GBT MEMO 213

Phase Center Locations For GBT Gregorian Feeds

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Abstract

This memo tabulates measured locations for the phase center points of GBT Gregorian feeds on their flanges. Transformations are given to generate receiver house system coordinates of these points when the flanges are mounted onto the receiver house turret and are at Gregorian focus position. The variation of position of each phase center point upon its receiver operating frequency is given, where this information is available.

Coordinates for the receiver house position and orientation and the receiver house Gregorian focus position, as determined by the Contractor's alignment survey at rigging elevation are also given. in the main reflector coordinate system determined by the alignment survey

A comparison is made of the locations of the receiver house coordinate reference frame and Gregorian focus point as found by two independent surveys: the Contractor's alignment survey, and a previous NRAO survey which used optical plummet and micrometer translation stage measurements to generate the receiver house reference frame. The two surveys give good agreement with one another, and indicate that the receiver house has been erected on the antenna close to its ideal design position.

Introduction:

This memo tabulates the locations of receiver feed phase center points relative to their mount flange geometry, for GBT feeds which mount on the receiver house roof. On the telescope, a phase center point's position in space can be described by its location relative to the *design telescope's* Gregorian focal plane and focal point. To specify this infromation, three types of data must be supplied. First, the position of the feed's phase center point must be specified with respect to the geometry of the flange to which it mounts. Second, locantion and orientation of the center of that flange, when on the receiver room turret, should be available in terms of its coordinates relative to a local receiver house frame of reference. Third the location of the receiver house reference frame must be determined relative to the main reflector reference frame of the telescope when the tipping structure sits at the telescope's rigging elevation angle. The receiver house location depends of course on the elevation angle of the telescope and will have to be determined in general, if not by direct survey or ranging measurements, by use of a mathematical model, such as the telescope's Finite Element Model. In this note we address all but the last of these tasks.

Towards this end this we review and analyze COMSAT Corporation report 121960 [Gurney-1]. This report describes the contractor's alignment procedure and results of aligning the GBT optics at the reference rigging elevation. We also calculate the as-erected orientation, at rigging elevation, of the receiver house, referenced to the GBT main reflector coordinate system. The main reflector coordinate system was determined by the contractor's analysis of photogrammetry and total station measurements of the main dish surface. We will check out the Contractor's measured values of the main reflector coordinates of the receiver house's Gregorian focus point.

Given the house's orientation and Gregorian focus point coordinates at rigging elevation and given the feed phase center coordinates relative to the receiver house reference frame, one can then calculate the phase center's main reflector system coordinates at rigging elevation. Equations are provided in Appendix A to do this calculation. At arbitrary telescope elevation one uses the telescope's Finite Element Model to correct the calculations.

Two independent surveys of the receiver house geometry were made when the receiver house was at ground level. An NRAO survey [Goldman-2] was made using a precision optical plummet mounted on a 2-dimensional micrometer translation stage to measure the turret flange center positions relative to the house structure. Flange template centers were located relative to the roof hard casters which are the primary rigidly-located structural elements on the house. The reproducibility of the flange center locations generated when the house detent pin was engaged to lock the turret was found to be $\simeq \pm 0.44$ mm; which may be assumed to be the accuracy of positioning the feeds. The positions of the flange template centers relative to one another were also measured to 0.05 mm, using the plummet and translation stages. These measurement results appear in Table 1.1 .

During this survey, bushings to hold 4 roof survey targets (FT1$\cdots$FT4) were aligned into position and welded to the receiver house roof. The lines of sight from FT2 to FT1 and from FT3 to FT4 respectively, when projected to the common join plane of the roof flanges (which is the Gregorian focal plane of the receiver house structure) intersect at the Gregorian focal point of the receiver house, at the center point of the lower surface plane of the template for flange N7. The projection onto the common flange plane of the line from FT2 to FT1 defines the $\widehat{X}_{hg}$ unit basis vector of the receiver house reference frame. The perpendicular to $\widehat{X}_{hg}$ in the common join plane, towards the receiver room door, defines the $\widehat{Z}_{hg}$ unit basis vector of this frame, and their common perpendicular, outwards from the turret. defines the $\widehat{Y}_{hg}$ unit basis vector of this frame. (Figure 4). The origin point of this frame is nominally at the turret axis and lies in the common flange join plane. The Gregorian focal point defined by the N7 flange is defined to have coordinates $\widehat{X}_{hg} \equiv 56$ inches, $\widehat{Y}_{hg} \equiv 0$ inches, $\widehat{Z}_{hg} \equiv 0$ inches.

The $(\widehat{X}_{hg}, \widehat{Y}_{hg}, \widehat{Z}_{hg})$ receiver house reference frame can then be located by observing survey targets which can be placed in the bushings welded to the house roof. After the feeds are mounted onto the house turret we can supply the positions of the individual feed phase centers with respect to the receiver house coordinate system (whose coordinate axes will lie parallel to the flange-frame's basis vectors when the flange is at Gregorian focus position) and whose origin is well-defined with respect to a reference point fixed with respect to the roof structure and serves as a nominal Gregorian focus point attached to the receiver house. In the NRAO receiver house survey, the point assigned to be the nominal "Gregorian focus point attached to the receiver house" was the center of flange template N7, when pinned at the Gregorian focus position. (Flange N7 has not yet been assigned to a receiver; in the near term it can be used for survey targeting). Receiver house coordinates of the physical centers of several other flanges were also located relative to those of the center of flange template N7, when the flanges were rotated and st respectively to Gregorian focus position [Goldman-2].

Given local *flange-frame* coordinates of the phase center point, PC_{ij}, of feed number j on turret flange Ni we can compute *receiver house* coordinates: $X_{hg}(PC_{ij})$, $Y_{hg}(PC_{ij})$, $Z_{hg}(PC_{ij})$ of this point, at rigging elevation. We can then compute the *main reflector* system coordinates of these phase centers, for rigging elevation. Transformation equations to do this, equations (A 4.2), are derived and appear in Appendix A of this memo.

Another independent ground survey of the receiver house was made during COMSAT's alignment survey of the telescope. That survey's procedures and results are reported in [Gurney-1]. As part of the survey, the contractor generated his own receiver house reference frame and coordinate system. The contractor used center points of 4 survey targets, FF1 $\cdots$FF4 near the front face of the house, as survey control points to generate his reference frame. We will refer to the contractor's coordinate system as the (x, y, z) turret system. In this report we will compare the positions of the two receiver house reference frames. Positions of roof targets FT1 $\cdots$ FT4 were determined in both the NRAO and COMSAT surveys; a comparison of measured roof target positions provides a check on the accuracy of determining the receiver house frame. The contractor's alignment procedure employed template flange N5 as his reference flange; however the relative positions of flange templates N5 and N7 were determined by the NRAO receiver house survey, and one has the choice of assigning either of these flange centers as reference point.

We also evaluate the as-built displacement and orientation of the receiver house relative to its design position after its erection onto the telescope, with the telescope set to rigging elevation. (The contractor's reported value for the as-built rigging elevation is 50.23°). We do this by comparing computed theoretical locations of targets FF1 $\cdots$ FF4 at the *design location* of the erected house to their *measured* locations (telescope at rigging elevation) referenced to survey control targets permanently mounted to the rim of the main reflector structure. That is, the orientation of the receiver house reference frame is known relative to survey targets mounted near the front face of the house, and the orientation of the house may be subsequently deduced from these targets' locations after the house has been erected onto the feed arm and the tipping structure set at rigging elevation.

The location of the phase center point for a feed depends in general not only on the feed's geometric location, but also on the frequency of the signal to be received. Specification of a phase center point's location is a process done in several steps. A feed's phase center position with respect to its mount flange is initially measured on the GBT antenna range. The phase center point is characterized by the criterion that received signal phase variation from a far field source is minimal for off-boresight illumination. Each feed's phase center coordinates are tabulated with respect to a local coordinate frame on the flange on which it sits; a table is provided which reports measured phase center coordinates of the feed (relative to its flange) for several frequencies. At other frequencies, coordinates are interpolated from tabulated values. The flange's local coordinate system is then tied to the receiver house coordinate frame, which is common to all flanges.

To locate a feed's phase center in space, with respect to the design Gregorian focus point of the telescope, the receiver house reference frame must be located with respect to the telescope's main reflector reference frame. To accomplish this one needs the offset of the as-built receiver house from its design configuration at rigging elevation and the change of offset when moved to arbitrary elevation. The incremental motion of the receiver house frame relative to the main reflector frame can be provided by the Finite Element Model of the telescope.

Because of bending and twisting of the telescope's vertical feed arm during telescope operation, the feed turret will, in general, also bend and twist with respect to the main reflector reference frame as the telescope varies in elevation. Any relative translation and rotation of the feed house with respect to the main reflector frame should be found, either by measurement or calculation, because this displacement is also a displacement of the Gregorian focal plane and focal point from its design position. When the subreflector is to be moved in position to image the incident signal from the instantaneous telescope prime focus to the instantaneous Gregorian focus, the relative displacement of the Gregorian focus point (which is to be the phase center of the active feed) and of the Gregorian focal plane (which is the feed's lower flange plane) should be known, in order to make allowance for these in setting the subreflector into position. The various coordinate frames and FEM calculations are described in [Goldman-1]. Definitions of the reference frames and thir coordinates systems are reviewed in Appendix A.

The purpose of providing the phase center data is, finally, to allow the determination, for each receiver feed and frequency, of a conjugate image point corresponding to the telescope's prime focus point as object point (which varies with tipping structure elevation), so that the subreflector optic may be moved to an appropriate position to image from the momentary prime focus of the telescope to the phase center of the receiver feed then in use at the Gregorian focus position of the telescope.

In this memo, we assume that the Gregorian focal plane of the design telescope coincides with the plane of the inner surface of each feed's mounting flange, when the receiver room turret is rotated to bring the flange to Gregorian focus position. All feeds were designed and constructed so that the feed's boresight axis lies perpendicular to the bottom plane surface of its mounting flange.

For Ku-band operation, 12-15.4 GHz, two separate feeds are mounted on receiver flange N4 [Masterman-2].

For K-band operation, 18-26.5 GHz, four separate feeds are mounted on receiver flange N6 [Masterman-1].

For Q-band operation, 40-52 GHz, four feeds are mounted on receiver flange N3.

Note: During feed testing on the antenna range, signals illuminating the feed from a far field source are transduced from the circularly symmetric feed to linearly polarized coaxial probes via a feed-to-waveguide-to-coaxial transition. E-field antenna pattern measurements are made by sweeping the feed along an arc lying in a plane containing both the E-field vector and the feed's boresight axis. H-field pattern measurements are made by sweeping the feed in an arc lying in a plane perpendicualar to that defined for the E-field measurements and containing the boresight axis. For each sweep, when the center of the rotation arc is the feed phase center, variation of received signal phase with illumination angle is minimal. When the feed is on the GBT, the polarized signal entering a receiver channel depends on the orientation of its feed transition's probe with respect to the feed flange.

The COMSAT Alignment Survey:

COMSAT's alignment survey of the GBT is reported in [Gurney-1]. Extracts from the survey results: data sheets W1, W2, W6, W8, W9 and report drawings (A through E), and comment sheets (Sheet 1 and Sheet 2) which describe the receiver house survey, are appended to this report.

The following sections of [Gurney-1] are relevant to the determination of the receiver phase center locations:

- 5.1 Determination of Feed Turret Position,
- 6.0 Determination of Best Fit Focal Point,
- 7.1 Initial Alignment of Feed Room,
- 9.1 Final Alignment of the Feed Room.

The receiver house survey control network uses five fiducial reference points. The fiducial points are the centers of four permanent targets FT1,···,FT4 on the front face of the receiver house, and a Gregorian focus point fixed relative to the house. A reference frame and (x,y,z)-coordinate system fixed to the receiver house was established, and the coordinates of the fiducial points are known relative to this coordinate system.

Receiver house survey control was established by the following steps:

(1) Total station targets were placed on the front and top of the receiver house (Figure A).

(2) Feed 5's tooling plate was bolted onto the $N5$ turret flange. A total station target was inserted into the center hole of the tooling plate. The target's center point was 2.000 inches above the plate's bottom surface.

(3) The turret was rotated and the turret detent pin was engaged so the $N5$ turret flange was locked at Gregorian focus position.

(4) The feed plate target center point location was measured using a total station.

(5) The turret was rotated counterclockwise to bring the next roof flange to Gregorian focus position. The feed plate target's position was remeasured. This step was repeated with each turret flange moved to Gregorian focus position.

(6) A best plane was fitted through the eight target point locations measured in the previous two steps.

(7) A circle of radius 56 inches, lying in the previously fitted plane, was best fitted to the eight target point locations, together with **x** and **y** coordinate axes in that plane fitted by least squares fit of the successive survey points from theoretical angles: 0, -40, -80, -120, 180, 120, 80, 40 degrees respectively to the **y**-axis.

(8) The fitted coordinate plane was translated downwards along its normal's direction by 2.000 inches. The translated plane is defined to be the "Gregorian focal plane of the receiver house." The origin of the coordinate system in this plane is defined to be the "Gregorian focal point of the receiver house." The coordinate system was extended to a right-handed (**x**,**y**,**z**)-coordinate system by defining the **z**-axis as normal to the (**x**,**y**)-plane through the origin point (**x**=0, **y**=0).

(9) The four front face total station target locations were surveyed and their locations were converted into (**x**,**y**,**z**)-coordinates.

Ideally, the receiver house is to be erected into position on the telescope so that *when the telescope has been set to its rigging angle of elevation*:

(1) The (**y**,**z**)-plane coincides with the left-right midplane $((Y_{rg}, Z_{rg})$-plane) of the telescope as determined by a total station survey of the receiver house with

respect the main reflector reference frame of the telescope.

(2) The Gregorian focus point of the receiver house lies at the design telescope's Gregorian focal point.

(3) The normal to the Gregorian focal plane of the receiver house (the **z**-axis) makes an angle of $-12.329°$ to the main reflector paraboloid axis (Z_{rg}-axis) of the telescope, as determined by photogrammetric survey of the main reflector surface. Survey control of the main reflector and establishment of the main reflector reference frame is achieved by a control network of six survey targets: $R1, \cdots, R6$ permanently located at the rim of the main reflector structure.

Flange and Phase Center Coordinate Data:

Table 1.1. Flange Center Offsets At Gregorian Foci (From NRAO Receiver House Survey).

Flange ($\mathrm{N}i$)		$X_{hg}(\mathrm{N}i) - X_{hg}(\mathrm{N7})$ (mm)	$Z_{hg}(\mathrm{N}i) - Z_{hg}(\mathrm{N7})$ (mm)
N1		not available	not available
N2		-1.00	+1.35
N3		+0.45	-0.05
N4		+0.60	+0.85
N5		-0.05	+0.65
N6		+0.20	+0.15
N7		0.00	0.00
N8		-0.50	-0.10

Table 1.2. Receiver House Frame Offsets From Design Value, At Rigging Elevation.

Offset from Design Gregorian Focus Point, of Center of Template for Flange N5, Telescope at Rigging Elevation.		Measured COMSAT Survey Value (mm).
$\mathbf{X}(\mathrm{N5}) - X_{rg}(\mathrm{F1})$		-2.337
$\mathbf{Y}(\mathrm{N5}) - Y_{rg}(\mathrm{F1})$		-4.479
$\mathbf{Z}(\mathrm{N5}) - Z_{rg}(\mathrm{F1})$		17.103

where

$X_{rg}(F1) \equiv 0.0$,

$Y_{rg}(F1) \equiv -11\,\mathrm{m} \times \sin 5.570^\circ = -1067.6797\,\mathrm{mm} = -42.0346\,\mathrm{inches}$,

$Z_{rg}(F1) \equiv 60\,\mathrm{m} - [11\,\mathrm{m} \times \sin 5.570^\circ] = 49,051.938\,\mathrm{mm} = 1931.1787\,\mathrm{inches}$,

and

$\mathbf{X}(\mathrm{N5})$, $\mathbf{Y}(\mathrm{N5})$, $\mathbf{Z}(\mathrm{N5})$ are survey values for the main reflector coordinates of the intersection point of the N5 template flange bottom plane with the flange center hole axis (flange set to Gregorian focus).

The receiver house coordinate differences listed in Table 1.1 were measured during the NRAO ground survey of the receiver house [Goldman-2] using an optical plummet on a 2-dimensional micrometer translation stage to measure the relative separations of survey targets centered in the roof flange templates, which were bolted and pinned to the receiver house turret.

The measured displacement components of the receiver house Gregorian focus point for flange N5, from its ideal design value was obtained by a reduction and adjustment of the COMSAT survey of the erected receiver house observed from a total station instrument near the center of the main reflector while the telescope tipping structure was at the rigging elevation of 50.23°. Observation of the survey targets R1 $\cdots$ R6 at the main reflector structure's rim by the total station instrument provided survey control for the main reflector coordinate system frame of reference.

Table 1.3 Receiver House Orientation Direction Cosines.

Direction Cosines of Receiver House Coordinate Axes to Main Reflector Axes	Theoretical Values at Rigging Elevation (12.329 Degree Coordinate Rotation)	Values Calculated from COMSAT Survey Results at Rigging Elevation
$\hat{x} \cdot \hat{X}$	1.0	1.000000
$\hat{x} \cdot \hat{Y}$	0.0	-0.000035
$\hat{x} \cdot \hat{Z}$	0.0	-0.000517
$\hat{y} \cdot \hat{X}$	0.0	0.000067
$\hat{y} \cdot \hat{Y}$	0.9769376	0.976763
$\hat{y} \cdot \hat{Z}$	0.2135248	0.214322
$\hat{z} \cdot \hat{X}$	0.0	-0.000099
$\hat{z} \cdot \hat{Y}$	-0.2135248	-0.214322
$\hat{z} \cdot \hat{Z}$	0.9769376	0.976764

Gregorian Feed Data:

Table 2.1. Gregorian Focus Front Ends.

Band (GHz)	Band Name	Turret Flange	Status	Notes
1.15-1.73	L	N1	Done	Single beam, dual lin/circ polarization
1.73-2.60	S	N5	Done	Single beam, dual lin/circ polarization
3.95-5.85	C	N2	Done	Single beam, dual lin/circ polarization
8.0-10.0	X	N8	Done	Single beam, dual circular polarization
12.0-15.4	Ku	N4	Done	Dual beam, dual circular polarization
18.0-22.0	K	N6	Done	Dual beam, dual circular polarization
22.0-26.5	K	N6	Done	Dual beam, dual circular polarization
26.0-40.0	Ka		Proposed	Dual beam, dual linear polarization
40.0-52.0	Q	N3	Under Test	Four beam, dual circular polarization
68-95			Proposed	Future development
95-116	F		Proposed	Future development
	3 mm		Proposed	3CAM Bolometer Array
		N7		Band not yet assigned to flange.

Table 2.2. Flange-Frame Local Coordinates.

Band (GHz)	Band Name	Flange No. N_i	Feed j	Z_{ij} (inch)	X_{ij} (inch)	Z_{ij} (mm)	X_{ij} (mm)
1.15-1.73	L	N1	1	0.0	0.0	0.0	0.0
1.73-2.60	S	N5	1	0.0	0.0	0.0	0.0
3.95-5.85	C	N2	1	0.0	0.0	0.0	0.0
8.0-10.0	X	N8	1	0.0	0.0	0.0	0.0
12.0-15.4	Ku	N4	1 (A)	4.243	4.243	107.772	107.772
12.0-15.4	Ku	N4	2 (B)	-4.243	-4.243	-107.772	-107.772
18.0-22.0	K	N6	1 (B)	3.250	3.250	82.550	82.550
18.0-22.0	K	N6	2 (A)	-3.250	-3.250	-82.550	-82.550
22.0-26.5	K	N6	3 (D)	3.250	-3.250	82.550	-82.550
22.0-26.5	K	N6	4 (C)	-3.250	3.250	-82.550	82.550
26.0-40.0	Ka	Future	Future				
40.0-52.0	Q	N3	1				
40.0-52.0	Q	N3	2				
40.0-52.0	Q	N3	3				
40.0-52.0	Q	N3	4				

Axial phase center coordinates are listed, where available, in tables 2.3 through 2.9 which follow. Values appearing in these tables were measured by Gary Anderson and S. Srikanth. A more complete compilation will appear, after measurements are completed, in GBT Drawing No. D3522M038, entitled "*GBT Project Turret Feed Layout.*"

Table 2.3. **GBT L-Band Phase Center Axial Coordinate.**

Frequency f (GHz)		Y_{11} (inch)	Y_{11} (mm)
1.10		+17.63	+447.802
1.15			
1.20		0.00	0.00
1.30		-7.69	-195.326
1.40			
1.50			
1.60		-14.19	-360.426
1.70			
1.73			

Table 2.4. GBT S-Band Phase Center Axial Coordinate.

Frequency f (GHz)		Y_{51} (inch)	Y_{51} (mm)
1.60		+4.939	+125.451
1.70		+3.895	+98.933
1.80		+1.615	+41.021
1.90		-0.708	-17.983
2.00		-3.355	-85.217
2.10		-6.134	-155.804
2.20		-9.093	-230.962
2.30		-9.335	-237.109
2.40		-13.429	-341.097
2.50		-14.531	-369.087
2.60		-15.555	-395.097
2.70		-16.587	-421.310

Table 2.5. GBT C-Band Phase Center Axial Coordinate.

Frequency f (GHz)		Y_{21} (inch)	Y_{21} (mm)
3.95		$34.90 - 34\frac{15}{16} = -0.0375$	-0.9525
4.20			
4.40			
4.60			
4.90		$34.90 - 34\frac{15}{16} = -0.0375$	-0.9525
5.20			
5.40			
5.60			
5.85		$34.90 - 35\frac{3}{16} = -0.2875$	-7.3025

Table 2.6. GBT X-Band Phase Center Axial Coordinate.

Frequency f (GHz)		Y_{81} (inch)	Y_{81} (mm)	Y_{81} (inch)	Y_{81} (mm)
		E-plane	E-plane	H-plane	H-plane
7.500		-0.360	-9.144	-0.350	-8.890
8.000		-0.570	-14.478	-0.575	-14.605
8.500		-0.670	-17.018	-0.683	-17.348
9.000		-0.670	-17.018	-0.683	-17.348
9.500		-0.670	-17.018	-0.683	-17.348
10.000		-0.773	-19.634	-0.785	-19.939
10.500		-0.875	-22.225	-0.785	-19.939

The above table is computed assuming a distance of 15.56 inches from the front of the feed aperture to the bottom surface of the feed flange.

Table 2.7. GBT Ku-Band Phase Center Axial Coordinate.

Frequency f (GHz)	Y_{41} (inch) E-plane A feed	Y_{41} (mm) E-plane A feed	Y_{42} (inch) E-plane B feed	Y_{42} (mm) E-plane B feed	Y_{41} (inch) H-plane A-feed	Y_{41} (mm) H-plane A-feed	Y_{42} (inch) H-plane B-feed	Y_{42} (mm) H-plane B feed
11.500	0.630	16.002	0.630	16.002	0.630	16.002	0.630	16.002
12.000	0.630	16.002	0.630	16.002	0.630	16.002	0.630	16.002
13.000	0.384	9.754	0.384	9.754	0.384	9.754	0.384	9.754
14.000	0.384	9.754	0.384	9.754	0.384	9.754	0.384	9.754
15.000	0.219	5.563	0.219	5.563	0.219	5.563	0.219	5.563
15.400	0.219	5.563	0.219	5.563	0.219	5.563	0.219	5.563
16.000	0.219	5.563	0.219	5.563	0.219	5.563	0.219	5.563

Table 2.8. GBT K-Band Phase Center Axial Coordinate.

Frequency f (GHz)	Y_{61} (inch) B-feed	Y_{61} (mm)	Y_{62} (inch) A feed	Y_{62} (mm)	Y_{63} (inch) D feed	Y_{63} (mm)	Y_{64} (inch) C feed	Y_{64} (mm)
22.0								
22.5								
23.0								
23.5								
24.0								
24.5								
25.0								
25.5								
26.0								
26.5								

Table 2.9. GBT Q-Band Phase Center Axial Coordinate.

Frequency f (GHz)	Y_{31} (inch)	Y_{31} (mm)	Y_{32} (inch)	Y_{32} (mm)	Y_{33} (inch)	Y_{33} (mm)	Y_{34} (inch)	Y_{34} (mm)
42.0								
43.0								
44.0								
45.0								
46.0								
47.0								
48.0								
49.0								
50.0								
51.0								
52.0								

The tabulated quantities can be used to compute receiver house coordinates and rigging elevation main reflector coordinates of the feed phase center points by use of equations (A 2.1) and (A 4.2) respectively, of Appendix A.

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Appendix A. Coordinate Reference Frames.

- **Local Feed Flange Frames:**

Unit basis vectors for flange Ni frame: $\widehat{X}_{ni}$, $\widehat{Y}_{ni}$, $\widehat{Z}_{ni}$ $(i = 1, \ldots, 8)$.

Coordinates of phase center point PC_{ij} : X_{ij} , Y_{ij} , Z_{ij}.

Description of frames: For each feed flange, Ni $(i = 1, \ldots, 8)$ which mounts on the receiver house turret there is a local reference frame and a right hand Cartesian coordinate system to describe the location of feed phase center points (Fig. 2). The X_{ni} and Y_{ni} coordinate axes lie in the lower (inside) plane of the flange. The Z_{ni} axis is perpendicular to this plane and points outwards. The frame coordinate system origin lies at the center of the flange's bolt circle in the lower flange plane. In this plane, the center points of two diametrically opposite bolt hole circles define the X_{ni}-axis. Details of flange design are given in Fig.1 and [Norrod-3,4,5].

The phase center point of feed number j of flange Ni : $PC_{ij}(f)$ has local flange coordinates $X_{ni}(PC_{ij})$, $Y_{ni}(PC_{ij}(f))$, $Z_{ni}(PC_{ij})$. The axial coordinate $Z_{ni}(PC_{ij})$ depends on the observing frequency, f, of the received signal, but the transverse coordinates are almost independent of frequency.

- **Receiver House Frame:**

Unit frame basis vectors: $\widehat{X}_{hg}$, $\widehat{Y}_{hg}$, $\widehat{Z}_{hg}$.

Coordinates of a point P: $X_{hg}(P)$, $Y_{hg}(P)$, $Z_{hg}(P)$.

Frame Origin Point, H_g: $(X_{hg}(H_g) = 0,\ Y_{hg}(H_g) \equiv 0,\ Z_{hg}(H_g) = 0)$.

Nominal Gregorian Focus Point Attached To Receiver House: M_{hg}
$(X_{hg}(M_{hg}) = 56" = 1422.4\,\text{mm},\ Y_{hg}(M_{hg}) = 0,\ Z_{hg}(M_{hg}) = 0)$.

Receiver house coordinates of Flange N7 template center

(from optical plummet survey):

$$(X_{hg}(N7) = 1422.4 \pm 0.3,\ Y_{hg}(N7) = 0.0,\ Z_{hg}(N7) = 1.5 \pm 0.7) \text{ mm.}$$

This is the intersection point of the flange axis with the flange's bottom plane.

Description of frame: This frame is rigidly embedded in the structure of the receiver house. The receiver house turret is constrained to roll in contact with two steel "hard casters." These 6" diameter casters are designed to have accurately parallel rotation axes and are at fixed spacing from one another. Two additional "preload casters" press the turret against the hard casters, so the turret actually contacts the hard casters. The four caster centers nominally lie on a square of diagonal 180 inches. The turret's axis of rotation is designed to be 90 inches from each hard caster axis. The feed mount flanges are flat circular plates; the flange axes are designed to lie on a 56" radius circle centered at the turret's nominal axis of rotation.

The (X_{hg}, Y_{hg})-coordinate plane is normal to the plane through the hard caster axes and is equidistant to these axes; the (X_{hg}, Z_{hg})-coordinate plane is the common parting plane of the receiver house turret-mounted flanges with their mating feed flanges (Fig. 3).

Four plates containing bushings were welded to the receiver house roof. A surveyor's target can be inserted into each bushing's hole. (Hubbs Machine & Manufacturing, Inc., Theodolite Target). The bushings were surveyed into place and located so that pairs of their targets' center points generate lines parallel to the $\widehat{X}_{hg}$ and $\widehat{Y}_{hg}$ directions, and intersect at the location of the axis of flange N7. The receiver house targets were subsequently surveyed by the contractor during his initial survey of the telescope configuration at rigging elevation. Their locations were determined relative to main reflector dish reference target control points during that survey.

When the turret is rotated to place flange Ni at the secondary focus position, the $\widehat{X}_{ni}$ frame vector points parallel to the $\widehat{X}_{hg}$ receiver house frame vector, and the $\widehat{Y}_{ni}$ frame vector points parallel to the $\widehat{Y}_{hg}$ receiver house frame vector.

- **Main Reflector Frame:**

Unit frame basis vectors: $\widehat{X}_{rg}$, $\widehat{Y}_{rg}$, $\widehat{Z}_{rg}$; $\widehat{\mathbf{X}}$, $\widehat{\mathbf{Y}}$, $\widehat{\mathbf{Z}}$ (Contractor's notation).

Coordinates of a point P : $X_{rg}(P)$, $Y_{rg}(P)$, $Z_{rg}(P)$.

Frame Origin Point: R_g , $X_{rg}(R_g) = 0$, $Y_{rg}(R_g) = 0$, $Z_{rg}(R_g) = 0$.

Point R_g is the vertex of the parent paraboloid.

Design Gregorian Focus Point Relative To The Parent Paraboloid: $F1$,

$X_{rg}(F1) = 0.0$, $Y_{rg}(F1) = -1067.6797$, $Z_{rg}(F1) = 49,051.938)$ mm.

$[Y_{rg}(F1) \equiv -11\,\text{meters} \times \sin 5.570°]$,

$[Z_{rg}(F1) \equiv 60\,\text{meters} - 11\,\text{meters} \times \cos 5.570°]$.

In inches:

$(X_{rg}(F1) = 0.0$, $Y_{rg}(F1) = -42.034634$, $Z_{rg}(F1) = 1931.1787$) inches.

Reported survey coordinates of template center for flange N5

(from COMSAT Data Sheet W8):

$(\mathbf{X}(N5) = -0.092$, $\mathbf{Y}(N5) = -42.211$, $\mathbf{Z}(N5) = 1931.852)$ inches.

In millimeters:

$(\mathbf{X}(N5) = -2.337$, $\mathbf{Y}(N5) = -1072.159$, $\mathbf{Z}(N5) = 49,069.041)$ mm.

These coordinates are obtained by analysis of total station measurements by the contractor, for a survey which includes reference control targets at the main reflector dish and reference survey targets on the receiver house.

Description of frame: Prior to photogrammetric survey of the main reflector dish, with the dish set to rigging elevation as indicated by the elevation encoder and surface panel actuators set at mid-position, the surface panels were adjusted initially to approximate the design parent paraboloid. This initial panel adjustment was made with a total station survey instrument. The main reflector frame was attached to a fit of a 60-meter focus paraboloid at rigging elevation to the as-built reflector surface points determined by photogrammetry, which are the panel photogrammetry target centers.

During photogrammetry, six contractor's survey targets were set at the dish periphery to provide survey control. The target centers constituted a local control network of reference points to describe locations of the panel photogrammetry target centers, and thereby subsequently provide physical reference control points for the fitted paraboloid. These targets were also used later to provide control points to reference surveyed target locations on the subreflector and receiver house structures when these structures were set to their nominal design positions on the telescope (at rigging elevation). By analyzing surveys which simultaneously included both the six dish rim control targets and targets on the receiver house and subreflector structures, the actual locations of the house and subreflector reference frames can be determined relative to the main reflector frame. The contractor's procedure for the initial GBT optics alignment is described in a contractor's specification document [Gurney-1]. The alignment results reported by the contractor (giving main reflector coordinates of survey targets on the receiver house and subreflector) were claimed to corresponded to a survey with the telescope elevation set to a rigging angle of 50.29°.

The (Y_{rg}, Z_{rg})-plane is the fitted symmetry plane of the dish photogrammetry targets. The Z_{rg}-axis is the axis of the fitted paraboloid, and is directed upwards from the vertex towards the prime focus. The Y_{rg}-axis is directed from the vertex towards the main reflector dish. The X_{rg}-axis is $\perp$ to the dish symmetry plane and is directed from this plane towards the right side of the telescope (the stair side).

The coordinate system used by the contractor in his alignment survey was a right-handed Cartesian coordinate system with axes labeled **X,Y,Z**. These axes correspond respectively to the main reflector frame coordinate axes as follows:

$$\text{Positive } \mathbf{X}\text{-axis} \Longleftrightarrow \text{Positive } X_{rg}\text{-axis}\,,$$

$$\text{Positive } \mathbf{Y}\text{-axis} \Longleftrightarrow \text{Positive } Y_{rg}\text{-axis}\,,$$

$$\text{Positive } \mathbf{Z}\text{-axis} \Longleftrightarrow \text{Positive } Z_{rg}\text{-axis}\,.$$

The (**X**,**Y**,**Z**)-coordinate system is illustrated in contractor's Figure E, "Final Alignments."

- **The Ideal Turret Frame:**

  Unit frame basis vectors: $\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$, $\widehat{\mathbf{z}}$.

  Coordinates of a point P: $\mathbf{x}(P)$, $\mathbf{y}(P)$, $\mathbf{z}(P)$.

  Frame Origin Point: M_g , $\mathbf{x}(M_g)=0$, $\mathbf{y}(M_g)=0$, $\mathbf{z}(M_g)=0$).

  M_g is the receiver house design Gregorian focus point.

  The ideal turret system is used by the contractor as an ideal in his alignment survey for the receiver house. It is a right-handed Cartesian (**x**,**y**,**z**)-coordinate system. Its axes are parallel to the receiver-house-system axes, but are displaced, and correspond to them as follows:

  Positive **x**-axis $\Longleftrightarrow$ Positive Z_{hg}-axis displaced 56" along the X_{hg}-axis,

  Positive **y**-axis $\Longleftrightarrow$ Positive X_{hg}-axis displaced 56" along the X_{hg}-axis,

  Positive **z**-axis $\Longleftrightarrow$ Positive Y_{hg}-axis displaced 56" along the X_{hg}-axis.

  This coordinate frame is used to describe the ideal positions of a survey target placed into the template for flange $N5$. The coordinate origin is the design receiver house Gregorian focal point. The contractor has used a survey target to generate this frame as follows. The target was designed and implemented so that the target center lies precisely on the axis of the center hole in the $N5$ template flange when inserted into this flange, and lies precisely 2.000 inches above the lower surface of the flange. After the $N5$ template is bolted onto the receiver house, the turret is first positioned so that the $N5$ roof flange sits at the receiver house Gregorian focus location (M_g); the turret detent pin is engaged and the target position is surveyed. Call this the "slot 1" position. The turret is then rotated counter-clockwise to the successive receiver flange positions at turret rotation angles: -40°, -80°, -120°, +180°, +120°, +80°, +40°, at slot positions 2, 3,· · ·, 8. The target center point will, ideally, move precisely on a circle of 56" radius. The design ideal values for this target's center point are given in Table A1 (cf. Fig. 1):

Table A1. Ideal N5 Survey-Target Turret-System Coordinates.

Position of Target Center	x	y	z
Slot 1	0.0	0.0	2.0
Slot2	56"·sin(-40°)	-56"·(1-cos(-40°))	2.0
Slot3	56"·sin(-80°)	-56"·(1-cos(-80°))	2.0
Slot4	56"·sin(-120°)	-56"·(1-cos(-120°))	2.0
Slot5	56"·sin(180°)	-56"·(1-cos(180°))	2.0
Slot6	56"·sin(120°)	-56"·(1-cos(120°))	2.0
Slot7	56"·sin(80°)	-56"·(1-cos(80°))	2.0
Slot8	56"·sin(40°)	-56"·(1-cos(40°))	2.0

The numerical values for these ideal coordinates are given below:

Table A1. N5 Target Center Ideal (x,y)-Coordinates.

Position of Target Center	x	y	z
Slot 1	0.0	0.0	2.0
Slot2	−35.9961	−13.1015	2.0
Slot3	−55.1492	−46.2757	2.0
Slot4	−48.4974	−84.0000	2.0
Slot5	0.0	−112.0000	2.0
Slot6	−48.4974	−84.0000	2.0
Slot7	55.1492	−46.2757	2.0
Slot8	35.9961	−13.1015	2.0

A physical embodiment of this reference frame is created in real 3-space, referenced to the physical receiver house, by the Contractor's alignment survey of the $N5$ template target at each of its turret settings. The front face receiver house target center points are also located by this survey. Total station position measurements are made for these 12 survey points. A best plane is least-squares fitted to the eight template-target-center survey locations, and subsequently a coordinate origin and (x,y)-coordinate axes in this plane are fitted by least squares minimization of the distances of these eight survey points from their theoretical coordinates (given in Table A2). The coordinate values for the flange-target-center survey points relative to the fitted turret coordinate system are given on Sheet 1 of Contractor's document [Gurney-1]. They are summarized in Table A3.

Table A3. Survey-Fitted N5 Target Center (x,y)-Coordinates.

Position of Target Center	x	y	z	z − 2.000"
	(inches)	(inches)	(inches)	(inches)
Slot 1	0.0	-0.009	2.001	+0.001
Slot2	−36.006	−13.118	1.996	-0.004
Slot3	−55.144	−46.280	2.003	+0.003
Slot4	−48.510	−83.954	2.000	+0.000
Slot5	-0.047	−111.958	1.999	-0.001
Slot6	−48.4974	−84.0000	2.001	+0.001
Slot7	55.143	−46.257	1.997	-0.003
Slot8	35.984	−13.095	2.002	+0.002

Position of Target Center	$\sqrt{x^2 + (y + 56)^2}$
	Target Radius (inches)
Slot 1	55.991
Slot2	55.994
Slot3	55.994
Slot4	55.988
Slot5	55.958
Slot6	56.000
Slot7	55.997
Slot8	55.997

- **Coordinate Transformations.**

The main-reflector-system coordinates of a point P are related to its coordinates in the receiver house system by the transformation:

$$\text{(A 1.0)} \quad \begin{bmatrix} X_{rg}(P) \\ Y_{rg}(P) \\ Z_{rg}(P) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \cos 12.329^\circ & -\sin 12.329^\circ & 0 \\ \sin 12.329^\circ & \cos 12.329^\circ & 0 \end{bmatrix} \begin{bmatrix} X_{hg}(P) \\ Y_{hg}(P) \\ Z_{hg}(P) \end{bmatrix} + \\ + \begin{bmatrix} X_{rg}(H_g) \\ Y_{rg}(H_g) \\ Z_{rg}(H_g) \end{bmatrix} .$$

The angle 12.329° is the design angle $(17.899^\circ - 5.570^\circ)$ between the normal to the Gregorian focal plane (the common join plane of the feed flanges to the turret) and the axis of the design telescope's parent paraboloid. The design coordinates of the receiver house system's origin (the nominal intersection of the turret axis with the Gregorian focus plane) are:

$$\text{(A 1.1)} \quad \begin{bmatrix} X_{rg}(H_g) \\ Y_{rg}(H_g) \\ Z_{rg}(H_g) \end{bmatrix} = \begin{bmatrix} 0.0 \\ Y_{rg}(F1) - (56\text{''} \times \cos 12.329^\circ) \\ Z_{rg}(F1) - (56\text{''} \times \sin 12.329^\circ) \end{bmatrix} \text{ which gives}$$

$$\text{(A 1.2)} \quad \begin{bmatrix} X_{rg}(H_g) \\ Y_{rg}(H_g) \\ Z_{rg}(H_g) \end{bmatrix} = \begin{bmatrix} 0.0 \\ -2,457.758 \text{ mm} \\ 48,748.220 \text{ mm} \end{bmatrix} = \begin{bmatrix} 0.0 \text{ inches} \\ -96.7621 \text{ inches} \\ 1,919.221 \text{ inches} \end{bmatrix} .$$

When receiver flange Ni is at Gregorian focus position, and the observing frequency is f, the receiver house coordinates of the phase center of feed j will be

$$\text{(A 2.1)} \quad \begin{aligned} X_{hg}(PC_{ij}) &= X_{hg}(Ni) + X_{ij} , \\ Y_{hg}(PC_{ij}) &= Y_{hg}(Ni) + Y_{ij}(f) , \\ Z_{hg}(PC_{ij}) &= Z_{hg}(Ni) + Z_{ij} . \end{aligned}$$

If main reflector coordinates of any flange center are available, for example from the contractor's survey of the receiver house from the main reflector surface, it will be simple to work with coordinate differences. Suppose that the main reflector coordinates of the center of the template for flange Nk have been measured by a total station survey to be

$$\text{(A 3.1)} \qquad \begin{aligned} X_{rg}(Nk) &= \mathbf{X}(Nk)\ , \\ Y_{rg}(Nk) &= \mathbf{Y}(Nk)\ , \\ Z_{rg}(Nk) &= \mathbf{Z}(Nk)\ , \end{aligned} \qquad \text{at the rigging elevation.}$$

Measured coordinates are given for the case $Nk = N5$ in [Gurney-1], Data Sheet W8. Using (A 1.0) and (A 2.1) gives

(A 4.1)

$$X_{rg}(PC_{ij}) = \mathbf{X}(Nk) + [Z_{ij} + Z_{hg}(Ni) - Z_{hg}(Nk)]\ ,$$

$$\begin{aligned} Y_{rg}(PC_{ij}) = \mathbf{Y}(Nk) &+ (\cos 12.329°) \cdot [X_{ij} + X_{hg}(Ni) - X_{hg}(Nk)] \\ &-(\sin 12.329°) \cdot [Y_{ij}(f) + Y_{hg}(Ni) - Y_{hg}(Nk)]\ , \end{aligned}$$

$$\begin{aligned} Z_{rg}(PC_{ij}) = \mathbf{Z}(Nk) &+ (\sin 12.329°) \cdot [X_{ij} + X_{hg}(Ni) - X_{hg}(Nk)]+ \\ &+(\cos 12.329°) \cdot [Y_{ij}(f) + Y_{hg}(Ni) - Y_{hg}(Nk)]\ . \end{aligned}$$

We now use the fact that $Y_{hg}(Ni) \equiv 0$ for each flange. The equations above, which give main reflector coordinates of the feed phase center points, at rigging elevation, then, for the case $Nk = N5$, reduce to:

(A 4.2)

$$X_{rg}(PC_{ij}) = \mathbf{X}(N5) + [Z_{ij} + Z_{hg}(Ni) - Z_{hg}(N5)]\ ,$$

$$Y_{rg}(PC_{ij}) = \mathbf{Y}(N5) + (\cos 12.329°) \cdot [X_{ij} + X_{hg}(Ni) - X_{hg}(N5)] - (\sin 12.329°) \cdot Y_{ij}(f)$$

$$Z_{rg}(PC_{ij}) = \mathbf{Z}(N5) + (\sin 12.329°) \cdot [X_{ij} + X_{hg}(Ni) - X_{hg}(N5)] + (\cos 12.329°) \cdot Y_{ij}(f)$$

Values for local flange-frame transverse coordinates X_{ij} and Z_{ij} of the feed phase centers PC_{ij} appear in Table 2.2. Measured values for the local flange-frame axial coordinate $Y_{ij}(f)$ of the feed phase centers PC_{ij} are given in Tables 2.3 to 2.9, for selected values of the frequency, f. The axial coordinate $Y_{ij}(f)$ is a smooth function of frequency and can be interpolated for non-tabulated frequency values.

Measured COMSAT alignment survey values for $\mathrm{X(N5)}, \mathrm{Y(N5)}, \mathrm{Z(N5)}$ at rigging elevation appear in Table 1.2. Optical plummet measurements [Goldman-2] for receiver-house-system coordinate differences appear in Table 1.1. At the time of survey measurements could not be made and values are not available for flange template $N1$. It is believed however that the following estimates are valid: $X_{hg}(N1) - X_{hg}(N7) = 0.0 \pm 1.0\,\mathrm{mm}$, $Z_{hg}(N1) - Z_{hg}(N7) = 0.0 \pm 1.0\,\mathrm{mm}$.

Given the above information, one uses the tabulated results and substitutes them into (A 4.2) to compute the main-reflector-system coordinates of the phase center points of the Gregorian feeds at the rigging angle of telescope elevation. Main reflector coordinates of these points are computed at other telescope elevations by using the available Finite Element Model of the telescope.

The Receiver House Orientation, As Erected.

For the ideal telescope at rigging elevation, the main reflector frame unit basis vectors are related to the unit basis vectors of the receiver house frame after the receiver house is erected to its design position, by the transformation:

$$\text{(A 5.1)} \qquad \begin{bmatrix} \widehat{X}_{rg} \\ \widehat{Y}_{rg} \\ \widehat{Z}_{rg} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \cos 12.329^\circ & \sin 12.329^\circ & 0 \\ -\sin 12.329^\circ & \cos 12.329^\circ & 0 \end{bmatrix} \begin{bmatrix} \widehat{X}_{hg} \\ \widehat{Y}_{hg} \\ \widehat{Z}_{hg} \end{bmatrix} .$$

If we agree that the main reflector frame and coordinate system, of the as-built telescope at rigging elevation, is established to be the Contractor's $\widehat{\mathbf{X}}$, $\widehat{\mathbf{Y}}$, $\widehat{\mathbf{Z}}$ frame which is generated during the COMSAT alignment survey (using centers of targets at the main reflector rim as survey control points) then we will have

$$\text{(A 5.2)} \qquad \begin{bmatrix} \widehat{X}_{rg} \\ \widehat{Y}_{rg} \\ \widehat{Z}_{rg} \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{X}} \\ \widehat{\mathbf{Y}} \\ \widehat{\mathbf{Z}} \end{bmatrix} .$$

The receiver house frame unit basis vectors, rigidly embedded in the receiver house structure are the $\widehat{\mathrm{x}}$, $\widehat{\mathrm{y}}$, $\widehat{\mathrm{z}}$ vectors determined by the survey of the N5 template flange target locations spanned by rotation of the receiver house turret.

The relation between the two sets of orthonormal basis vectors is given in general by a matrix equation of the form:

$$\text{(A 5.3)} \qquad \begin{bmatrix} \widehat{\mathrm{x}} \\ \widehat{\mathrm{y}} \\ \widehat{\mathrm{z}} \end{bmatrix} = [\mathbf{R}] \cdot \begin{bmatrix} \widehat{\mathbf{X}} \\ \widehat{\mathbf{Y}} \\ \widehat{\mathbf{Z}} \end{bmatrix} , \qquad \text{where the}$$

matrix $[\mathbf{R}]$ is an orthonormal 3×3 rotation matrix of direction cosines:

$$\text{(A 5.4)} \qquad [\mathbf{R}] = \begin{bmatrix} \widehat{\mathrm{x}} \cdot \widehat{\mathbf{X}} & \widehat{\mathrm{x}} \cdot \widehat{\mathbf{Y}} & \widehat{\mathrm{x}} \cdot \widehat{\mathbf{Z}} \\ \widehat{\mathrm{y}} \cdot \widehat{\mathbf{X}} & \widehat{\mathrm{y}} \cdot \widehat{\mathbf{Y}} & \widehat{\mathrm{y}} \cdot \widehat{\mathbf{Z}} \\ \widehat{\mathrm{z}} \cdot \widehat{\mathbf{X}} & \widehat{\mathrm{z}} \cdot \widehat{\mathbf{Y}} & \widehat{\mathrm{z}} \cdot \widehat{\mathbf{Z}} \end{bmatrix} .$$

The theoretical value of the matrix [**R**] for the case of a receiver house erected to its ideal design position is just the matrix appearing in equation (A 5.1). The matrix [**R**] for the receiver house as-erected is determined by two sets of alignment survey observations: (1) the receiver house coordinates of targets FF1 to FF4 with respect to the house coordinate system generated by the N5 target during turret rotation, while the house is on the ground, and (2) observations of the coordinates of FF1 to FF4 with respect to the main reflector frame after the house has been erected onto the telescope and the telescope is at rigging elevation. Assuming that the house has remained rigid during rigging onto the telescope, the house frame basis vectors can be reconstructed from the four sets of target coordinates measured after the house has been erected onto the telescope.

In calculating the basis vectors for the receiver house frame, an intermediate triple of (non-orthogonal) unit basis vectors has been used to tie the house frame on the ground to the house frame as erected at rigging elevation. These three unit vectors are: $\widehat{\mathbf{f}}_3$ in the direction from FF1 to FF2, $\widehat{\mathbf{f}}_1$ in the direction from FF4 to FF2,and $\widehat{\mathbf{f}}_2$ in the direction of their cross product $\widehat{\mathbf{f}}_3 \times \widehat{\mathbf{f}}_1$. The basis vectors $\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$, $\widehat{\mathbf{z}}$ found by ground survey are expressed as a linear sum over the vectors $\widehat{\mathbf{f}}_1$, $\widehat{\mathbf{f}}_2$, and $\widehat{\mathbf{f}}_3$. If the images of these six vectors under erection of the house are $\widehat{\mathbf{x}}_M$, $\widehat{\mathbf{y}}_M$, $\widehat{\mathbf{z}}_M$ and $\widehat{\mathbf{F}}_1$, $\widehat{\mathbf{F}}_2$, and $\widehat{\mathbf{F}}_3$ respectively then the coefficients of the linear transformation between the frame basis vectors of the erected house and the auxiliary basis vectors generated by the erected targets are the same as for the ground survey, if the house remains rigid during erection. The basis vectors for the erected house frame can then be determined from the locations of FF1 to FF4 determined by COMSAT survey observations from the main reflector surface.

The computations are carried out in Appendix D, and are reported in Table 1.3. The computations were performed using MATHCAD computer codes. Use of these codes, which do not carry a full set of mathematical text fonts, has necessitated use of a different notation scheme. The notation is defined during the computations. Instead of the basis vector $\widehat{\mathbf{x}}$, for example, the notation [x] is used. The vector $\widehat{\mathbf{x}}_M$ determined by survey measurements is denoted by [xM].

Appendix B. Transform Of Reported Target Center Coordinates In Contractor's Main Reflector System (Work Sheet W1) To Contractor's Receiver Room System, Using Transform Parameters Employed By The Contractor.

S = sin 12.329 deg. C = cos 12.329 deg.
S' = sin 12. 328713 deg. C' = cos 12.328713 deg.

FF1, FF2, FF3, FF4 are receiver house front targets.
FT1, FT2, FT3, FT4 are receiver house roof targets.

X(P), Y(P), Z(P) are Contractor's main reflector system coordinates of a point P. All coordinates are given in inches!

x(P), y(P), z(P) are Contractor's receiver house system coordinates of P. This system's origin is claimed to be at N5.

Xrg(P), Yrg(P), Zrg(P) are main reflector coordinates of point P.

For the ground alignment survey, Data Sheet W1 reports:

$i := 1, 2 .. 9$

			Target
$X_1 := 0.030$	$Y_1 := -3.391$	$Z_1 := 1941.522$	FT1 (i=1)
$X_2 := 0.030$	$Y_2 := -190.824$	$Z_2 := 1900.565$	FT2 (i=2)
$X_3 := -95.903$	$Y_3 := -42.472$	$Z_3 := 1932.997$	FT3 (i=3)
$X_4 := 95.968$	$Y_4 := -42.339$	$Z_4 := 1932.726$	FT4 (i=4)
$X_5 := 152.515$	$Y_5 := 23.716$	$Z_5 := 1787.315$	FF1 (i=5)
$X_6 := 152.693$	$Y_6 := 11.575$	$Z_6 := 1931.365$	FF2 (i=6)
$X_7 := -152.432$	$Y_7 := 23.305$	$Z_7 := 1787.492$	FF3 (i=7)
$X_8 := -152.213$	$Y_8 := 11.334$	$Z_8 := 1931.232$	FF4 (i=8)
$X_9 := 0.0$	$Y_9 := -42.047$	$Z_9 := 1931.181$	N5 (Ideal), from Data Sheet W8

$S' := 0.2135199$ $C' := 0.9769387$

From sheets 1 and 2, the reported Contractor's coordinate transform is:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} := \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & C' & S' \\ 0.0 & -S' & C' \end{pmatrix} \cdot \begin{bmatrix} X_i \\ Y_i + 42.047 \\ Z_i - 1931.181 \end{bmatrix}$$

Applying this gives:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{pmatrix} -0.03 \\ 39.9726 \\ 1.8487 \end{pmatrix} \quad \text{FT1} \qquad \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{pmatrix} 0.03 \\ -151.8831 \\ 1.8569 \end{pmatrix} \quad \text{FT2}$$

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{pmatrix} -95.903 \\ -0.0274 \\ 1.8649 \end{pmatrix} \quad \text{FT3} \qquad \begin{bmatrix} x_4 \\ y_4 \\ z_4 \end{bmatrix} = \begin{pmatrix} 95.968 \\ 0.0446 \\ 1.5717 \end{pmatrix} \quad \text{FT4}$$

$$\begin{bmatrix} x_5 \\ y_5 \\ z_5 \end{bmatrix} = \begin{pmatrix} 152.515 \\ 33.5282 \\ -154.59 \end{pmatrix} \quad \text{FF1} \qquad \begin{bmatrix} x_6 \\ y_6 \\ z_6 \end{bmatrix} = \begin{pmatrix} 152.693 \\ 52.4247 \\ -11.2696 \end{pmatrix} \quad \text{FF2}$$

$$\begin{bmatrix} x_7 \\ y_7 \\ z_7 \end{bmatrix} = \begin{pmatrix} -152.432 \\ 33.1644 \\ -154.3293 \end{pmatrix} \quad \text{FF3} \qquad \begin{bmatrix} x_8 \\ y_8 \\ z_8 \end{bmatrix} = \begin{pmatrix} -152.213 \\ 52.1609 \\ -11.3481 \end{pmatrix} \quad \text{FF4}$$

$$\begin{bmatrix} x_9 \\ y_9 \\ z_9 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{matrix} \text{Ideal Receiver House} \\ \text{Gregorian Focus Point } \left(M_{hg}\right) \end{matrix}$$

The Contractor's coordinate transform (sheets 1 and 2) is:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} := \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & C' & S' \\ 0.0 & -S' & C' \end{pmatrix} \cdot \begin{bmatrix} X_i \\ Y_i + 42.047 \\ Z_i - 1931.181 \end{bmatrix}$$

This transformation, from the main reflector to the receiver house coordinate systems differs slightly from the theoretical design transformation. The contractor assumed a rotation angle of the receiver house of 12.328713 degrees to the design paraboloid axis, and main reflector system design coordinates for the ideal Gregorian focus point, F1:

X(F1) = 0.0 inches, Y(F1) = -42.047 inches, Z(F1) = 1931.181 inches.

The ideal design value of the receiver house rotation angle is exactly 12.329 degrees and the ideal design main reflector coordinates of F1 are:

X(F1) = 0.0 inches, Y(F1) = -42.0346 inches, Z(F1) = 1931.1787 inches.

It is not clear why the contractor deviated from the precise ideal design parameters in setting up his transformation. The discrepancies in coordinate values calculated by the Contractor's transformation, compared to those obtained from the ideal transformation, are however at the level of 0.01 inches. The survey errors are at the level of 1mm. The error in using the above Contractor's transformation may, then, be considered to be of minor importance.

The values of the Contractor's receiver house coordinates computed for the targets: FT1...FT4, FF1...FF4, on the previous page, agree with the Contractor's values reported for the table "GBT Feed Room" on Contractor's Sheet 1, with the exception of the value x1 for FT1, which is a trivial data entry error and should be corrected to -0.030 on Sheet 1.

Appendix C. Comparison Of The Receiver House Frame And Coordinate System Generated By The Roof Targets, To That Generated By Observing The N5 Template Target As The Turret Is Rotated.

FT1, FT2, FT3, FT4 are receiver house roof targets.

x(P), y(P), z(P) are Contractor's receiver house system coordinates of the center point, P, of a receiver house target.

The Contractor's receiver house coordinates of the roof targets, as determined by the alignment survey, are given below. All values are in inches.

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} := \begin{pmatrix} -0.030 \\ 39.9726 \\ 1.8487 \end{pmatrix} \quad \mathrm{FT1} \qquad \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} := \begin{pmatrix} 0.030 \\ -151.8831 \\ 1.8569 \end{pmatrix} \quad \mathrm{FT2}$$

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} := \begin{pmatrix} -95.903 \\ -0.0274 \\ 1.8649 \end{pmatrix} \quad \mathrm{FT3} \qquad \begin{bmatrix} x_4 \\ y_4 \\ z_4 \end{bmatrix} = \begin{pmatrix} 95.968 \\ 0.0446 \\ 1.5717 \end{pmatrix} \quad \mathrm{FT4}$$

The projection points of the roof target center points onto the Contractor's receiver house Gregorian focus plane have (Contractor's receiver house) coordinates:

$$\begin{bmatrix} x_1 \\ y_1 \\ pz_1 \end{bmatrix} = \begin{pmatrix} -0.030 \\ 39.9726 \\ 0.0 \end{pmatrix} \quad \mathrm{FT1projection} \qquad \begin{bmatrix} x_2 \\ y_2 \\ pz_2 \end{bmatrix} = \begin{pmatrix} 0.030 \\ -151.8831 \\ 0.0 \end{pmatrix} \quad \mathrm{FT2projection}$$

$$\begin{bmatrix} x_3 \\ y_3 \\ pz_3 \end{bmatrix} = \begin{pmatrix} 95.903 \\ -0.0274 \\ 0.0 \end{pmatrix} \quad \mathrm{FT3projection} \qquad \begin{bmatrix} x_4 \\ y_4 \\ pz_4 \end{bmatrix} = \begin{pmatrix} 95.968 \\ 0.0446 \\ 0.0 \end{pmatrix} \quad \mathrm{FT4projection}$$

The displacement vectors of the projections of the displacement vectors from FT2 to FT1 and from FT3 to FT4 are given below, together with the lengths of these vectors.

$$\begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ pz_1 - pz_2 \end{bmatrix} = \begin{pmatrix} -0.06 \\ 191.8557 \\ 0 \end{pmatrix} \qquad d_{12} := \sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}$$

$$d_{12} := 191.8557$$

$$\begin{bmatrix} x_4 - x_3 \\ y_4 - y_3 \\ pz_4 - pz_3 \end{bmatrix} = \begin{pmatrix} 191.871 \\ 0.072 \\ 0 \end{pmatrix} \qquad d_{43} := \sqrt{\left(x_4 - x_3\right)^2 + \left(y_4 - y_3\right)^2}$$

$$d_{43} := 191.8710$$

The unit vectors from FT2 to FT1 and from FT3 to FT4 , projected to the receiver house Gregorian plane are then, respectively:

$$\left(\frac{1}{d_{12}}\right) \cdot \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ pz_1 - pz_2 \end{bmatrix} = \begin{pmatrix} -0.0003127 \\ 1 \\ 0 \end{pmatrix} \qquad \left(\frac{1}{d_{43}}\right) \cdot \begin{bmatrix} x_4 - x_3 \\ y_4 - y_3 \\ pz_4 - pz_3 \end{bmatrix} = \begin{pmatrix} 1 \\ 0.0003753 \\ 0 \end{pmatrix}$$

These vectors lie parallel to the receiver house coordinate axes established by optical plummet and theodolite measurements relative to the hard castor references of the receiver house. These axis directions lie very close to the Contractor's y- and x-axes determined by the turret rotation survey (within 1.4 arc-minutes of their respective axes).

Rename the projected unit vectors as:

$$\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} := \begin{pmatrix} -0.0003127 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} := \begin{pmatrix} 0.9999999 \\ 0.0003753 \\ 0 \end{pmatrix}$$

Their vector cross product is:

$$\begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} := \overrightarrow{\begin{pmatrix} 0.9999999 \\ 0.0003753 \\ 0 \end{pmatrix} \times \begin{pmatrix} -0.0003127 \\ 1 \\ 0 \end{pmatrix}}$$

which gives

$$\overrightarrow{\begin{pmatrix} 0.9999999 \\ 0.0003753 \\ 0 \end{pmatrix} \times \begin{pmatrix} -0.0003127 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The dot product of the first two unit vectors is 0.000063 . They lie within 15 arc-seconds of perpendicularity to one another.

The intersection point coordinates of the projections of the rays from FT2 to FT1 and from FT3 to FT4, after projection onto the receiver house Gregorian focal plane, are the coordinates of the receiver house Gregorian focal point expected to be found as a result of the optical plummet and theodolite survey of the receiver house [Goldman-2]. Its coordinates derived from the COMSAT alignment survey results (Contractor's receiver house coordinates) are computed below.

Let the slopes of these two projected rays be

$$m_{12} := \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{43} := \frac{y_4 - y_3}{x_4 - x_3}$$

and let the receiver house coordinates of their intersection points be ξ and η

Solving for the coordinates of the intersection point in terms of the four target projection point coordinates gives

$$\xi := \frac{y_1 - y_4 + (m_{43} \cdot x_3 - m_{12} \cdot x_1)}{m_{43} - m_{12}} \qquad \eta := y_4 + m_{43} \cdot (\xi - x_3)$$

which gives $\xi = -0.0175244$ $\eta = 0.0805812$

The alignment survey result intersection point coordinates, in inches, are then:

$x(N7) := -0.0175$ $y(N7) := 0.0806$

The Contractor's alignment survey results give receiver house coordinates (that is, x and y coordinates) for the N5 target:

$x(N5) := 0.0$ $y(N5) := -0.009$ inches.

The NRAO receiver house survey gave the result (Table 1.1),

x(N7) - x(N5) = -0.05 mm = - 0.0020 inches,
y(N7) - y(N5) = 0.65 mm = 0.0256 inches,

The Contractor's alignment survey gave,

x(N7) - x(N5) = -0.0175 inches,
y(N7) - y(N5) = 0.0896 inches.

There is a difference of -0.0155 inches in the x-coordinate displacement and a difference of 0.064 inches in the y-coordinate displacement of the N7 and N5 flange focal point positions for the two surveys. In the Contractor's alignment survey the coordinates for N5 were determined by reducing a set of turret target rotation observations, whereas the coordinates for N7 were determined by reduction of observations of the four roof targets.

These differences are compatible with the target setting accuracies and standard errors for the two surveys, and the results of the two surveys should be considered to be in very good agreement with one another.

Appendix D. Determination Of The Receiver House Orientation As Erected, With Respect To The Main Reflector Frame And Coordinate System Generated By COMSAT's Alignment Survey.

FF1, FF2, FF3, FF4 are front face targets on the receiver house.

x(P), y(P), z(P) are Contractor's receiver house system coordinates of the reference (center point) of receiver house target, P.

Vector notation is also used to represent the receiver house coordinates for the target P:

$$\begin{pmatrix} Px \\ Py \\ Pz \end{pmatrix}$$

The Contractor's receiver house coordinates of the front face targets, determined by the COMSAT alignment survey, are given below. All values are in inches.

$$\begin{pmatrix} FF1x \\ FF1y \\ FF1z \end{pmatrix} = \begin{pmatrix} 152.515 \\ 33.528 \\ -154.590 \end{pmatrix} \quad FF1 \qquad \begin{pmatrix} FF2x \\ FF2y \\ FF2z \end{pmatrix} = \begin{pmatrix} 152.693 \\ 52.425 \\ -11.269 \end{pmatrix} \quad FF2$$

$$\begin{pmatrix} FF3x \\ FF3y \\ FF3z \end{pmatrix} = \begin{pmatrix} -152.432 \\ 33.165 \\ 154.329 \end{pmatrix} \quad FF3 \qquad \begin{pmatrix} FF4x \\ FF4y \\ FF4z \end{pmatrix} = \begin{pmatrix} -152.213 \\ 52.161 \\ -11.348 \end{pmatrix} \quad FF4$$

Theoretical coordinates of the front face targets, when the house has been lifted and set on the telescope to its ideal design location are, in the Contractor's main reflector coordinates (from Data Sheet W8):

$$\begin{pmatrix} FF1X \\ FF1Y \\ FF1Z \end{pmatrix} = \begin{pmatrix} 152.515 \\ 23.716 \\ 1787.315 \end{pmatrix} \qquad \begin{pmatrix} FF2X \\ FF2Y \\ FF2Z \end{pmatrix} = \begin{pmatrix} 152.693 \\ 11.575 \\ 1931.365 \end{pmatrix}$$

$$\begin{pmatrix} FF3X \\ FF3Y \\ FF3Z \end{pmatrix} = \begin{pmatrix} -152.432 \\ 23.305 \\ 1787.492 \end{pmatrix} \qquad \begin{pmatrix} FF4X \\ FF4Y \\ FF4Z \end{pmatrix} = \begin{pmatrix} -152.213 \\ 11.334 \\ 1931.232 \end{pmatrix}$$

These coordinates are in the Contractor's main reflector system.

The actual survey values, as measured and adjusted for the main reflector system coordinates of the front face house targets, as erected onto the tipping structure at rigging elevation (from Data Sheet W8) are:

$$\begin{pmatrix} FF1MX \\ FF1MY \\ FF1MZ \end{pmatrix} := \begin{pmatrix} 152.593 \\ 23.675 \\ 1787.460 \end{pmatrix} \qquad \begin{pmatrix} FF2MX \\ FF2MY \\ FF2MZ \end{pmatrix} := \begin{pmatrix} 152.758 \\ 11.424 \\ 1931.407 \end{pmatrix}$$

$$\begin{pmatrix} FF3MX \\ FF3MY \\ FF3MZ \end{pmatrix} := \begin{pmatrix} -152.264 \\ 23.278 \\ 1787.752 \end{pmatrix} \qquad \begin{pmatrix} FF4MX \\ FF4MY \\ FF4MZ \end{pmatrix} := \begin{pmatrix} -151.997 \\ 11.194 \\ 1931.431 \end{pmatrix}$$

We now define three unit vectors which are considered to be rigidly oriented relative to the receiver house structure:

We first compute the distances between the reference points of targets FF2 and FF1, and between those of FF2 and FF4 respectively.

$$d_{21} = \sqrt{(FF2x - FF1x)^2 + (FF2y - FF1y)^2 + (FF2z - FF1z)^2}$$

$$d_{42} = \sqrt{(FF2x - FF4x)^2 + (FF2y - FF4y)^2 + (FF2z - FF4z)^2}$$

$$d_{21} = 144.5615 \qquad d_{42} = 304.9062$$

We define the unit vectors, [f3] directed from FF2 to FF1, and [f1] directed from FF4 to FF2, and give their coordinates in the Contractor's receiver house system.

$$\begin{pmatrix} f3x \\ f3y \\ f3z \end{pmatrix} := \left(\frac{1}{d_{21}}\right) \cdot \begin{pmatrix} FF2x - FF1x \\ FF2y - FF1y \\ FF2z - FF1z \end{pmatrix}$$ Vector [f3]

$$\begin{pmatrix} f1x \\ f1y \\ f1z \end{pmatrix} := \left(\frac{1}{d_{42}}\right) \cdot \begin{pmatrix} FF2x - FF4x \\ FF2y - FF4y \\ FF2z - FF4z \end{pmatrix}$$ Vector [f1]

Their vector cross product is

$$\begin{pmatrix} ux \\ uy \\ uz \end{pmatrix} = \begin{pmatrix} f3y \cdot f1z - f1y \cdot f3z \\ f3z \cdot f1x - f1z \cdot f3x \\ f3x \cdot f1y - f1x \cdot f3y \end{pmatrix}$$ Vector [f3] x [f1]

The amplitude of this vector is

$$du = \sqrt{(ux)^2 + (uy)^2 + (uz)^2}$$

The unit vector in the direction of the cross product [f3] x [f1] is

$$\begin{pmatrix} f2x \\ f2y \\ f2z \end{pmatrix} = \left(\frac{1}{du}\right) \cdot \begin{pmatrix} ux \\ uy \\ uz \end{pmatrix}$$ Vector [f2]

We have now defined a triple of unit vectors: [f1], [f2], [f3] which are fixed rigidly with respect to the receiver house reference frame. Vectors [f2] and [f3] are perpendicular to [f1] but not to one another. Vectors [f1], [f2], [f3] in that order form a right hand triple.

We define the matrix, f , and its inverse f_inv, as:

$$f = \begin{pmatrix} f1x & f1y & f1z \\ f2x & f2y & f2z \\ f3x & f3y & f3z \end{pmatrix} \qquad \text{with} \qquad f\_inv = f^{-1}$$

Numerically we get:

$$\begin{pmatrix} f1x \\ f1y \\ f1z \end{pmatrix} = \begin{pmatrix} 0.999999 \\ 0.000866 \\ 0.000259 \end{pmatrix} \qquad [f1]$$

$$\begin{pmatrix} f2x \\ f2y \\ f2z \end{pmatrix} = \begin{pmatrix} -0.000825 \\ 0.991419 \\ -0.130718 \end{pmatrix} \qquad [f2]$$

$$\begin{pmatrix} f3x \\ f3y \\ f3z \end{pmatrix} = \begin{pmatrix} 0.001231 \\ 0.130719 \\ 0.991419 \end{pmatrix} \qquad [f3]$$

These vectors can be expressed in the form:

[f1] = f1x [x] + f1y [y] + f1z [z]

[f2] = f2x [x] + f2y [y] + f2z [z]

[f3] = f3x [x] + f3y [y] + f3z [z] .

We can express the unit basis vectors [x], [y], [z] in terms of the unit vectors [f1], [f2], [f3],

[x] = xf1 [f1] + xf2 [f2] + xf3 [f3]

[y] = yf1 [f1] + yf2 [f2] + yf3 [f3]

[z] = zf1 [f1] + zf2 [f2] + zf3 [f3] ,

where the matrix of coefficients xf is the inverse of the matrix f :

$$xf = f\_inv$$

$$xf = \begin{pmatrix} 1 & -0.000825 & -0.00037 \\ 0.000657 & 0.991419 & 0.130718 \\ -0.001329 & -0.130718 & 0.991421 \end{pmatrix}$$

We now use the COMSAT alignment survey results (Data Sheet W8) to express vectors [f1], [f2], [f3] for the as-erected receiver house in terms of their components relative to the Contractor's main reflector frame unit basis vectors: [X], [Y], [Z] .

As before we compute the distances between the target reference points FF2 and FF1, and FF2 and FF4 respectively. But now we use their main reflector coordinates corresponding to the house as-erected, when the telescope is at rigging elevation.

$$dm_{21} := \sqrt{(FF2MX - FF1MX)^2 + (FF2MY - FF1MY)^2 + (FF2MZ - FF1MZ)^2}$$

$$dm_{42} := \sqrt{(FF2MX - FF4MX)^2 + (FF2MY - FF4MY)^2 + (FF2MZ - FF4MZ)^2}$$

$dm_{21} = 144.467481$ $\quad$ $dm_{42} = 304.755088$

$dm_{21} - d_{21} = -0.094019$ (inches) $\quad$ $dm_{42} - d_{42} = -0.151112$ (inches)

One expects the differences in the line above to be zero, for a perfect survey and a rigid receiver house. The differences seem a bit large for the case of survey error alone, and may also reflect some receiver house deformation.

We next define unit vectors, [FM3] directed from FF2 to FF1, and [FM1] directed from FF4 to FF2, and give their coordinates in the Contractor's receiver house system.

$$\begin{pmatrix} F3MX \\ F3MY \\ F3MZ \end{pmatrix} = \left(\frac{1}{dm_{21}}\right) \cdot \begin{pmatrix} FF2MX - FF1MX \\ FF2MY - FF1MY \\ FF2MZ - FF1MZ \end{pmatrix}$$ Vector [FM3]

$$\begin{pmatrix} F1MX \\ F1MY \\ F1MZ \end{pmatrix} = \left(\frac{1}{dm_{42}}\right) \cdot \begin{pmatrix} FF2MX - FF4MX \\ FF2MY - FF4MY \\ FF2MZ - FF4MZ \end{pmatrix}$$ Vector [FM1]

Their vector cross product is

$$\begin{pmatrix} UX \\ UY \\ UZ \end{pmatrix} = \begin{pmatrix} F3MY \cdot F1MZ - F1MY \cdot F3MZ \\ F3MZ \cdot F1MX - F1MZ \cdot F3MX \\ F3MX \cdot F1MY - F1MX \cdot F3MY \end{pmatrix}$$ Vector [FM3] x [FM1]

The amplitude of this vector is

$$dU := \sqrt{(UX)^2 + (UY)^2 + (UZ)^2}$$

The unit vector in the direction of the cross product is

$$\begin{pmatrix} F2MX \\ F2MY \\ F2MZ \end{pmatrix} := \left(\frac{1}{dU}\right) \cdot \begin{pmatrix} UX \\ UY \\ UZ \end{pmatrix} \qquad \text{Vector [FM2]}$$

We get, numerically:

$$\begin{pmatrix} F1MX \\ F1MY \\ F1MZ \end{pmatrix} = \begin{pmatrix} 1 \\ 0.000755 \\ -0.000079 \end{pmatrix} \quad \text{[FM1]} \qquad \begin{pmatrix} F2MX \\ F2MY \\ F2MZ \end{pmatrix} = \begin{pmatrix} -0.000745 \\ 0.996398 \\ 0.084802 \end{pmatrix} \quad \text{[FM2]}$$

$$\begin{pmatrix} F3MX \\ F3MY \\ F3MZ \end{pmatrix} = \begin{pmatrix} 0.001142 \\ -0.084801 \\ 0.996397 \end{pmatrix} \quad \text{[FM3]}$$

Unit vectors [FM1], [FM2], [FM3] are the images of unit vectors [f1], [f2], [f3] respectively, when the receiver house is erected and the telescope is at rigging elevation (as defined by the readout of the elevation encoder).

Call the images of the receiver house basis vectors [x], [y], [z] after the house erection and setting of the telescope to rigging elevation: [xM], [yM], [zM].

We can express these image unit basis vectors in terms of the unit vectors [FM1], [FM2], [FM3] generated by the erected front face targets:

[xM] = xf1 [FM1] + xf2 [FM2] + xf3 [FM3]

[yM] = yf1 [FM1] + yf2 [FM2] + yf3 [FM3]

[zM] = zf1 [FM1] + zf2 [FM2] + zf3 [FM3] .

We note that if the receiver house erection has been a rigid motion house, then the transformation matrix for the house basis vectors in terms of the unit vectors directed between front face targets is unchanged. In this case the transformation matrix is again xf.

Expanding the previous equations into their X, Y, Z components, we get:

$$xMX = (xf1)(F1MX) + (xf2)(F2MX) + (xf3)(F3MX)$$

$$xMY = (xf1)(F1MY) + (xf2)(F2MY) + (xf3)(F3MY)$$

$$xMZ = (xf1)(F1MZ) + (xf2)(F2MZ) + (xf3)(F3MZ),$$

$$yMX = (yf1)(F1MX) + (yf2)(F2MX) + (yf3)(F3MX)$$

$$yMY = (yf1)(F1MY) + (yf2)(F2MY) + (yf3)(F3MY)$$

$$yMZ = (yf1)(F1MZ) + (yf2)(F2MZ) + (yf3)(F3MZ),$$

$$zMX = (zf1)(F1MX) + (zf2)(F2MX) + (zf3)(F3MX)$$

$$zMY = (zf1)(F1MY) + (zf2)(F2MY) + (zf3)(F3MY)$$

$$zMZ = (zf1)(F1MZ) + (zf2)(F2MZ) + (zf3)(F3MZ).$$

Define the matrix FM as:

$$FM := \begin{pmatrix} F1MX & F1MY & F1MZ \\ F2MX & F2MY & F2MZ \\ F3MX & F3MY & F3MZ \end{pmatrix}$$

Numerically,

$$FM = \begin{pmatrix} 1 & 0.000755 & -0.000079 \\ -0.000745 & 0.996398 & 0.084802 \\ 0.001142 & -0.084801 & 0.996397 \end{pmatrix}$$

Define the matrix of direction coefficients xMiXj as:

$$xMiXj := xf \cdot FM$$

Call

$$\begin{pmatrix} xMdotX & xMdotY & xMdotZ \\ yMdotX & yMdotY & yMdotZ \\ zMdotX & zMdotY & zMdotZ \end{pmatrix} = xMiXj$$

Then numerically,

$$\begin{pmatrix} xMdotX & xMdotY & xMdotZ \\ yMdotX & yMdotY & yMdotZ \\ zMdotX & zMdotY & zMdotZ \end{pmatrix} = xMiXj = \begin{pmatrix} 1 & -0.000035 & -0.000517 \\ 0.000067 & 0.976763 & 0.214322 \\ -0.000099 & -0.214322 & 0.976764 \end{pmatrix}$$

This is the matrix of direction cosines, of the frame vectors [xM], [yM], [zM] of the erected receiver house at tipping elevation, with respect to the main reflector frame basis vectors [X], [Y], [Z] of the main reflector frame generated by COMSAT's alignment survey.

The elements of this matrix are listed in Table 1.3, where they are compared to their values expected for the ideal design telescope. The agreement is excellent, and indicates that the erected house has been set to its design orientation relative to COMSAT's main reflector frame to a few minutes of arc for each house basis vector.

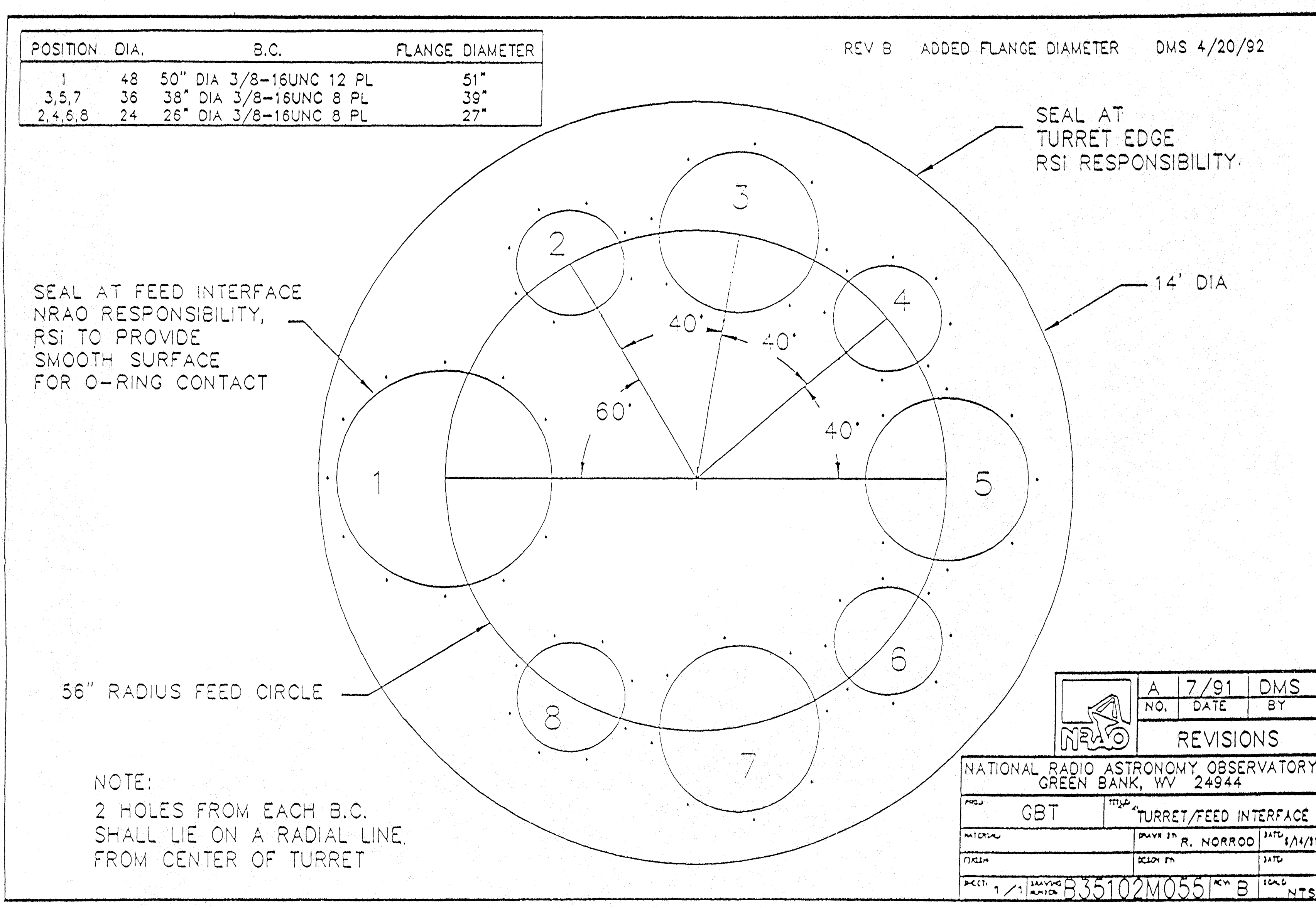


Figure 1. Receiver House Flange Configuration.

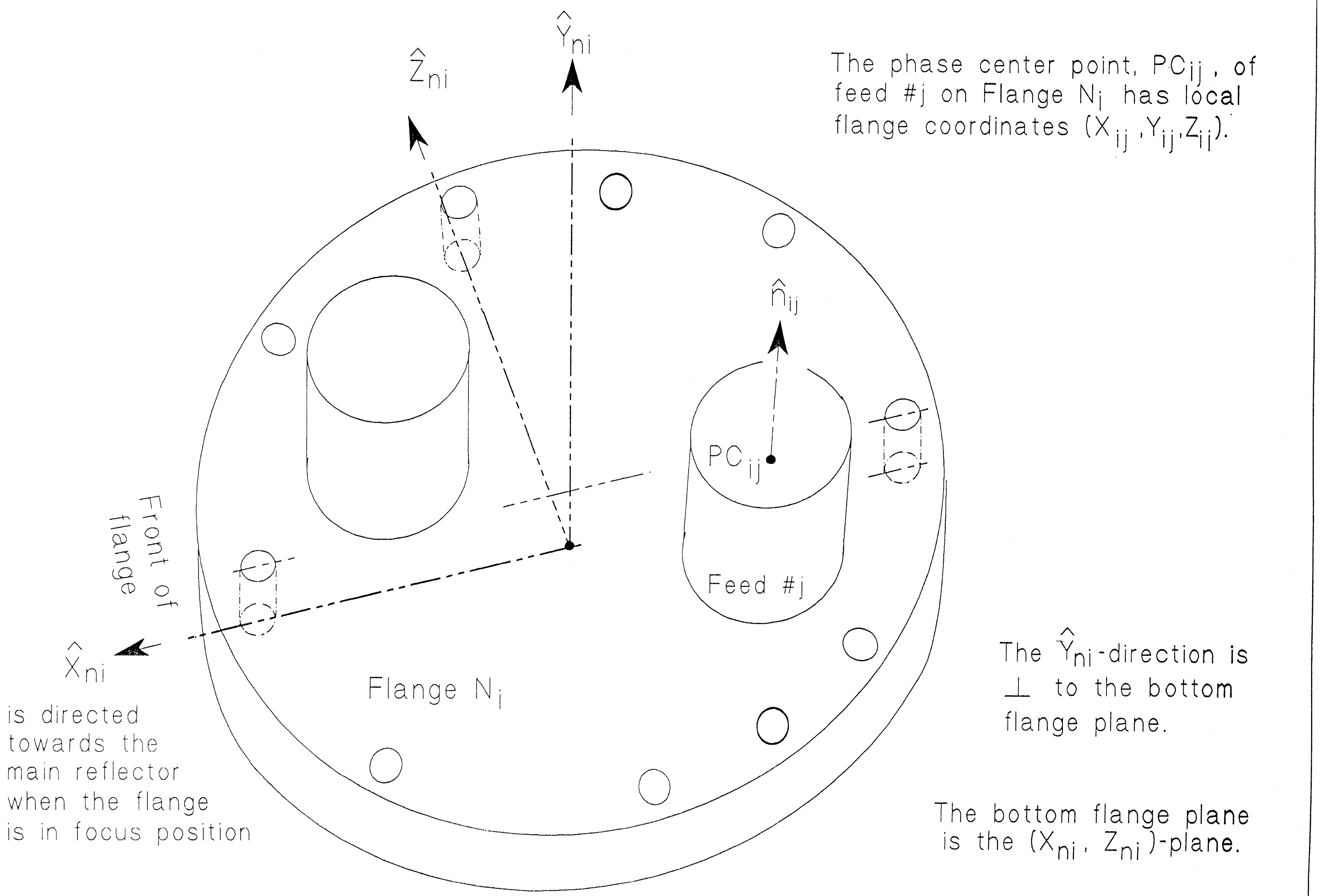


Figure 2. Flange Reference Frame For Receiver Flange N_i

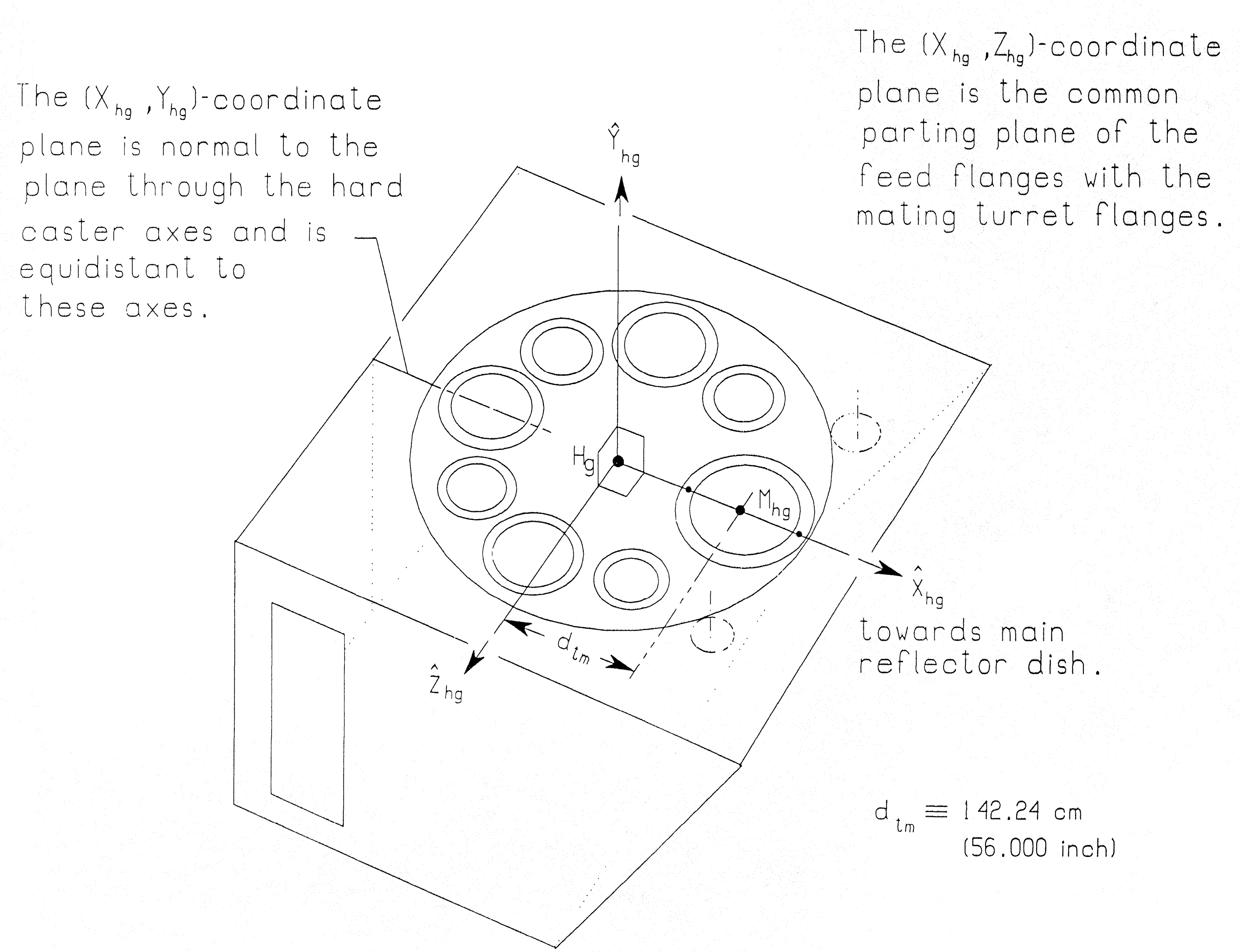
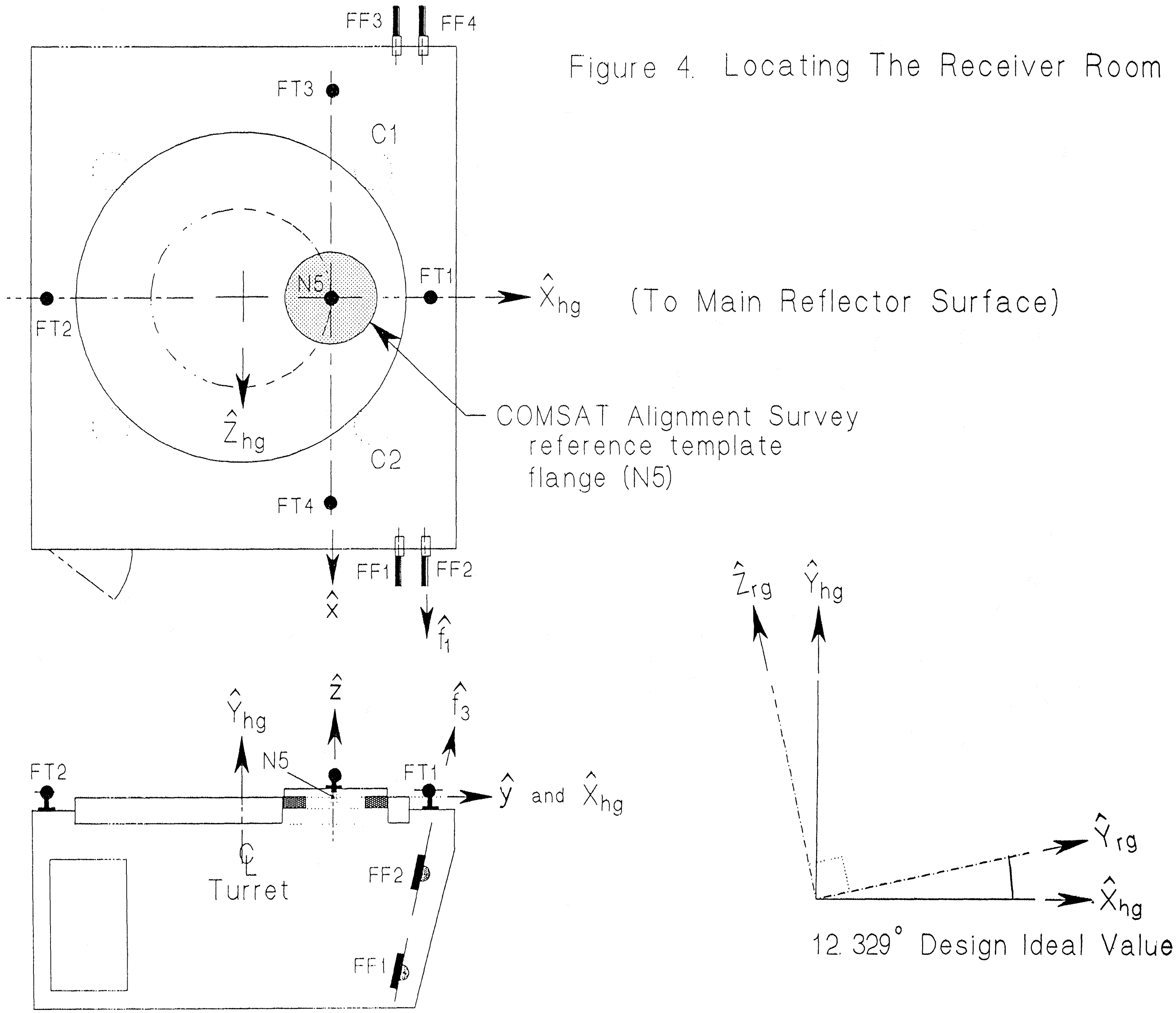
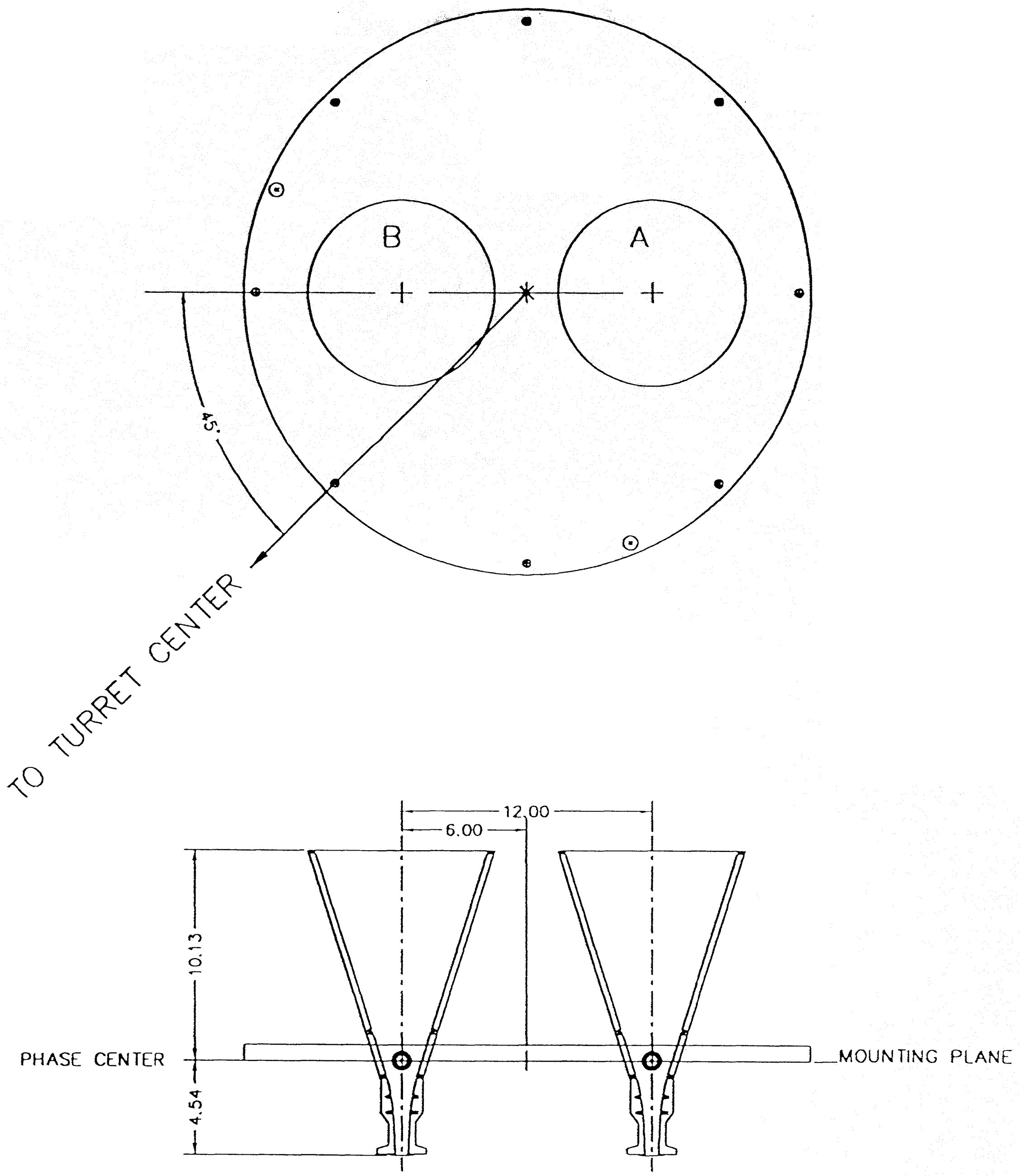


Figure 3. The Receiver House Frame.

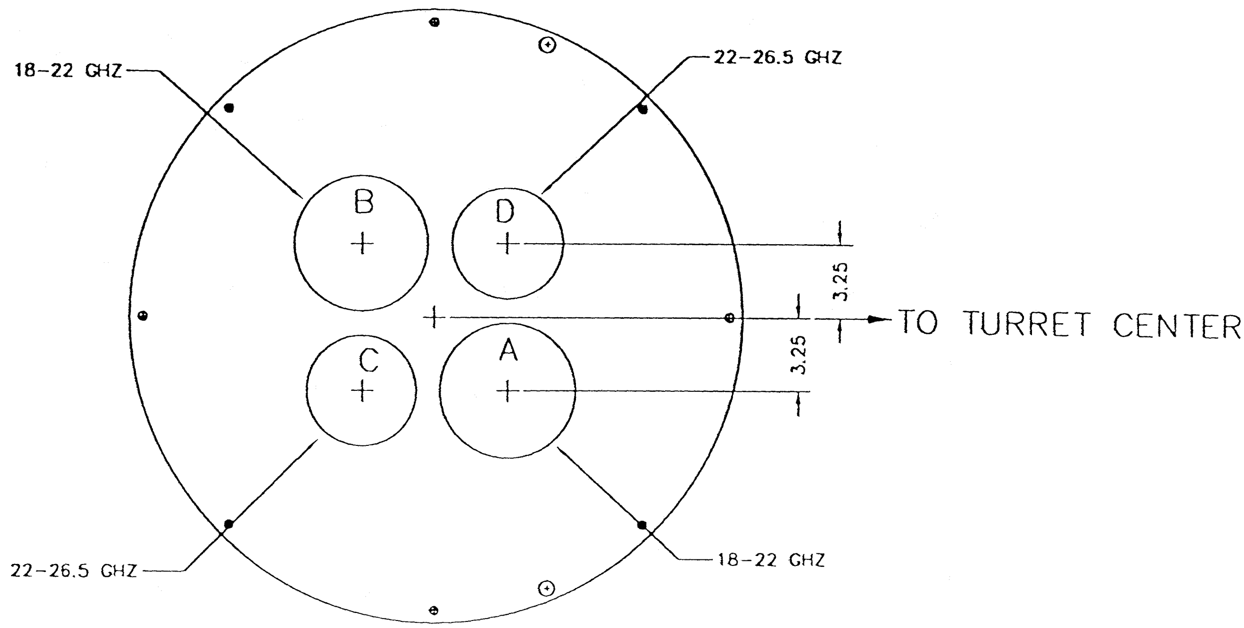
Figure 4. Locating The Receiver Room

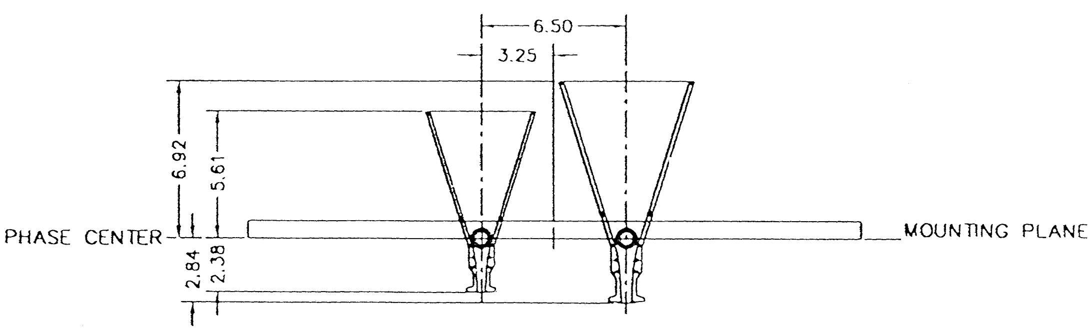




PHASE CENTER LOCATIONS
KU-BAND FEEDS
12-15.4 GHZ

NOTE: PHASE CENTER LIES ON FEED CENTERLINE.
IF RECEIVER IS MOUNTED ON FEED ROTATOR MECHANISM, PHASE CENTERS SHIFT UP TOWARD SUBREFLECTOR 2.0 INCHES.





ONLY A AND C FEEDS SHOWN FOR CLARITY

PHASE CENTER LOCATIONS
K-BAND FEEDS
18-26.5 GHZ

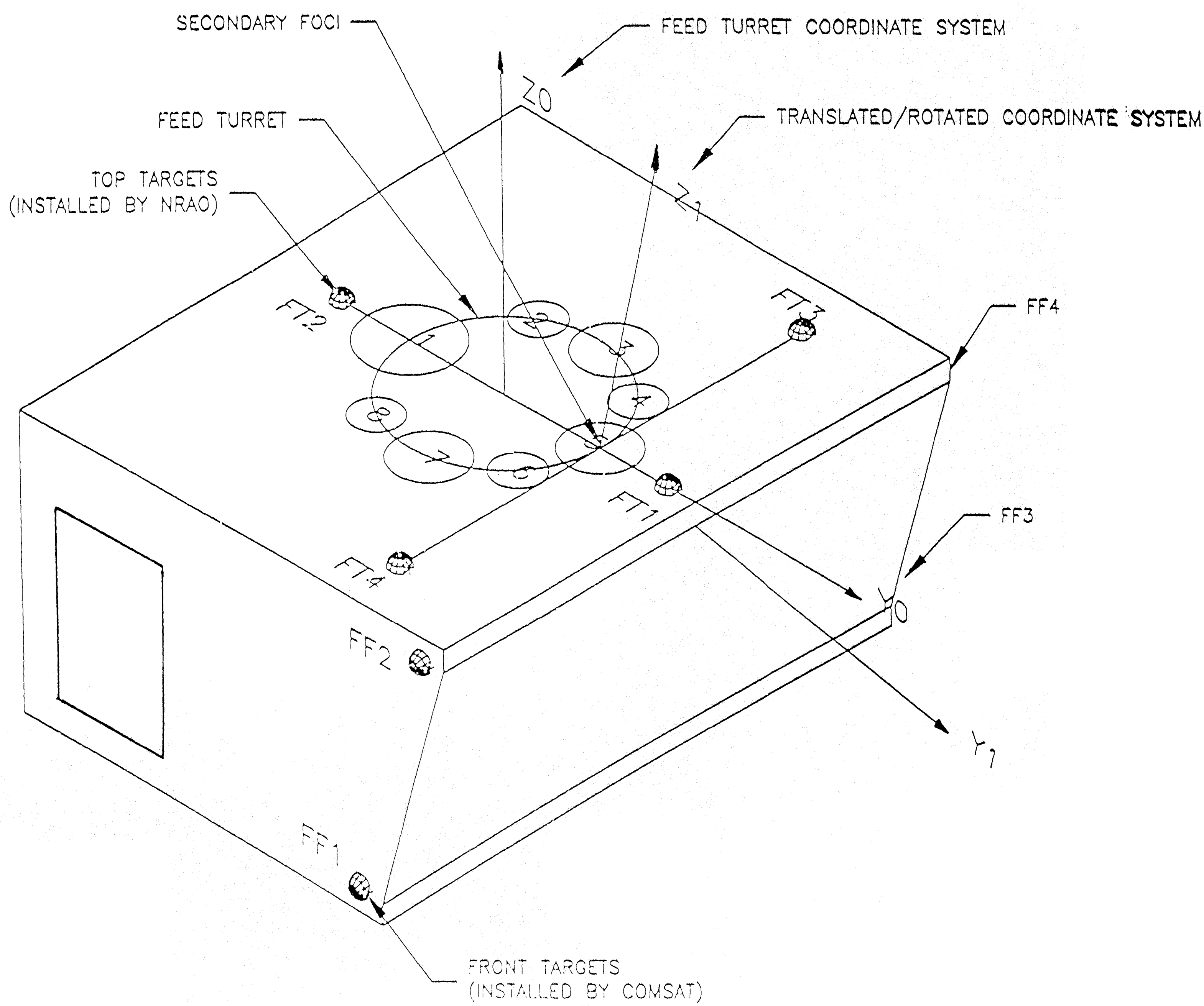


FIGURE A. FEED ROOM TARGETS

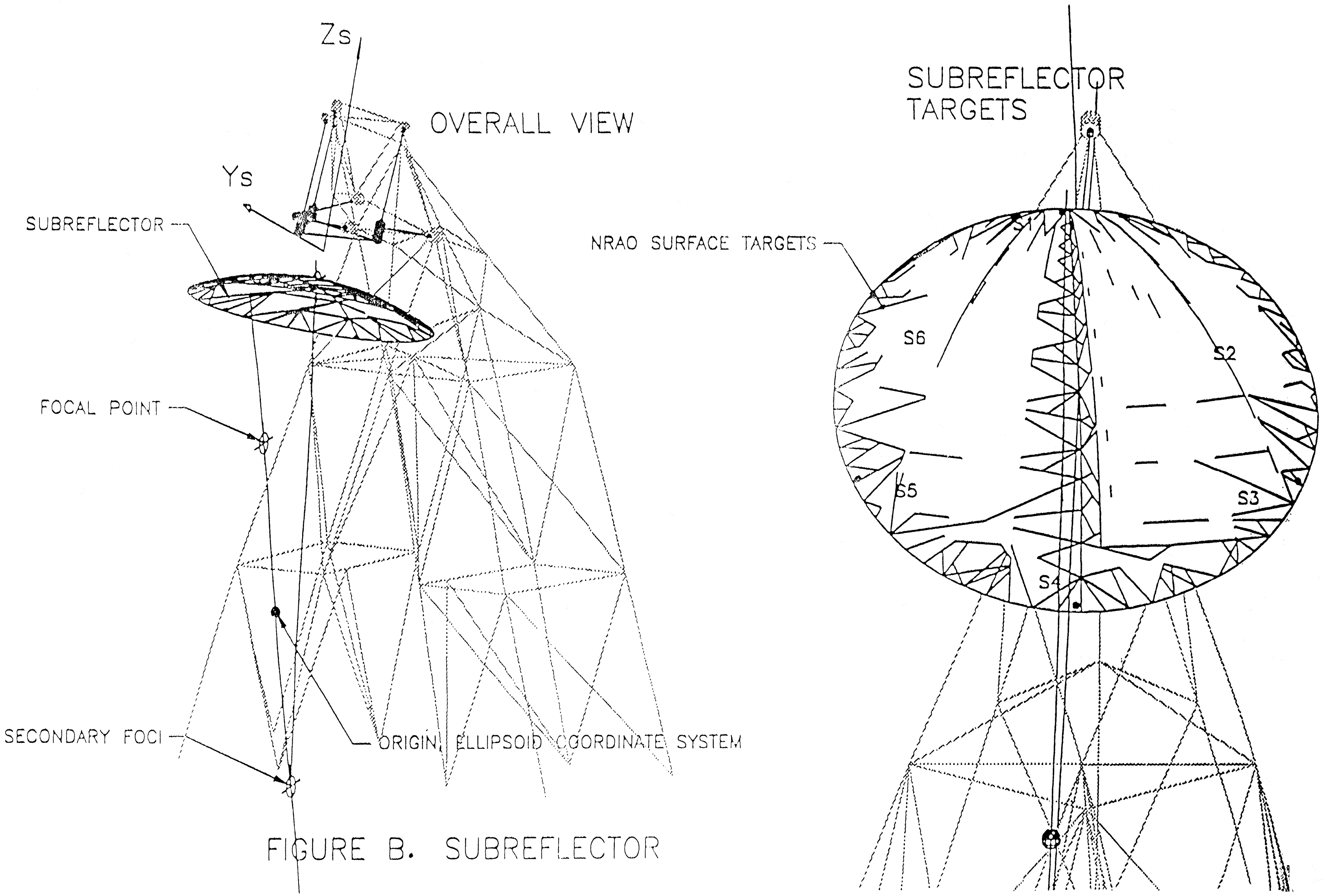


FIGURE B. SUBREFLECTOR

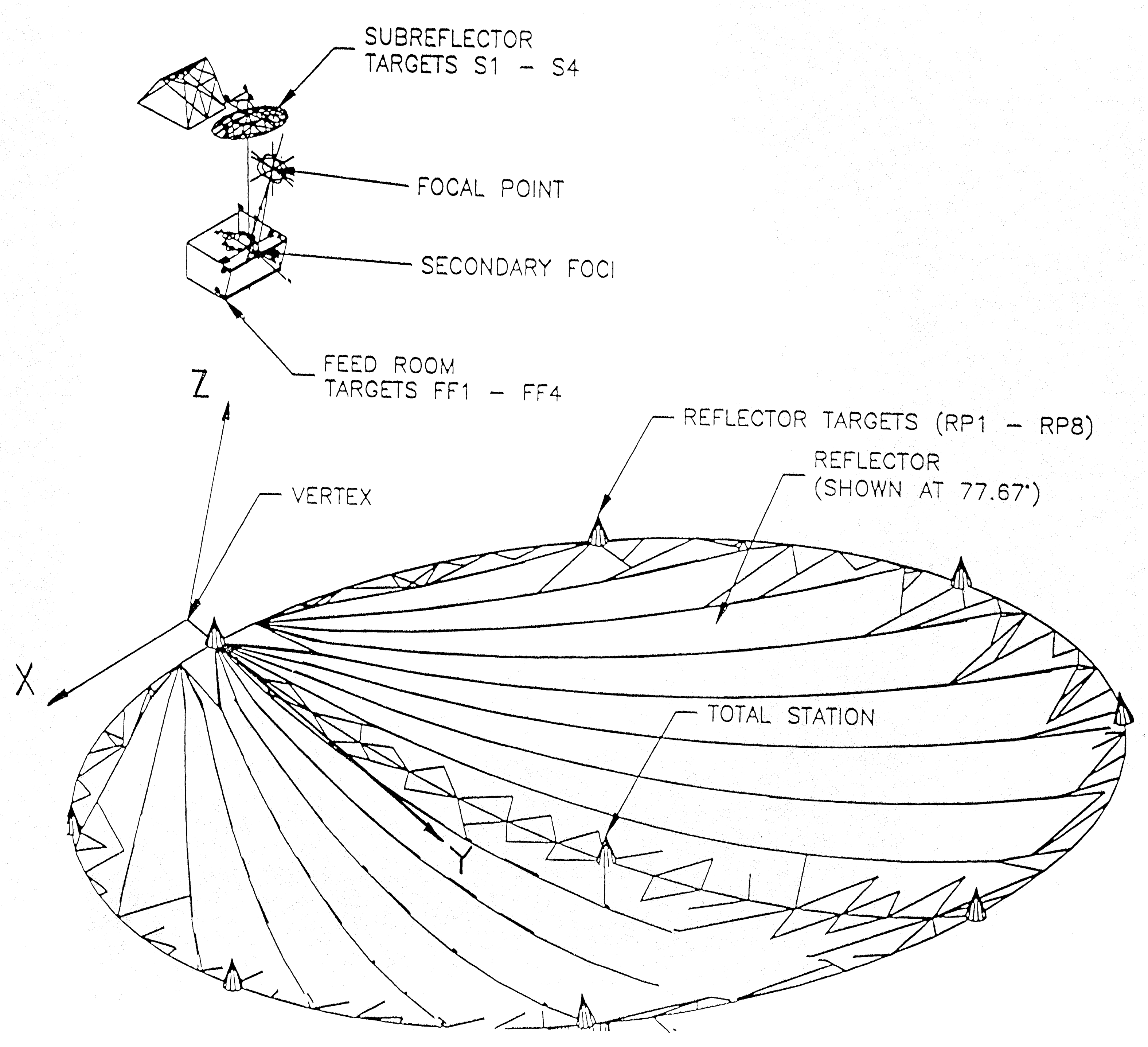


FIGURE E. FINAL ALIGNMENTS

Sheet 1

Avg roo
u - .009

GBT Feed Room

	Node	X	Y	Z
	SLOT 1	0.000	-0.009	2.001
	SLOT 2	-36.006	-13.118	1.996
	SLOT 3	-55.144	-46.280	2.003
	SLOT 4	-48.510	-83.954	2.000
	SLOT 5	-0.047	-111.958	1.999
	SLOT 6	48.498	-83.964	2.001
	SLOT 7	55.143	-46.257	1.997
	SLOT 8	35.984	-13.095	2.002
FF2	RTTOP	152.693	52.425	-11.269
FF1	RTBOT	152.515	33.528	-154.590
FF4	LFTOP	-152.213	52.161	-11.348
FF3	LFBOT	-152.432	33.165	-154.329
FT2	BKCTR	0.030	-151.883	1.857
FT3	LEFT	-95.903	-0.028	1.885
FT1	FRNTCTR	-0.013	39.972	1.849
FT4	RIGHT	95.968	0.045	1.571
	Operational	0.000	0.000	0.000

Measured on top of 0.50" plate w/ target w/ 1.50" offset

Measured w/ targets w/ 1.50" offset

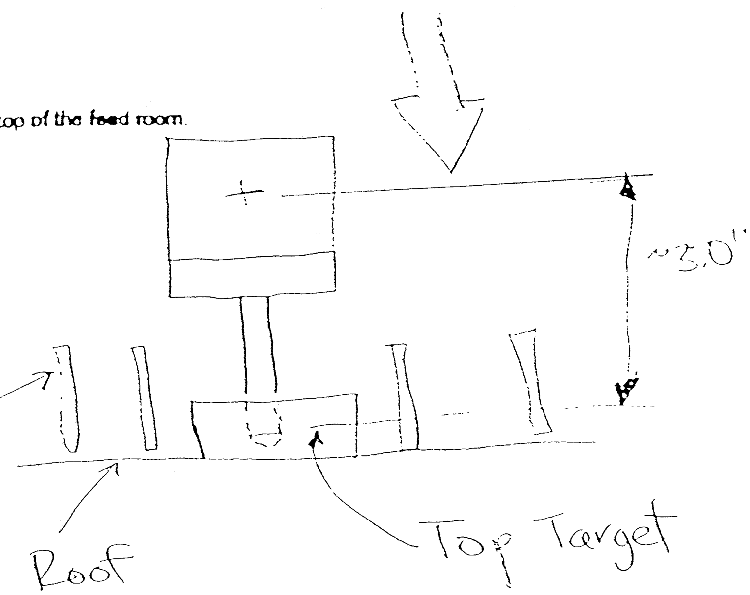
Measured w/ special targets w/ nominal 3.0" offset

Coordinate System Description

Z increases up and is perpendicular to a plane through the top of the feed room.

Y increase along R0 when standing at the vertex.

The origin is at the operational feed.



GBT Feed Room

Sheet 2

	Node	X	Y	Z
	SLOT 1	0.000	-0.436	1.953
	SLOT 2	-36.006	-13.242	-0.851
	SLOT 3	-55.144	-45.641	-7.925
	SLOT 4	-48.510	-82.445	-15.972
	SLOT 5	-0.047	-109.803	-21.952
	SLOT 6	48.498	-82.455	-15.973
	SLOT 7	55.143	-45.617	-7.928
	SLOT 8	35.984	-13.220	-0.840
FF2	RTTOP	152.693	53.622	0.184
FF1	RTBOT	152.515	65.763	-143.856
FF4	LFTOP	-152.213	53.381	0.051
FF3	LFBOT	-152.432	65.352	-143.689
FT2	BKCTR	0.030	-148.777	-30.616
FT3	LEFT	-95.803	-0.425	1.816
FT1	FRNTCTR	-0.013	38.656	10.341
FT4	RIGHT	95.968	-0.292	1.545
	Operational	0.000	0.000	0.000

Sheet 1 after 12.32° rotation.

Coordinate System Description

Z increases up and is perpendicular to a plane through the top of the feed room after being rotated 12.328713 degrees.

Y increase along R0 when standing at the vertex

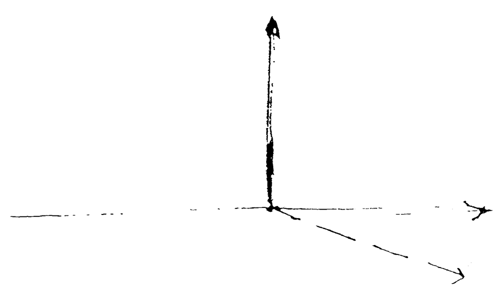
The origin is at the center of the operational feed.

$x' = x$

$y' = y\cos\alpha + z\sin\alpha$

$z' = z\cos\alpha - y\sin\alpha$

where $\alpha = -12.328713°$



	Node	X	Y	Z
	SLOT 1	0.000	-42.483	1933.134
	SLOT 2	-36.006	-55.289	1930.330
	SLOT 3	-55.144	-87.688	1923.256
	SLOT 4	-48.510	-124.492	1915.209
	SLOT 5	-0.047	-151.850	1909.229
	SLOT 6	48.498	-124.502	1915.208
	SLOT 7	55.143	-87.664	1923.255
	SLOT 8	35.984	-55.267	1930.341
FF2	RTTOP	152.693	11.575	1931.365
FF1	RTBOT	152.515	23.716	1787.315
FF4	LFTOP	-152.213	11.334	1931.232
FF3	LFBOT	-152.432	23.305	1787.492
FT2	BKCTR	0.030	-190.824	1900.565
FT3	LEFT	-95.903	-42.472	1932.997
FT1	FRNTCTR	-0.013	-3.391	1941.522
FT4	RIGHT	95.968	-42.339	1932.726
	Operational	0.000	-42.047	1931.181

— Theoretical

Sheet 2 after translatu to put operational fee phase center @ theoret location.

Coordinate System Description

Z increases up and is perpendicular to a plane through the top of the feed room after being rotated 12.328713 degrees.

Y increase along R0 when standing at the vertex.

The origin is at the origin of the main parabola.

$X'' = X'$

$Y'' = Y' - 42.047$

$Z'' = Z' - 1931.181$

DATA SHEET W1
GROUND ALIGNMENT FEED TURRET

1.	Total Station Set-up	Yes	No	
	a. Feed plates installed in Feeds 5 position	X		
	b. Feed plate measured in all 8 positions	X		
	c. Coordinate system established to best fit circular plane	X		
	d. Coordinate system translated to secondary focus	X		
	e. Coordinate system rotated by 12.328713°	X		
	f. Coordinate system translated to global system	X		
2.	Top targets	X	Y	Z
	FT1	-.030	-3.391	1941.52
	FT2	0.030	-190.824	1900.56
	FT3	-95.903	-42.472	1932.99
	FT4	95.968	-42.339	1932.72
3.	Supplemental targets installed			
4.	Front targets	X	Y	Z
	FF1	152.515	23.716	1787.31
	FF2	152.693	11.575	1931.36
	FF3	-152.432	23.305	1787.49
	FF4	-152.213	11.334	1931.23
Acceptance Criteria		Yes	No	
1.	2. Top targets measured in global coordinate system	X		
2.	4. Front targets measured in global coordinate system	X		

COMSAT John W. [signature] DATE 10/10/00

NRAO __________ DATE __________

DATA SHEET W8

FINAL ALIGNMENT FEED ROOM

1.	Offset Secondary Focus Location	X	Y	Z
	a. Theoretical at 50.29° rigging angle	0	-42.047	1931.181
	b. Measured (using hidden point method)	-0.092	-42.211	1931.852
2.	Turret Slope	Theoretical	Meas.	Error
	a. Front-Back	0°	0°	0°
	b. Sideways	0°	0°	0°
3.	Final Measurements Front Targets			
	a. Theoretical at rigging (from W1, 4)	X	Y	Z
	FF1	152.515	23.716	1787.315
	FF2	152.693	11.575	1931.365
	FF3	-152.432	23.305	1787.492
	FF4	-152.213	11.334	1931.232
	b. Measured	X	Y	Z
	FF1	152.593	23.675	1787.460
	FF2	152.758	11.424	1931.407
	FF3	-152.264	23.278	1787.752
	FF4	-151.997	11.194	1931.431
	e. Errors (3b – 3a)	X	Y	Z
	FF1	0.078	-0.041	0.145
	FF2	0.065	-0.151	0.042
	FF3	0.168	-0.027	0.260
	FF4	0.216	-0.140	0.199
	Average errors	0.132	-0.092	0.162
Acceptance Criteria		Yes	No	
1.	2a and b Turret slope errors ≤ ± 0.1°	X		
2.	4c Errors ≤ ± 0.50 in.	X		

COMSAT John W. [illegible] DATE 10/10/00

NRAO ____________________ DATE ____________