GBT MEMO 212

Rangefinder Target Visibility Codes

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See L0633 — "Rangefinder target visibility codes — background, program listing, and software" for additional materials.

Abstract

Codes for computing rangefinder target visibility have been developed. For arbitrary GBT azimuth and elevation, given a list of potential rangefinder targets, the visibility of each target for ranging by each of the rangefinders is determined. An output list or file of rangefinder-target pairs having mutual visibility is generated for each antenna orientation.

These codes have three uses. First, they can be used to model metrology scan scenarios to determine whether sufficient lines of sight are available for target location by trilateration. Second, they can be used to model and improve pointing of the optical axis for targets mounted on the GBT elevation structure, to achieve good availability of sight lines for metrology. Third they can be used as an active pre-processing filter for the metrology scan scheduler; potential scans to unavailable targets are eliminated by preprocessing by these codes.

0.1. Introduction.

The GBT rangefinder metrology system will be used to measure and characterize the telescope antenna. Both static and dynamic scans of the telescope will be made to effect these measurements. Advance knowledge of the mutual visibility between system rangefinders and retroreflector targets is needed for several different reasons.

During dynamical ranging of the antenna limited scan time is available to carry out measurements. Abortive scans, for which no mutual visibility between rangefinder and target exists and for which no range data can be acquired, should not be attempted. One can use the visibility codes to prepare tables of visibility of each target-rangefinder pair over the azimuth and elevation range of the telescope, at (typically) one degree angle increments. These tables can be made part of the rangefinder system's scan scheduler. The scheduler will look up the tables to eliminate abortive scans when arranging scans.

When using the rangefinder system for location of a retroreflector target, one should know in advance which rangefinders have the possibility of measuring range to that target. In particular, if the target location is to be found by trilateration, when the GBT is at a given azimuth and elevation, one should know if enough range paths are available to achieve the trilateration. At minimum, three paths are needed; four are needed if target coordinate error estimates are needed. The visibility codes allow one to determine which rangefinders can be used, and if they are sufficient in number to achieve trilateration, at this telescope orientation.

The usefulness of a target is depends on how it is oriented when it is mounted on the telescope. The target has only a limited solid angle from which it can be illuminated and will retro reflect. The target should be mounted in orientation with respect to its local telescope environment, so that it allows trilateration over a useful and wide range of telescope orientations. The visibility codes allow one to estimate the regions of telescope Azimuth $\times$ Elevation space allowing range scans between any target-rangefinder pair. By using the visibility codes to model the accessible region of this orientation space, for any target, one can determine target mounting orientations which will produce an acceptably large view region.

0.2. Target Visibility Criteria.

If a target's location is to be measured by range trilateration, at an arbitrary antenna orientation, sufficient lines of sight between that target and the collection of rangefinders must be found to tie down the target's location. For each potential ranging scenario, one wants to know if sufficient lines of sight are available to trilaterate the target location. If so, what rangefinders should make the scan measurements?

Rangefinders and targets may not be visible to one another for either of two reasons. First, the illuminating radiation from the rangefinder may be entrant on the target at too large an angle of incidence to allow retroreflection. Second, the radiation from rangefinder to target may be blocked by the surface of a material body. In the first case, there is a limited solid angle of acceptance of each target, for incident illumination, which allows retroreflection from the target. In the second case, lines of sight between rangefinder and target may be blocked by physical obstacles. Examples of such obstacles are: the main dish surface panels, the scattering-shield on the vertical feed arm, the walls of the receiver feed room, the platform bases of the feed arm rangefinders, the underside of the horizontal feed arm.

Each target has a unique axis ray associated with it. The solid angle for acceptance of target illumination which allows retroreflection can be described with respect to this axis ray. For a cube corner prism it is the ray directed outward from the prism's reference point through the prism's exit face, having equal direction cosines withthe three corner edges. For the cat's-eye reflectors having the geometry of two joined half-balls it is the ray directed from the mutual ball center point outwards through the small half-ball's surface, and perpendicular to the half-ball junction plane of the two half-balls. For feed arm rangefinders used as targets this axis is the rangefinder local optical axis: directed from the scanning mirror's reference stationary scan point towards the center point of the photodetector diode.

The criterion for retroreflection from a cube corner prism, of BK-7 optical glass, is assumed to be the following The direction of the ray from the prism reference point to the rangefinder scan point lies within 25 degrees of the prism axis. The criterion for retroreflection from the cat's-eye ball reflector is assumed to be: the direction of the ray from the mutual ball center point to the rangefinder scan point lies within 60 degrees of the reflector's axis. For a feed arm rangefinder, used either as an illumination source or a target, the criterion for an available view line between this rangefinder and either a target or an illuminating second rangefinder is slightly more complicated, but is analytically tractable and simple.

0.3. Blockage Surface And Ray Intersection Geometry.

For our purposes, we can consider all blockage surfaces to be of two shapes. One shape is the portion of a paraboloid bounded by a rim curve which is the intersection of the paraboloid with a plane ("the bounding plane"). The second shape is a the interior of plane symmetric trapezoid surface. This includes rectangle and isosceles triangle interiors as special cases.

A paraboloid is used to model ray blockage by the main dish. Rays between ground rangefinders and main dish rim ball targets can be blocked by the dish, as can rays between feed arm rangefinders and ground benchmark targets or ground rangefinders used as targets. At low elevations, ball targets beneath the elevation bearing platforms can be blocked from some ground rangefinders by the main dish.

The bottom, roof, and sides of the receiver feed room may act as ray blocking surfaces. The feed arm laser platforms also can be blocking surfaces. These are modeled by rectangles. The vertical feed arm tip is modeled by a trapezoid, as is the horizontal feed arm.

0.3.1. Geometry of ray intersection with the main dish paraboloid.

In GBT main reflector system coordinates, the equation of the main reflector dish design paraboloid is:

$$(1.01) \qquad X_r^2 + Y_r^2 = 4f\,Z_r\,, \quad \text{where} \quad f = 60.000\,meter.$$

The rim curve of the dish (Fig. 1) is the intersection of this paraboloid with the plane

$$(1.02) \qquad Z_r = (\frac{1}{15}) + (Y_r - 4)\left(\frac{45\frac{1}{15} - \frac{1}{15}}{104 - 4}\right) = 0.45Y_r - 1.7333\ (meters).$$

The blockage surface is the compact portion of the paraboloid bounded by that rim curve. That is, the main dish surface is the set of points satisfying both (1.01) and the inequality: $Z_r \leq 0.45Y_r - 1.7333\ (meters)$.

We assume that we are given two points in space: S and T with main reflector system coordinates (S_{xr}, S_{yr}, S_{zr}) and (T_{xr}, T_{yr}, T_{zr}) respectively. These points represent the scan reference point of a rangefinder and the fiducial reference point of a retroreflector target, respectively. The line connecting S and T is represented parametrically by the equation:

$$(1.03) \qquad P(t) = (P_{xr}, P_{yr}, P_{zr}) =$$

$$((1-t)S_{xr} + (t)T_{xr}\,,\ (1-t)S_{yr} + (t)T_{yr}\,,\ (1-t)S_{zr} + (t)T_{zr})\,.$$

where $P(t)$ is a general point on the line ST and t is a real parameter generating points on ST. Point lies between S and T if and only if $0 < t < 1$. One can also represent the points as displacement vectors from the origin point of the ground reference coordinate frame; the equation of the line is then:

$$(1.04) \qquad \overrightarrow{P(t)} = (1-t)\,\overrightarrow{S} + (t)\,\overrightarrow{T}\,.$$

A *line* ST may have several distinct types of intersection with the paraboloid, which is an unbounded analytical surface. These are illustrated in Fig. 2:

It may not intersect the paraboloid at all: (1), (5).

It may intersect the paraboloid external to the main reflector bounded surface patch: (2).

It may have intersection points with the main reflector bounded surface patch, but not between S and T: *(3)*.

It may have intersection points on the bounded surface patch and also lying between S and T: *(4)*.

Only in the last case does the paraboloidal surface patch block the ray from T to S. To compute intersection points of line ST with the paraboloid substitute (1.03) into (1.01):

(1.05)

$$((1-t)S_{xr} + (t)T_{xr})^2 + ((1-t)S_{yr} + (t)T_{yr})^2 = (4f)((1-t)S_{zr} + (t)T_{zr})\,.$$

After some manipulation this gives:

$$(1.06) \qquad (aa)t^2 - 2(bb)t + cc = 0 \ , \qquad \text{where}$$

$$aa = (S_{xr} - T_{xr})^2 + (S_{yr} - T_{yr})^2 \ ,$$

$$bb = (S_{xr})(S_{xr} - T_{xr}) + (S)(S - T) + (2f)(T_{zr} - S_{zr}) \ ,$$

$$cc = (S_{xr})^2 + (S_{yr})^2 - 4fS_{zr} \, .$$

Calling $dd = (bb)^2 - (aa)(cc)$, the two parameter values corresponding to intersection points are

$$(1.06) \qquad t_1 = \frac{bb - \sqrt{dd}}{aa} \quad , \quad t_2 = \frac{bb + \sqrt{dd}}{aa} \, .$$

If $dd < 0$ there are no real intersection points and no blockage. When $dd \geq 0$ there will be blockage if and only if both

$((0 < t < 1)$ *and* $(P_{zr}(t) \leq 0.45Y_r - 1.7333))$ for either $t = t_1$ or $t = t_2$.

0.3.2. Geometry of ray intersection with a symmetric trapezoid.

We examine here the intersection geometry of the line ST with a plane symmetric trapezoid. We assume that a plane trapezoid is given in Euclidean 3-space. Its vertices in cyclic order are points $P1$, $P2$, $P3$, $P4$. Its base is $P1P2$ and side $P4P3$ is parallel to the base. Base angles $\angle P4P1P2$ and $\angle P3P2P1$ are assumed equal to one another (Fig.3).

Referring to Fig. 3, the trapezoid vertices are related by

$$(2.01) \qquad \overrightarrow{P3} = \overrightarrow{P4} + \left(\overrightarrow{P1} - \overrightarrow{P2} - 2\overrightarrow{u}\right) \qquad \text{where}$$

$$(2.02) \qquad \overrightarrow{u} = \left(\overrightarrow{P2} - \overrightarrow{P1}\right) \left\{ \frac{\left(\overrightarrow{P4} - \overrightarrow{P1}\right) \cdot \left(\overrightarrow{P2} - \overrightarrow{P1}\right)}{\left(\overrightarrow{P2} - \overrightarrow{P1}\right) \cdot \left(\overrightarrow{P2} - \overrightarrow{P1}\right)} \right\} = A \cdot \left(\overrightarrow{P2} - \overrightarrow{P1}\right)$$

since $\overrightarrow{u}$ is parallel to $\overrightarrow{P2} - \overrightarrow{P1}$. We then also get

$$(2.03)\qquad |\overrightarrow{u}| = \left|\overrightarrow{P4} - \overrightarrow{P1}\right| \cdot (\cos\theta) \qquad \text{where} \qquad \theta = \angle P4P1P2\,,$$

which is the projection of $\overrightarrow{P4} - \overrightarrow{P1}$ on $\overrightarrow{P2} - \overrightarrow{P1}$.

We use main reflector frame coordinates to specify the locations of the trapezoid vertices. We define

$$(2.04)\qquad \overrightarrow{Pi} = (P_{ixr},\, P_{iyr},\, P_{izr}) \quad (i = 1, 2, 3, 4)\,.$$

The subscript r indicates that coordinates refer to the main reflector coordinate system. Expressed in terms of coordinates:

$$(2.05)\qquad \overrightarrow{P4} - \overrightarrow{P1} = (P_{4xr} - P_{1xr}\,,\, P_{4yr} - P_{1yr}\,,\, P_{4zr} - P_{1zr})\,,$$

$$(2.06)\qquad \overrightarrow{P2} - \overrightarrow{P1} = (P_{2xr} - P_{1xr}\,,\, P_{2yr} - P_{1yr}\,,\, P_{2zr} - P_{1zr})\,,$$

$$(2.07)\qquad A = \left\{ \frac{\left(\overrightarrow{P4} - \overrightarrow{P1}\right) \cdot \left(\overrightarrow{P2} - \overrightarrow{P1}\right)}{\left(\overrightarrow{P2} - \overrightarrow{P1}\right) \cdot \left(\overrightarrow{P2} - \overrightarrow{P1}\right)} \right\} =$$

$$\frac{(P_{4xr} - P_{1xr})\,(P_{2xr} - P_{1xr}) + (P_{4yr} - P_{1yr})\,(P_{2yr} - P_{1yr}) + (P_{4zr} - P_{1zr})\,(P_{2zr} - P_{1zr})}{(P_{2xr} - P_{1xr})^2 + (P_{2yr} - P_{1yr})^2 + (P_{2zr} - P_{1zr})^2}\,,$$

$$(2.08)\qquad \overrightarrow{u} = A \cdot (P_{2xr} - P_{1xr}\,,\, P_{2yr} - P_{1yr}\,,\, P_{2zr} - P_{1zr})\,, \quad \text{and}$$

$$(2.09)\qquad \overrightarrow{P3} = \overrightarrow{P4} + (1 - 2A)\left(\overrightarrow{P2} - \overrightarrow{P1}\right).$$

Given two points in space,

$$(2.10)\qquad S = \overrightarrow{S} = (SX_r\,,\, SY_r\,,\, SZ_r) \quad \text{and} \quad T = \overrightarrow{T} = (TX_r\,,\, TY_r\,,\, TZ_r)\,,$$

the points on the line ST through these points is given by the parametric equation

$$(2.10)\qquad \overrightarrow{P(t)} = (1 - t)\,\overrightarrow{T} + (t)\,\overrightarrow{S} \quad \text{in the real parameter } t.$$

The equation of the plane through the trapezoid can be expressed as:

$$(2.11) \qquad aX_r + bY_r + cZ_r = d\,.$$

The intersection of line ST with the trapezoid plane occurs at the point on ST having the parameter value

$$(2.12) \quad t = t_1 = \frac{d - (a \cdot TX_r + b \cdot TY_r + c \cdot TZ_r)}{a \cdot (SX_r - TX_r) + b \cdot (SY_r - TY_r) + c \cdot (SZ_r - TZ_r)}\,.$$

The point $\overrightarrow{P}(t_1)$ lies between S and T if and only if $0 < t_1 < 1$. If an intersection of the ray from T to S is found with the plane of the trapezoid, one must checwhether the intersection point belongs to the trapezoid. This involves a little agebraic manipulation.

We proceed as follows. We parametrize the symmetric trapezoid (Fig. 4) using two real numerical parameters: u and v. Points on the trapezoid circumference are specified by the condition that either u or v is equal to 1, and both $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Interior points of the trapezoid are specified by the simultaneous conditions: $0 < u < 1$ and $0 < v < 1$. The plane of the trapezoid is parametrized by the equation:

$$(2.13) \quad \overrightarrow{P}(u,v) = (1-v)\left\{(1-u)\overrightarrow{P1} + (u)\overrightarrow{P4}\right\} + (v)\left\{(1-u)\overrightarrow{P2} + (u)\overrightarrow{P3}\right\}.$$

Substitute (2.09) into (2.13). After a bit of manipulation, one gets,

$$(2.14) \quad \overrightarrow{P}(u,v) = \overrightarrow{P1} \cdot (1 - v - u + 2uvA) + \overrightarrow{P2} \cdot (v - 2uvA) + \overrightarrow{P4} \cdot (u)\,.$$

To simplify the notation let

$$(2.15) \quad \mathcal{A} = 1 - u - v + 2uvA\,, \qquad \mathcal{B} = v - 2uvA\,, \qquad \mathcal{D} = u\,.$$

Here $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}$ are functions of the parameters u and v.

We find the values of the parameters u and v which correspond to the intersection point $\overrightarrow{P}(t_1)$ of the ray from T to S with the plane of the trapezoid $\overrightarrow{P1}$, $\overrightarrow{P2}$, $\overrightarrow{P3}$, $\overrightarrow{P4}$ by setting

$$(2.16) \qquad \vec{P}(t_1) = \vec{P}(u,v)\,. \qquad \text{This gives:}$$

$$(2.17) \qquad \left(\vec{P}(t_1)\right)_{xr} = P_{1xr}\cdot(1-u-v+2uvA) + P_{2xr}\cdot(v-2uvA) + P_{4xr}\cdot(u)\ ,$$

$$(2.18) \qquad \left(\vec{P}(t_1)\right)_{yr} = P_{1yr}\cdot(1-u-v+2uvA) + P_{2yr}\cdot(v-2uvA) + P_{4yr}\cdot(u)\ ,$$

$$(2.19) \qquad \left(\vec{P}(t_1)\right)_{zr} = P_{1zr}\cdot(1-u-v+2uvA) + P_{2zr}\cdot(v-2uvA) + P_{4zr}\cdot(u)\ ,$$

which are three linear equations in the unknowns $\mathcal{A}$,$\mathcal{B}$,$\mathcal{D}$.
The left sides of the above equations can be directly computed, since t_1 has been found, and the main reflector coordinates of the four corner points are input data. The equations can be solved for $\mathcal{A}$,$\mathcal{B}$ and $\mathcal{D}$, which then allows one to solve for parameters u and v in terms of the coordinates of the rangefinder's scan point, the target's reference point, and the trapezoid corner points. The algebra is straightforward, but messy.

From the defining equations (2.15) we get

$$(2.20) \qquad 1-u \;=\; (1-v-u+2uvA) + (v-2uvA) \;=\; \mathcal{A}+\mathcal{B}\ , \quad \mathcal{D} = u\,.$$

These give the identity

$$(2.21) \qquad \mathcal{A}+\mathcal{B}+\mathcal{D} \equiv 1\ .$$

In order for ray ST to hit the trapezoid,

$$(2.22) \qquad \text{Intersection} \longleftrightarrow\ 0 \le\ u\,,\, v\ \le\ 1\ .$$

Substituting $\mathcal{D} = u$ into the second equation of (2.15) gives the parameter values

$$(2.23) \qquad v = \frac{\mathcal{B}}{1-2A\mathcal{D}} \quad \text{and} \quad u = \mathcal{D} \quad \text{with} \quad 0 \le u\,,\, v \le 1\,.$$

We now give the detailed algebra to compute the numerical values of unknowns a, b, c, d of (2.11) and $\mathcal{A}$,$\mathcal{B}$,$\mathcal{D}$ of (2.17)-(2.19).

The equation of the plane (2.11) through points $P1$, $P2$, $P4$ may also be written in terms of the coordinates of three points through which it passes:

$$
(2.24) \qquad \begin{vmatrix} X_r - P_{1xr} & Y_r - P_{1yr} & Z_r - P_{1zr} \\ P_{2xr} - P_{1xr} & P_{2yr} - P_{1yr} & P_{2zr} - P_{1zr} \\ P_{4xr} - P_{1xr} & P_{4yr} - P_{1yr} & P_{4zr} - P_{1zr} \end{vmatrix} = 0\,.
$$

Expanding this determinant and comparing the coefficients to the constants appearing in (2.11) gives

(2.25)

$$a = (P_{2yr} - P_{1yr}) \cdot (P_{4zr} - P_{1zr}) - (P_{2zr} - P_{1zr}) \cdot (P_{4yr} - P_{1yr})$$

$$b = (P_{2zr} - P_{1zr}) \cdot (P_{4xr} - P_{1xr}) - (P_{2xr} - P_{1xr}) \cdot (P_{4zr} - P_{1zr})$$

$$c = (P_{2xr} - P_{1xr}) \cdot (P_{4yr} - P_{1yr}) - (P_{2yr} - P_{1yr}) \cdot (P_{4xr} - P_{1cr})$$

$$d = a \cdot P_{1xr} + b \cdot P_{1yr} + c \cdot P_{1zr}\,.$$

We may rewrite equations (2.17)-(2.19) as

(2.26)

$$P_{t1xr} = P_{1xr} \cdot \mathcal{A} + P_{2xr} \cdot \mathcal{B} + P_{4xr} \cdot \mathcal{D}$$

$$P_{t1yr} = P_{1yr} \cdot \mathcal{A} + P_{2yr} \cdot \mathcal{B} + P_{4yr} \cdot \mathcal{D}$$

$$P_{t1zr} = P_{1zr} \cdot \mathcal{A} + P_{2zr} \cdot \mathcal{B} + P_{4zr} \cdot \mathcal{D}$$

These are solved by determinants. Set

$$
(2.27) \qquad \text{Delta1} = \begin{vmatrix} P_{t1xr} & P_{2xr} & P_{4xr} \\ P_{t1yr} & P_{2yr} & P_{4yr} \\ P_{t1zr} & P_{2zr} & P_{4zr} \end{vmatrix}\,,
$$

$$
(2.28) \qquad \text{Delta2} = \begin{vmatrix} P_{1xr} & P_{t1xr} & P_{4xr} \\ P_{1yr} & P_{t1yr} & P_{4yr} \\ P_{1zr} & P_{t1zr} & P_{4zr} \end{vmatrix}\,,
$$

$$(2.29)\qquad \text{Delta3} = \begin{vmatrix} P_{1xr} & P_{2xr} & P_{t1xr} \\ P_{1yr} & P_{2yr} & P_{t1yr} \\ P_{1zr} & P_{2zr} & P_{t1zr} \end{vmatrix},$$

$$(2.30)\qquad \text{Delta4} = \begin{vmatrix} P_{1xr} & P_{2xr} & P_{4xr} \\ P_{1yr} & P_{2yr} & P_{4yr} \\ P_{1zr} & P_{2zr} & P_{4zr} \end{vmatrix}.$$

Then

$$(2.31)\quad \mathcal{A} = \frac{\text{Delta1}}{\text{Delta4}}, \quad \mathcal{B} = \frac{\text{Delta2}}{\text{Delta4}}, \quad \mathcal{D} = \frac{\text{Delta3}}{\text{Delta4}}.$$

Given these coefficients, the parameters u and v can be computed and examined to see if they lie in the unit interval.

0.4. Scan Visibility Of Feed Arm Rangefinders.

The viewing solid angle of a feed arm rangefinder is, in part, limited by the rangefinder itself. With reference to Fig. 5, the line of sight from the rangefinder's scan reference point S is limited by the extended base of the rangefinder's platform base. It can not penetrate the platform base. The line of sight is also constrained to lie within an angle $14.44^\circ < \mathit{View\ Angle} < (14.44^\circ + 91.84^\circ)$ of the rangefinder optical axis, which is the local platform $\tilde{z}$-axis. These constraints should be included in the visibility computation code. This is accomplished by making each platform base a potential blocking rectangle and setting maximum and minimum acceptable bounds for the direction cosine of the line of sight with respect to the $\tilde{z}$-axis.

Neglecting structural gravity deformations of the telescope, which should be acceptable for the visibility computations, the local rangefinder platform coordinate frame unit vectors are related to those of the main reflector coordinate frame by a known rotation:

$$(3.01)\qquad \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix}_S = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}_S \cdot \begin{bmatrix} \widehat{X}_r \\ \widehat{Y}_r \\ \widehat{Z}_r \end{bmatrix}$$

The vector of unit length directed from a feed arm rangefinder's scan reference point S to a target reference point T can be expressed as

$$(3.02) \qquad \widehat{ST} = (-1) \cdot \left\{ \alpha_r \widehat{X}_r + \beta_r \widehat{Y}_r + \gamma_r \widehat{Z}_r \right\} \quad \text{where}$$

$$(3.03) \quad \alpha_r(S,T) = \frac{S_{xr} - T_{xr}}{|ST|}\ , \quad \beta_r(S,T) = \frac{S_{yr} - T_{yr}}{|ST|}\ , \quad \gamma_r(S,T) = \frac{S_{zr} - T_{zr}}{|ST|}\ .$$

These give,

$$(3.04) \quad \cos(\text{View Angle}) = \widehat{ST} \cdot \tilde{z} = (-1)\left\{ A_{31} \cdot \alpha_r + A_{32} \cdot \beta_r + A_{33} \cdot \gamma_r \right\}\ ,$$

which is the cosine of the angle "View Angle," between the view line from S to T and the optic axis, $\tilde{z}$, of the rangefinder platform. The matrix coefficients A_{ij} are constants which will be measured and provided for each rangefinder platform. Referring to Fig.5, we see, for sight line ST, if that

$$(3.05a) \qquad \cos(\text{View Angle}) \geq \cos(14.44^\circ)) \longrightarrow \text{Blockage} \quad \text{and}$$

$$(3.05b) \qquad \cos(\text{View Angle}) \leq \cos(14.44^\circ + 91.84^\circ) \longrightarrow \text{Blockage.}$$

In terms of rangefinder local coordinates and unit local frame vectors, the corner points of the rangefinder platform ray-blocking rectangle are, (cf Fig. 5):

(3.06)

$$\begin{aligned}
\overrightarrow{P1} &= \overrightarrow{S} - (\frac{W}{2})\tilde{x} + (-a)\tilde{y} + (-L_1)\tilde{z} & a &= 0.1214\,m \\
\overrightarrow{P2} &= \overrightarrow{S} - (\frac{W}{2})\tilde{x} + (-a)\tilde{y} + (L - L_1)\tilde{z} & L &= 0.9652\,m \\
\overrightarrow{P3} &= \overrightarrow{S} + (\frac{W}{2})\tilde{x} + (-a)\tilde{y} + (-L_1)\tilde{z} & W &= 0.4206\,m \\
\overrightarrow{P4} &= \overrightarrow{S} + (\frac{W}{2})\tilde{x} + (-a)\tilde{y} + (L - L_1)\tilde{z} & L_1 &= 0.2350\,m\,.
\end{aligned}$$

Use (3.01) to get the corner point coordinates in the main reflector coordinate system.

(3.07)

$$P_{1xr} = S_{xr} - (\frac{W}{2})A_{11} + (-a)A_{21} + (-L_1)A_{31} \ , \quad P_{2xr} = P_{1xr} + (L)A_{31}$$

$$P_{1yr} = S_{yr} - (\frac{W}{2})A_{12} + (-a)A_{22} + (-L_1)A_{32} \ , \quad P_{2yr} = P_{1yr} + (L)A_{32}$$

$$P_{1zr} = S_{xr} - (\frac{W}{2})A_{13} + (-a)A_{23} + (-L_1)A_{33} \ , \quad P_{2zr} = P_{1zr} + (L)A_{33}$$

$$P_{3xr} = P_{1xr} + (W)A_{11} + (L)A_{31} \ , \qquad P_{4xr} = P_{1xr} + (W)A_{11}$$

$$P_{3yr} = P_{1yr} + (W)A_{12} + (L)A_{32} \ , \qquad P_{4xr} = P_{1xr} + (W)A_{12}$$

$$P_{3xr} = P_{1xr} + (W)A_{13} + (L)A_{33} \ , \qquad P_{4xr} = P_{1xr} + (W)A_{13} \ .$$

0.5. Structure Of The Visibility Codes.

The visibility codes are written in C++ language. A header code: VIEWLINE.H contains class definitions for rangefinder scan points and retrotarget reference points (ScanPt and TargetPt respectively). This file also contains the vertex points for the blocking tapezoid surfaces, in the main reflector coordinate system, and also contains several telescope dimension constants.

An input data file: VIEWFIDS.DAT contains coordinates of the rangefinder scan points and target fiducial points and optical axis direction cosines. The input data is given in the ground coordinate system for ground-based rangefinders and benchmark targets. The data is given in alidade coordinates for alidade-based targets and is givenin main reflector system coordinates for elevation structure mounted rangefinders and targets.

The main computational file is VIEWLINE.CPP. This code reads data input from VIEWFIDS.DAT, performs the geometric computations to judge mutual visiblity of rangefinders and targets and generates an output file: VIEWLINE.DAT which is a list of mutually visible rangefinder-target pairs At present the code is configured to request an antenna azimuth and an elevation. by command line input from a monitor keyboard. (Subsequent versions of the code will be configured to generate data base tables). An output check code VIEWLINE.PRM is also generated as an aid to confirming the validity of the computations. This code prints out the input coordinates and their conversions to other coordinate systems, and also prints out geometric coordinate transformation matrices as checks on the computations.

Figure 1. The Main Reflector Surface And Its Rim Boundary Plane

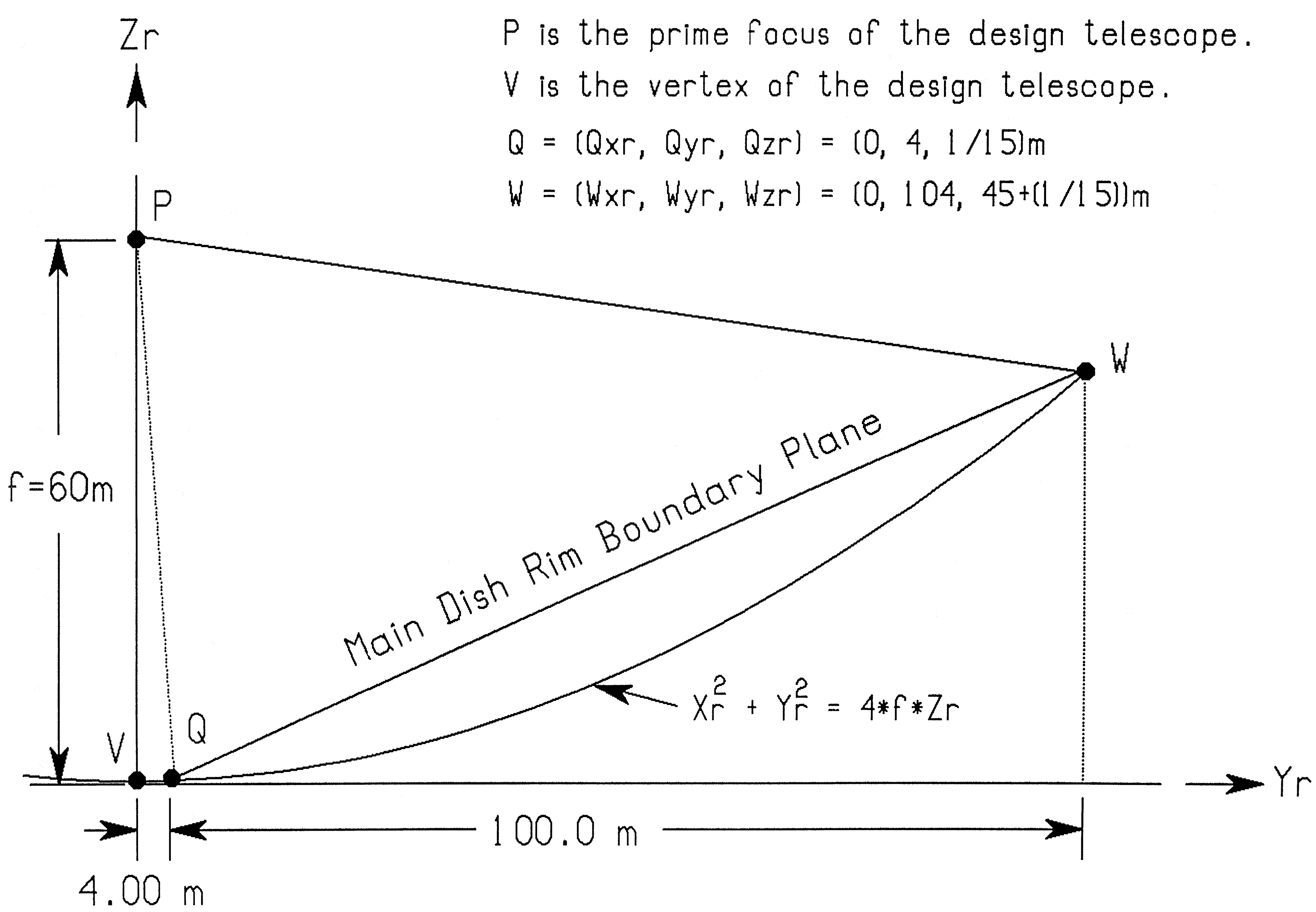


Figure 2. Possible Ray Blockage Scenarios.

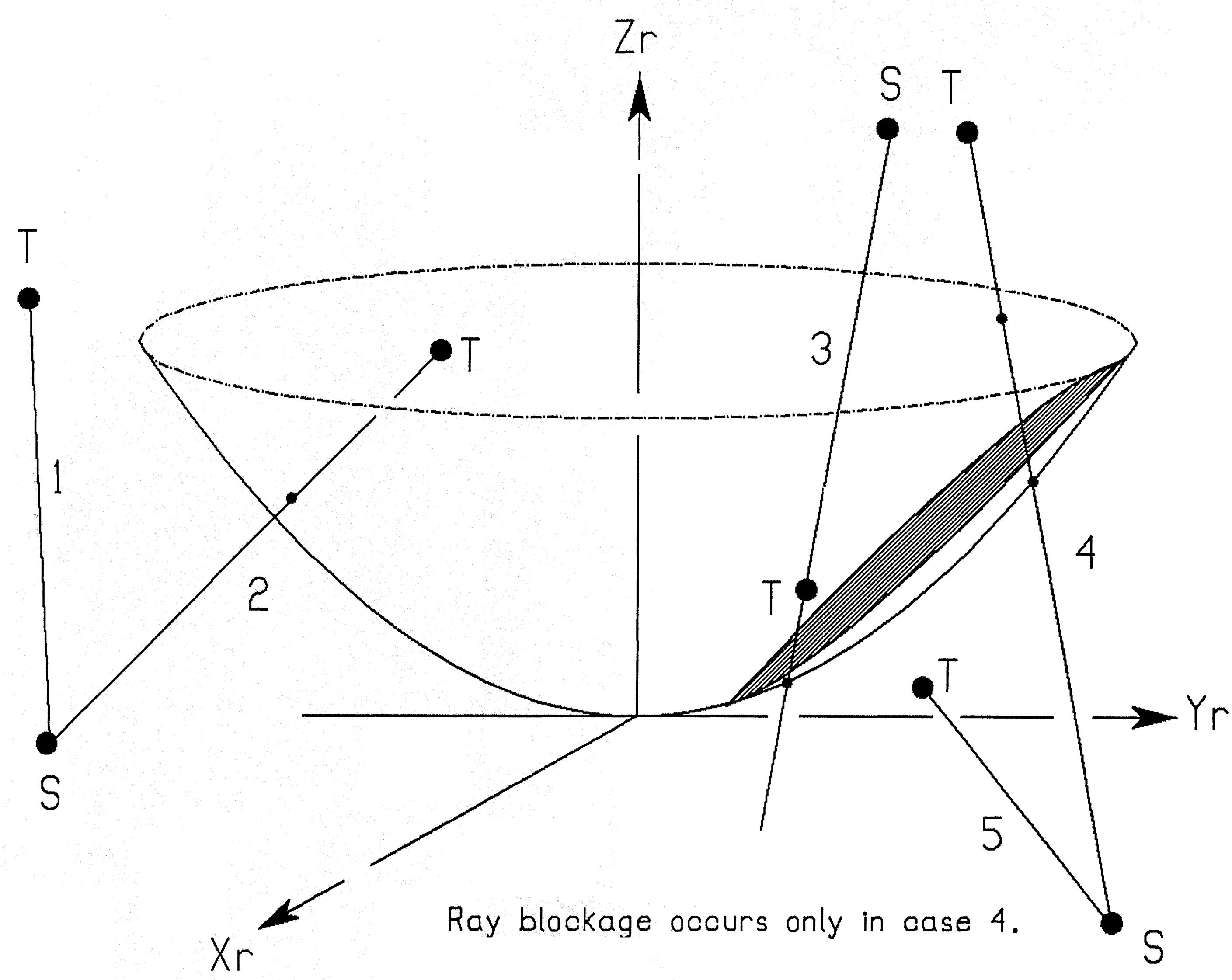


Figure 3. Geometry Of The Symmetric Trapezoid.

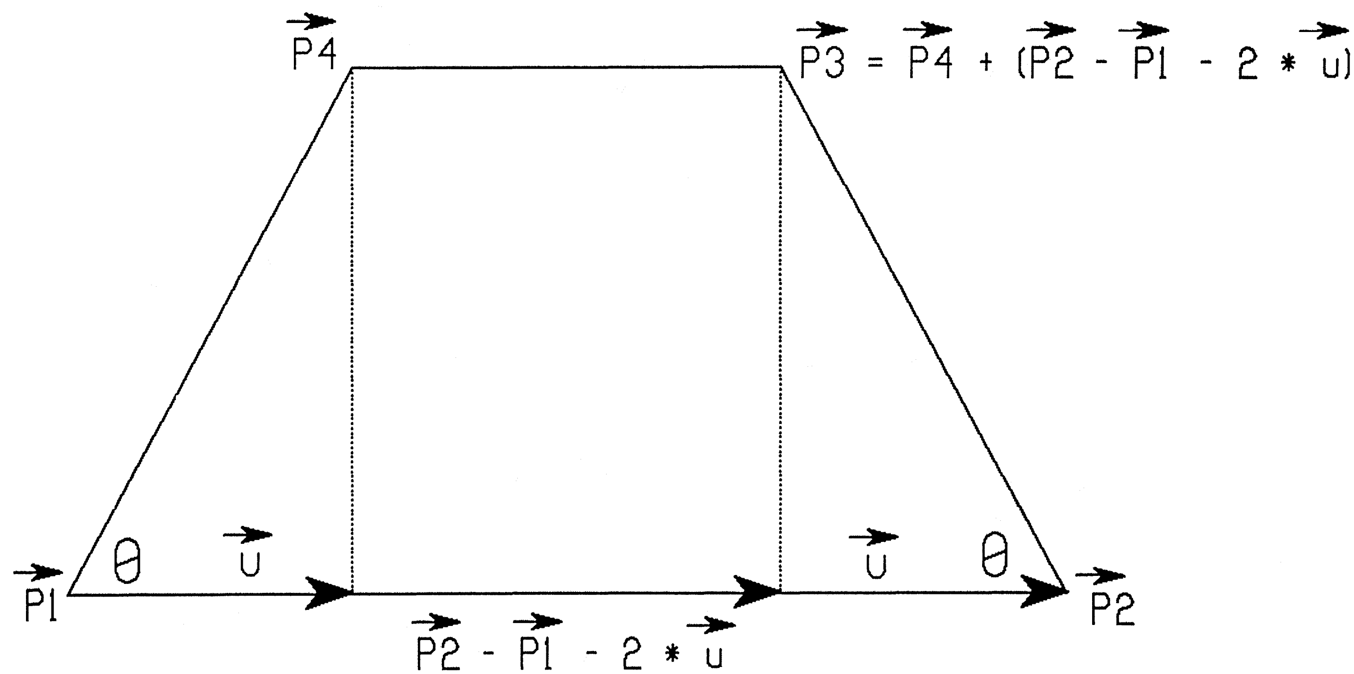
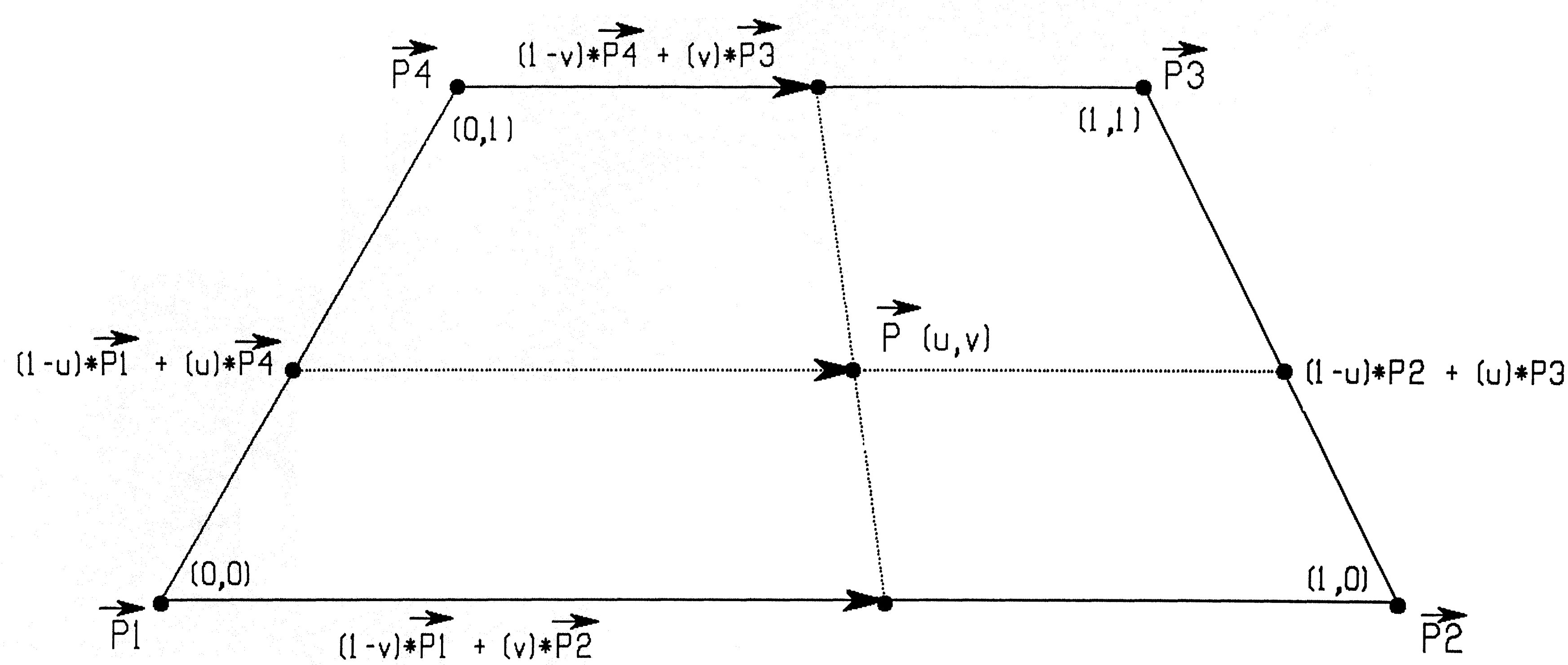


Figure 4. Parametric Representation For The Symmetric Trapezoid.



$$\vec{P}(u,v) = (1-v)*[\,(1-u)*\vec{P1} + (u)*\vec{P4}\,] + (v)*[\,(1-u)*\vec{P2} + (u)*\vec{P3}\,]$$

$$\vec{P}(u,v) = (1-v)*(1-u)*\vec{P1} + v*(1-u)*\vec{P2} + v*u*\vec{P3} + (1-v)*u*\vec{P4}$$

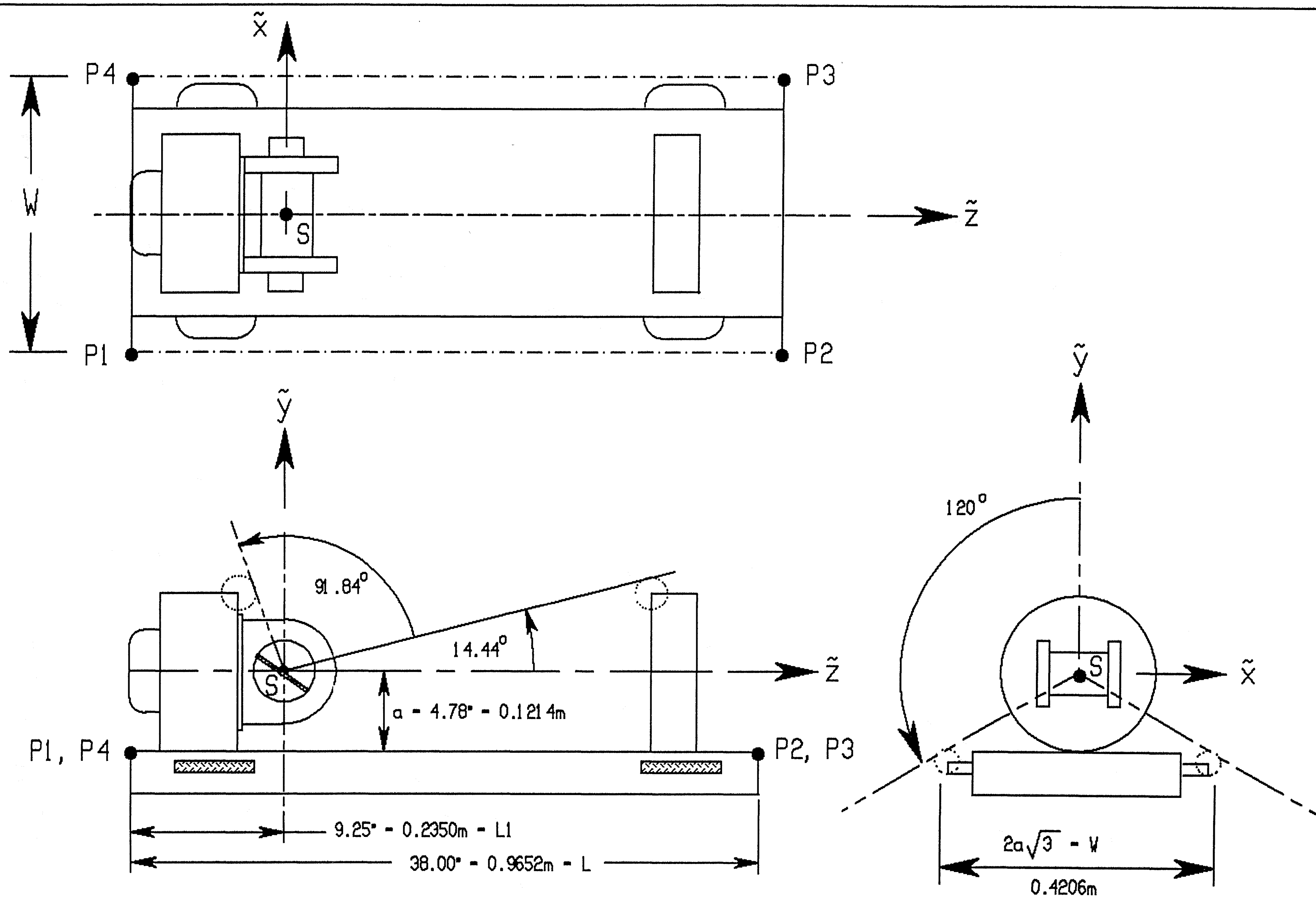


Figure 5. The Feed Arm Rangefinder Blocking Rectangle And Acceptance Cone Geometry.