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Range Scan Program

Static Check Scan, Main Reflector Surface

The measurement problem.

Measurements are to be made are to verify the main reflector surface shape and orientation at commanded telescope elevation EL_{com} and azimuth AZ_{com} during the time interval from $t = t^0$ to $t = t^0 + t_{scan}$. The telescope is in fixed position during these measurements. One checks the surface by scanning a canonical set of 100 surface cube prism retroreflectors by each of the feed arm rangefinders $ZY_{13}, \ldots, ZY_{18}$. These rangefinders also scan one another, to establish a local frame of reference. Ranges are adjusted, and coordinates are established for the measured surface points. A best fit of the measured points is established to a pre-calculated surface shape, corresponding to the telescope's elevation, and distance residuals of the measured points from this surface are obtained.

The surface orientation is then found by measuring the orientation of the feed arm laser scan point configuration with respect to the ground reference coordinate frame.

We assume that, before performing this scan program, a data base of approximate coordinates of the feed arm ranger scan reference points exists, relative to the telescope's geometric elevation reference frame, with a standard error for each coordinate. (The table is generated using either design or measured coordinates; it is later updated as measurement results become available.) We also assume that two tables of reference point coordinates in this frame are available for type ZPG targets. One table gives coordinates corresponding to the surface rigging elevation angle of the telescope, when all surface actuators are set at positions to give parent paraboloid shape to the surface. The other table gives nominal target fiducial coordinates corresponding to the current actuator settings. The latter table is used for aiming the feed arm rangefinder beams.

To initiate measurements, a request for execution of the range scan program "**Static Check Scan, Main Reflector Surface**" is made to the server for the SCHEDULER FOR RETRORE-FLECTOR SCANS. This is done by keyboard input from the telescope control room computer (with the aid of a graphic user monitor interface) to the server for TELESCOPE OPERATOR INITIATED SCHEDULING REQUESTS. The request includes a clock start time t^0 for program execution . The scan scheduler generates the list of {rangefinder, target} pairs, a sequence of target measurements, and internal timing triggers for the scans .

Timing triggers for the program generate a buffer time interval between each scan-command trigger and the command to begin laser illumination for that scan. During this interval the scan-ranger and target-ranger ZY-computers signal to the ZIY computer that their scan mirror aiming motions are complete; the ZIY computer acknowledges readiness by returning a scan-permission signal to the scan-ranger's ZY-computer.

The measurement procedure consists of several steps. The n'th step occurs during the time interval from $t^0 + t^{(n)}$ to $t^0 + t^{(n+1)}$, and takes $t^{(n+1)} - t^{(n)}$ seconds to complete. If n_{max} is the total number of measurement steps, then $t^{(n_{max})} - t^0 = t_{scan}$.

The first step in the program is to make reference path calibrations for all rangers and then locate feed arm ranger scan points relative to one another (to generate a local survey control network on the vertical feed arm). Each feed arm ranger, in sequence, aims to and scans the remaining rangers. Each target ranger mirror is set normal to the incoming scan laser beam, flipped to expose its retroreflector prism to the beam (anti-autocollimation configuration). Thirty 1-way paths are available, to measure 15 distances between ranger scan points. One measures the feed arm scan-point to scan-point distances during the time interval t^0 to $t^0 + t^{(1)}$. For each ranger, the comparison reference path electronic phase is entered into a buffer file of initial input data for range computation. For each laser scan pair the range path electronic phase is also entered into this input data file.

[Note: Cube corner prisms on ranger scan mirrors are classed as a separate type: ZMG, "Mirror Mounted Retroreflector", because special scan instructions and range corrections are needed.]

To aim a feed arm rangefinder, elevation frame coordinates of its scan point are requested from the TIPPING STRUCTURE APPROXIMATE COORDINATE DATABASE. This data (including a flag to indicate that ranger-to-ranger measurements will be made) is passed to the FEEDARM RANGEFINDER ATTITUDE PROCESSOR, which computes scan mirror drive settings needed to aim each ranger to each of the other rangers' scan points. When flagged, it also computes mirror drive settings required to flip each ranger's scan mirror to anticollimation position, relative to each of the other feed arm rangers, and expose the attached prism as a range target. These requests are made prior to the start time t^0.

Each range to be measured needs correction parameters for distance calculations. To look up these parameters, the reference path cube prism depth $(D)_i$ and prism correction constant magnitude $|P_c|_i$, scan-mirror to reference-prism distance $(L_{spc})_i$ and scan-point to prism-fiducial-point distance $(H)_i$ for each feed arm ranger are requested from the COMPARISON TARGET RETROREFLECTOR AND COMMON MODE PATH DATABASE in the scan ranger's ZY-computer. A trigger signal is then sent to the LASER RANGE REDUCTION COMPUTATIONAL PROCESSOR to compute the nominal distance between the scan ranger's scan point and each target ranger's scan point. For each {scan ranger, target ranger} pair these values are entered as a data vector (together with a flag indicating ranger-to-ranger measurement) into a buffer file of initial input data for computation of range for that pair. The rangers are then commanded to aim, acquire electronic path phase, compute measured range, and pass the measured range output vector to a buffer file of unadjusted ranges. The measured scan-point to scan-point distances define the positions of the ranger scan points with respect to one another, for determination of the feed arm scan point control network.

To determine a surface target's position with respect to the feed arm control network, one performs trilaterations. There are 15 available combinations of 4 feed arm rangefinders to triangulate a surface target; 6 trilaterations using 5 rangefinders. A main-reflector surface prism target can be trilaterated with respect to the feed arm control network (this includes getting a standard error) only if one or more sets of 4 or more feed arm rangefinders have lines of sight to that target. Not all of the 600 paths between canonical surface targets and feed arm rangers are available. Visibility of each surface target from each feed arm ranger is time-independent, and needs determination only once. A lookup table of rangefinder sets available to trilaterate each target must be generated. This needs to be done only once; afterwards the table is always available.

The second step in our measurement program is to acquire ranges needed to locate the feed arm rangers' scan reference points with respect to the telescope's ground frame of reference, by scanning from feed arm lasers to benchmark retroreflectors on stable ground monuments (type ZBG targets), during the time interval $t^0 + t^{(1)}$ to $t^0 + t^{(2)}$. Visibility and incidence angle corrections depend on the current (stationary) values of EL_{com} and AZ_{com}. The ground benchmark targets visible to each feed arm rangefinder are computed by the "LINE OF SIGHT VISIBILITY COMPUTATIONAL PROCESSOR" in the ranger's ZY-computer. For each visible benchmark target, it computes the modular rangefinder nominal distance in integer modulation half wavelengths, $\mathcal{N}_{ij} \cdot (\lambda_{mod}/2)$, using the APPROXIMATE RANGE COMPUTATIONAL MODULE in the rangefinder's ZY-computer and also requests and computes the angle of incidence of the scan beam to the target. The target prism depth $(D)_j$ and prism correction constant amplitude $|P_c|_j$; look up the rangefinder reference prism depth $(D)_i$ and prism correction constant amplitude $|P_c|_i$ and scan mirror to reference prism distance $(L_{spc})_i$ are looked up in the available retroreflector data base. For each rangefinder, these pairs and the computed and lookup values are entered into a buffer file for computation of the benchmark target ranges to that rangefinder. This buffer file holds the a-priori range information parameters needed to compute rangefinder to benchmark target range, after rangefinder and reference path phases are measured. (The file is a data input file for the LASER RANGE REDUCTION COMPUTATIONAL PROCESSOR, code in the rangefinder's ZY-computer, which computes the measured range.)

The third step in the measurement program is to measure ranges from feed arm and ground rangers to telescope rim triplet retroreflector assemblies, during the time interval $t^0 + t^{(2)} < t < t^0 + t^{(3)}$. The pair of ranges, from each ranger to the two cube corner reflectors of a rim assembly, give data for computation of the distance from that ranger to the center of the triplet's ball retroreflector. (The computation is done in the range reduction processor of the rangefinder). Data output vectors for the reduced ranges from feed arm and ground rangers to the ball center of each rim triplet are then passed to the unadjusted measured range data buffer in the ZIY-computer. The ranges to rim ball centers, together with ranges from feed arm rangers to ground benchmark targets obtained in step two and ranges between feed arm rangers obtained in step one, are then least-squares-adjusted using the RANGE ADJUSTMENT CODES in the ZIY-computer. The range adjustment codes also compute ground frame coordinates of the ranger scan points and rim target reference points. The adjusted output ranges and ground frame coordinates are then passed to the ADJUSTED RANGE AND FIDUCIAL POINT COORDINATE DATA OUTPUT FILE BUFFER in the ZIY-computer.

The fourth step in the measurement program is to scan the surface retroreflectors. The feed arm rangers are commanded to scan the canonical set of surface targets during the time interval $t^0 + t^{(3)} < t < t^0 + t^{(4)}$. If each of 100 targets is scanned to trilaterate its position, one acquires between 400 to 600 ranges for each surface scan.

The range reduction processor is requested to compute the target ranges and send an output data vector: {ranger ID, rangefinder class ID, target ID, target class ID, measurement phase, comparison path phase, computed range, time, date, air refractive index, measurement program ID} to a measurement data output file in the UNADJUSTED MEASURED RANGES DATA BUFFER in the ZIY-computer.

The set of measured ranges of step four, together with feed arm ranger-to-ranger distances from step one and feed arm ranger reference point coordinates found in step three are used as input data for adjusting ground-frame coordinates of the surface target reference points. To do this a trigger signal requests execution of the range adjustment codes and transmission of the output data of the adjustment to the ADJUSTED RANGE AND FIDUCIAL POINT COORDINATE DATA OUTPUT FILE BUFFER and the RANGE DATA LOG ARCHIVE FILE.

After obtaining the main-reflector-surface target reference point coordinates, one may wish to convert each target's fiducial point coordinates to the coordinates of the main reflector surface point associated with that prism target. Mathematical expressions are available, together with data bases of nominal surface normal direction cosines and cube prism geometry parameters, which allow such conversions to be performed.

Required Calibration And Refractometer Scans.

Most of the measurements in the present scan program are feed arm ranger scans. During steps 1, 2 and 4 of the program, refractometer range scans and refractometer calibration scans will be requested between ground rangers and ground benchmark targets, independent of any feed arm scans. The refractometer scans can be made concurrent with and independent of the feed arm range scans. During step 4, requested refractometer calibration and range scans can interleave with requested ground ranger to rim ball up-scans.

Required {rangefinder, target} pairs.

Scans are made for the following sets of {rangefinder, target} pairs, in this measurement program:

Scan Sources	Scan Targets	Comments
$\{ZY_i\}_{i=13\cdots18}$	$\{ZMG_j \mid i \neq j\}$	Feed arm ranger-to-ranger scan.
$\{ZY_i\}_{i=13\cdots18}$	$\subset \{ZBG\}$	Feed arm ranger to ground benchmark targets.
$\{ZY_i\}_{i=13\cdots18}$	$\{ZUG_j\}_{j=1,\ldots,20}$	Down-scans to rim cube prisms.
$(ZY_i)_{i=1\cdots18}$	(ZRG_i)	Reference path calibrations.
$\{ZY\}_{1,\ldots,12}$	$\{ZBG_j \mid \mathrm{refr}(i,j)=1\}$	Refractometer paths.
$\{ZY\}_{1,\ldots,12}$	$\{ZDG_j \mid \mathrm{vis}(i,j)=1\}$	Up-scans to visible rim balls.
$\{ZY_i\}_{i=13\cdots18}$	$\{ZPG_j \mid \mathrm{vis}(i,j)=1\}$	Scan of main reflector surface.

Scan sequence.

The scan times below refer to mid-scan time of the individual scans, measured after start time t^0. That is, clock time is the time listed below, plus t^0.

Scan	Scan Subinterval, From t^0	Scan Interval
Feed arm reference path calibrations:		
$(ZY_i)_{13\cdots18} \rightarrow (ZRG_i)$	$0 \rightarrow \Delta t_{rs}$	$0 \rightarrow \Delta t_{ts}$
Scans between feed arm rangers.		
$ZY_{13} \rightarrow ZMG_{14\ldots18}$	$t^{(0,0,m)} \rightarrow t^{(0,0,m+1)} \;\; (m=0\ldots4)$	$\Delta t_{ts} \rightarrow t^{(0,1)}$
$ZY_{14} \rightarrow ZMG_{13,15\ldots18}$	$t^{(0,1)} + t^{(0,1,m)} \rightarrow t^{(0,1)} + t^{(0,1,m+1)}$	$t^{(0,1)} \rightarrow t^{(0,2)}$
$ZY_{15} \rightarrow ZMG_{13,14,16\ldots18}$	$t^{(0,2)} + t^{(0,2,m)} \rightarrow t^{(0,2)} + t^{(0,2,m+1)}$	$t^{(0,2)} \rightarrow t^{(0,3)}$
$ZY_{16} \rightarrow ZMG_{13\ldots15,17,18}$	$\cdots$	$t^{(0,3)} \rightarrow t^{(0,4)}$
$ZY_{17} \rightarrow ZMG_{13\ldots16,18}$	$\cdots$	$t^{(0,4)} \rightarrow t^{(0,5)}$
$ZY_{18} \rightarrow ZMG_{13\ldots17}$	$t^{(0,5)} + t^{(0,5,m)} \rightarrow t^{(0,2)} + t^{(0,5,m+1)}$	$t^{(0,5)} \rightarrow t^{(1)}$
Down-scans to benchmark retroreflectors:		
$\{ZY_i\}_{i=13\cdots18} \rightarrow \{ZBG_j \mid \mathrm{vis}(i,j)=1\}$		$t^{(1)} \rightarrow t^{(2)}$
Down-scans to rim cube retroreflectors:		
$\{ZY_i\}_{i=13\cdots18} \rightarrow \{ZUG_j\}_{j=1\cdots10}$	$t^{(1)} + 2jh - 2h \rightarrow t^{(1)} + 2jh - h$	$t^{(2)} \rightarrow t^{(3)}$
$\{ZY_i\}_{i=13\cdots18} \rightarrow \{ZUG_j\}_{j=11\cdots21}$	$t^{(1)} + 2jh - 21 \cdot h \rightarrow t^{(1)} + 2jh - 20 \cdot h$	$t^{(2)} \rightarrow t^{(3)}$
Ground reference path calibrations:		
$(ZY_i)_{1\cdots12} \rightarrow (ZRG_i)$	$t^{00}_{comp} + p \cdot \Delta t_{cy} \rightarrow t^{00}_{comp} + p \cdot \Delta t_{cy} + h$	$0 \rightarrow t^{(0,2)}$
Refractometer path measurement:		
$(ZY_i)_{1\cdots12} \rightarrow \{ZBG_j \mid \mathrm{refr}(i,j)=1\}$	$t^{00}_{refr} + p \cdot \Delta t_{cy} \rightarrow t^{00}_{refr} + p \cdot \Delta t_{cy} + h$	$0 \rightarrow t^{(0,2)}$
Visible rim ball up-scan.		
$(ZY_i)_{1\cdots12} \rightarrow \{ZDG_j \mid \mathrm{vis}(i,j)=1\}$		$t^{(2)} \rightarrow t^{(3)}$
Down-scan to main reflector surface:		
$\{ZY_i\}_{i=13\cdots18} \rightarrow \{ZPG_j \mid \mathrm{vis}(i,j)=1\}$		$t^{(3)} \rightarrow t^{(4)}$
Refractometer path measurement:		
$(ZY_i)_{1\cdots12} \rightarrow \{ZBG_j \mid \mathrm{refr}(i,j)=1\}$		$t^{(3)} \rightarrow t^{(4)}$.

The time interval, h , between successive scans of a feed arm rangefinder is partitioned as follows. After start of laser illumination by the scan rangefinder, it illuminates its target retroreflector for a time interval Δt_i. The integration time for a calibration reference scan, $(\Delta t_i)_{comp}$, should in general be the same as that for a range scan. After a buffer time interval Δt_{buf} the ranger's scan mirror drive is commanded to aim to the next target; aiming and settling of the scan mirror is allowed for a time interval Δt_{md} . After this interval, a permissive request for confirmation of aiming readiness is sent to the scanning and target rangefinders is initiated; an additional handshake time interval Δt_{hs} is allowed, to notify the ZIY-computer that the scan mirror and target are in place for ranging illumination, and to return permission to the scan ranger's ZY-computer to start target illumination. The free buffer time between scans available for other activities is: $\Delta t_{buf} = h - \Delta t_i - \Delta t_{md} - \Delta t_{hs}$.

The following values are suggested for these time intervals:

$$\Delta t_i \quad = \quad 64 \text{ milliseconds,}$$
$$(\Delta t_i)_{comp} \quad = \quad 64 \text{ milliseconds,}$$
$$\Delta t_{md} \quad = \quad 90 \text{ milliseconds,}$$
$$\Delta t_{hs} \quad = \quad 10 \text{ milliseconds,}$$
$$h \quad = \quad 200 \text{ milliseconds, giving}$$
$$\Delta t_{buf} \quad = \quad 36 \text{ milliseconds free.}$$

The duration of the scan subintervals $t^{(0,k,m+1)} - t^{(0,k,m)}$ should also be set equal to h.

The integration time, $(\Delta t_i)_{refr}$, for refractometer scans and the time interval, $(h)_{refr}$, between successive refractometer scans by ground rangefinders along refractometer paths can be made longer, because these scans can be concurrent with the feed arm inter-rangefinder scans.

The 6 feed arm reference path calibrations are done simultaneously. Using the values above, we can choose $\Delta_{ts} = 200$ milliseconds.

One can scan up to 3 distinct directed ranger-to-ranger paths simultaneously. It is not possible to scan all 30 paths in 10 scan periods, but is possible in 11 periods.

Using the notation $(i \rightarrow j)$ to represent a scan from ranger ZY_i to target ranger ZY_j , and $[(\ \ ),(\ \ ),(\ \ )]$ to denote three simultaneous scans, one possible scan sequence is the following, requiring 2200 milliseconds to complete:

$$[(13 \rightarrow 18), (16 \rightarrow 17), (14 \rightarrow 15)] \quad ,$$
$$[(13 \rightarrow 14), (15 \rightarrow 16), (17 \rightarrow 18)] \quad ,$$
$$[(13 \rightarrow 15), (15 \rightarrow 17), (14 \rightarrow 16)] \quad ,$$
$$[(13 \rightarrow 16), (15 \rightarrow 18), (14 \rightarrow 17)] \quad ,$$
$$[(13 \rightarrow 17), (15 \rightarrow 14), (16 \rightarrow 18)] \quad ,$$
$$[(18 \rightarrow 13), (17 \rightarrow 16), (14 \rightarrow 18)] \quad ,$$
$$[(14 \rightarrow 13), (16 \rightarrow 15), (18 \rightarrow 17)] \quad ,$$
$$[(15 \rightarrow 13), (17 \rightarrow 15), (16 \rightarrow 14)] \quad ,$$
$$[(16 \rightarrow 13), (18 \rightarrow 15), (17 \rightarrow 14)] \quad ,$$
$$[(17 \rightarrow 13), (18 \rightarrow 14), (15, 16 \rightarrow ?)] \quad ,$$
$$[(18 \rightarrow 16), (13, 14, 15, 17 \rightarrow ?)] \quad .$$

Visibility Computations And Data Handling.

For telescope attitude (AZ, EL), given a ranger reference point S_i and a scan target reference point T_j we express their mutual visibility by a 2-valued function of 4 variables: $\text{vis}(S_i, T_j; AZ, EL) = 1$ if they are mutually visible and $= 0$ if no line-of-sight exists between them. For certain ranger-target pairs, visibility does not depend on telescope orientation, it depends only on the scan endpoints. For such pairs, we use a modified visibility function: $\text{vis}(S_i, T_j) = 1$ if they are mutually visible and $= 0$ if no line-of-sight exists. As part of the scan program, the SCHEDULER FOR RETROREFLECTOR SCANS in each ZY_i computer generates a request to compute $\text{vis}(S_i, T_j; AZ, EL)$ for each target T_j scanned by a rangefinder, S_i. As part of the routine used to compute the 4-variable visibility function, the scan pair list is flagged whenever a {ranger,target} pair belongs to a class of scan pairs having constant visibility. If this is the case the pair's visibility value is obtained from a lookup table; otherwise visibility is computed using an appropriate geometric algorithm. In general, one uses commanded values AZ_{com}, EL_{com} when calling for computation of the visibility function.

For each telescope AZ, EL pair used in the scan program, the list of targets scanned by S_i is modified to include those of visibility 1 and exclude those of visibility 0.

Scan Sources	Scan Targets	Comments	Visibility
$\{ZY_i\}_{i=13\cdots18}$	$\{ZMG_j \mid i \neq j\}$	Between feed arm rangers.	$\text{vis}(ZY_i, ZMG_j) \equiv 1$
$\{ZY_i\}_{i=13\cdots18}$	$\{ZUG_j\}_{j=1,\ldots,20}$	Down-scan to rim prism.	$\text{vis}(ZY_i, ZUG_j) \equiv 1$
$(ZY_i)_{i=1\cdots18}$	(ZRG_i)	Reference path calibration.	$\text{vis}(ZY_i, ZRG_j) \equiv 1$
$\{ZY\}_{1,\ldots,12}$	$\{ZBG_j \mid \text{refr}(i,j) = 1\}$	Refractometer path.	$\text{vis}(ZY_i, ZB_j \mid \text{refr}(i,j) = 1)) \equiv 1$
$\{ZY\}_{1,\ldots,12}$	$\{ZDG_j \mid \text{vis}(i,j) = 1\}$	Up-scan to visible rim ball..	$\text{vis}(ZY_i, ZDG_j; EL, AZ))$ to be computed for each (EL, AZ).
$\{ZY_i\}_{i=13\cdots18}$	$\{ZPG_j \mid \text{vis}(i,j) = 1\}$	Main surface scan.	Target sets to be computed.

For surface cube prism retroreflectors, significant return signal is expected when the angle of incidence of the ray, from ranger to target, to the prism front face normal is $\simeq 25$ degrees or less. (Return versus incidence angle is reported in F. Schwab, GBT Memo No. 64).

For up-scans from ground rangefinders to main reflector rim balls the following criteria determine visibility. The ray from ranger scan point to ball center must not intersect the main reflector surface or the convex hull of the feed arm structure. The outward axis of the half-cone of visibility of the ball target points from the centroid of the feedarm ranger scan points to the ball center. The half-angle of the ball's forward half-cone of visibility is $\simeq 60$ degrees. The ray from the ground ranger's scan point to the ball center must lie inside this half-cone.

For benchmark target scans similar criteria apply. The ray from feed arm ranger to target must lie within the visibility half-cones of target and ranger, and not intersect the main reflector surface or convex hull of the feedarm structure.

Data Logging Protocols.

The result of a measurement is treated as a data list or a structure, whose members are strings or numbers containing the significant information relating to the measurement. The particular presentation, format, and notation for the measurement result can vary, but it should contain the following information (not necessarily in order given):

Data Entry	(Example)
Data Type	Unadjusted Range
Measurement Program	Static Check Scan Main Refl Surf
Rangefinder ID	ZY14
Rangefinder Type	Feed Arm.
Target Retroreflector ID	ZPG06+240
Target Type	ZPG_G
Range, Unadjusted (mm)	
Mid-Scan-Time (UTC)	22_37_55.158
Julian Date (and/or)	
Local Clock Time	
Calendar Date	11_07_99
Data Source	Range Reduction Processor
Data Destination	Unadjusted Measured Range Buffer

Another example is that of input data for range computations. The ZY_i-computer for a ranger holds the following rangefinder-specific data needed to compute range for a scan:

Data Entry	Comments
ZY_i	Ranger ID. (For range output data to ZIY).
$X_e(S_i)$	Approximate (elevation frame) coordinates of
$Y_e(S_i)$	ranger mirror's scan reference point (mm).
$Z_e(S_i)$	
$(D)_i$	Ranger reference prism depth (mm).
$(D_{sm})_i$	Ranger scan mirror's prism depth.
$\mid P_c \mid_i$	Magnitude of reference prism correction constant (mm).
$\mid P_{c,sm} \mid_i$	Magnitude of scan mirror prism correction constant.
$(L_{spc})_i$	Distance, scan mirror ref. point to ref. prism face.
H	Distance, scan prism face to scan mirror surface.
$\mathbf{I}_{ii}$	Calibration reference path cycle slip index.
$(D_{cmm})_i$	Approx. common mode optical path residue modulo $(\lambda_{\mathrm{mod}}/2)$.

Besides the rangefinder-specific information, the following target-related and scan-specific information is supplied:

Data Entry	Comments
Scan Flag	1=Ordinary Scan, 2=Ranger-Ranger, 3=Calibration Ref. Path.
T_j	Target retroreflector ID.
Target type	(Examples are: ZPG, ZRG, ZBG, ZUG, $\cdots$).
$X_e(T_j)$	Current approximate elevation frame
$Y_e(T_j)$	coordinates of target's fiducial point.(mm).
$Z_e(T_j)$	
$\mid P_c \mid_j$	Magnitude of target retroreflector correction constant (mm).
η	Group refractive index of air.
t^{scan}	Mid-scan measurement time (IRIG clock time, or equivalent).
Δt_i	Scan integration interval (sec).
$EL_{com}(t^{scan})$	Commanded telescope elevation at mid-time of scan.
I_{inc}	Incidence angle (radian).
Data Destination	Approximate Range Computational Module.

The incidence angle is zero for the cases of a ball (ZDG) retroreflector, a calibration scan, or a rangefinder-to-rangefinder scan. The approximate coordinates will depend on the tipping structure elevation assumed for the scan, and will be computed prior to generation of this list.

After pre-processing of this data to compute range correction terms, in the APPROXIMATE RANGE COMPUTATIONAL MODULE, the range correction terms are transferred as an input data list to the RANGE REDUCTION COMPUTATIONAL PROCESSOR.

Range Interpolation And Extrapolation.

For this static scan program, interpolation and extrapolation of measured range data are not required. One can perform trilaterations, range adjustments and coordinate determinations using ranges measured at arbitrary times, because the ranges do not vary with time.

Range Computation Algorithms.

For rangefinder to target distances compute:

$$\mathcal{N}_{ij} = \text{Int} \left\{ \frac{\eta \cdot \left[d_{ij}(AZ, EL) + |P_c|_j \right] + (D_{cmm})_i}{(\lambda_m/2)} \right\}$$

$$\mathbf{I}_{ii} = \text{Int} \left[(D_{cmm})_i + \frac{\eta \cdot (L_{spc})_i + \eta_{glass} \cdot (D)_i}{(\lambda_m/2)} \right]$$

The range distance is:

$$d_{ij} = \left(\mathcal{N}_{ij} + \frac{((\overline{\Delta\Phi})_{ii})_{\text{electr}} - ((\overline{\Delta\Phi})_{ij})_{\text{electr}}}{2\pi} - \mathbf{I}_{ii} \right) \cdot \left(\frac{\lambda_m}{2} \right)$$

$$- |P_c|_j - \Delta R(I_{in}) + (L_{spc})_i + n \cdot (D)_i \ .$$

For rangefinder to rangefinder distances compute:

$$\mathcal{N}_{ii'} = \text{Int} \left\{ \frac{\eta \cdot \left[d_{S_i S_{i'}}(AZ, EL) - (H)_{i'} + \frac{(D_{sm})_{i'}}{n} + |P_c|_{i'} \right] + (D_{cmm})_i}{(\lambda_m / 2)} \right\}$$

$$\mathbf{I}_{ii} = \text{Int} \left[(D_{cmm})_i + \frac{\eta \cdot (L_{spc})_i + \eta_{glass} \cdot (D)_i}{(\lambda_m / 2)} \right]$$

The range distance is:

$$|S_i S_{i'}| = \left(\mathcal{N}_{ii'} + \frac{((\overline{\Delta\Phi})_{ii})_{\text{electr}} - ((\overline{\Delta\Phi})_{ii'})_{\text{electr}}}{2\pi} - \mathbf{I}_{ii} \right) \cdot \left(\frac{\lambda_m}{2} \right)$$

$$+ \left[(H)_{i'} - \frac{(D_{sm})_{i'}}{n} \right] - |P_c|_{i'} + (L_{spc})_i + n \cdot (D)_i \ .$$

Approximate Rangefinder And Target Coordinates.

Carry out the following steps to calculate the approximate elevation frame coordinates of the rangefinder scan reference point and target fiducial reference point, and the approximate distance between them, $d_{ij}(AZ, EL)$, for commanded values of azimuth and elevation ($AZ = AZ_{com}$, $EL = EL_{com}$).

1. For each ZY_i feed arm rangefinder ($113 \leq i \leq 118$), look up the tabulated elevation frame coordinates $X_{eg}^0(S_i)$, $Y_{eg}^0(S_i)$, $Z_{eg}^0(S_i)$ for S_i corresponding to the reference (surface rigging angle) elevation in the "TIPPING STRUCTURE APPROX. COORDINATE DATABASE", when S_i is a node of the Finite Element Model. If it is not an FEM node, look up the reference coordinates $X_{eg}^0(\mathfrak{N}_i)$, $Y_{eg}^0(\mathfrak{N}_i)$, $Z_{eg}^0(\mathfrak{N}_i)$ of its associated nearby node $\mathfrak{N}_i$. Also look up (in this database) the displacement components of S_i from $\mathfrak{N}_i$: $\xi_{S_i, \mathfrak{N}_i} = \xi$, $\eta_{S_i, \mathfrak{N}_i} = \eta$, $\zeta_{S_i, \mathfrak{N}_i} = \zeta$.

2. For each ZY_i ground rangefinder ($101 \leq i \leq 112$), look up the (fixed) base frame coordinates $X(S_i)$, $Y(S_i)$, $Z(S_i)$ of S_i in the "GROUND RANGEFINDER & GROUND RETROREFLECTOR DATABASE".

3. If T_j is a ground benchmark target (type ZBG), look up the fixed base frame coordinates $X(T_j)$, $Y(T_j)$, $Z(T_j)$ of T_j in the "GROUND RANGEFINDER & GROUND RETROREFLECTOR DATABASE".

4. If T_j is a main-reflector-surface target (type ZPG), look up the reference coordinates $X_{eg}^0(T_j)$, $Y_{eg}^0(T_j)$, $Z_{eg}^0(T_j)$ corresponding to the surface rigging elevation in the "TIPPING STRUCTURE APPROX. COORDINATE DATABASE", for the surface actuator set at mid position. Also look up the differential displacement coordinates ξ_1, η_1, ζ_1 of T_j. Increment the reference coordinates by $\xi_1 \cdot L$, $\eta_1 \cdot L$, $\zeta_1 \cdot L$ respectively where L is the current surface actuator displacement of the target's associated node $\mathfrak{N}_j$. L is looked up in the "SURFACE ACTUATOR DATABASE".

5. If T_j is a main-reflector rim triplet cube prism target (type ZUG), look up the reference elevation frame coordinates for T_j corresponding to the reference (surface rigging angle) elevation in the "RIM RETROREFLECTOR TRIPLET DATABASE".

6. If T_j is a main-reflector-rim ball target (type ZDG), look up the reference elevation frame coordinates for T_j corresponding to the reference (surface rigging) elevation in the "RIM RETROREFLECTOR TRIPLET DATABASE".

7. Convert base frame coordinates of ground benchmark (ZBG) targets $X(T_j)$, $Y(T_j)$, $Z(T_j)$ $\rightarrow$ $X_{eg}(T_j; AZ, EL)$, $Y_{eg}(T_j; AZ, EL)$, $Z_{eg}(T_j; AZ, EL)$ and ground ($101 \leq i \leq 112$) rangefinders $X(S_i)$, $Y(S_i)$, $Z(S_i)$ $\rightarrow$ $X_{eg}(S_i; AZ, EL)$, $Y_{eg}(S_i; AZ, EL)$, $Z_{eg}(S_i; AZ, EL)$ to elevation frame coordinates by the transformation:

$$\begin{bmatrix} X_{eg} \\ Y_{eg} \\ Z_{eg} \end{bmatrix} = \begin{bmatrix} X \cdot \cos AZ - Y \cdot \sin AZ \\ (X \cdot \sin AZ + Y \cdot \cos AZ) \cdot \sin EL + (h_e - Z) \cdot \cos EL \\ (X \cdot \sin AZ + Y \cdot \cos AZ) \cdot \cos EL - (h_e - Z) \cdot \sin EL \end{bmatrix}.$$

Here $h_e = 48.2600$ meter. The coordinates obtained by this transformation are the elevation frame coordinates for the ground-fixed reference points, and need no additional corrections.

8. For nodes associated with feed arm scan reference points or main reflector surface or main reflector rim targets or other tipping structure targets, compute the node displacement vector and the node joint rotation vector. To do these operations first look up the twelve Finite Element Model matrix elements for each of these nodes in the "FINITE ELEMENT MODEL DATABASE". (These are the ZDelta's, HDelta's, ZTilt's and HTilt's). For each tipping structure target or scan-reference-point associated node $\mathfrak{N}$, and also the node (#1000) for the elevation axis midpoint E_g compute

$$
\begin{bmatrix} \Delta X_{eg}(\mathfrak{N}) \\ \Delta Y_{eg}(\mathfrak{N}) \\ \Delta Z_{eg}(\mathfrak{N}) \end{bmatrix} = \begin{bmatrix} ZDeltaX & HDeltaX \\ ZDeltaY & HDeltaY \\ ZDeltaZ & HDeltaZ \end{bmatrix}_{\mathfrak{N}} \cdot \begin{bmatrix} \sin EL - \sin EL_{surf\_rig} \\ \cos EL - \cos EL_{surf\_rig} \end{bmatrix} .
$$

For surface rigging reference elevation $50.8° = EL_{surf\_rig}$

$$
\begin{bmatrix} \sin EL - \sin EL_{surf\_rig} \\ \cos EL - \cos EL_{surf\_rig} \end{bmatrix} = \begin{bmatrix} \sin EL - 0.7749445 \\ \cos EL - 0.6320293 \end{bmatrix} .
$$

Then compute:

$$
\begin{bmatrix} t_x(EL) \\ t_y(EL) \\ t_z(EL) \end{bmatrix}_{\mathfrak{N}} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}_{\mathfrak{N}} =
$$

$$
\left[\begin{bmatrix} ZTiltX & HTiltX \\ ZTiltY & HTiltY \\ ZTiltZ & HTiltZ \end{bmatrix}_{\mathfrak{N}} - \begin{bmatrix} ZTiltX & HTiltX \\ ZTiltY & HTiltY \\ ZTiltZ & HTiltZ \end{bmatrix}_{E_g} \right] \cdot \begin{bmatrix} \sin EL - \sin EL_{surf\_rig} \\ \cos EL - \cos EL_{surf\_rig} \end{bmatrix} .
$$

9. Finally, compute the elevation frame coordinates for tipping structure reference points:

$$
\begin{bmatrix} X_{eg}(S_i; EL) \\ Y_{eg}(S_i; EL) \\ X_{eg}(S_i; EL) \end{bmatrix} = \begin{bmatrix} X_{eg}^0(S_i) \\ Y_{eg}^0(S_i) \\ Z_{eg}^0(S_i) \end{bmatrix} + \begin{bmatrix} \Delta X_{eg} \\ \Delta Y_{eg} \\ \Delta X_{eg} \end{bmatrix}_{\mathfrak{N}_i} + \begin{bmatrix} t_y \cdot \zeta - t_z \cdot \eta \\ t_z \cdot \xi - t_z \cdot \zeta \\ t_x \cdot \eta - t_y \cdot \xi \end{bmatrix}_{\mathfrak{N}_i, S_i} .
$$

$$
\begin{bmatrix} X_{eg}(T_j; AZ, EL) \\ Y_{eg}(T_j; AZ, EL) \\ X_{eg}(T_j; AZ, EL) \end{bmatrix} = \begin{bmatrix} X_{eg}^0(T_j) \\ Y_{eg}^0(T_j) \\ Z_{eg}^0(T_j) \end{bmatrix} + \begin{bmatrix} \Delta X_{eg} \\ \Delta Y_{eg} \\ \Delta X_{eg} \end{bmatrix}_{\mathfrak{N}_j} + \begin{bmatrix} t_y \cdot \zeta - t_z \cdot \eta \\ t_z \cdot \xi - t_z \cdot \zeta \\ t_x \cdot \eta - t_y \cdot \xi \end{bmatrix}_{\mathfrak{N}_j, S_j} .
$$

Feed Arm Rangefinder Aiming Coordinates.

1. Look up the elements of the rangefinder platform direction cosine matrices $[A]_{ZY_i}$ $(113 \leq i \leq 118)$ in the "FEEDARM LASER PLATFORM DIRECTION COSINE DATABASE".

2. For target-rangefinder pair $\{T_j , S_i\}$, compute the displacement vector from the rangefinder scan reference point to target fiducial point:

$$\begin{bmatrix} X_{eg}(j,i; AZ , EL) \\ Y_{eg}(j,i; AZ , EL) \\ Z_{eg}(j,i; AZ , EL) \end{bmatrix} = \begin{bmatrix} X_{eg}(T_j; AZ , EL) - X_{eg}(S_i; EL) \\ Y_{eg}(T_j; AZ , EL) - Y_{eg}(S_i; EL) \\ Z_{eg}(T_j; AZ , EL) - Z_{eg}(S_i; EL) \end{bmatrix}.$$

3. Compute the distance $d_{ij}(AZ , EL)$ $(= d_{ij}$ in abbreviated notation):

$$d_{ij} = \sqrt{X_{eg}^2(j,i; AZ , EL) + Y_{eg}^2(j,i; AZ , EL) + Z_{eg}^2(j,i; AZ , EL)}.$$

4. Compute local rangefinder platform cartesian coordinates (with origin at S_i) of target point T_j. (Use the t-components for S_i).

$$\widetilde{x}_{T_j} = X_{eg}(j,i; AZ , EL) \cdot (A_{11} + t_y \cdot A_{13} - t_z \cdot A_{12})$$

$$+ Y_{eg}(j,i; AZ , EL) \cdot (A_{12} + t_z \cdot A_{11} - t_x \cdot A_{13}) + Z_{eg}(j,i; AZ , EL) \cdot (A_{13} + t_x \cdot A_{12} - t_y \cdot A_{11})$$

$$\widetilde{y}_{T_j} = Z_{eg}(j,i; AZ , EL) \cdot (A_{21} + t_y \cdot A_{23} - t_z \cdot A_{22})$$

$$+ Y_{eg}(j,i; AZ , EL) \cdot (A_{22} + t_z \cdot A_{21} - t_x \cdot A_{23}) + Z_{eg}(j,i; AZ , EL) \cdot (A_{23} + t_x \cdot A_{22} - t_y \cdot A_{21})$$

$$\widetilde{z}_{T_j} = X_{eg}(j,i; AZ , EL) \cdot (A_{31} + t_y \cdot A_{33} - t_z \cdot A_{32})$$

$$+ Y_{eg}(j,i; AZ , EL) \cdot (A_{32} + t_z \cdot A_{31} - t_x \cdot A_{33}) + Z_{eg}(j,i; AZ , EL) \cdot (A_{33} + t_x \cdot A_{32} - t_y \cdot A_{31})$$

5. Compute the beam aiming angles of target T_j from rangefinder ZY_i.

$$\Phi (i,j; AZ , EL) = \cos^{-1} \left(\frac{\widetilde{z}_{T_j}}{d_{ij}} \right).$$

$$\Theta (i,j; AZ , EL) = (\text{atan}_2) \left(\frac{\widetilde{y}_{T_j}}{\widetilde{x}_{T_j}} \right).$$