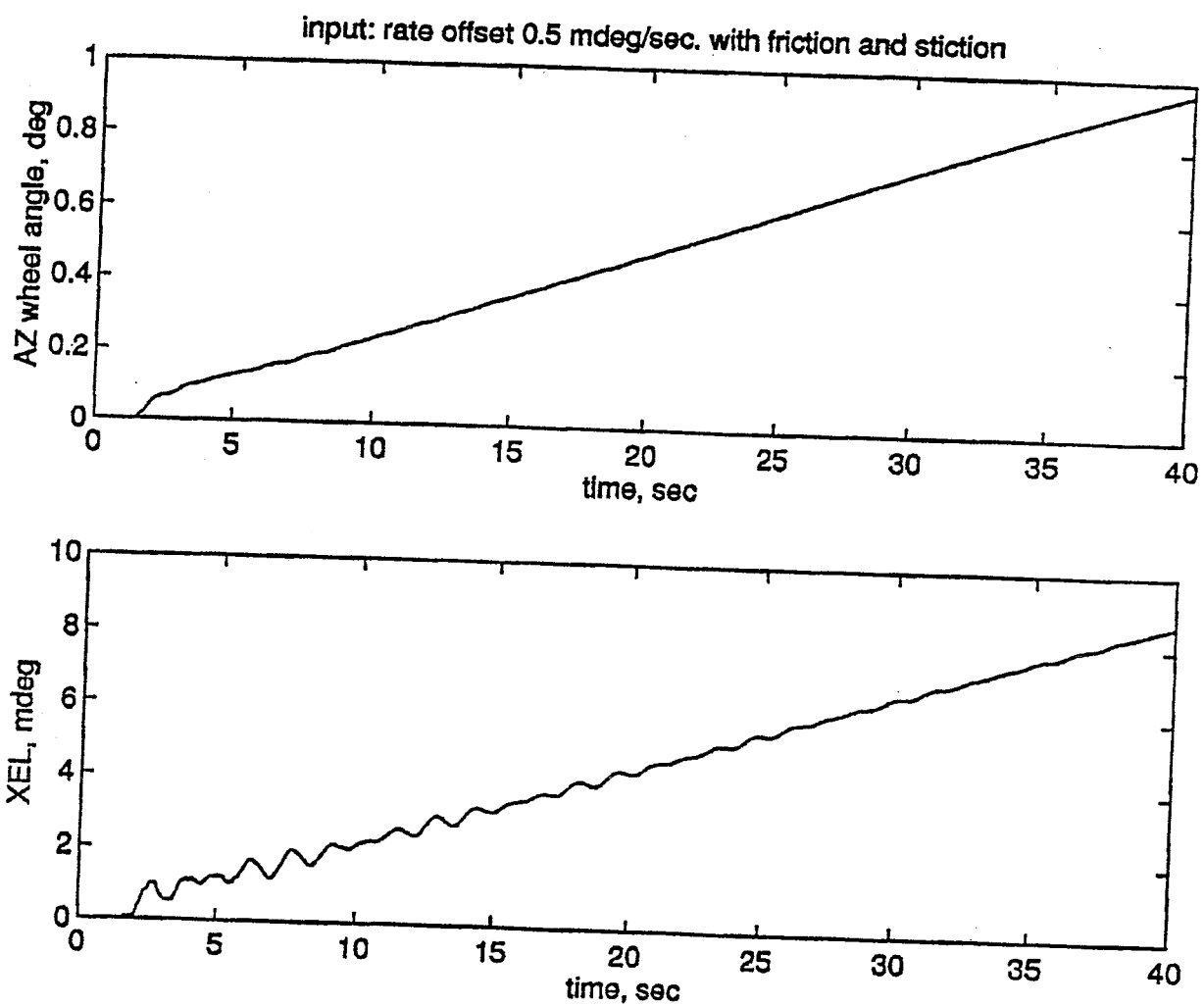


Fig.2. Simulations for the rate offset of 1.0 mdeg/sec, a) azimuth wheel angle, b) cross-elevation position.

Fig.2a,b



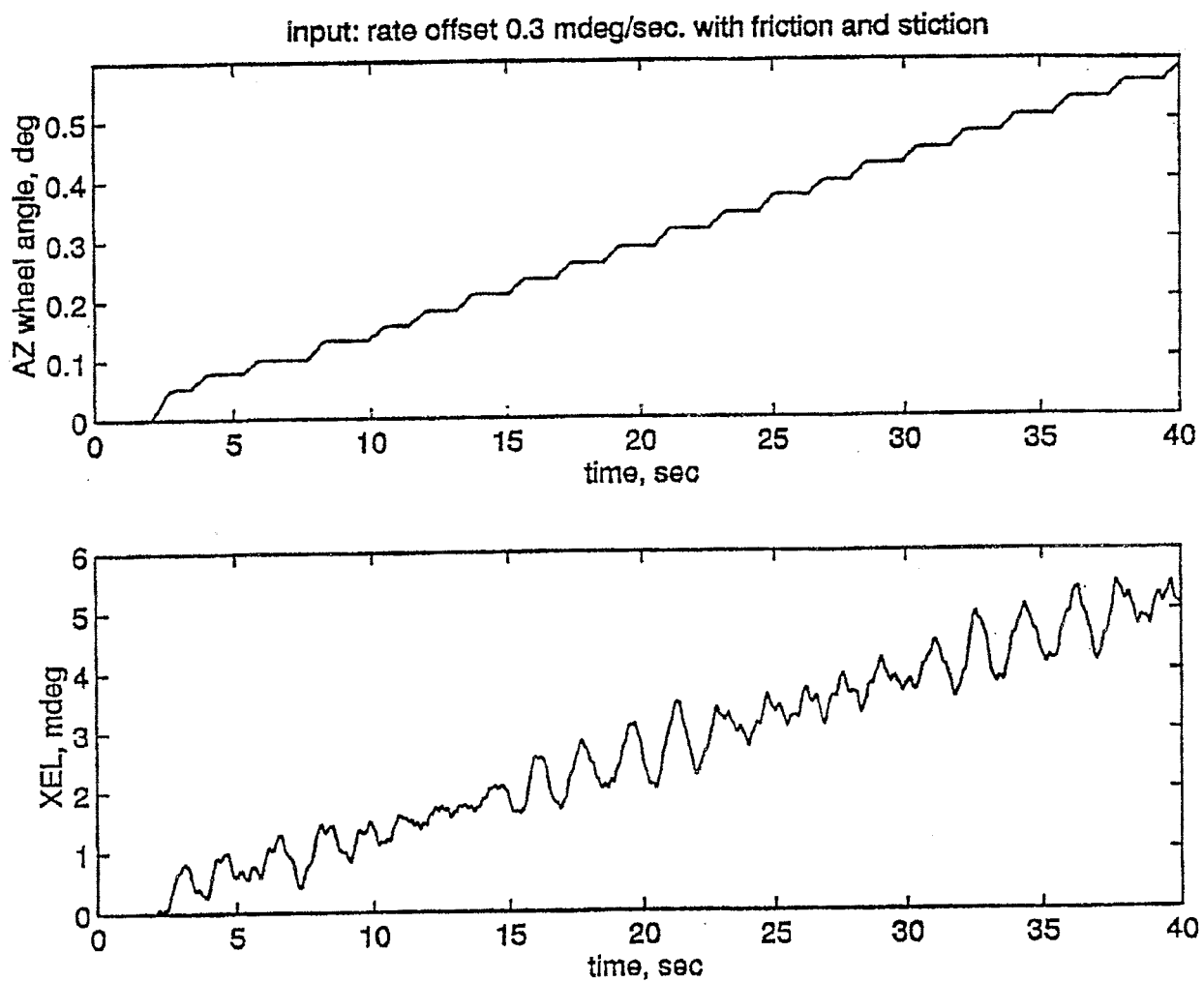


Fig.3. Simulations for the rate offset of 0.3 mdeg/sec, a) azimuth wheel angle, b) cross-elevation position.

Fig.3a,b