GBT MEMO 217

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Range Scan Program

Rangefinder Aiming

Theory:

Here we give computations required to find fiducial point positions at arbitrary elevation and azimuth of the tipping structure. The analysis and notation follow that of GBT Memo 165, M. Goldman, "GBT Coordinates And Coordinate Transformations," February 1997. Design distance parameters and transformation matrices appear in an appendix to that memo. The telescope is to be set to command azimuth AZ_{com} and command elevation angle EL_{com}. We first calculate positions of fiducial reference points of targets and rangefinder scan reference points on the tipping structure, with respect to the geometric elevastion reference frame of the telescope, when the tipping structure is at elevation EL_{com}. Subsequently, we find the local aiming angles to aim from feed arm rangefinders to target retroreflectors on the tipping structure or the main reflector surface. We also find the local aiming angles to aim from feed arm rangefinders to benchmark retroreflectors fixed to ground monuments.

Let point A be the origin of the ground coordinate reference frame of the telescope. Let the orthogonal unit reference vectors of the ground frame be $\widehat{X}, \widehat{Y}, \widehat{Z}$ where $\widehat{Z}$ is directed upwards along the local gravity vertical at A and $\widehat{Y}$ is directed horizontally North.

Let point E_g be the midpoint of the elevation axis. It is assumed to be at

(A 1.0) $$\overrightarrow{E}_g = \overrightarrow{A} + h_e\widehat{Z} \,,$$

independent of the telescope elevation angle and azimuth.

In terms of the unit vectors: $\widehat{X}_{eg}(AZ_{com}\,,\, EL_{com})$, $\widehat{Y}_{eg}(AZ_{com}\,,\, EL_{com})$, $\widehat{Z}_{eg}(AZ_{com}\,,\, EL_{com})$ which define the *geometric elevation reference frame* with origin at $\overrightarrow{E}_g$ the optically significant points of the geometric telescope are, with respect to the ground frame of reference:

(A 2.0) $\overrightarrow{E}_g = (0)\widehat{X}_{eg} + (0)\widehat{Y}_{eg} + (0)\widehat{Z}_{eg} + h_e\widehat{Z}$, the elevation axis midpoint (node #1000),

(A 2.1) $\overrightarrow{R}_g = (0)\widehat{X}_{eg} + (-d_{re})\widehat{Y}_{eg} + (h_{re})\widehat{Z}_{eg} + h_e\widehat{Z}$, the vertex of the parent paraboloid (node #100000),

(A 2.2) $\overrightarrow{P}_g = (0)\widehat{X}_{eg} + (-d_{re})\widehat{Y}_{eg} + (h_{pe})\widehat{Z}_{eg} + h_e\widehat{Z}$, the prime focus point of the parent paraboloid (node #50000),

(A 2.3) $\overrightarrow{M}_g = (0)\widehat{X}_{eg} + (-d_{re} - d_{mp})\widehat{Y}_{eg} + (h_{pe} - h_{mp})\widehat{Z}_{eg} + h_e\widehat{Z}$, the gregorian focus point of the geometric telescope (node #40700),

(A 2.4) $\overrightarrow{S}_g = (0)\widehat{X}_{eg} + (-d_{re} - d_{sp})\widehat{Y}_{eg} + (h_{pe} + h_{sp})\widehat{Z}_{eg} + h_e\widehat{Z}$, the mid-ray point of the ellipsoidal subreflector surface (node #50005).

(A 2.5) $\overrightarrow{CE}_g = (0)\widehat{X}_{eg} + (-d_{re} - (\frac{1}{2})d_{mp})\widehat{Y}_{eg} + (h_{pe} - (\frac{1}{2})h_{mp})\widehat{Z}_{eg} + h_e\widehat{Z}$, the center point of the subreflector ellipsoid.

When the tipping structure is at command azimuth and elevation, the matrix of direction cosines relating the unit basis vectors of the elevation and ground frames is:
(A 3.0)

$$\begin{array}{lll}
\widehat{X}_{eg} \cdot \widehat{X} = \cos AZ_{com} & \widehat{Y}_{eg} \cdot \widehat{X} = \sin AZ_{com} \cdot \sin EL_{com} & \widehat{Z}_{eg} \cdot \widehat{X} = \sin AZ_{com} \cdot \cos EL_{com} \\
\widehat{X}_{eg} \cdot \widehat{Y} = -\sin AZ_{com} & \widehat{Y}_{eg} \cdot \widehat{Y} = \cos AZ_{com} \cdot \sin EL_{com} & \widehat{Z}_{eg} \cdot \widehat{Y} = \cos AZ_{com} \cdot \cos EL_{com} \\
\widehat{X}_{eg} \cdot \widehat{Z} = 0 & \widehat{Y}_{eg} \cdot \widehat{Z} = -\cos EL_{com} & \widehat{Z}_{eg} \cdot \widehat{Z} = \sin EL_{com} .
\end{array}$$

The design values of the telescope distances, in meters, are:

$$
\text{(A 4.0)} \quad
\begin{array}{lcll}
h_e & = & 48.26000\,m & (1900.0000") \\
h_{re} & = & 04.99999 & (\ 196.8500") \\
h_{pe} & = & 64.99999 & (2559.0547") \\
h_{rp} & = & 60.00000 & (2362.2047") \\
d_{re} & = & 54.83911 & (2159.0158") \\
h_{mp} & = & 10.94806 & (\ 431.0260") \\
d_{mp} & = & 01.06768 & (\ 042.0346") \\
h_{sp} & = & 03.80287 & (\ 149.7195") \\
d_{sp} & = & 04.29173 & (\ 168.9656") \\
|P_g\ M_g\ | & = & 11.00000 & (\ 433.0709") \\
|P_g\ S_g\ | & = & 15.09916 & (\ 594.4551") \\
|P_g\ CE_g| & = & 05.50000 & (\ 216.5354")\,.
\end{array}
$$

We wish to compute the change of position of target and rangefinder fiducial reference points, *relative to the geometric elevation frame of reference*, caused by change of gravity loading on the telescope, when tipping structure elevation changes from the reference rigging angle $EL_{surf\_rig} = 50.8°$ to the command elevation angle EL_{com}, using the current finite element structural model of the tipping structure.

When the telescope moves from reference elevation EL_{surf_rig} to elevation EL_{com} the structural nodes undergo strain shift displacement and joint rotation.

If the geometric elevation frame coordinates of tipping structure node $\mathfrak{N}$ are:

$$
\text{(A 5.0)} \quad
\begin{array}{lcl}
X_{eg}(\mathfrak{N};\ EL_{surf\_rig}) & \equiv & X^{\circ}_{eg}(\mathfrak{N}) \\
Y_{eg}(\mathfrak{N};\ EL_{surf\_rig}) & \equiv & Y^{\circ}_{eg}(\mathfrak{N}) \\
Z_{eg}(\mathfrak{N};\ EL_{surf\_rig}) & \equiv & Z^{\circ}_{eg}(\mathfrak{N})\ ,
\end{array}
$$

when the tipping structure is at the reference rigging elevation, the initial position vector of $\mathfrak{N}$ with respect to the ground frame origin is:

$$
\text{(A 5.1)} \quad \overrightarrow{X^{o}}(\mathfrak{N};\ EL_{surf\_rig}) = h_e \cdot \widehat{Z} + \overrightarrow{X^{o}}_{eg}(\mathfrak{N};\ EL_{surf\_rig})\ , \quad \text{where}
$$

$$
\text{(A5.1.1)} \quad \overrightarrow{X^{o}}_{eg}(\mathfrak{N};\ EL) \equiv X^{\circ}_{eg}(\mathfrak{N}) \cdot \widehat{X}_{eg}(\mathfrak{N};\ EL) + Y^{\circ}_{eg}(\mathfrak{N}) \cdot \widehat{Y}_{eg}(\mathfrak{N};\ EL) + Z^{\circ}_{eg}(\mathfrak{N}) \cdot \widehat{Z}_{eg}(\mathfrak{N};\ EL)\ ,
$$

The reference rigging angle node coordinates $X^{\circ}_{eg}(\mathfrak{N})$, $Y^{\circ}_{eg}(\mathfrak{N})$, $Z^{\circ}_{eg}(\mathfrak{N})$ are looked up in the "TIPPING STRUCTURE APPROX. COORDINATES DATABASE". They are constants, but can be updated as knowledge of the telescope evolves.

By moving the tipping structure to elevation EL_{com}, the position of node $\mathfrak{N}$ moves to a point with geometric elevation frame coordinates

$$\text{(A 5.2)} \quad \overrightarrow{X}(\mathfrak{N};\ EL_{com}) = h_e \cdot \widehat{Z} + \overrightarrow{X^{\circ}_{eg}}(\mathfrak{N};\ EL_{com}) + \{\Delta X_{eg}(\mathfrak{N}) - \Delta X_{eg}(E_g)\}\widehat{X}_{eg}$$

$$+\{\Delta Y_{eg}(\mathfrak{N}) - \Delta Y_{eg}(E_g)\}\widehat{Y}_{eg} + \{\Delta Z_{eg}(\mathfrak{N}) - \Delta Z_{eg}(E_g)\}\widehat{Z}_{eg}$$

where the geometric elevation frame basis vectors in the equation above have been rotated to correspond to the command elevation, as given by (A 3.0). The node's coordinate displacement components are given by the finite element model of the tipping structure:

$$\text{(A 6.0)} \quad \begin{bmatrix} \Delta X_{eg}(\mathfrak{N}) \\ \Delta Y_{eg}(\mathfrak{N}) \\ \Delta Z_{eg}(\mathfrak{N}) \end{bmatrix} = \begin{bmatrix} ZDeltaX & HDeltaX \\ ZDeltaY & HDeltaY \\ ZDeltaZ & HDeltaZ \end{bmatrix}_{\mathfrak{N}} \begin{bmatrix} \sin EL_{com} - \sin EL_{surf\_rig} \\ \cos EL_{com} - \cos EL_{surf\_rig} \end{bmatrix}.$$

The node matrix on the right side is given by the finite element model. Matrix elements are looked up in the "FINITE ELEMENT MODEL DATABASE".

The joint at the node is also rotated. Its rotation vector, relative to that at E_g, is also obtained from the finite element model:

$$\text{(A 6.1)} \quad \begin{bmatrix} t_x(EL_{com}) \\ t_y(EL_{com}) \\ t_z(EL_{com}) \end{bmatrix}_{\mathfrak{N}} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}_{\mathfrak{N}} =$$

$$\left[\begin{bmatrix} ZTiltX & HTiltX \\ ZTiltY & HTiltY \\ ZTiltZ & HTiltZ \end{bmatrix}_{\mathfrak{N}} - \begin{bmatrix} ZTiltX & HTiltX \\ ZTiltY & HTiltY \\ ZTiltZ & HTiltZ \end{bmatrix}_{E_g} \right] \cdot \begin{bmatrix} \sin EL_{com} - \sin EL_{surf\_rig} \\ \cos EL_{com} - \cos EL_{surf\_rig} \end{bmatrix}$$

where

$$\text{(A 6.2.1)} \qquad [\text{Rot } (EL_{com})]_{\mathfrak{N}} = t_x\widehat{X}_{eg} + t_y\widehat{Y}_{eg} + t_z\widehat{Z}_{eg} \;=\; \mathbf{t}_{\mathfrak{N}}$$

is a clockwise rotation about the direction of $\mathbf{t}_{\mathfrak{N}}$, by t radians, where

(A 6.2.1) $\qquad t = | \, \mathbf{t}_{\mathfrak{N}}| = \sqrt{t_x^2 + t_y^2 + t_z^2}\ .$

The matrix elements in (A6.2.1) are also looked up from the "FINITE ELEMENT MODEL DATABASE".

For 50.8° rigging elevation and commanded elevation EL_{com},

$$\text{(A 6.3)} \quad \begin{bmatrix} \sin EL_{com} - \sin EL_{surf\_rig} \\ \cos EL_{com} - \cos EL_{surf\_rig} \end{bmatrix} = \begin{bmatrix} \sin EL_{com} - 0.7749445 \\ \cos EL_{com} - 0.\ 6320293 \end{bmatrix} .$$

The points which we wish to locate in space are not nodes of the tipping structure but are reference points which, in a broad sense, are tied rigidly to the tipping structure. A fiducial reference point, $\mathfrak{F}$, is specified by giving its displacement, $\overrightarrow{D}(\mathfrak{F};\, \mathfrak{N})$, from a nearby structural node, $\mathfrak{N} = \mathfrak{N}(\mathfrak{F})$, with respect to the geometric elevation frame when the tipping structure is at the rigging elevation. When telescope elevation shifts to EL_{com}, the displacement vector of the fiducial point from its nearby node is rotated by $\mathbf{t}_{\mathfrak{N}}$ with respect to the elevation frame coordinate frame.

To make this precise, let us assume that an optical target whose center point is defined to be the fiducial reference point $\mathfrak{F}$ is cemented to the tipping structure near structural node point $\mathfrak{N}$. We also assume that the initial displacement vector from $\mathcal{N}$ to $\mathcal{F}$ is given, at the rigging angle elevation, by the relation

$$\text{(A 6.4)} \quad \begin{bmatrix} X^{\circ}_{eg}(\mathfrak{F}) \\ Y^{\circ}_{eg}(\mathfrak{F}) \\ Z^{\circ}_{eg}(\mathfrak{F}) \end{bmatrix} = \begin{bmatrix} X^{\circ}_{eg}(\mathfrak{N}) + \xi_{\mathfrak{F},\mathfrak{N}} \\ Y^{\circ}_{eg}(\mathfrak{N}) + \eta_{\mathfrak{F},\mathfrak{N}} \\ Z^{\circ}_{eg}(\mathfrak{N}) + \zeta_{\mathfrak{F},\mathfrak{N}} \end{bmatrix} = \begin{bmatrix} X^{\circ}_{eg}(\mathfrak{N}) \\ Y^{\circ}_{eg}(\mathfrak{N}) \\ Z^{\circ}_{eg}(\mathfrak{N}) \end{bmatrix} + \overrightarrow{D}(\mathfrak{F};\, \mathfrak{N}) \quad \text{where}$$

$$\text{(A 6.5)} \qquad \overrightarrow{D}(\mathfrak{F};\, \mathfrak{N}) = \xi \cdot \widehat{X}_{eg} + \eta \cdot \widehat{Y}_{eg} + \zeta \cdot \widehat{Z}_{eg}\, .$$

For simplicity of notation we omit the explicit dependence of the displacement components on the node and fiducial point.

The component values $\xi_{\mathfrak{F},\mathfrak{N}} = \xi$, $\eta = \eta_{\mathfrak{F},\mathfrak{N}}$, $\zeta = \zeta_{\mathfrak{F},\mathfrak{N}}$ are, in general, looked up in the "FINITE ELEMENT MODEL DATABASE," except for the case of surface actuator retroreflector targets. For surface panel retroreflectors the components vary linearly with the extension of the surface panel actuator at the node from its reference position

(A6.5.1) $\overrightarrow{D}(\mathfrak{F};\ \mathfrak{N}) = (\xi_0 + \xi_1 \cdot L_{\mathfrak{N}}) \cdot \widehat{X}_{eg} + (\eta_0 + \eta_1 \cdot L_{\mathfrak{N}}) \cdot \widehat{Y}_{eg} + (\zeta_0 + \zeta_1 \cdot L_{\mathfrak{N}}) \cdot \widehat{Z}_{eg}\,.$

For surface targets, the six displacement parameters and the surface actuator elongation are looked up in the "SURFACE ACTUATOR DATABASE".
For surface actuator targets we rewrite (A6.4) as:

(A 6.6) $\quad \overrightarrow{X^0}(\mathfrak{F}) = \overrightarrow{X^0}(\mathfrak{N}) + \overrightarrow{D^0}(\mathfrak{F};\ \mathfrak{N}) + L_{\mathfrak{N}} \cdot \overrightarrow{D^1}(\mathfrak{F};\ \mathfrak{N})\ .$

When the tipping structure moves to elevation angle EL_{com}, $\mathfrak{F}$ moves to

(A 6.7) $\quad \overrightarrow{X}(\mathfrak{F};\ EL_{com}) \ = \ \overrightarrow{X}(\mathfrak{N};\ EL_{com}) + [Rot(EL_{com})]_{\mathfrak{N}} \cdot \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N})\ .$

Using (A 5.2) we get

(A 6.8)
$$\overrightarrow{X}(\mathfrak{F};\ EL_{com}) = \overrightarrow{X^{\circ}}(\mathfrak{F}) + \{\ [\mathrm{Rot}\ (EL_{com})]_{\mathfrak{N}} \cdot \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N}) - \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N})\ \}$$
$$+\ \{\Delta X_{eg}(\mathfrak{N}) - \Delta X_{eg}(E_g)\}\widehat{X}_{eg} + \{\Delta Y_{eg}(\mathfrak{N}) - \Delta Y_{eg}(E_g)\}\widehat{Y}_{eg}$$
$$+\ \{\Delta Z_{eg}(\mathfrak{N}) - \Delta Z_{eg}(E_g)\}\widehat{Z}_{eg}\ .$$

For $0<t<<1$ radian the term in braces can be shown to be:

(A 6.9) $\quad [\mathrm{Rot}\ (EL_{com})]_{\mathfrak{N}} \cdot \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N}) - \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N})\ =$

$$\left[\left(1-\frac{t^2}{6}\right)(t_y\zeta - t_z\eta) + \left(\frac{t_x}{2}\right)(t_x\xi + t_y\eta + t_z\zeta) - (\xi)\left(\frac{t^2}{2}\right)\right]\widehat{X}_{eg} +$$
$$\left[\left(1-\frac{t^2}{6}\right)(t_z\xi - t_x\zeta) + \left(\frac{t_y}{2}\right)(t_x\xi + t_y\eta + t_z\zeta) - (\eta)\left(\frac{t^2}{2}\right)\right]\widehat{Y}_{eg} +$$
$$\left[\left(1-\frac{t^2}{6}\right)(t_x\eta - t_y\xi) + \left(\frac{t_z}{2}\right)(t_x\xi + t_y\eta + t_z\zeta) - (\zeta)\left(\frac{t^2}{2}\right)\right]\widehat{Z}_{eg}\ .$$

Small joint rotations occur, and we may use, with negligible error:

(A6.9.1) $\quad [\mathrm{Rot}\ (EL_{com})]_{\mathfrak{N}} \cdot \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N}) - \overrightarrow{D}(\mathfrak{F};\ \mathfrak{N})\ =$

$$(t_y \cdot \zeta - t_z \cdot \eta)\widehat{X}_{eg} + (t_z \cdot \xi - t_x \cdot \zeta)\widehat{Y}_{eg} + (t_x \cdot \eta - t_y \cdot \xi)\widehat{Z}_{eg}.$$

We remember here that the rotation vector is associated with node $\mathfrak{N}$, and the displacement vector is associated with reference fiducial point $\mathfrak{F}$ and node $\mathfrak{N}$.

With respect to the ground frame of reference, when the telescope's tipping structure is moved to the commanded elevation from the rigging elevation, gravity loading on the telescope causes a shift of position of fiducial point $\mathfrak{F}$ from its arrival point as calculated by the geometric telescope model. The position shift of $\mathfrak{F}$ is given in ground reference frame coordinates by

$$\text{(A 6.10)} \quad \begin{aligned} \Delta X(\mathfrak{F}) &= \left(\overrightarrow{X}(\mathfrak{F};\, EL_{com}) - \overrightarrow{X^{\circ}}(\mathfrak{F})\right) \cdot \widehat{X} \\ \Delta Y(\mathfrak{F}) &= \left(\overrightarrow{X}(\mathfrak{F};\, EL_{com}) - \overrightarrow{X^{\circ}}(\mathfrak{F})\right) \cdot \widehat{Y} \\ \Delta Z(\mathfrak{F}) &= \left(\overrightarrow{X}(\mathfrak{F};\, EL_{com}) - \overrightarrow{X^{\circ}}(\mathfrak{F})\right) \cdot \widehat{Z}\ . \end{aligned}$$

The numerical values can be calculated explicitly from the finite element model coefficients by using (A 6.0), $\cdots$, (A 6.8), and inserting the values

$$\widehat{X}_{eg} \cdot \widehat{X} = \cos AZ_{com} \qquad \widehat{Y}_{eg} \cdot \widehat{X} = \sin AZ_{com} \cdot \sin EL_{com} \qquad \widehat{Z}_{eg} \cdot \widehat{X} = \sin AZ_{com} \cdot \cos EL_{com}$$

$$\widehat{X}_{eg} \cdot \widehat{Y} = -\sin AZ_{com} \qquad \widehat{Y}_{eg} \cdot \widehat{Y} = \cos AZ_{com} \cdot \sin EL_{com} \qquad \widehat{Z}_{eg} \cdot \widehat{Y} = \cos AZ_{com} \cdot \cos EL_{com}$$

$$\widehat{X}_{eg} \cdot \widehat{Z} = 0 \qquad \widehat{Y}_{eg} \cdot \widehat{Z} = -\cos EL_{com} \qquad \widehat{Z}_{eg} \cdot \widehat{Z} = \sin EL_{com}$$

into (A 3.0).

We note that these computations of tipping structure fiducial and node point shifts are made assuming that the telescope's azimuth axis does not move, when the antenna tilts from access elevation to surface rigging elevation.

When fiducial point $\mathfrak{F}$ is a retroreflector target point T_j, having associated reference node $\mathfrak{N}_j = \mathfrak{N}\,(T_j)$, and which is on the tipping structure or main reflector surface, we compute:

$$\text{(A6.11)} \quad \begin{bmatrix} X_{eg}(T_j\,;\, EL_{com}) \\ Y_{eg}(T_j\,;\, EL_{com}) \\ Z_{eg}(T_j\,;\, EL_{com}) \end{bmatrix} = \begin{bmatrix} X^{o}_{eg}(T_j) \\ Y^{o}_{eg}(T_j) \\ Z^{o}_{eg}(T_j) \end{bmatrix} \quad +$$

$$\begin{bmatrix} ZDeltaX & HDeltaX \\ ZDeltaY & HDeltaY \\ ZDeltaZ & HDeltaZ \end{bmatrix}_{T_j} \begin{bmatrix} \sin EL_{com} - \sin EL_{surf\_rig} \\ \cos EL_{com} - \cos EL_{surf\_rig} \end{bmatrix} + \begin{bmatrix} (t_y \cdot \zeta - t_z \cdot \eta)_{T_j, \mathfrak{N}_j} \\ (t_z \cdot \xi - t_x \cdot \zeta)_{T_j, \mathfrak{N}_j} \\ (t_x \cdot \eta - t_y \cdot \xi)_{T_j, \mathfrak{N}_j} \end{bmatrix}.$$

When point $\mathfrak{F}$ is a rangefinder scan point S_i having an associated reference node $\mathfrak{N}_i = \mathfrak{N}(S_i)$, and which is on the tipping structure or main reflector surface, we compute:

$$\text{(A6.12)} \quad \begin{bmatrix} X_{eg}(S_i;\ EL_{com}) \\ Y_{eg}(S_i;\ EL_{com}) \\ Z_{eg}(S_i;\ EL_{com}) \end{bmatrix} = \begin{bmatrix} X^o_{eg}(S_i) \\ Y^o_{eg}(S_i) \\ Z^o_{eg}(S_i) \end{bmatrix} +$$

$$\begin{bmatrix} ZDeltaX & HDeltaX \\ ZDeltaY & HDeltaY \\ ZDeltaZ & HDeltaZ \end{bmatrix}_{S_i} \begin{bmatrix} \sin EL_{com} - \sin EL_{surf\_rig} \\ \cos EL_{com} - \cos EL_{surf\_rig} \end{bmatrix} + \begin{bmatrix} (t_y \cdot \zeta - t_z \cdot \eta)_{S_i, \mathfrak{N}i} \\ (t_z \cdot \xi - t_x \cdot \zeta)_{S_i, \mathfrak{N}_i} \\ (t_x \cdot \eta - t_y \cdot \xi)_{S_i, \mathfrak{N}_i} \end{bmatrix}.$$

At elevation angle EL_{com} the displacement vector to the target fiducial reference point T_j from the scan point S_i is:

$$\text{(A6.13)} \quad \begin{bmatrix} X_{eg}(j, i;\ EL_{com}) \\ Y_{eg}(j, i;\ EL_{com}) \\ Z_{eg}(j, i;\ EL_{com}) \end{bmatrix} = \begin{bmatrix} X_{eg}(T_j;\ EL_{com}) - & X_{eg}(S_i;\ EL_{com}) \\ Y_{eg}(T_j;\ EL_{com}) - & Y_{eg}(S_i;\ EL_{com}) \\ Z_{eg}(T_j;\ EL_{com}) - & Z_{eg}(S_i;\ EL_{com}) \end{bmatrix}.$$

The distance from S_i to T_j is:

$$\text{(A6.14)} \quad d_{ij}(AZ_{com}, EL_{com}) =$$

$$\sqrt{X^2_{eg}(j, i;\ EL_{com}) + Y^2_{eg}(j, i;\ EL_{com}) + Z^2_{eg}(j, i;\ EL_{com})}\ .$$

This is the calculated range distance which is used as an input data parameter in computations of the modular range count integer $\mathfrak{N}_{ij}$.

Let us define the direction cosines of the ray from S_i to T_j by:

$$\text{(A6.15)} \qquad \begin{aligned} \alpha_{eg}(j,i;\ EL_{com}) &\equiv \frac{X_{eg}(j,i;\ EL_{com})}{d_{ij}(AZ_{com}\ ,\ EL_{com})} \\ \beta_{eg}(j,i;\ EL_{com}) &\equiv \frac{Y_{eg}(j,i;\ EL_{com})}{d_{ij}(AZ_{com}\ ,\ EL_{com})} \\ \gamma_{eg}(j,i;\ EL_{com}) &\equiv \frac{Z_{eg}(j,i;\ EL_{com})}{d_{ij}(AZ_{com}\ ,\ EL_{com})} \end{aligned} .$$

These quantities are used in computations of line-of-sight visibility between rangefinder and target. They have been defined above for the case that both rangefinder scan point and target are on the tipping structure or main reflector surface. The geometric elevation coordinates of those reference points depend weakly on EL_{com} and are independent of $AZ_{com.}$ The definitions (A6.15) can be extended to the more general case where T_j is a ground benchmark target or a ground rangefinder reference point, whose geometric elevation frame coordinates have been computed as functions of EL_{com} and AZ_{com}.

$$\text{(A6.17)} \qquad \begin{aligned} \alpha_{eg}(j,i;\ AZ_{com}\ ,\ EL_{com}) &\equiv \frac{X_{eg}(j,i;\ AZ_{com}\ ,\ EL_{com})}{d_{ij}(AZ_{com}\ ,\ EL_{com})} \\ \beta_{eg}(j,i;\ AZ_{com}\ ,\ EL_{com}) &\equiv \frac{Y_{eg}(j,i;\ AZ_{com}\ ,\ EL_{com})}{d_{ij}(AZ_{com}\ ,\ EL_{com})} \\ \gamma_{eg}(j,i;\ AZ_{com}\ ,\ EL_{com}) &\equiv \frac{Z_{eg}(j,i;\ AZ_{com}\ ,\ EL_{com})}{d_{ij}(AZ_{com}\ ,\ EL_{com})} \end{aligned} \quad , \text{ where}$$

$$\text{(A6.18)} \qquad d_{ij}(AZ_{com}\ ,\ EL_{com}) =$$

$$\sqrt{X_{eg}^2(j,i;\ AZ_{com}\ ,\ EL_{com}) + Y_{eg}^2(j,i;\ AZ_{com}\ ,\ EL_{com}) + Z_{eg}^2(j,i;\ AZ_{com}\ ,\ EL_{com})}\ .$$

When T_j is a ground-fixed fiducial, the elevation frame coordinates of T_j to be substituted into (A6.13) are also functions of the azimuth angle AZ_{com}. One uses, in this case:

$$\text{(A6.13.1)} \quad \begin{bmatrix} X_{eg}(j,i;\ AZ_{com}, EL_{com}) \\ Y_{eg}(j,i;\ AZ_{com}, EL_{com}) \\ Z_{eg}(j,i;\ AZ_{com}, EL_{com}) \end{bmatrix} = \begin{bmatrix} X_{eg}(T_j;\ AZ_{com}, EL_{com}) - X_{eg}(S_i;\ EL_{com}) \\ Y_{eg}(T_j;\ AZ_{com}, EL_{com}) - Y_{eg}(S_i;\ EL_{com}) \\ Z_{eg}(T_j;\ AZ_{com}, EL_{com}) - Z_{eg}(S_i;\ EL_{com}) \end{bmatrix}.$$

Elevation frame coordinates of T_j are calculated using the ground reference frame coordinates: $X(T_j)$, $Y(T_j)$, $Z(T_j)$ of T_j , which are looked up in the "GROUND RANGEFINDER & GROUND RETROREFLECTOR DATABASE." For the case that T_j is ground-fixed, the coordinate transformation equations are:

$$\text{(A6.19)} \quad \begin{bmatrix} X_{eg}(T_j;\ AZ_{com}, EL_{com}) \\ Y_{eg}(T_j;\ AZ_{com}, EL_{com}) \\ Z_{eg}(T_j;\ AZ_{com}, EL_{com}) \end{bmatrix} =$$

$$\begin{bmatrix} X(T_j) \cdot \cos AZ_{com} - Y(T_j) \cdot \sin AZ_{com} \\ (X(T_j) \cdot \sin AZ_{com} + Y(T_j) \cdot \cos AZ_{com}) \cdot \sin EL_{com} + (h_e - Z(T_j)) \cdot \cos EL_{com} \\ (X(T_j) \cdot \sin AZ_{com} + Y(T_j) \cdot \cos AZ_{com}) \cdot \cos EL_{com} - (h_e - Z(T_j)) \cdot \sin EL_{com} \end{bmatrix}.$$

Aiming Of Feed Arm Rangefinders:

Aiming of feed arm rangefinders is calculated as follows. The feed arm rangefinder ZY$_i$ has a local right-handed Cartesian reference frame: $\widetilde{x}_i$, $\widetilde{y}_i$, $\widetilde{z}_i$ which is defined relative to the rangefinder platform and Kelvin mount balls attached thereto. The $\widetilde{z}_i$-axis is parallel to the platform base plane (as defined by the centers of the three Kelvin ball centers) and is both the scan mirror azimuth axis and the rangefinder's optical axis. The $\widetilde{y}_i$-axis is perpendicular to the platform base plane, looking outward. The origin of the local frame is at the scan reference point S_i of the rangefinder, which is the intersection of the local azimuth and elevation axes of the rangefinder. The $\widetilde{x}_i$-axis is parallel to the platform base plane, passes through S_i, and coincides with the local elevation azis of the rangefinder. Relative to this local coordinate system the coordinates of a target retroreflector fiducial point T_j are:

$$\text{(A6.20)} \quad \begin{bmatrix} \widetilde{x}_{Tj} \\ \widetilde{y}_{Tj} \\ \widetilde{z}_{Tj} \end{bmatrix} = \begin{bmatrix} d_{ij} \cdot \sin\Phi \cdot \cos\Theta \\ d_{ij} \cdot \sin\Phi \cdot \sin\Theta \\ d_{ij} \cdot \cos\Phi \end{bmatrix} \quad \text{where}$$

Θ and Φ are rangefinder local spherical polar coordinates:

$$\text{(A6.21.1)} \qquad \Phi = \cos^{-1}\left(\frac{\widetilde{z}_{Tj}}{d_{ij}}\right) , \quad 0 \le \Phi < \pi ,$$

$$\text{(A6.21.2)} \qquad \Theta = (\text{atan}_2)\left(\frac{\widetilde{y}_{Tj}}{\widetilde{x}_{Tj}}\right) , \quad -\frac{\pi}{2} \le \Theta < \frac{3\pi}{2} ,$$

$$\text{(A6.21.3)} \qquad d_{ij} = \sqrt{(\widetilde{x}_{Tj})^2 + (\widetilde{y}_{Tj})^2 + (\widetilde{z}_{Tj})^2} .$$

(Note: Equations 5.27.1 and 5.27.2 of GBT Memo 165 have typographical errors. The correct equations are given above.)

According to a note of F.R. Schwab, "Laser Rangefinder Locations And Orientations," March 6, 1996, when translated to the present local coordinates, the rangefinder's laser beam is unobstructed over the ranges:

$$\text{(A6.27)} \qquad 20^\circ < \left(\frac{360^\circ}{2\pi}\right) \cdot \Phi < 105^\circ \quad , \quad -28^\circ < \left(\frac{360^\circ}{2\pi}\right) \cdot \Theta < 208^\circ .$$

Algorithms are presently available to compute the rangefinder encodecommanded count settings once the local beam polar angles have been specified. We must now calculate Φ and Θ, given the geometric elevation frame coordinates of the scan and target reference points, corresponding to tipping structure elevation EL_{com}. To achieve this, we must first specify the rangefinder platform orientation relative to the geometric elevation reference frame.

We start with a given reference set of direction cosines specifying the orientation of the two reference frames to one another at the surface rigging angle. These 9 direction cosines are looked up in in the "FEEDARM LASER PLATFORM DIRECTION COSINE LOOKUP TABLE DATABASE." At the surface rigging angle, we may write:

$$\text{(A6.28)} \qquad \begin{bmatrix} \widetilde{X} \cdot \widehat{X^0_{eg}} & \widetilde{X} \cdot \widehat{Y^0_{eg}} & \widetilde{X} \cdot \widehat{Z^0_{eg}} \\ \widetilde{Y} \cdot \widehat{X^0_{eg}} & \widetilde{Y} \cdot \widehat{Y^0_{eg}} & \widetilde{Y} \cdot \widehat{Z^0_{eg}} \\ \widetilde{Z} \cdot \widehat{X^0_{eg}} & \widetilde{Z} \cdot \widehat{Y^0_{eg}} & \widetilde{Z} \cdot \widehat{Z^0_{eg}} \end{bmatrix}_{ZY_i} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}_{ZY_i} = [A]_{ZY_i} .$$

Here $\widetilde{X}$, $\widetilde{Y}$, $\widetilde{Z}$ are unit vectors along the local rangefinder axes. The A's are the direction cosines looked up in the above database.

When the tipping structure is at orientation AZ_{com}, EL_{com} the platform frame unit vectors are rotated with respect to their reference orientation with respect to the geometric elevation reference frame. Let $\mathfrak{N}_i$ be the node associated with the scan point S_i of rangefinder ZY_i. Let $\mathbf{t}_{\mathfrak{N}_i} = (t_x, t_y, t_z)$ be the joint rotation associated with this node, at the commanded azimuth and elevation, and calculated using (A 6.1). From (A6.9.1) it follows that

$$\text{(A6.29.1)} \qquad \widetilde{X} = (A_{11} + t_y \cdot A_{13} - t_z \cdot A_{12})\widehat{X}_{eg} +$$

$$+ (A_{12} + t_z \cdot A_{11} - t_x \cdot A_{13})\widehat{Y}_{eg} + (A_{13} + t_x \cdot A_{12} - t_y \cdot A_{11})\widehat{Z}_{eg} \ ,$$

$$\text{(A6.29.2)} \qquad \widetilde{Y} = (A_{21} + t_y \cdot A_{23} - t_z \cdot A_{22})\widehat{X}_{eg} +$$

$$+ (A_{22} + t_z \cdot A_{21} - t_x \cdot A_{23})\widehat{Y}_{eg} + (A_{23} + t_x \cdot A_{22} - t_y \cdot A_{21})\widehat{Z}_{eg} \ ,$$

$$\text{(A6.29.3)} \qquad \widetilde{Z} = (A_{31} + t_y \cdot A_{33} - t_z \cdot A_{32})\widehat{X}_{eg} +$$

$$+ (A_{32} + t_z \cdot A_{31} - t_x \cdot A_{33})\widehat{Y}_{eg} + (A_{33} + t_x \cdot A_{32} - t_y \cdot A_{31})\widehat{Z}_{eg} \ ,$$

Noting that the displacement vector from scan point to target point can be written in the form

$$\text{(A6.30)} \qquad (\widetilde{x}_{Tj})\widetilde{X} + (\widetilde{y}_{Tj})\widetilde{Y} + (\widetilde{z}_{Tj})\widetilde{Z} =$$

$$X_{eg}(i,j;\ AZ_{com}\ ,\ EL_{com})\widehat{X}_{eg} + Y_{eg}(i,j;\ AZ_{com}\ ,\ EL_{com})\widehat{Y}_{eg} + Z_{eg}(i,j;\ AZ_{com}\ ,\ EL_{com})\widehat{Z}_{eg}$$

and taking the dot product with the unit local platform frame vector $\widetilde{X}$ and using mutual perpendicularity of the platform frame vectors gives

$$\text{(A6.31)} \qquad \widetilde{x}_{Tj} = X_{eg}(i,j;\ AZ_{com}\ ,\ EL_{com})(\widehat{X}_{eg} \cdot \widetilde{X}) +$$

$$+ Y_{eg}(i,j;\ AZ_{com}\ ,\ EL_{com})(\widehat{Y}_{eg} \cdot \widetilde{X}) + Z_{eg}(i,j;\ AZ_{com}\ ,\ EL_{com})(\widehat{Z}_{eg} \cdot \widetilde{X}).$$

Other rangefinder platform frame coordinates of the beam scan vector are found in similar manner. The rangefinder platform frame local coordinates of the beam scan vector are then:

(A6.32.1)
$$\widetilde{x}_{Tj} = X_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{11} + t_y \cdot A_{13} - t_z \cdot A_{12}) + \\ + Y_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{12} + t_z \cdot A_{11} - t_x \cdot A_{13}) + \\ + Z_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{13} + t_x \cdot A_{12} - t_y \cdot A_{11}) \ .$$

(A6.32.2)
$$\widetilde{y}_{Tj} = X_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{21} + t_y \cdot A_{23} - t_z \cdot A_{22}) + \\ + Y_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{22} + t_z \cdot A_{21} - t_x \cdot A_{23}) + \\ + Z_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{23} + t_x \cdot A_{22} - t_y \cdot A_{21}) \ .$$

(A6.32.3)
$$\widetilde{z}_{Tj} = X_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{31} + t_y \cdot A_{33} - t_z \cdot A_{32}) + \\ + Y_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{32} + t_z \cdot A_{31} - t_x \cdot A_{33}) + \\ + Z_{eg}(i,j;\, AZ_{com}\,,\, EL_{com}) \cdot (A_{33} + t_x \cdot A_{32} - t_y \cdot A_{31}) \ .$$

The local rangefinder beam aiming angles are computed using:

(A6.21.1)
$$\Phi = \cos^{-1}\left(\frac{\widetilde{z}_{Tj}}{d_{ij}}\right) = \Phi\,(i,j;\, AZ_{com}\,,\, EL_{com})\ ,$$

(A6.21.2)
$$\Theta = (\mathrm{atan}_2)\left(\frac{\widetilde{y}_{Tj}}{\widetilde{x}_{Tj}}\right) = \Theta\,(i,j;\, AZ_{com}\,,\, EL_{com})\ .$$

These are our final equations for the rangefinder aiming angles.