Target Position Corrections Due To Non-Perpendicularity Of Laser Scan Axes

Michael A. Goldman

May 20, 1996

Abstract

Corrections are given to the laser scanner position coordinates of retroreflector targets to be tracked by a GBT laser ranger as a function of the deviation from perpendicularity of the elevation and azimuth scan axes.

1. Definition Of The Scan Tracking Problem

The task of tracking a target retroreflector on the Green Bank Telescope may be stated in the following way:

The scan mirror of each laser range station mounts on an elevation rotor shaft which is coupled, through bearings, to an elevation rotation yoke which is part of a machined azimuth rotor shaft. The azimuth rotor is coupled via bearings to the base plate of the station, to which other optical components are attached. The relative orientation of the elevation and azimuth scan axes is determined by the quality of machining of the azimuth rotor piece, because the bearing seats for the rotation shaft bearings are machined into this piece.

Given a tracking program describing the coordinates of a target retroreflector mounted on the Green Bank Telescope, relative to some ground-fixed coordinate system, a set of laser mirror elevation scan angle and azimuth angle coordinates is computed for each laser range station. The elevation and azimuth scan angle

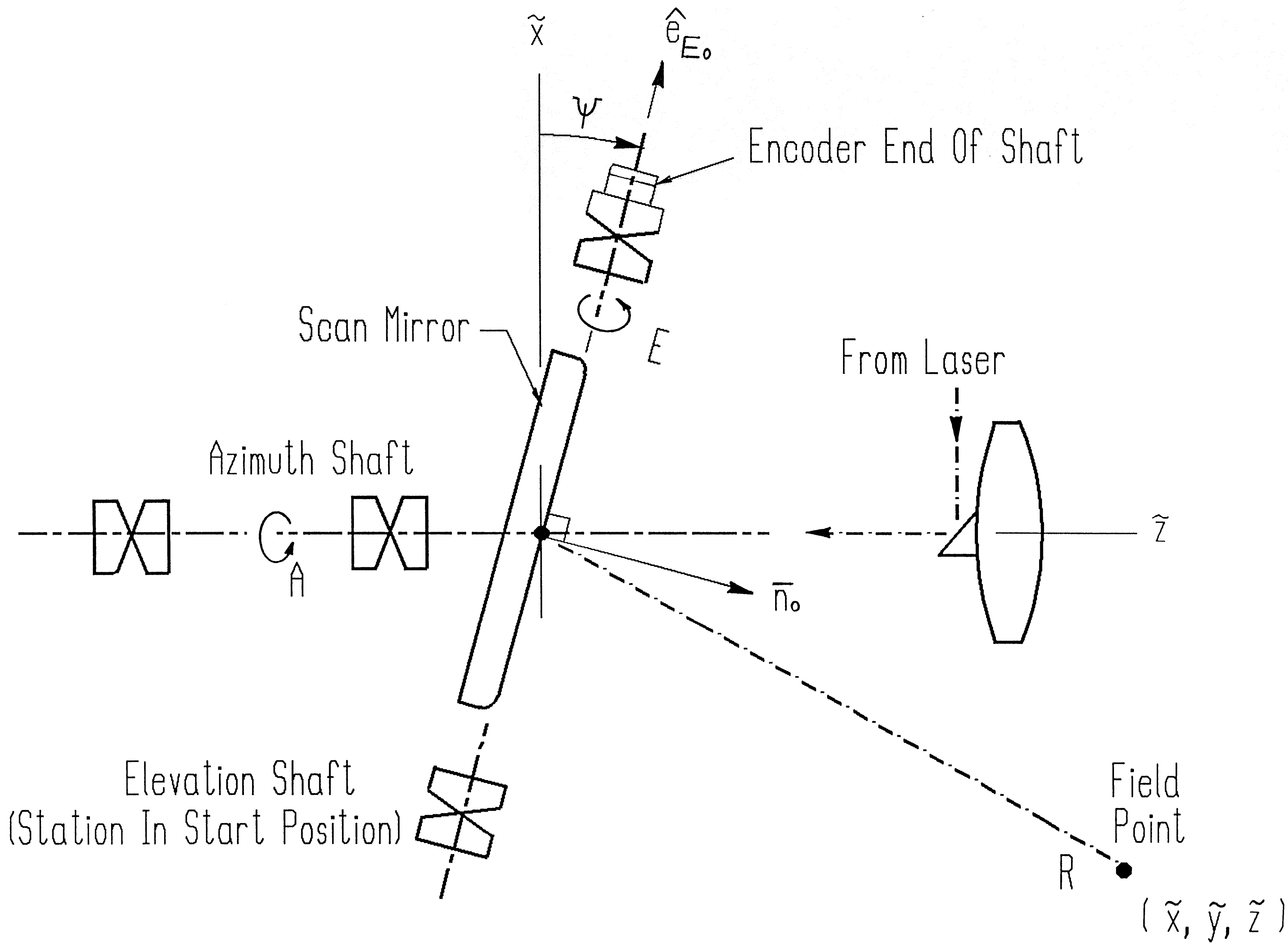

Figure 1. Plan View Of The Scan Geometry.

coordinates are used as input data to drive elevation and azimuth servomechanisms to point the laser beam of each station, from the center of its scan mirror towards the retroreflector target.

Each ranging station is provided with a local station-based coordinate system. This system is also a ground-fixed system for those stations that are used as ground stations, after they are surveyed into place and aligned with respect to a survey control network. The relation of feed arm stations to earth-fixed coordinates is determined dynamically. In the present discussion, it is assumed that laser pointing is described by a local station-based cartesian coordinate system, which can be related to ground-based coordinates, by means which we do not mention explicitly. The local station coordinate system is a cartesian ($\widetilde{x}$,$\widetilde{y}$, $\widetilde{z}$) system. This coordinate system is defined in GBT drawing D35420M051, and shown in Fig.1. The $\widetilde{z}$-coordinate is directed along the rotation axis of the azimuth rotor, pointing from the scan mirror towards the detector, through the detector lens. The optical axis of the detector lens is coincident with the $\widetilde{z}$-axis. The $\widetilde{x}$- and $\widetilde{y}$-coordinates are perpendicular to the $\widetilde{z}$-axis, and unit vectors in the $\widetilde{x}$,$\widetilde{y}$,$\widetilde{z}$ directions in that order form a right-hand triple. The $\widetilde{x}$-axis lies parallel to the base plane of the station. For ground stations, the azimuth rotation axis will be set close to horizontal when the station is mounted into position at its ground location; the $\widetilde{y}$-axis will be set to lie close to vertical upwards.

The origin of this coordinate system is defined to be the intersection point of the azimuth and elevation scan axes. We assume here that the scan axes, which are defined mechanically by two sets of bearing seats machined into the azimuth rotor piece do, in fact, intersect. This definition of the local coordinate system origin is independent of any assumptions made regarding perpendicularity of the azimuth and elevation axes.

We assume that the station scan mirror is mounted so that its plane of reflection contains the elevation axis.

The axes of elevation and azimuth for each station are determined by the two sets of bearing seats machined into the azimuth rotor piece. The two axes should be, ideally, mutually perpendicular to one another and intersect. If the axes intersect, but deviate from perpendicularity, software corrections will have to be made to the target's scan coordinates computed assuming perpendicularity. Errors of the order of an arc-minute will lead to several centimeters error, for target distances near 100 meters. The present memo is a computation of the target pointing errors to be corrected for when the azimuth and elevation axes intersect, but deviate from perpendicularity.

2. Computation Of Target Coordinates.

In this section we compute the retroreflector target's local station-based coordinates as a function of the station elevation and azimuth angles and the offset angle of the azimuth and elevation axes from perpendicularity. The scan geometry is shown in Figure 1.

We use the following notation to describe the scan geometry and kinematics:

The local cartesian coordinate system at a given ranging station is defined to be an ($\tilde{x}$,$\tilde{y}$,$\tilde{z}$) coordinate system, with $\tilde{z}$-axis along the azimuth rotor axis, and the origin at the intersection point of the azimuth and elevation axes.

Motion of the azimuth rotor is described by a rotation angle A about the positive scan azimuth axis. The "start position of the azimuth rotor" is defined to be the rotor position such that the elevation axis lies in the ($\tilde{x}$,$\tilde{z}$)-plane and the elevation axis' positive direction (from scan mirror center towards the angle encoder) points in the sense of the positive x-axis. (When the scan azimuth and elevation axes are accurately perpendicular, the positive elevation axis will point along the positive $\tilde{x}$-axis). The azimuth angle $A \equiv 0$ when the azimuth rotor is in its start position.

The "start position of the scanning station" is defined by the conditions that the azimuth rotor is in its start position and, simultaneously, the plane of the scan mirror is perpendicular to the (x,z)-plane.

Motion of the elevation rotor is described by a rotation angle E about the positive elevation axis. Elevation angle $E \equiv 0$ when the scanning station is in its start position. We will say that "the elevation rotor will be in its start position" in the general case that $E = 0$, for arbitrary values of A .

We note that the station will be in its starting position when both the azimuth and elevation rotors are in their start positions.

We assume that the optical position encoders used to sense the rotor elevation angle E and azimuth rotor angle A will be set in the following manner when the scanning station is in general position (A, E):

$$(1.1a) \qquad N_a(A) = Nao + \frac{10^5(A + \frac{\pi}{2})}{2\pi} \quad ;$$

$$(1.1b) \quad N_e(E) = Neo + \frac{10^5(-E)}{2\pi} \ .$$

The encoders each have a count increment of 10^5 counts per rotor shaft revolution.

The allowed range for scanning will be limited to the intervals:

$$(1.2a) \quad -\pi \leq A < \pi$$

$$(1.2b) \quad -\frac{\pi}{2} \leq E \leq 0 \ .$$

We call $\widehat{e_x}$, $\widehat{e_y}$, $\widehat{e_z}$ the unit vectors along the positive $\widetilde{x}$- , $\widetilde{y}$ - and $\widetilde{z}$-axes.

When the azimuth and elevation rotors are both in home position, we call the unit vector pointing along the positive elevation axis $\widehat{e_{Eo}}$, and call a unit vector pointing along the azimuth axis $\widehat{e_A}$. Since the azimuth axis is fixed with respect to the $(\widetilde{x},\widetilde{y},\widetilde{z})$ coordinate frame, $\widehat{e_A} = \widehat{e_z}$. not only when the azimuth rotor is in start position, but also in general position $A \neq 0$.

When the station and rotors are in start position. the elevation axis lies in the $(\widetilde{x},\widetilde{z})$-plane. We define the angle between the $\widetilde{x}$-axis and the positive elevation axis in home position to be ψ . This angle is the deviation from perpendicularity of the two scan axes. The geometry is illustrated in Figure 1.

We let $\widehat{e_A}$ be the unit vector pointing in the positive direction of the azimuth axis. This is a fixed vector and

$$(2.1) \qquad \widehat{e_A} = -\widehat{e_z} \ .$$

We call $\widehat{e_{Eo}}$ the unit vector pointing in the positive direction along the elevation axis, in the case that the station is in start position.

We call $\widehat{e_{no}}$ the unit normal to the scan mirror surface, in the case that the station is in start position. This vector lies in the (x,z)-plane.

When the scan mirror is in general position, we define the unit vector pointing along the elevation axis to be $\widehat{e_E}(A, E, \psi)$.

When the elevation rotor is in its start position, $A \equiv 0$; then $\widehat{e_E}\,(A = 0, E = 0, \psi) = \widehat{e_{Eo}}$. We have,

$$(2.2) \qquad \widehat{e_{Eo}} = (\cos\psi)\widehat{e_x} + (\sin\psi)\widehat{e_z} \ .$$

When the scan mirror is in general position we define the unit vector pointing along the normal to the mirror to be $\widehat{e_n}(A, E, \psi)$.

When the elevation rotor is in its home position, $\phi \equiv 0$; then $\widehat{e_n}\,(A = 0, E = 0, \psi) \ = \ \widehat{e_{no}}$. We have,

$$(2.3) \qquad \widehat{e_{no}} = (-\sin\psi)\widehat{e_x} + (\cos\psi)\widehat{e_z} \ .$$

We now rotate the mirror in azimuth, about $\widehat{e_A} = \widehat{e_z}$, by angle A .

The mirror unit normal and unit elevation axis direction vector are then moved to:

$$(2.4) \qquad \widehat{e_{no}} \ \longrightarrow \ \widehat{e_{nA}} \ \equiv \ [Rot(\widehat{e_A}\ ,\ A)]\,\widehat{e_{no}}$$

$$(2.5) \qquad \widehat{e_{Eo}} \ \longrightarrow \ \widehat{e_{EA}} \ \equiv \ [Rot(\widehat{e_A}\ ,\ A)]\,\widehat{e_{Eo}} \ .$$

We use the result (A2.1) of Appendix II to compute these vectors.

Starting from (2.4) , and using (2.1) and (2.3) we have

$$(2.6)\quad \widehat{e_{nA}} = (\cos A)\widehat{e_A} + (1-\cos A)(\widehat{e_A}\cdot\widehat{e_{no}})\widehat{e_{no}} + (\sin A)(\widehat{e_A}\times\widehat{e_{no}}) \quad \text{giving}$$

$$(2.7)\quad \widehat{e_{nA}} = [(-\sin\psi)(\cos A)\widehat{e_x} + (\cos\psi)(\cos A)\widehat{e_z}] + [(1-\cos A)(\cos\psi)\widehat{e_z}] + [(\sin A)(\widehat{e_z}\times\widehat{e_{no}})] \ ,$$

which simplifies to

$$(2.8)\quad \widehat{e_{nA}} = (-\sin\psi)(\cos A)\widehat{e_x} + (-\sin\psi)(\sin A)\widehat{e_y} + (\cos\psi)\widehat{e_z} \ .$$

Starting from (2.5) , and using (2.1) and (2.2) we have

$$(2.9)\quad \widehat{e_{EA}} = (\cos A)\widehat{e_A} + (1-\cos A)(\widehat{e_A}\cdot\widehat{e_{Eo}})\widehat{e_{Eo}} + (\sin A)(\widehat{e_A}\times\widehat{e_{Eo}}) \quad \text{giving}$$

$$(2.10)\quad \widehat{e_{EA}} = [(\cos\psi)(\cos A)\widehat{e_x} + (\sin\psi)(\cos A)\widehat{e_z}] + [(1-\cos A)(\sin\psi)\widehat{e_z}] + [(\sin A)(\widehat{e_z}\times\widehat{e_{Eo}})] \ ,$$

which simplifies to

$$(2.11)\quad \widehat{e_{EA}} = (\cos\psi)(\cos A)\widehat{e_x} + (\cos\psi)(\sin A)\widehat{e_y} + (\sin\psi)\widehat{e_z} \ .$$

We now rotate the mirror by an elevation angle E about the elevation axis $\widehat{e_{EA}}$. The elevation rotor axis remains fixed in direction, that is

$$(2.12)\qquad \widehat{e_{EA}} \;\rightarrow\; \widehat{e_E} \;\equiv\; [Rot(\widehat{e_{EA}}\ ,\ E)]\,\widehat{e_{EA}} \;=\; \widehat{e_{EA}} \ .$$

Under this rotation, the mirror normal now points in the direction of the unit vector $\widehat{e_n}$ where,

$$(2.13)\qquad \widehat{e_{nA}} \;\rightarrow\; \widehat{e_n} \;\equiv\; [Rot(\widehat{e_{nA}}\ ,\ E)]\,\widehat{e_{nA}} \quad \text{is computed using (A2.1):}$$

$$(2.14)\qquad \widehat{e_n} = (\cos E)\widehat{e_{nA}} + (1-\cos E)(\widehat{e_{EA}}\cdot\widehat{e_{nA}})\widehat{e_{EA}} + (\sin E)(\widehat{e_{EA}}\times\widehat{e_{nA}}) \ .$$

The middle term in equation (2.14) vanishes because $\widehat{e_{EA}}$ and $\widehat{e_{nA}}$ are perpendicular to one another. After some algebraic manipulation one gets:

$$(2.15)\quad \widehat{e_n} = [(\sin E)(\sin A) - (\cos E)(\cos A)(\sin\psi)]\widehat{e_x}$$

$$+[(-\sin E)(\cos A) + (\cos E)(\sin A)(\sin\psi)]\widehat{e_y} + (\cos E)(\cos\psi)\widehat{e_z} .$$

In Appendix I the reflected unit ray, $\widehat{e_r}$. from a plane mirror is computed in terms of the incident ray $\widehat{e_i}$ and the unit normal $\widehat{e_n}$ to the mirror,

$$(\text{A1.2})\qquad \widehat{e_r} = \widehat{-e_z} + 2(\widehat{e_n}\cdot\widehat{e_z})\widehat{e_n} .$$

We now compute the unit vector in the direction of the reflected laser beam from the scan mirror.

Using (A1.2), with $\widehat{e_n}\cdot\widehat{e_z} = (\cos\psi)(\cos E)$, we get

$$(2.16)\quad \widehat{e_r} = \widehat{-e_z} + 2(\cos E)(\cos\psi)\{\ [(\sin E)(\sin A) - (\cos E)(\cos A)(\sin\psi)]\ \widehat{e_x}$$

$$+[(-\sin E)(\cos A) + (\cos E)(\sin A)(\sin\psi)]\ \widehat{e_y} + [(\cos E)(\cos\psi)]\ \widehat{e_z}\} .$$

Consider the situation where the scan mirror has been moved from its home position to an azimuth angle A and elevation angle E . A laser beam directed initially along the negative z axis towards the mirror will reflect at the scan center point, which is the intersection of the scan azimuth and elevation axes, in the direction of $\widehat{e_r}$. A field target point along the reflected ray, at range distance R from the scan center point, will have local $(\tilde{x},\tilde{y},\tilde{z})$ station coordinates:

$$(2.17)\ \tilde{x}(A,E,\psi) = (\widehat{e_r}\cdot\widehat{e_x})R = [(\sin 2E)(\sin A)(\cos\psi) - (\cos^2 E)(\cos A)(\sin 2\psi)]R$$

$$(2.18)\ \tilde{y}(A,E,\psi) = (\widehat{e_r}\cdot\widehat{e_y})\ R = [(-\sin 2E)(\cos A)(\cos\psi) + (\cos^2 E)(\sin A)(\sin 2\psi)]R ,$$

$$(2.19)\ \tilde{z}(A,E,\psi) = (\widehat{e_r}\cdot\widehat{e_z})\ R = [2(\cos^2 E)(\cos^2\psi) - 1]R ,\quad \text{where}$$

$$(2.20)\qquad R^2 = \tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2 .$$

If the azimuth rotor is properly machined. so that $\psi = 0$, then these reduce to:

$$(2.21)\quad \tilde{x}_o \equiv \tilde{x}(A,E,\psi=0) = [\,(\sin 2E)(\sin A)\,]R ,$$

(2.22) $\tilde{y}_o \equiv \tilde{y}(A, E, \psi = 0) = [\,(-\sin 2E)(\cos A)\,]R$.

(2.23) $\tilde{z}_o \equiv \tilde{z}(A, E, \psi = 0) = \quad (\cos 2E)\, R$, where

(2.24) $R^2 = \tilde{x}_o^2 + \tilde{y}_0^2 + \tilde{z}_o^2$.

3. Computation Of Corrected Azimuth And Elevation Coordinates.

We solve the following pointing problem, using the results of Section 2.

We wish to point the laser beam of a ranging station at a distant target, given the local station coordinates $(\tilde{x}_t\,,\, \tilde{y}_t\,,\, \tilde{z}_t)$ of the target center. If the elevation and azimuth axes determined by the azimuth rotor piece are mutually perpendicular, then the pointing azimuth angle A_t and the pointing elevation angle E_t of the target are computed using (2.21) through (2.23).

For each station, the deviation angle from perpendicularity of the azimuth and elevation axes, ψ , is measured. Typically ψ is found to range from 0.2 to 1.5 arc- minutes. A one minute arc of a 100 meter radius circle has an arc length of $\pi \div (0.6 \times 1.8)$ cm = 2.909 cm. For targets at 100 to 200 meters distance, which are typical for laser ranging , the beam landing at target range could typically be several centimeters off target systematically, if values A_t and E_t computed using equations (2.21)-(2.23) were used to set the pointing of the station's scan mirror.

Instead, pointing angles, A_p and E_p, should be used to set the scan mirror, where A_p and E_p satisfy equations (2.17)-(2.19) using the given values of ψ and $\tilde{x}_t$, $\tilde{y}_t$, $\tilde{z}_t$.

Equation (2.19) can be solved explicitly for E_p . Using $R^2 = \tilde{x}_t^2 + \tilde{y}_t^2 + \tilde{z}_t^2$, we get

(2.25) $E_p = -(1/2)\{\cos^{-1}[-1 + (R + \tilde{z}_t)(\sec^2 \psi)/2R]\}$;

here we choose the solution where E_p is negative ($-\pi/2 \leq E_p \leq 0$).

Equations (2.17) and (2.18) are linear in $\sin A_p$ and $\cos A_p$, after E_p and R have been computed from the known values of $\tilde{x}_t, \tilde{y}_t, \tilde{z}_t$ and ψ .

Call:

$$
\begin{array}{lll}
 & b_1 = \tilde{x}_t/R & b_2 = \tilde{y}_t/R \\
(2.28) & a_{11} = (\sin 2E_p)(\cos\psi) & a_{12} = (-\cos^2 E_p)(\sin 2\psi) \\
 & a_{21} = (\cos^2 E_p)(\sin 2\psi) & a_{22} = (-\sin 2E_p)(\cos\psi)
\end{array}
$$

Then

$$
(2.29) \qquad \begin{aligned} b_1 &= a_{11}(\sin A_p) + a_{12}(\cos A_p) \\ b_2 &= a_{21}(\sin A_p) + a_{22}(\cos A_p) \end{aligned}
$$

where $a_{21} = -a_{12}$ and $a_{22} = -a_{11}$,

which is solved directly to give:

$$
(2.30) \qquad \begin{aligned} \sin A_p &= \frac{b_1 a_{11} + b_2 a_{12}}{a_{11}^2 - a_{12}^2} \\ \cos A_p &= \frac{b_1 a_{12} + b_2 a_{11}}{a_{12}^2 - a_{11}^2} \, . \end{aligned}
$$

The angle A_p is computed from the relation

$$
(2.31) \qquad A_p = \text{atan2}\ (\sin A_p\ ,\ \cos A_p\ ).
$$

Angles E_p and A_p are in radians, and convert to shaft encoder counts by substituting them into equations (1.1a) and (1.1b).

4. Small Angle Correction.

Since the angle ψ is small, typically below 2 arc-minutes, it is convenient to compute the uncorrected pointing angles A_t and E_t from the cartesian target coordinates $\tilde{x}_t, \tilde{y}_t, \tilde{z}_t$ using the equations (2.21)-(2.24) which hold for the case of perpendicular rotor axes, and making a first order small correction in ψ . This is allowable, provided the elevation angle of the target is not near zero. As an example, it is easily seen geometrically that for a target situated on the azimuth axis, one would have to rotate the azimuth angle by a large value, $\pi/2$, as an azimuth correction, before making a small elevation angle correction to point the beam, when $\psi \neq 0$. In this case the first order correction in ψ of the azimuth angle is divergent. Due to scan angle limiting in the scanner structure such a case will not occur physically, and a first order correction in ψ of the pointing angles A_t and E_t is adequate.

We compute the first order corrections as follows:

Given a target point $T = (\widetilde{x}_t, \widetilde{y}_t, \widetilde{z}_t)$, where $\widetilde{x}_t^2 + \widetilde{y}_t^2 + \widetilde{z}_t^2 = R^2$, we call the scan angle coordinates required to point the laser beam, from the coordinate origin (the scan axis intersection point) to this point, A_p and E_p . Let

(4.1) $\xi = \widetilde{x}_t/R\,, \quad \eta = \widetilde{y}_t/R\,, \quad \zeta = \widetilde{z}_t/R\,.$

Then from (2.17)-(2.19),

(4.2) $\xi = (\sin 2E_p)(\sin A_p)(\cos\psi) - (\cos^2 E_p)(\cos A_p)(\sin 2\psi) \quad ,$

(4.3) $\eta = (\sin 2E_p)(\cos A_p)(-\cos\psi) + (\cos^2 E_p)(\sin A_p)(\sin 2\psi) \quad ,$

(4.4) $\zeta = 2(\cos^2 E_p)(\cos^2\psi) - 1\,,$ where

(4.5) $\xi^2 + \eta^2 + \zeta^2 \equiv 1\,.$

As ψ is departs from zero the scan angles A_p and E_p required to point to the fixed target point T become functions of ψ. That is, $A_p = A_p(\psi)$ and $E_p = E_p(\psi)$. The derivatives of these functions, with respect to ψ are determined by the conditions that ξ , η , and ζ should not change as ψ varies. That is:

(4.6) $d\xi/d\psi = 0\,,\ d\eta/d\psi = 0\,,\ d\zeta/d\psi = 0\,.$

The last equation of (4.6) also follows from the first two and (4.5).

We then have:

(4.7a) $\quad 0 = (\partial\xi/\partial A_p)\dfrac{dA_p}{d\psi} + (\partial\xi/\partial E_p)\dfrac{dE_p}{d\psi} + (\partial\xi/\partial\psi) \quad .$

(4.7b) $\quad 0 = (\partial\eta/\partial A_p)\dfrac{dA_p}{d\psi} + (\partial\eta/\partial E_p)\dfrac{dE_p}{d\psi} + (\partial\eta/\partial\psi) \quad .$

For small $|\psi|$, when $E_p \neq 0$, we have the first order corrections:

(4.8a) $\quad A_p(\psi) = A_t + \psi\left(\dfrac{dA_p}{d\psi}\right)_{\psi=0} \quad ,$

(4.8a) $\quad E_p(\psi) = E_t + \psi\left(\dfrac{dE_p}{d\psi}\right)_{\psi=0} \quad .$

The derivatives appearing in equations (4.8) are computed by calculating the partial derivatives of equations (4.2) and (4.3) to obtain the coefficients appearing in equations (4.7), then solving the pair of equations (4.7) for $\dfrac{dA_p}{d\psi}$ and $\dfrac{dE_p}{d\psi}$, using $A_p(\psi = 0) = A_t$ and $E_p(\psi = 0) = E_t$.

After some extended manipulation and trigonometric simplification, we get:

(4.9a) $\quad \left(\dfrac{dA_p}{d\psi}\right)_{\psi=0} = (1 + \sec 2E_t)(\cos 2A_t)(\cot 2E_t) \quad ,$

(4.9b) $\quad \left(\dfrac{dE_p}{d\psi}\right)_{\psi=0} = \left(\dfrac{1}{2}\right)(1 + \sec 2E_t)(\sin 2A_t) \quad .$

The shaft angles for pointing are, to first order in ψ (ψ in radians), then:

(4.10a) $\quad A_p(\psi) = A_t + \psi(1 + \sec 2E_t)(\cos 2A_t)(\cot 2E_t) \quad ;$

(4.10b) $\quad E_p(\psi) = E_t + \left(\dfrac{\psi}{2}\right)(1 + \sec 2E_t)(\sin 2A_t) \quad .$

In Appendix III it is demonstrated that the pointing angles A_t and E_t , required when the scan axes are perpendicular, can be expressed in a very simple way, in terms of the spherical polar coordinates of the target point T .

(A3.3.1) $\quad A_t = \Theta - \dfrac{\pi}{2},$

(A3.3.2) $\quad E_t = -\dfrac{\Phi}{2} \; .$

The pointing angles then become, to first order in ψ, in terms of the target point spherical angle coordinates Θ and Φ:

$$(4.11a) \quad A_p(\psi) = \ \Theta - \frac{\pi}{2} + \psi(1+\sec\Phi)(\cos 2\Theta)(\cot\Phi) \quad ,$$

$$(4.11b) \quad E_p(\psi) = -\frac{\Phi}{2} - \left(\frac{\psi}{2}\right)(1+\sec\Phi)(\sin 2\Theta) \quad .$$

The shaft encoder setting counts become, to first order in ψ, in terms of the target point spherical angle coordinates Θ and Φ:

$$(4.12a) \quad N_a(\Theta\,,\,\Phi\,,\,\psi) = N_{ao} + \frac{10^5(\Theta + \psi[(1+\sec\Phi)(\cos 2\Theta)(\cot\Phi)\,])}{2\pi} \quad ,$$

$$(4.12b) \quad N_e(\Theta\,,\,\Phi\,,\psi) = N_{eo} + \frac{10^5(\Phi + \psi[(1+\sec\Phi)(\sin 2\Theta)])}{4\pi} \quad .$$

Equations (4.12) are the results we are seeking.

The count increments to be added to the encoder settings are then:

$$(4.13a) \quad \Delta N_a(\Theta\,,\,\Phi\,,\,\psi) = \frac{10^5\psi[(1+\sec\Phi)(\cos 2\Theta)(\cot\Phi)\,]}{2\pi} \quad ,$$

$$(4.13b) \quad \Delta N_e(\Theta\,,\,\Phi\,,\psi) = \frac{10^5\psi[(1+\sec\Phi)(\sin 2\Theta)]}{4\pi} \quad \text{, where } \psi \text{ is in radians.}$$

If ψ is given in arc-minutes (1 arc-minute $= \left(\frac{\pi}{180}\right)\left(\frac{1}{60}\right)$radian), then the encoder count increments are:

$$(4.13a) \quad \Delta N_a(\Theta\,,\,\Phi\,,\,\psi) = 4.63(\psi_{\mathbf{minutes}})[(1+\sec\Phi)(\cos 2\Theta)(\cot\Phi)\,] \quad ,$$

$$(4.13b) \quad \Delta N_e(\Theta\,,\,\Phi\,,\psi) = 4.63(\psi_{\mathbf{minutes}})[(1+\sec\Phi)(\sin 2\Theta)] \quad .$$

Equations (4.13) give the count increments to be added to the shaft encoders, to correct for non-perpendicularity of the scan angle axes.

5. Discussion.

The corrections (4.13) for the scanner shaft angles should, in principle, improve the laser pointing and tracking of target reflectors. In practice this will depend strongly upon correct knowledge of the initial angle encoder counts Nao and Neo. These count values are typically determined by laboratory optical bench measurements using one or more autocollimators. One measures encoder counts when the station is in start position, or when one or both shafts are 180 degrees from start position, and notes changes in the autocollimated return spot. The measurements are complicated by the fact that the scan mirror is in a two axis gimbal mount, when mounted in the scanning station, and one has to adjust four angles properly to set up the measurements. The encoder offset counts N_{ao} and N_{eo} must be determined independently of the first order angle corrections, which are invalid when E is near zero. To do the laboratory measurements of N_{ao} and N_{eo}, one must be able to set the azimuth rotor's axis coincident with or parallel to a known $\widetilde{z}$-axis, also set the plane of the azimuth and elevation rotor axes coincident with or parallel to a known $(\widetilde{x}\,,\,\widetilde{z})$-plane, and set the scan mirror to both contain the elevation axis and lie perpendicular to the $(\widetilde{x}\,.\,\widetilde{z})$-plane. Careful attention to the technique of measurement is needed. Offset error in N_{ao} or N_{eo} will produce a constant shift in the pointing azimuth or elevation angle, respectively, which may compare in size with the computed counts corrections (4.13). We do not discuss any details of the initial scan mirror and angle encoder alignment and setting measurements in this note. This is a topic for a separate, extended treatment. But it is important to observe that errors in setting the scan mirror or shaft angle encoders can produce pointing errors large as or larger than the error corrected in this note.

The corrections (4.13) are easily entered into the software controlling the scanning unit's pointing commands. The angle ψ is available for each scanning unit, as a single measured number. The angles Θ and Φ are provided to command the scanner. Instead of commanding the scanner to rotate its shafts to generate the count signals $N_a(\Theta\,,\,\Phi\,)$ and $N_e(\Theta\,,\,\Phi\,)$ specified by equations (A3.4) one commands the scanner to move its shafts to give the count signals $N_a(\Theta\,,\,\Phi\,,\,\psi)$ and $N_a(\Theta\,,\,\Phi\,,\,\psi)$ given by equations (4.12), a modification of two lines of code.

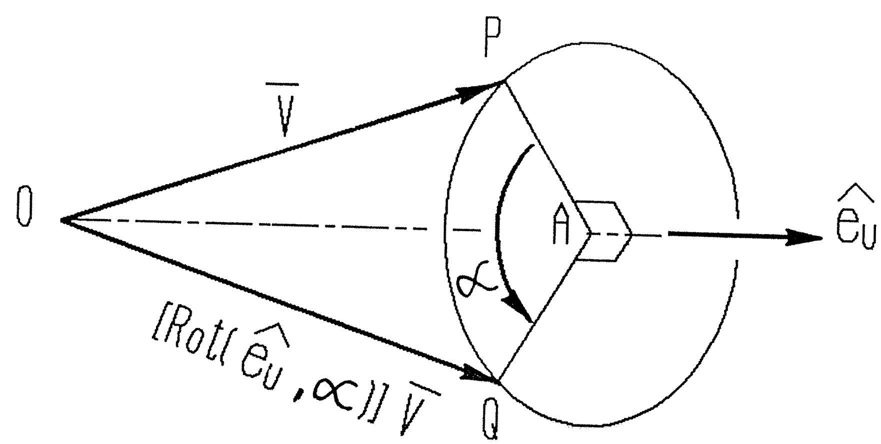

Figure 3.

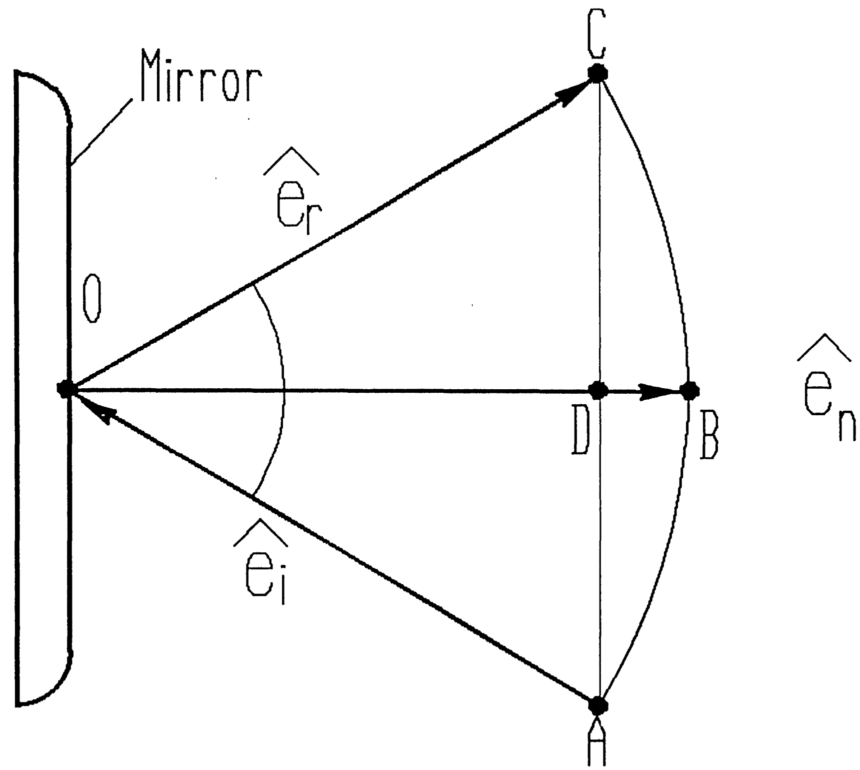

Figure 2.

6. Appendix I. Ray Reflection From A Plane Mirror.

Here we compute the unit vector along the reflected ray when an incident ray is reflected from a plane mirror. We assume that $\widehat{e_n}$ is the unit outward normal to the plane mirror surface and $\widehat{e_i}$ is the unit vector along the incident ray direction.

We call $\widehat{e_r}$ the unit reflection ray vector. Referring to Fig. 2,

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{OA} + 2(\overrightarrow{AD}) . \qquad \text{Also } \widehat{e_r} = \overrightarrow{OC} .$$

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} .$$

$$\overrightarrow{OD} = [(\widehat{e_n}) \cdot (-\widehat{e_i})]\widehat{e_n} , \quad \overrightarrow{AO} = \widehat{e_i} .$$

These give:

$$\text{(A1.1)} \qquad \widehat{e_r} = \widehat{e_i} - 2(\widehat{e_n} \cdot \widehat{e_i})\widehat{e_n} .$$

In the present memo we assume that, always, $\widehat{e_i} = -\widehat{e_z}$, giving

$$\text{(A1.2)} \qquad \widehat{e_r} = \widehat{-e_z} + 2(\widehat{e_n} \cdot \widehat{e_z})\widehat{e_n} ,$$ which is our desired result.

7. Appendix II. Rigid Body Rotation With One Body Point Fixed.

We use the following result from vector analysis in computing the motion of the normal to the scan mirror and elevation axis.

We are given a rigid body with one point, O, fixed and a rotation of the body about an axis OA , through O, by an angle α . (Figure 3).

Let $\widehat{e_u}$ be a unit vector along the rotation axis. Let P be a point in the rigid body.

The point P will be moved to a point Q. The vector $\overrightarrow{OP} \equiv \overrightarrow{V}$ will be transformed by the rotation ($\widehat{e_u}$, α) to the vector $\overrightarrow{OQ} \equiv [Rot(\widehat{e_u} , \alpha)]\overrightarrow{V}$.

The following result holds:

$$\text{(A2.1)} \qquad [Rot(\widehat{e_u}\,,\alpha\,)]\,\overrightarrow{V} = (\cos\alpha)\,\overrightarrow{V} + (1-\cos\alpha)(\widehat{e_u}\cdot\overrightarrow{V}\,)\widehat{e_u} + (\sin\alpha)(\widehat{e_u}\times\overrightarrow{V}\,)\,.$$

This result is given in H. Goldstein, Classical Mechanics.

8. Appendix III. Target Polar Coordinates.

On occasion it will be convenient to specify the location of a target point in spherical coordinates. In the $(\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ coordinate system let the coordinates of a scan target point T be $(\widetilde{x_t}\,,\,\widetilde{y_t}\,,\,\widetilde{z_t})$, where $\tilde{x}_t^2+\tilde{y}_t^2+\tilde{z}_t^2=R^2$. We express the coordinates of T in spherical polar coordinates R , Θ , and Φ :

(A3.1.1) $\widetilde{x_t}=R(\sin\Phi)(\cos\Theta)$

(A3.1.2) $\widetilde{y_t}=R(\sin\Phi)(\sin\Theta)$

(A3.1.3) $\widetilde{z_t}=R(\cos\Phi)$

If the azimuth scan rotor were machined perfectly, so that $\psi=0$, then the scan angles of the target, A_t and E_t in this case, would be determined, from (2.21)-(2.24), by the relations:

(A3.2.1) $\widetilde{x_t}=R(\sin 2E_t)(\cos A_t)$

(A3.2.2) $\widetilde{y_t}=R(\sin 2E_t)(-\cos A_t)$

(A3.2.3) $\widetilde{z_t}=R(\cos 2E_t)$.

In the case where the azimuth and elevation scan axes are accurately perpendicular (so that $\psi=0$), the target scan angles are easily found in terms of the target point spherical coordinates, by comparing equations (A3.1) and (A3.2):

(A3.3.1) $A_t=\Theta-\dfrac{\pi}{2}$,

(A3.3.2) $E_t=-\dfrac{\Phi}{2}$.

The shaft encoder readings can then, when $\psi=0$, be set in terms of the polar coordinates Θ and Φ of the target point T to be:

(A3.4a) $N_a(\Theta\,,\,\Phi\,) = N_{ao} + \dfrac{10^5\Theta}{2\pi}$,

(A3.4b) $N_e(\Theta\,,\,\Phi\,) = N_{eo} + \dfrac{10^5\Phi}{4\pi}$, from (1.1a) and (1.1b) .

The scan ranges are then

(A3.5a) $\dfrac{\pi}{2} \leq \Theta \leq \dfrac{3\pi}{2}$,

(A3.5b) $0 \leq \Phi \leq \pi$.

We see that the azimuth shaft's rotation angle A_t and encoder count setting N_a depend only on the polar coordinate Θ , while the elevation shaft's rotation angle E_t and encoder count setting N_e depend only on the polar coordinate Φ , when the scan axes are perpendicular.

This separability is not available when the scan axes are non-perpendicular. For a given target point $T = (\widetilde{x_t}\,,\,\widetilde{y}_t\,,\,\widetilde{z}_t\,)$, whose spherical polar coordinates R , Θ , and Φ are given by (A3.1.1)-(A3.1.3), when $\psi \neq 0$, the shaft rotation angles and encoder count settings become functions of all three variables Θ , Φ , and ψ . We let the new shaft rotation angles, in azimuth and elevation respectively, required to point the laser beam from the scan center point to point T be :

(A3.6) $A_p = A_p(T\,;\psi) = A_p(\Theta,\,\Phi\,,\psi\,)$,

(A3.7) $E_p = E_p(T\,;\psi) = E_p(\Theta,\,\Phi\,,\psi\,)$,

The angles A_p and E_p are the corrected shaft angles to be used for pointing, when the scan axes are non-perpendicular. We introduce angle corrections from the
ideal settings:

(A3.8) $A_p = A_t + \Delta A = \Theta - \dfrac{\pi}{2} + \Delta A(\Theta,\,\Phi\,,\psi\,)$,

(A3.9) $E_p = E_t + \Delta E = -\dfrac{\Phi}{2} + \Delta E(\Theta,\,\Phi\,,\psi\,)$.

The corrected shaft angles are computed in Section 3, and approximated to first order in ψ in Section 4. The corrected encoder counts then become:

$$\text{(A3.10a)} \quad N_a(\Theta\,,\,\Phi\,,\,\psi) = N_{ao} + \frac{10^5(\Theta + \Delta A(\Theta,\,\Phi\,,\psi\,))}{2\pi} \quad ,$$

$$\text{(A3.10b)} \quad N_e(\Theta\,,\,\Phi\,,\psi) = N_{eo} + \frac{10^5(\Phi - 2\Delta E(\Theta,\,\Phi\,,\psi\,))}{4\pi} \quad .$$

The count increments to be added to the encoder settings when $\psi \neq 0$ are:

$$\text{(A3.11a)} \quad \Delta N_a = \frac{10^5 \Delta A(\Theta,\,\Phi\,,\psi\,)}{2\pi} \quad ,$$

$$\text{(A3.11b)} \quad \Delta N_e = \frac{-10^5 \Delta E(\Theta,\,\Phi\,,\psi\,)}{2\pi} \quad .$$